Methods of choosing the most profitable real assets among options that require different amounts for various periods of time

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Methods of Choosing The Most Profitable Real Assets Among Options That Require Different Amounts For Various Periods of Time

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ABSTRACT

The article deals with the indicators used when making investment decisions in situations when it is necessary to invest different amounts of money in alternative real assets for different periods of time. The author suggests two new indicators: “the indicator of the speed of specific increment in value” (IS) and “conditional bank deposit” (CBD). The first indicator is used to choose the best investments that bring some profit. The second indicator is used to choose the best investments that do not directly bring profit, but are required for the operation of the company.

The existing methods used to choose the best variant for real investments have been developed during several decades. To solve this task, such indicators as net present value (NPV), internal rate of return (IRR), profitability index (PI), modified internal rate of return ($MIRR$), payback period (Pb), equivalent annual annuity (EAA), equivalent annual cost (EAC). All these indicators have been thoroughly developed for three cases:

1) several alternative real assets are compared, in which the same amount is invested for the same period of time;

2) several options are compared, in which the same amount is invested but for different periods of time\terms;

3) several options are compared, in which different amounts are invested for the same period of time.

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In the meanwhile, there is a fourth case when it is necessary to compare alternatives where **both** amounts **and** terms are different. All the above mentioned indicators do not apply in such a situation. Let us specify that this work analyzes making of investment decisions in a commercial company. That means that the investment project is described by the net cash flow (NCF) that does not include macroeconomic effects.

To do further analysis, let's divide real investments into two categories: money-gaining and money-losing investments. Money-gaining investments bring in returns (for example, a company buys some equipment to manufacture building products with their subsequent sale). Money-losing investments do not participate in the formation of the profit directly, but they are required in the business (for example, the company buys a car for the transportation of its employees). Both money-gaining and money-losing investments can require different amounts for different periods of time. Managers often have to compare such real assets and choose from the alternative: "expensive, economical, long-term" and "inexpensive, uneconomical, short-term".

Such a situation is described in the work of Richard A. Brealey, Stewart C. Myers (2003, 2006): "suppose the firm is forced to choose between two cars, A and B. The two cars are designed differently but have the same capacity and do exactly the same job. Car A costs \$15,000 and will last three years. It costs \$5,000 per year to run. Car B is an economy model having the price of only \$10,000, but it will last only two years and costs \$6,000 per year to run." Let's call this company "user" and use this example to illustrate our further reasoning, assuming that the work of these authors is well-known and available to a wide range of specialists.

Let's decide upon the formulas and designations, used in this work. Net present value is calculated according to the following formula:

$$NPV = \sum_{t=0}^{n} \frac{NCF_t}{(1+k)^t} = \sum_{t=0}^{n} NCF_t \times PVIF_{k,t} \qquad (1)$$

where k is the discount rate;

t – certain period of time;

n – the period during which the investments are used;

$PVIF_{k,t}$ – present value interest factor, that is calculated according to the following formula:

$$PVIF_{k,t} = \frac{1}{(1+k)^t} \tag{2}$$

PI is calculated as follows:

$$PI = \frac{\sum_{t=0}^{n} CIF_t \times PVIF_{k,t}}{\sum_{t=0}^{n} |COF_t| \times PVIF_{k,t}} \tag{3}$$

where COF_t is cash outflow – negative elements of $NCF;$

CIF_t – cash inflow (positive elements of NCF).

Equivalent annual annuity (EAA) is determined according to the formula:

$$EAA = \frac{NPV}{PVIFA_{k,n}} \tag{4}$$

where $PVIFA_{k,n}$ is the present value interest factor of an annuity, that is calculated according to the following formula:

$$PVIFA_{k,n} = \sum_{t=1}^{n} \frac{1}{(1+k)^t} = \frac{(1+k)^n - 1}{k(1+k)^n} \tag{5}$$

EAA shows what the annual proceeds have to be for their current value to equal NPV of the evaluated project. The proceeds arise annually, of equal size, during n years. The alternative where EAA is higher, wins.

Equivalent annual costs (EAC) are calculated in a similar way:

$$EAC = \frac{PV_{costs}}{PVIFA_{k,n}} \quad (6)$$

where PV_{costs} is the current value of all expenses for purchasing and operating the equipment.

However, interpretation of EAC differs greatly from that of EAA. R. Brealey and S. Myers suggest the following interpretation of the economic meaning of EAC: that is the amount of the rent that has to be paid to the equipment owner if the decision about rent is made. "You can think of the equivalent annual cost of car A or B as an annual rental charge." Of several alternatives, the one that has the smallest EAC is more profitable. At the discount rate of 6%, the best one is car A, because it has the smallest value of EAC: \$10,612 against \$11,454 for car B.

It should be noted that a new subject with predetermined functions and standards of behavior is introduced in this definition of the economic meaning of EAC – "owner". But appearance of such a subject requires substantiation. Let's check with the help of numbers whether it will be profitable for the owner to lease car A. Let's assume that the operating costs are paid by the user, and, as a result, the owner gets the difference between EAC and the operating costs, which is \$10,612 – \$5,000 = \$5,612 annually. It turns out that the owner invests in objects with zero NPV (the calculations are given in Table 1). This is the second assumption in the economic meaning of EAC which does not look very reliable. Moreover, this assumption does not coincide with one of axioms of the investment analysis according to which money should be invested in some objects providing positive (not zero) NPV. Thus, the variant of the economic interpretation of EAC suggested above requires clarification.

Table I

Cash flows of the owner connected with purchasing of the car and its leasing

Indicator	*Period*			
	0	*1*	*2*	*3*
1. Car purchase, $	*15,000*			
2. Income of the owner, $		*5,612*	*5,612*	*5,612*
3. NCF_t, $	*-15,000*	*5,612*	*5,612*	*5,612*
4. $PVIF_{6\%,t}$	*1*	*0.9434*	*0.890*	*0.8396*
5. Discounted NCF_t, $	*-15,000*	*5,294*	*4,994*	*4,712*
6. NPV, $	*0.000*			

The author of this work proposes another indicator to compare money-losing investments that require different amounts for different periods of time. The most profitable alternative can be chosen based on the analysis of two strategies: (1) the company regularly chooses equipment of type A, (2) the company regularly chooses equipment of type B. The most profitable strategy will show which equipment is most profitable.

These strategies can be evaluated if we calculate what amount we have to deposit with the bank today to buy equipment infinitely (as it becomes unserviceable) and finance the operating costs. The strategy that requires smallest investments is the most profitable one. Let's call this amount a conditional bank deposit, CBD. Thus, CBD of the strategy will consist of two elements: a deposit to finance the purchase chain (CBD^{buying}) and a deposit to finance the chain of the operating costs ($CBD^{op.costs}$):

$$CBD = CBD^{buying} + CBD^{op.costs} \tag{7}$$

The author of this work proposes to compare money-losing investments that require different amount for different periods of time based on this indicator. Let's assume that we make the payments to purchase the equipment (at the price $Price^{equipment}$) and to cover operating costs (annual value of $Op.Costs$) in the beginning of the period (*prenumerando*). These are perpetual cash flows. We will do the regular purchase of the equipment once every l years (l is the performance life of the equipment). The amount to be deposited with the bank is determined according to the following formula:

$$CBD^{buying} = Price^{equipment} \times PVIF_{k,l,\infty}^{buying} \tag{8}$$

where $PVIF_{k,l,\infty}^{buying}$, the coefficient of the current cost for the perpetual cash flow for purchasing the equipment, is determined according to the following formula:

$$PVIF_{k,l,\infty}^{buying} = \frac{(1+k)^l}{(1+k)^l - 1} \tag{9}$$

To finance the operating costs, it will be necessary to deposit with the bank some amount that is calculated according to the following formula:

$$CBD^{op.costs} = Op.Costs \times PVIF_{k,\infty}^{op.costs} \tag{10}$$

where $PVIF_{k,\infty}^{op.costs}$, the coefficient of the current cost for the infinite cash flow used to cover the operating costs, is calculated according to the following formula:

$$PVIF_{k,\infty}^{op.costs} = \frac{1}{k} + 1 \tag{11}$$

Let's calculate CBD for the above example. The calculations are given in Table 2.

Table II

Calculation of CBD for cars A and B

Indicator	Car A	Car B
1. Discount rate (k), %	6	6
2. Performance life of the equipment (l), year	3	2
3. $Price^{equipment}$, $	15,000	10,000
4. $Op.Costs$, $ per year	5,000	6,000
5. $PVIF_{k,l,\infty}^{buying}$	6.24	9.09
6. $PVIF_{k,\infty}^{op.costs}$	17.7	17.7
7.CBD^{buying}, $	93,527	90,906
8. $CBD^{op.costs}$, $	88,333	106,000
9. Total CBD, $	181,861	196,906

The choice for EAC and CBD in this example coincides - car is the best one. But some situations are possible, when these indicators give some opposite result. For example, let's compare "expensive, economical, long-term" equipment C with "inexpensive, uneconomical, short-term" equipment D and E (their characteristics are given in Table 3).

Table III

Economic characteristics of equipment C, D, E.

Indicator	Equipment C	Equipment D	Equipment E
1. $Price^{equipment}$, $	64,000	45,600	45,600
2. Performance life of the equipment (l), year	10	7	7
3. $Op.Costs$, $ per year	2,000	2,400	2,500
4. EAC, $	10,696	10,569	10,669
5. CBD, $	180,259	178,543	180,309

Comparing C and D, we come to the conclusion that D is better – its both EAC are CBD are smaller. But if we change the amount of annual operating costs for D from $2,400 to $2,500, we will see a different picture (that is characteristics of equipment E). As for indicator EAC, equipment E is better than C, as for CBD - it's vice versa: equipment C is more cost-efficient than equipment E. In such a way, evaluation of money-losing investments that require different amounts for different periods of time can be opposite based on these indicators. The author of this work thinks it necessary to make the choice based on CBD in this case.

Let's get back to the question how an owner that will get profit from his purchase and operation of some equipment can choose between the "expensive, economical, long-term" car A and the "inexpensive, uneconomical, short-term" car B? Usually, to compare money-gaining investments that require different amounts for different periods of time, the method of chain repetition with the calculation of NPV of chain, PI, EAA is recommended.

In Table 1 we made the calculations based on the fact that owner's profit means the difference between EAA and operating costs. But why does he not lease A and B at the same price!? Let us consider the situation if the annual payment is set in the amount of $13,200. The owner will annually get $13,200 – $5,000 = $8,200 from each car A and $13,200 – $6,000 = $7,200 from each car B.

As it is necessary to invest A and B different amounts for different periods of time, let us compare these alternatives using the chain repetition method. Let's assume that the owner has $30,000 to buy 2 cars A or 3 cars B. We have made the amounts the same in such a way. Let's also assume that the owner will buy cars A two times and B three times. As a result, the terms are the same and the alternatives are comparable. The cash flows in these two chains are given in Table 4 and Table 5.

Table IV

Table 4. Cash flow of the owner that is connected with the purchase and lease of two cars A that was done twice (NCF_t^{2A+2A})

Indicator	Period						
	0	1	2	3	4	5	6
1. NCF_t^{2A+2A}, $, including:	-30,000	16,400	16,400	-13,600	16,400	16,400	16,400
1.1. NCF_t^{2A} (first purchase)	-30,000	16,400	16,400	16,400			
1.2. NCF_t^{2A} (second purchase)				-30,000	16,400	16,400	16,400
2. $PVIF_{6\%,t}$	1	0.9434	0.890	0.8396	0.7921	0.7473	0.7050
3. Discounted NCF_t^{2A+2A}, $	-30,000	15,472	14,596	-11,419	12,990	12,255	11,561

Table V

Table 5. Cash flow of the owner that is connected with the purchase and lease of three cars B that was done three times ($NCF_t^{3B+3B+3B}$)

Indicator	Period						
	0	1	2	3	4	5	6
1. $NCF_t^{3B+3B+3B}$, $, including:	-30,000	21,600	-8,400	21,600	-8,400	21,600	21,600
1.1. NCF_t^{3B} (first purchase)	-30,000	21,600	21,600				
1.2. NCF_t^{3B} (second purchase)			-30,000	21,600	21,600		
1.3. NCF_t^{3B} (third purchase)					-30,000	21,600	21,600
2. $PVIF_{6\%,t}$	1	0.9434	0,890	0.8396	0.7921	0.7473	0.7050
3. Discounted $NCF_t^{3B+3B+3B}$, $	-30,000	20,377	-7,476	18,136	-6,654	16,141	15,227

With these data, NPV^{2A+2A} = $25,456, $NPV^{3B+3B+3B}$ = $25,752, and it seems that it is more cost-efficient to invest in equipment B. However, despite the attempt to make these alternatives comparable by means of chain repetition, the analyzed cash flows have a significant difference: sums of their cash outflows (COF) differ. To purchase A we spend \$30,000 + \$13,600 = \$43,600, to purchase B we spend \$30,000 + \$8,400 + \$8,400 = \$46,800. The present values are also different. Discounted COF for A will be \$30,000 + \$11,419=\$41,419; discounted COF for B will be \$30,000 + \$7,476 + \$6,654=\$44,130. It is obvious that we should use PI to compare alternatives with the different amounts but the same terms. For equipment A this indicator is 1,61 \$/\$, and for equipment B it will be 1,58 \$/\$. Thus there is a contradiction: NPV shows that project B is better, and PI shows that project A is better!

Let us note that PI is not good for comparing investments with the same amounts but different periods of time. For example, the owner is deciding on whether to buy car A or F with NCF^F= \$-15,000, \$6,943, \$6,943, \$6,943, \$6,943. The same amount of money is invested, but car F will last a year longer. NPV for A and F is the same and is \$9,057. PI is also the same and is 1.6 \$/\$. But it is obvious that it is more cost-efficient to buy car A, since the result will be obtained a year earlier in this case. In other words, the annual amount of NPV will be bigger. A simple indicator can be suggested for comparing investments that require the same amounts for different periods of time. Let us call it "an average annual amount of NPV":

$$\text{Average yearly sum of NPV} = \frac{NPV}{n}, \$/per\ year \tag{12}$$

Among several alternatives the one with the highest indicator is the most cost-efficient. If the owner always buys cars of type A, he will gain more benefits in comparison with systematic purchase of cars of type F. An additional argument for choosing cars A can be given: EAA^A= \$3,388 dollars, while EAA^B= \$2,614 dollars.

It is commonly believed that EAA can be used for comparing investments with different amounts and different periods of time. The author of the present paper thinks that this indicator can give incorrect results. Let us prove it by the following discussion. According to its economic interpretation, NPV is an ***increment*** of invested money that along with this money gives planned rate (k) with planned dynamics of results (CIF). Let us show this statement in table 6.

Table VI

Table 6. Economic interpretation of NPV^{C}

Indicator	Period			
	0	1	2	3
1. Cash inflow, $		9,000	9,000	9,000
2. Rate, %		6%	6%	6%
3. $PVIF_{6\%,t}$		0.9434	0.890	0.8396
4. Discounted cash inflow, $		8,491	8,010	7,557
5. Amount of investments in the object, $	15,000			
6. NPV, $	9,057			
7. Money in the object (at the beginning of the period), $		24,057	16,501	8,491
8. Profit generated by the object (within the period), $		1,443	990	509
9. Amounts received by the investor (at the end of the period), $		9,000	9,000	9,000
10. Money in the object (at the end of the period), $	24,057	16,501	8,491	0

Since NPV means increment, this indicator cannot be considered in isolation from the amount of invested money (I). However in calculating EAA we don't take into account I, and this reduces the reliability of this indicator. Therefore, for reliable evaluation it is necessary to take into account NPV, invested money and a period of time. All these are combined in the "indicator of the speed of specific increment in value" (IS) suggested by the author of the present paper:

$$IS = \frac{NPV}{I \times n} \tag{13}$$

This indicator integrates two principles: "faster" and "more" and shows *dollars* of the net present value of the project that are obtained *annually per an invested dollar*. Among several alternatives the one with the biggest indicator is the most cost-efficient. The strong point of IS is that it is simple and realistic. This indicator does not require transforming cash flows. Let us use IS to compare cars A and B. The owner's cash flows related to purchase and use of these cars are given in tables 7 and 8.

Table VII

Table 7. Owner's cash flows related to purchase and use of car A

Indicator	Period			
	0	1	2	3
1.Car price, $	15,000			
2.Car use income, $		13,200	13,200	13,200
3.Operation costs, $		5,000	5,000	5,000
4.NCF_t^A, $	-15,000	8,200	8,200	8,200
5.$PVIF_{6\%,t}$	1	0.9434	0.890	0.8396
6.Discounted NCF_t^A, $	-15,000	7,736	7,298	6,885

Table VIII

Table 8. Owner's cash flows related to purchase and use of car B

Indicator	Period		
	0	1	2
1. Car price	10,000		
2. Car use income		13,200	13,200
3. Operation costs		6,000	6,000
4. NCF_t^B	-10,000	7,200	7,200
5. $PVIF_{6\%,t}$	1	0.9434	0.890
6. Discounted NCF_t^B	-10,000	6,792	6,408

NPV and EAA show that car A is better: NPV^A= \$6,919, NPV^B= \$3,200; EAA^A = \$2,588; EAA^B = \$1,746. However IS shows that car B is better: IS^A= 0.15 \$/\$ annually, IS^B= 0.16 \$/\$ annually:

$$IS^A = \frac{\$6{,}919}{\$15{,}000 \times 3 year} = 0.15\$/\$\ per\ year \tag{14}$$

$$IS^B = \frac{\$3{,}200}{\$10{,}000 \times 2 year} = 0.16\$/\$\ per\ year \tag{15}$$

IS can be also applied in a reverse situation. Let us assume that an engineer has developed a technical novelty which is an alternative to the existing equipment. The question is, what maximum price it is possible to sell this novelty at? The maximum price of the novelty will depend on the maximum the owner is ready to pay. It is he who will have to compare "expensive, economic, long-term" equipment with "inexpensive, uneconomical, short-term".

This task can be solved in the following way. Let us assume that equipment A with price I^A and service life n exists, and alternative equipment B with price I^B and service life m is developed. These two alternatives have different production capacities and operation costs, and thus different net cash flow: for equipment A it is NCF_t^A, and for equipment B this value will be NCF_t^B. If we equate IS^A to IS^B, we get the following equation which is a basis for determining characteristics of developed equipment:

$$\frac{\sum_{t=1}^{n} NCF_t^A \times PVIF_{k,t} - I^A}{n \times I^A} = \frac{\sum_{t=1}^{n} NCF_t^B \times PVIF_{k,t} - I^B}{m \times I^B} \tag{16}$$

On the basis of this equation we can determine boundary values of different characteristics of equipment B, for example, its maximum price (I^B) with specified NCF_t^B, m, IS^A. Let us assume that NCF_t^A and NCF_t^B are annuities, in this case a condition for determining the maximum price value of the developed equipment[2] is obtained from the above equation:

$$I^B < \frac{n \times NCF_t^B \times PVIFA_{k,m}}{m \times NCF_t^A \times \frac{PVIFA_{k,n}}{I^A} - m + n} \qquad (17)$$

Let us use the above example to consider the operation of this formula. Assume that the price of car A is specified, and the maximum price of car B is to be found. Using formula 17 and data from tables 7 and 8, we obtain:

$$I^B = \frac{3year \times \$7{,}200 \times 3.465}{2year \times \$8{,}200 \times \frac{2.673}{\$15{,}000} - 2year + 3year} = \$10{,}096 \qquad (18)$$

If \$10,096 is invested in car B, its purchase efficiency will be equal to car A purchase efficiency:

$$IS^B = \frac{\$3{,}104}{\$10{,}096 \times 2year} = 0.15\$/\$ \; per\ year \qquad (19)$$

If car B sells at a higher price, it will be more cost-efficient for the owner to buy car A.

So this paper discusses two indicators (CBD and IS) which can help make a right investment decision in a situation when different amounts for different periods of time are to be invested in alternative real assets. The application range of these indicators is very wide: from a small company buying an equipment unit to a state investing hundreds of billions of dollars. The suggested CBD and IS surely need a bit of criticism and if they stand up to it, we can consider that the task of comparing this type of investments is solved.

[2] Methods used to determine NCF_t^B and to take into account differences in production capacity of alternative equipment types and differences in operation costs and prices are considered in the paper Коган А.Б. Способы определения экономических характеристик инноваций // Сибирская финансовая школа. – №1. Новосибирск: САФБД, 2010, с.106-111.

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