Expectations-Based Reference-Dependent Preferences and Asset Pricing

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Abstract

This paper incorporates expectations-based reference-dependent preferences into the canonical Lucas-tree asset-pricing economy. Expectations-based loss aversion increases the equity premium and decreases the consumption-wealth ratio, because uncertain fluctuations in consumption are perceived to be more painful. Moreover, because unexpected cuts in consumption are particularly painful, the agent wants to postpone such cuts to let his reference point decrease. Thus, even though shocks are i.i.d., loss aversion induces variation in the consumption-wealth ratio, which generates variation in the equity premium, expected returns, and predictability. The level and variation in the equity premium and the predictability in returns match historical moments, but the associated variation in intertemporal substitution motives results in excessive variation in the risk-free rate. This effect can be partially offset with variation in expected consumption growth, heteroskedasticity in consumption growth, or time-variant disaster risk. As a key contribution, I show that the preferences resolve the equity-premium puzzle and simultaneously imply plausible risk attitudes towards small and large wealth bets.

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1 Introduction

Several leading asset-pricing models assume reference-dependent preferences that evaluate consumption relative to a reference point. Campbell and Cochrane (1999) assume habit-formation[1] preferences, and Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), and Yogo (2008) use prospect-theory[2] preferences. Each of these models assume that the reference point is backward-looking and formalize it in specific ways. Moreover, the prospect-theory models specify utility directly over financial wealth instead of consumption, which implies a narrow-framing[3] assumption. Koszegi and Rabin (2006, 2007, 2009) develop a new, generally-applicable model of reference-dependent preferences in a series of influential theory papers, which successfully explains an array of behavioral and experimental evidence. The preferences are based on consumption and offer a fully-endogenized reference-point specification, thereby eliminating one of the major degrees of freedom associated with prospect theory.

In an otherwise standard Lucas-tree model, expectations-based loss aversion intuitively implies a first-order shift and variation in the consumption-wealth ratio; the latter of which is a new and distinct prediction in the prospect-theory asset-pricing literature. As a result, the model matches historical levels of the equity premium, its volatility, and the degree of predictability in returns. Remarkably, these implications are independent of common assumptions in the literature, such as a separate process for dividends or narrow framing.[4] Moreover, I show that the preferences imply plausible risk attitudes towards small, medium, and large wealth bets and thus make a first step in explaining microeconomic evidence and resolving the equity-premium puzzle; this can be seen as a key contribution to the existing literature.

Expectations-based reference-dependent preferences consist of two components. "Consumption utility" is determined by consumption and corresponds to the standard model of utility. Contemporaneous and prospective "gain-loss utility" is determined by a comparison of current and future consumption with the reference point and corresponds to the prospect-theory model of utility. The latter component incorporates loss aversion; small losses are more painful than equal-sized gains are pleasurable. The reference point is stochastic and corresponds to the agent's fully probabilistic rational beliefs about current and future consumption formed in the previous period. Then, the agent

[1]Habit formation (Abel (1990)) is a preference theory saying that people's utility function depends on the change in consumption rather than the level of consumption.

[2]Prospect theory (Kahneman and Tversky (1979)) is a behavioral theory aimed to describe risk preferences elicited in experiments. The theory says that people care about gains and losses relative to a reference point, whereby small losses hurt more than equal-sized gains give pleasure, i.e., people are loss averse.

[3]Narrow framing refers to the phenomenon that people appear to evaluate an offered gamble in isolation, rather than mixing it with existing risk or considering its actual implications for consumption instead of financial wealth.

[4]A separate dividend process is typically assumed to reduce the contemporaneous correlation of consumption and returns; this, however, is not necessary in the basic model, which matches the contemporaneous correlation reasonably well.

compares consumption utility for each possible outcome under his updated beliefs with consumption utility for each possible outcome under his prior beliefs, and he experiences a corresponding sensation of gain or loss. Accordingly, the agent derives gain-loss utility from unexpected changes in present consumption and revisions in expectations over future consumption; therefore, gain-loss utility can be interpreted as utility over good and bad news.

This paper incorporates such "news-utility" preferences into an otherwise standard consumption-based asset-pricing model and solves for the rational-expectations equilibrium in closed form. The model environment is a simple endowment economy with log-normal consumption growth in the spirit of Lucas (1979). The Mehra and Prescott (1985) model – which shows that constant relative risk aversion preferences are inconsistent with basic financial market moments – is preserved as a special case.

As a stepping stone to describing the model's asset-pricing implications, I first explain two predictions about the model's consumption-wealth ratio.[5] First, the consumption-wealth ratio is shifted down relative to the standard model. Because the agent is loss averse, he anticipates uncertain fluctuations in gain-loss utility that are painful on average. But, these fluctuations are less painful on a less steep part of the utility curve, which introduces an additional precautionary-savings motive. Second, the consumption-wealth ratio varies, in contrast to the standard model and despite the i.i.d. environment. Because the agent is loss averse relative to his expectations, he finds unexpected reductions in consumption more painful than expected reductions in consumption; hence, the agent wants to postpone unexpected reductions in consumption until his expectations will have decreased. More precisely, reducing future consumption automatically decreases the future reference point, whereas the present reference point is fixed. Consequently, reductions in future consumption are relatively less painful than reductions in present consumption. Finally, these two effects on the consumption-wealth ratio are first order as they depend on loss aversion.

These findings drive the model's asset-pricing predictions. First, the shift of the consumption-wealth ratio is reflected in an increased mean equity premium. Because the agent is loss averse, he requires a high compensation for the painful fluctuations in consumption associated with uncertainty. Second, the variation in the consumption-wealth ratio is reflected in variation of expected returns. In bad times, the agent desires to consume more and save less. In general equilibrium, this desire increases the consumption-wealth ratio, decreases the price-consumption ratio, and thus increases expected returns. Accordingly, the model generates predictability: In bad times, a high consumption-wealth ratio predicts high future returns. Because high expected returns have high standard deviations, which increase the price of risk, expected excess returns are higher too. Thus, excess returns are predictable too.

I calibrate the news-utility preference parameters in line with microeconomic evidence and show that this calibration generates realistic attitudes towards small, medium,

[5]Koszegi and Rabin (2006, 2007, 2009) anticipate these consumption decision-making implications in a two-period, two-outcome consumption-savings problem.

and large wealth bets. Moreover, this calibration obtains a log equity premium of approximately six percent with a standard deviation of nineteen percent and thus matches historical stock market data, even though consumption equals dividends in the basic Lucas-tree model.[6] Moreover, I find variation in the consumption-wealth ratio around three percent and R^2s in the predictability regressions of approximately ten percent. These values match the empirical findings of Lettau and Ludvigson (2001), who document the medium-term predictability properties of the consumption-wealth ratio.[7] Besides, I show that such strong predictability of the consumption-wealth ratio on the return and excess return on the aggregate consumption claim is not generated by other leading asset-pricing models.

A misprediction of the model is strong variation in the risk-free rate, which is commonly predicted by habit-formation models but not borne out by the data. In the event of adverse shock realizations, the agent dislikes immediate reductions in consumption and is unwilling to substitute intertemporally, which increases both the expected risky and risk-free rate of return. Although not reflected in aggregate data, this underlying time-variation in substitution motives may not be implausible in practice. Indeed, because people are unwilling to substitute intertemporally in sometimes, they use credit cards and payday loans, thus borrowing at high interest rates. The intent of this paper is not to change the evidence-based utility function; rather, I take the variation in substitution motives seriously and explore three model-environment extensions that partly offset the strong intertemporal substitution effects on the risk-free rate.

First, I assume variation in expected consumption growth, as in Bansal and Yaron (2004). Second, I assume variation in consumption growth volatility, i.e., heteroskedasticity in the consumption process, as in Campbell and Cochrane (1999). Third, I add disaster risk to the consumption process, so that there is a small probability that the agent suffers a large loss in consumption, as in Barro (2006, 2009). I find that news-utility preferences amplify disaster risk, because they feature "left-skewness aversion", as opposed to standard prospect-theory preferences. The addition of heteroskedasticity or disaster risk introduces variation in the strength of the precautionary savings motive, which partly offsets the effects of the variation in substitution motives on the risk-free rate, adds variation in the price of risk, and generates long-horizon predictability.

Last, I quickly describe the model's welfare implications. News utility increases the costs of business cycle fluctuations, in the spirit of Lucas (1978), to realistic levels.

[6]The model's equity premium and its volatility are increasing in the model's simulation frequency, which, therefore, constitutes a major calibrational degree of freedom. Taking the calibration as given, I choose a one-and-a-half month frequency that happens to match both the historical equity premium and its volatility. The model's frequency matters due to the famous idea of myopic loss aversion (Benartzi and Thaler (1995)): In contrast to standard preferences, loss aversion implies that a cumulative lottery becomes inherently less attractive when its independent draws are evaluated more rather than less frequently. I am intrigued by the idea that people require a large compensation for risk, because they are subject to painful fluctuations in beliefs when they worry frequently about small fluctuations in their future consumption.

[7]Furthermore, Lustig, Nieuwerburgh, and Verdelhan (2012) and Hirshleifer and Yu (2011) document the volatility of the consumption-wealth ratio and the return on the aggregate consumption claim.

Moreover, the first welfare theorem does not hold, because the preferences are subject to a beliefs-based time inconsistency.[8]

After a literature review, I present the preferences, the model environment, and the Markovian rational-expectations equilibrium in sections 3.1 and 3.2. Then, in section 3.3, I explain the model's predictions about the consumption-wealth ratio. In section 4.1, I discuss the model's asset-pricing implications and calibrate the model to gauge its quantitative implications in section 4.2. In section 5, I extend the model to allow for time-variant expected consumption growth, time-variant volatility, and disaster risk. Section 6 explains the model's implications for welfare. Finally, section 7 concludes.

2 Comparison to the literature

In recent years, loss aversion became a widely accepted explanation for the equity-premium puzzle. I further this literature by showing that most results carry over to a new, micro-founded preference specification, which has been used in a variety of contexts to explain behavioral and experimental evidence.[9] As a result, major degrees of freedom associated with prospect theory can be eliminated: The reference point is fully endogenous, and tight ranges exist for all preference parameters. However, a different calibrational degree of freedom emerges, which did not receive much attention in static applications; the length of each time period, or the model's simulation frequency. Simulating the model at higher frequency increases the equity premium, because the loss averse agent finds many independent draws of a gamble less attractive than all these

[8] Lacking an appropriate commitment device, the agent optimizes in each period, taking his beliefs as given. Thus, he is inclined to positively surprise himself with extra consumption in each period. Consequently, the agent is forced to choose a sub-optimal consumption path, which differs from the expected-utility-maximizing path on which the agent jointly optimizes over consumption and beliefs.

[9] Expectations-based reference-dependent preferences have been incorporated into microeconomic models by many authors: For instance, Heidhues and Koszegi (2008, 2010); Herweg and Mierendorff (forthcoming); Rosato (2012) explore the implications for consumer pricing, which are tested by Karle, Kirchsteiger, and Peitz (2011). Herweg, Müller, and Weinschenk (2010) analyze the implications for optimal principal-agent contracts more generally Eisenhuth (2012) for mechanism design. Furthermore, an incomplete list of papers providing direct evidence for Koszegi and Rabin (2006, 2007, 2009) preferences is: Sprenger (2010) on implications of stochastic reference points, Abeler, Falk, Goette, and Huffman (forthcoming) on labor supply decision-making, Gill and Prowse (2012) on real-effort tournaments, Meng (2010) on the disposition effect, and Ericson and Fuster (2010) on the endowment effect (a finding which was contradicted by Heffetz and List (2011)). Barseghyan, Molinari, Donoghue, and Teitelbaum (2010) structurally estimate a model of insurance deductible choice. Suggestive evidence is provided by Crawford and Meng (2009) on labor supply decision-making, Pope and Schweitzer (2011) on golf player's performance, and Sydnor (2010) on deductible choice as well as a reading of the numerous conflicting papers on the endowment effect which can be reconciled with the idea of expectations determining the reference point. All of these papers consider the static model of Koszegi and Rabin (2006, 2007, 2009). But, because the dynamic preferences of Koszegi and Rabin (2009) are a straightforward extension, the evidence is equally valid for Koszegi and Rabin (2009) preferences. Moreover, because monetary payoffs in experiments reflect future consumption instead of contemporaneous consumption, all experiments that elicit preferences with monetary payoffs indirectly support the idea that agents are loss averse over news about future consumption.

gambles' cumulative outcome.[10] I calibrate the model in line with microeconomic evidence and then choose a one-and-a-half month frequency that matches both the equity premium and its volatility, i.e., the historical risk-return trade off. Moreover, I show that the preferences are tractable in a multi-period, continuous-outcome framework; this is not readily apparent given their high level of complexity.

While other models are equally able to match asset-pricing moments, news utility simultaneously explains behavior observed in microeconomic studies. I show that the preference parameters induce realistic attitudes towards small, medium, and large wealth bets, which are not well explained by other preference specifications.[11] Therefore, I take a step forward in developing a framework that can match both macroeconomic and microeconomic behavior. This improved micro foundation has desirable implications, namely variation in the consumption-wealth ratio, expected returns, and predictability, which matches the evidence provided by Lettau and Ludvigson (2001) better than those of the standard, habit-formation, or long-run risk models. But, the well-known problem that the risk-free rate responds strongly to intertemporal smoothing incentives is not resolved. Time-variant consumption growth, heteroskedasticity, or disaster risk is needed to offset some of the effects on the risk-free rate.

The pioneering prospect-theory asset-pricing papers, Barberis, Huang, and Santos (2001) and Benartzi and Thaler (1995), specify gain-loss utility directly over fluctuations in financial wealth. In so doing, the authors make an assumption about narrow framing. The agent narrowly frames the stock market, because he experiences gain-loss utility directly over financial wealth. In contrast, the news-utility agent experiences gain-loss utility over the implications of his financial wealth for contemporaneous and future consumption. The news-utility model yields a high equity premium without narrow framing, because the agent experiences gain-loss utility over fluctuations in contemporaneous consumption and the entire stream of future consumption, which makes uncertainty sufficiently painful. Yogo (2008) argues also that fluctuations in consumption rather than financial wealth are the relevant measure of risk. The author's preferences are a mixture of habit formation and prospect theory and yield a high equity premium; variation in the risk-free rate is mitigated via persistence in the habit process. The main difference with respect to Koszegi and Rabin (2009) preferences is that the reference point is backward-looking. In contrast, Andries (2011) incorporates loss aversion into a consumption-based asset pricing model and explains positive skewness premia and a flat security market line. The agent's value function features a kink at the expected value of consumption, which nicely captures forward-looking reference

[10]This result relates to myopic loss aversion and the Samuelson's colleague story (Benartzi and Thaler (1995)).

[11]Standard, habit-formation, or long-run risk preferences do not simultaneously match risk attitudes towards small and large wealth bets, because the agent is second-order risk averse. Similarly, the disappointment-aversion models do not robustly match such risk attitudes, because the agent is not necessarily "at the kink". The asset-pricing theories based on prospect theory imply plausible attitudes towards small and large wealth bets but not consumption bets and are thus inconsistent with the endowment-effect evidence (Kahneman, Knetsch, and Thaler (2009)).

dependence. However, because the agent exhibits reference-dependent preferences with respect to his future value rather than present consumption, the underlying preference mechanisms and predictions are very different from those I obtain.

Campbell and Cochrane (1999) show that habit formation matches a range of asset-pricing moments. Moreover, this paper's main prediction, the variation in the agent's willingness to substitute intertemporally, has also been emphasized by Campbell and Cochrane (1999). But, the authors exactly offset this variation in intertemporal substitution motives by a habit process that features variation in the agent's precautionary-savings motive. Furthermore, because the agent's habit increases the curvature of the value function, the agent's effective risk aversion is quite high and becomes the main variability-driving mechanism. The same holds true for Barberis, Huang, and Santos (2001), variation in the coefficient of loss aversion introduces predictability, whereas the additively separable gain-loss components over financial wealth yields a constant consumption-wealth ratio and risk-free rate. In the news-utility model, effective risk aversion is constant and equals the coefficient of relative risk aversion. The model retains the power utility property that the curvature of the value function is solely determined by the coefficient of relative risk aversion, as gain-loss utility is proportional to consumption utility.

Routledge and Zin (2010) assume generalized disappointment-aversion preferences and show that these are consistent with basic financial market moments. The model has been extended to long-run risk by Bonomo, Garcia, Meddahi, and Tedongap (2011). However, these models rely on high risk aversion in low states of the world when the agent is likely to be disappointed, as habit-formation preferences do.[12] Furthermore, Campanale, Castro, and Clementi (2010) assume disappointment-aversion preferences in a production economy. In this model the excessive volatility of the risk-free rate can be reduced by assuming a high intertemporal elasticity of substitution. However, the variation in returns is acyclical by construction, which rules out predictability.[13]

This paper contributes to the literature by incorporating a preference specification, which has proven to be consistent with an array of micro evidence in a variety of domains. Thus, I can relate microeconomic evidence to the model's asset-pricing implications and the intuition of which, pin down narrow ranges for all parameters, and simultaneously match risk attitudes over small, medium, and large stakes.

[12]Strong variation in effective risk aversion has problems to robustly match evidence on risk attitudes towards wealth bets and is contradicted by portfolio-choice data (Brunnermeier and Nagel (2008)).

[13]Epstein and Zin (1989) preferences are able to rationalize the equity premium with the addition of long-run risk or heterogeneous agents as shown by Bansal and Yaron (2004). Epstein and Zin (1989) preferences feature a constant elasticity of intertemporal substitution that can be chosen as an additional parameter in the model. A stark difference between this approach and my model is that the elasticity of intertemporal substitution is typically chosen to be above one to match financial market moments, whereas the asset-pricing implications of Koszegi and Rabin (2009) preferences and other micro evidence suggest a value below one.

3 The Model

3.1 Expectations-based reference-dependent preferences

I assume expectations-based reference-dependent preferences, as specified in Koszegi and Rabin (2009). Instantaneous utility in each period is the sum of consumption utility and gain-loss utility. The latter component consists of "contemporaneous" gain-loss utility about current consumption and "prospective" gain-loss utility about the entire stream of future consumption. Thus, total instantaneous utility in period t is given by

$$U_t = u(C_t) + n(C_t, F^{t-1}_{C_t}) + \gamma \sum_{\tau=1}^{\infty} \beta^{\tau} \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}). \tag{1}$$

The first term in equation (1) corresponds to consumption utility in period t, which is a power-utility function $u(c) = \frac{c^{1-\theta}}{1-\theta}$. The following terms are defined over both consumption and the agent's "beliefs" about consumption, which I explicitly define below. Throughout the paper, I assume rational expectations such that the agent's beliefs about any of the model's variables equal the objective probabilities determined by the economic environment.

Definition 1. Let I_t denote the agent's information set in some period $t \leq t+\tau$, then the agent's probabilistic beliefs about consumption in period $t+\tau$, conditional on period t information, is denoted by $F^{t}_{C_{t+\tau}}(c) = Pr(C_{t+\tau} < c|I_t)$ and $F^{t+\tau}_{C_{t+\tau}}$ is degenerate.

To understand the following terms in equation (1), first note that the reference point in period t are the fully probabilistic beliefs about consumption in period t and all future periods $t+\tau$, given the information available in period $t-1$. According to definition 1, the agent's beliefs formed in period $t-1$ about period $t+\tau$ consumption are denoted by $F^{t-1}_{C_{t+\tau}}$. Thus, the second term in equation (1), $n(C_t, F^{t-1}_{C_t})$, corresponds to gain-loss utility in period t over contemporaneous consumption. Gain-loss utility is determined by a piecewise linear value function $\mu(\cdot)$ with slope η and a coefficient of loss aversion λ, i.e., $\mu(x) = \eta x$ for $x > 0$ and $\mu(x) = \eta\lambda x$ for $x \leq 0$. The parameter $\eta > 0$ weights the gain-loss utility component relative to the consumption utility component and $\lambda > 1$ implies that losses are weighed more heavily than gains; the agent is loss averse. Because the agent compares his actual contemporaneous consumption with his prior beliefs, he experiences gain-loss utility over "news" about contemporaneous consumption as follows

$$\begin{aligned} n(C_t, F^{t-1}_t(C^{t-1}_t)) &= \int_0^{\infty} \mu(u(C_t) - u(c)) dF^{t-1}_{C_t}(c) \\ &= \eta \int_0^{C_t} (u(C_t) - u(c)) dF^{t-1}_{C_t}(c) + \eta\lambda \int_{C_t}^{\infty} (u(C_t) - u(c)) dF^{t-1}_{C_t}(c). \end{aligned} \tag{2}$$

The third term in equation (1), $\gamma \sum_{\tau=1}^{\infty} \beta^{\tau} \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}})$, corresponds to prospective gain-loss utility in period t over the entire stream of future consumption. Prospective gain-loss utility about period $t+\tau$ consumption, $\boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}})$, depends on $F^{t-1}_{C_{t+\tau}}$, the agent's beliefs with which he entered the period, and on $F^{t}_{C_{t+\tau}}$, the agent's updated beliefs about period $t+\tau$ consumption. $F^{t-1}_{C_{t+\tau}}$ and $F^{t}_{C_{t+\tau}}$ are correlated distribution functions, because future uncertainty is contained in both prior and updated beliefs about $C_{t+\tau}$. Thus, there exists a joint distribution, which I denote by $F^{t,t-1}_{C_{t+\tau}} \neq F^{t}_{C_{t+\tau}} F^{t-1}_{C_{t+\tau}}$. Because the agent compares his new beliefs with his prior beliefs, he experiences gain-loss utility over "news" about future consumption

$$\boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) = \int_0^\infty \int_0^\infty \mu(u(c) - u(r)) dF^{t,t-1}_{C_{t+\tau}}(c,r). \tag{3}$$

Both contemporaneous and prospective gain-loss utility correspond to an outcome-wise comparison as assumed in Koszegi and Rabin (2006, 2007).[14] Moreover, the agent discounts prospective gain-loss utility exponentially by β, the standard agent's consumption utility discount factor; and prospective gain-loss utility is subject to another discount factor γ relative to contemporaneous gain-loss utility, so that the agent puts a weight $\gamma\beta^{\tau} < 1$ on prospective gain-loss utility about consumption in period $t+\tau$.

Because both contemporaneous and prospective gain-loss utility are experienced over news, the preferences are referred to as "news utility".

3.2 The model environment and equilibrium

The model environment. I consider a Lucas (1979) tree model in which the sole source of consumption is an everlasting tree that produces C_t units of consumption each period t. I assume that consumption growth is log-normal, following Mehra and Prescott (1985). Thus, the endowment economy's exogenous consumption process is given by

$$log(\frac{C_{t+1}}{C_t}) = \mu_c + \varepsilon_{t+1} \text{ with } \varepsilon_{t+1} \sim N(0, \sigma_c^2). \tag{4}$$

The price of the Lucas tree in each period t is P_t. Moreover, there exists a risk-free asset in zero net supply with return R^f_{t+1}. The period $t+1$ return of holding the Lucas tree is then $R_{t+1} = \frac{P_{t+1}+C_{t+1}}{P_t}$. Each period t, the agent faces the price of the Lucas tree P_t and the risk-free return R^f_{t+1} and, acting as a price taker, optimally decides how much to consume C^*_t and how much to invest in the risky asset α^*_t.[15]

[14] The outcome-wise comparison of Koszegi and Rabin (2006, 2007) has been generalized to an ordered comparison in Koszegi and Rabin (2009), because the agent would otherwise experience gain-loss disutility over future uncertainty even if no update in information takes place. I circumvent this problem by explicitly noting that prior and new beliefs about consumption are correlated, i.e., I generalize the gain-loss formula of Koszegi and Rabin (2006, 2007)

$$n(F_c, F_r) = \int_0^\infty \int_0^\infty \mu(u(c) - u(r)) dF_r(r) dF_c(c) \quad \text{to} \quad \boldsymbol{n}(F_{c,r}) = \int_0^\infty \int_0^\infty \mu(u(c) - u(r)) dF_{c,r}(c,r).$$

The ordered comparison yields qualitatively and quantitatively similar results but the model's solution is not as tractable.

[15] A benefit of the Lucas-tree environment is that the correlation structure of consumption, which is left unspecified in equation (2) and (3), is fully determined by the exogenous market-clearing consump-

Equilibrium prices and definition. Because the agent fully updates his beliefs each period and the consumption process is i.i.d., I look for an equilibrium price and risk-free return process that is "Markovian" in the sense that the price-consumption ratio depends on the current shock only.

Definition 2. The price process $\{P_t\}_{t=0}^{\infty}$ and risk-free return process $\{R_{t+1}^f\}_{t=0}^{\infty}$ are Markovian if, in each period t, the price-consumption ratio $\frac{P_t}{C_t}$ and the risk-free return R_{t+1}^f depend on the realization of the shock ε_t only, such that $\frac{P_t}{C_t} = p(\varepsilon_t)$ and $R_{t+1}^f = r(\varepsilon_t)$ with the functions $p(\cdot)$ and $r(\cdot)$ being independent of calender time t or endowment C_t.

Facing Markovian prices and returns, the agent's maximization problem in period t is given by

$$max_{C_t}\{u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{\infty} \beta^{\tau} \boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1}) + E_t[\sum_{\tau=1}^{\infty} \beta^{\tau} U_{t+\tau}]\}. \tag{5}$$

The agent's wealth in the beginning of period t, W_t, is determined by the portfolio return R_t^p, which depends on the risky return realization R_t, the risk-free return R_t^f, and previous period's optimal portfolio share α_{t-1}. The budget constraint is

$$W_t = (W_{t-1} - C_{t-1})R_t^p = (W_{t-1} - C_{t-1})(R_t^f + \alpha_{t-1}(R_t - R_t^f)). \tag{6}$$

In each period t, the agent optimally decides how much to consume C_t^*, how much to invest $W_t - C_t^*$, and how much to invest in the risky asset α_t^*. In equilibrium, the price of the tree $P_t = W_t - C_t$ adjusts so that the single agent in the model always chooses to hold the entire tree, i.e., $\alpha_t^* = 1$ for all t, and consume the tree's entire payoff $C_t^* = C_t$ for all t as determined by the endowment economy's exogenous consumption process (4). In the following, I derive the "Markovian rational-expectations equilibrium" recursively; in the Lucas-tree model, it corresponds to the preferred-personal equilibrium, as defined in Koszegi and Rabin (2006).[16]

Definition 3. The Markovian rational-expectations equilibrium consists of a Markovian price process $\{P_t = C_t p(\varepsilon_t)\}_{t=0}^{\infty}$ and a risk-free return process $\{R_{t+1}^f = r(\varepsilon_t)\}_{t=0}^{\infty}$ such that the solution $\{C_t^*, \alpha_t^*\}_{t=0}^{\infty}$ of the price-taker's maximization problem (5) subject to the budget constraint (6) satisfies goods market clearing $\{C_t^* = C_t\}_{t=0}^{\infty}$ and asset market clearing $\{\alpha_t^* = 1\}_{t=0}^{\infty}$.

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tion process, i.e., $F_{C_{t+\tau}}^t(c) = Pr(C_{t+\tau} < c|I_t)$, $I_t = \{C_t, P_t, \varepsilon_t\}$, and $\frac{C_{t+1}}{C_t} = e^{\mu_c + \varepsilon_{t+1}} \sim log - N(\mu_c, \sigma_c^2)$ for any $t \in [0, \infty)$ such that $F_{C_{t+\tau}}^t = log - N(log(C_t) + \tau\mu_c, \tau^2\sigma_c^2)$ for any $\tau > 0$.

[16] The personal-equilibrium solution concept introduced by Koszegi and Rabin (2006) is the family of credible state-contingent plans, which the agent's beliefs are rationally based on. Moreover, among all credible state-contingent plans the agent chooses the plan that maximizes expected reference-dependent utility going forward, the preferred-personal equilibrium. Because the agent's plan is credible, his behavior is time consistent. The first-order condition is derived under the premise that the agent enters period t, takes his beliefs as given, and optimizes with respect to consumption. Moreover, he rationally expects to behave like this in the future so that behavior maps into correct beliefs and vice versa.

Equilibrium existence and structure.

Proposition 1. *A Markovian rational-expectations equilibrium exists.*

This and the following propositions' proofs can be found in appendices B.1 to E.1.

The equilibrium has a very simple structure and can be derived in closed form. In each period t, optimal consumption C_t^* is a fraction of current wealth W_t such that $C_t^* = W_t \rho_t$. As appendix B.2 shows, the consumption-wealth ratio ρ_t is

$$\rho_t = \frac{C_t^*}{W_t} = \frac{1}{1 + \frac{Q+\Omega+\gamma Q\Omega+\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}}. \tag{7}$$

Here, $F(\cdot)$ denotes the cumulative normal distribution function $N(0, \sigma_c)$ and Q and Ω are determined by exogenous parameters. Thus, ρ_t varies with the realization of ε_t, is i.i.d., independent of calender time t, or current endowment C_t. The price-consumption ratio is $\frac{P_t}{C_t} = \frac{1-\rho_t}{\rho_t}$. The agent's value function is proportional to the power utility of wealth $V_t = u(W_t)\Psi_t$. Ψ_t varies with the realization of ε_t, is i.i.d., independent of calendar time t, or current endowment C_t. I now explain the news-utility agent's first-order condition in detail to build intuition for Q and Ω and to clarify why and how ρ_t varies with ε_t.

3.3 Predictions about the consumption-wealth ratio

Before turning to the model's asset pricing implications, I describe the agent's first-order condition to provide intuition for two predictions about the agent's consumption-wealth ratio, which are formalized in propositions 2 and 3 and illustrated in figure 1. Although the first-order condition appears complicated, the terms can be easily understood one component at a time. The agent's consumption-wealth ratio ρ_t, equation 7, results from the model's first-order condition

$$\begin{aligned} &C_t^{-\theta}(1 \underbrace{+\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))}_{\textbf{contemporaneous gain-loss}}) \\ &\quad = (\frac{\rho_t}{1-\rho_t})^{1-\theta} (\underbrace{(W_t - C_t)^{-\theta}(Q + \Omega + \gamma\Omega Q}_{= -\frac{d\beta E_t[u(W_{t+1})\Psi_{t+1}]}{dC_t}} + \underbrace{\gamma Q(\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t)))}_{\textbf{prospective gain-loss}})). \end{aligned} \tag{8}$$

First, for $\eta = 0$, the model collapses to the standard consumption-based asset-pricing model with constant relative risk aversion and log-normal consumption growth assumed by Mehra and Prescott (1985) among many others. The first-order condition becomes

$$C_t^{-\theta} = (\frac{\rho^s}{1-\rho^s})^{1-\theta}(W_t - C_t)^{-\theta} Q \tag{9}$$

and results in a constant consumption-wealth ratio $\rho^s = \frac{1}{1+Q}$. Let me return to news utility and henceforth assume that $\eta > 0$ and $\lambda > 1$. In the following, I describe the news-utility agent's first-order condition, equation (8), to show that, in contrast to the standard model, the consumption-wealth ratio is shifted down and is not constant.

The shift in the consumption-wealth ratio. The left hand side of the first-order condition, equation (8), is simply determined by marginal consumption and gain-loss utility over contemporaneous consumption. Marginal gain-loss utility is given by the states that would have promised less consumption $F_{C_t}^{t-1}(C_t)$, weighted by η, or more consumption $1-F_{C_t}^{t-1}(C_t)$, weighted by $\eta\lambda$, i.e., $\frac{\partial n(C_t, F_{C_t}^{t-1})}{\partial C_t} = u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1 - F_{C_t}^{t-1}(C_t)))$. A key technical insight here allows me to simplify the marginal gain-loss utility term: In the Lucas-tree model, equilibrium consumption is determined by the realization of the shock ε_t, which allows me to simplify $F_{C_t}^{t-1}(C_t) = F(\varepsilon_t)$.

Let me turn to the right hand side of equation (8). The first term represents the marginal value of savings $-\frac{d\beta E_t[u(W_{t+1})\Psi_{t+1}]}{dC_t} = u'(W_t - C_t)(Q + \Omega + \gamma\Omega Q)$ with Q and Ω determined by exogenous parameters. In the standard model, the marginal value of savings is given by $u'(W_t - C_t)Q$. Thus, Q represents the discounted stream of future consumption utility, and Ω represents expected gain-loss utility; the marginal value of savings is determined by $Q + \Omega + \gamma\Omega Q$, the sum of expected consumption utility, expected contemporaneous gain-loss utility, and expected prospective gain-loss utility discounted by γ. Accordingly, if expected gain-loss disutility is positive $\Omega > 0$, then the marginal value of saving increases relative to the standard model. The underlying intuition is that the agent anticipates gain-loss disutility that is proportional to marginal consumption utility. Thus, fluctuations are less painful on a less steep part of the utility curve, and the agent has an additional incentive to increase savings. Moreover, it can be shown that the additional precautionary-savings motive is first-order, i.e., $\frac{\partial\Omega}{\partial\sigma_c}|_{\sigma_c=0} > 0$, because it depends on concavity of the utility curve rather than prudence as in the standard model.

However, if the agent discounts news about the future $\gamma < 1$ he has an additional reason to consume more today, because positive news about contemporaneous consumption are overweighted. Thus, the additional precautionary-savings motive results in the consumption-wealth ratio being lower than in the standard model, if the agent does not discount future news too highly $\gamma > \bar{\gamma}$. These ideas can be formalized in the following proposition.

Proposition 2. *If $\theta > 1$ and $\gamma > \bar{\gamma}$ with $\bar{\gamma} = \frac{\eta\lambda - \frac{\Omega}{Q}}{\Omega + \eta\lambda} < 1$ then, for all realizations of ε_t, the consumption-wealth ratio in the news-utility model is lower than in the standard model $\rho_t < \rho^s$. Moreover, $\bar{\gamma}$ is decreasing in the news-utility parameters $\frac{\partial\bar{\gamma}}{\partial\lambda}, \frac{\partial\bar{\gamma}}{\partial\eta} \leq 0$.*[17]

Koszegi and Rabin (2009) state in proposition 8 that news-utility introduces an additional first-order precautionary savings motive in a two-period two-outcome model. Proposition 8 carries over only for $\theta > 1$, because I consider multiplicative instead of additive shocks. Multiplicative shocks imply that savings increase the absolute value of tomorrow's wealth bet, which the news-utility agent dislikes. For $\theta < 1$, this effect dominates the intertemporal smoothing desire. For log utility $\theta = 1$, the two motives

[17] If $\theta > 0$ and $\frac{\eta - \frac{\Omega}{Q}}{\Omega + \eta} < \gamma < \bar{\gamma}$ then ρ^s and ρ_t cross at $\varepsilon_t = \bar{\varepsilon}_t$ and $\bar{\varepsilon}_t$ is decreasing in the news-utility parameters $\frac{\partial\bar{\varepsilon}_t}{\partial\lambda}, \frac{\partial\bar{\varepsilon}_t}{\partial\eta} \leq 0$.

exactly offset each other and $\Omega = 0$. Thus, if $\theta = 1$ and $\gamma = 1$, the news-utility model becomes observationally equivalent to the standard model.[18]

Variation in the consumption-wealth ratio. Let me move on to the second part on the right hand side in the first-order condition (8) that represents marginal prospective gain-loss utility. In the absence of expected gain-loss disutility $\Omega = 0$ and prospective gain-loss discounting $\gamma = 1$, marginal contemporaneous and prospective gain-loss utilities would cancel out. Then, I would be back in the standard model with a proportional response of consumption to wealth. However, contemporaneous marginal utility is driven above future marginal utility due to the additional marginal value of savings $\Omega > 0$ so that $Q + \Omega + \gamma Q \Omega \neq \gamma Q$. Thus, the consumption-wealth ratio ρ_t varies with the realization of ε_t.

Moreover, the consumption-wealth ratio is decreasing for $\theta > 1$. Because unexpected losses are particularly painful, the agent consumes relatively more of his wealth in the event of an adverse shock. I first outline a simplified intuition: If the agent encounters an adverse shock, decreasing consumption below expectations today is more painful than decreasing consumption tomorrow when the reference point will have decreased. If the agent encounters a positive shock he experiences less painful gain-loss fluctuations today relative to tomorrow when the reference point will have increased. Thus, the agent wants to delay the consumption response to shocks, which makes the consumption-wealth ratio variable. More formally, in the event of an adverse shock, present marginal gain-loss utility is high relative to future marginal gain-loss utility. Today's reference point is invariable, whereas tomorrow's reference point will have adjusted to today's shock. Thus, future marginal gain-loss utility is constant whereas present marginal gain-loss is high, and the agent wants to consume relatively more today and relatively less tomorrow.[19] The following proposition formalizes this idea.

Proposition 3. *If $\theta \neq 1$ news utility introduces variation in the consumption-wealth ratio $\frac{\partial \rho_t}{\partial \varepsilon_t} \neq 0$. Moreover, for $\theta > 1$ the consumption-wealth ratio is decreasing $\frac{\partial \rho_t}{\partial \varepsilon_t} < 0$.*

The model's implications are illustrated in figure 1, which displays the consumption-wealth ratio ρ_t as a function of the shock to consumption growth and contrasts it with the standard agent's ratio for two levels if σ_c. The corresponding calibration is given in table 3 with $\lambda = 2$. News-utility preferences predict a downward shift and specific variation in the consumption-wealth ratio. The shape is driven by marginal gain-loss utility, which depends on the shock distribution $\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t)) \in [\eta, \eta\lambda]$. As ε_t is characterized by a bell-shaped distribution, the variation in the consumption-wealth ratio is bounded. The agent experiences gain-loss utility over all other states he might have been in weighted by their probabilities. For extreme realizations of

[18] This result is analogous to a result for quasi-hyperbolic discounting obtained by Barro (1999).

[19] This prediction about consumption is loosely related to a result in Koszegi and Rabin (2009): The authors find that in the event of surprises about wealth the agent responds asymmetrically to gains and losses. In particular, sufficiently small unexpected gains are consumed entirely, whereas losses are delayed.

ε_t, the consumption-wealth ratio approaches its limits because the states near these realizations have very low probabilities. ρ_t and ρ^s are displayed for two levels of σ_c, which illustrates that, for a small increase in σ_c, the downward shift in ρ_t is larger than the downward shift in ρ^s, because the additional precautionary savings motive is a first-order effect, i.e., $\frac{\partial \rho_t}{\partial \sigma_c}|_{\sigma_c \to 0} > 0$, while the standard precautionary-savings motive is second order.

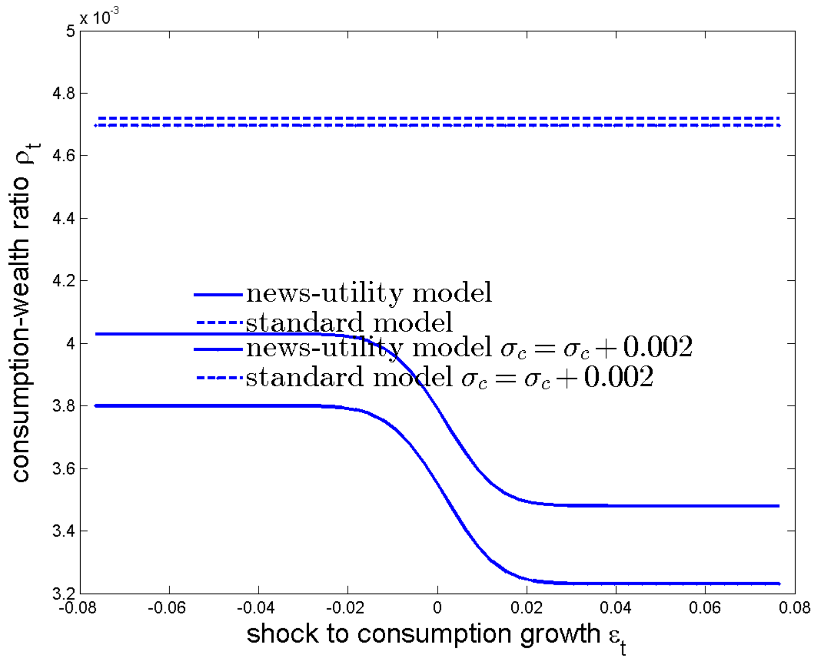

Figure 1: Consumption-wealth ratio ρ_t in the news-utility and standard models.

Furthermore, the steepness or responsiveness of the consumption-wealth ratio near the center of the distribution depends on the amount of economic uncertainty σ_c. The responsiveness of the consumption-wealth ratio is determined by the extent of pain or pleasure induced by gain-loss utility that is generally reduced for wider prior distributions. For example, a moderately bad realization feels less painful if the previously expected distribution was relatively less narrow; accordingly, the agent does not feel the need to respond as much. Finally, the consumption-wealth ratio is skewed in the sense that the agent underconsumes more in good times than he overconsumes in bad times. Adverse shocks are over-weighted and thus more effectively alleviated by previously expected uncertainty. Accordingly, the consumption-wealth ratio becomes more skewed when uncertainty increases.[20]

[20]This asymmetry can be illustrated by an increase in economic volatility σ_c. The consumption-wealth ratio shifts down and becomes more skewed. In the event of an overall negative surprise, $\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))$ will be reduced if the distribution of ε_t is wider because $1 - F(\varepsilon_t)$ decreases and is over-weighted by λ, therefore, ρ_t will be less variable. However, in the event of an overall

4 Asset Pricing

Now I turn to the model's asset-pricing implications. First, I derive the expected risky return, the risk-free return, and the equity premium. Then I illustrate the model's main asset-pricing predictions, namely variation in expected returns, the equity premium, and predictability. I aim to build intuition for these asset-pricing results by connecting them back to my prior theoretical results about the consumption-wealth ratio. Proposition 4 formalizes the main idea. In turn, I calibrate the model to gauge its quantitative performance in section 4.2 and compare them with the asset-pricing literature.

4.1 Predictions about expected returns and the equity premium

Expected returns and the equity premium. The return of holding the entire Lucas tree is $R_{t+1} = \frac{P_{t+1}+C_{t+1}}{P_t}$. I can rewrite the expected risky return in terms of the consumption-wealth ratio ρ_t and consumption growth $\frac{C_{t+1}}{C_t}$ by taking expectations and noting that $P_t = W_t - C_t = C_t \frac{1-\rho_t}{\rho_t}$, i.e.,

$$E_t[R_{t+1}] = \frac{\rho_t}{1-\rho_t} E_t[\frac{1}{\rho_{t+1}} \frac{C_{t+1}}{C_t}]. \tag{10}$$

$E_t[\frac{1}{\rho_{t+1}} \frac{C_{t+1}}{C_t}]$ is constant because consumption growth $\frac{C_{t+1}}{C_t} = e^{\mu_c + \varepsilon_{t+1}}$ and next period's consumption-wealth ratio ρ_{t+1} are i.i.d., as reported in definition 2 such that $\frac{P_{t+1}}{C_{t+1}} = p(\varepsilon_{t+1}) = \frac{1-\rho_{t+1}}{\rho_{t+1}}$. However, $E_t[R_{t+1}]$ varies with the consumption-wealth ratio ρ_t.

I can rewrite the first-order condition as $1 = E_t[M_{t+1} R_{t+1}]$, which gives rise to the agent's stochastic discount factor M_{t+1} derived in appendix B.2. The risk-free return is the inverse of the conditional expectation of the stochastic discount factor

$$R^f_{t+1} = \frac{1}{E_t[M_{t+1}]} = \frac{\rho_t}{1-\rho_t}(Q + \Omega + \gamma\Omega Q) E_t[\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-\theta}\Psi_{t+1}]^{-1}. \tag{11}$$

$E_t[\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-\theta}\Psi_{t+1}]^{-1}$ is constant because consumption growth $\frac{C_{t+1}}{C_t} = e^{\mu_c+\varepsilon_{t+1}}$, the next period's consumption-wealth ratio ρ_{t+1}, and the value function's proportionality factor Ψ_{t+1} are i.i.d. However, R^f_{t+1} varies with the consumption-wealth ratio ρ_t. The equity premium

$$E_t[R_{t+1}] - R^f_{t+1} = \frac{\rho_t}{1-\rho_t}(E_t[\frac{1}{\rho_{t+1}}\frac{C_{t+1}}{C_t}] - (Q+\Omega+\gamma\Omega Q)E_t[\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-\theta}\Psi_{t+1}]^{-1}) \tag{12}$$

is characterized by a constant price of risk. The price of risk and the conditional Sharpe ratio $S_t = \frac{E_t[R_{t+1} - R^f_t]}{\sigma_t(R_{t+1})}$ are constant, because the agent holds the entire stock market and thus faces the same risk each period. However, the quantity of risk $\sigma_t(R_{t+1})$ varies with the consumption-wealth ratio ρ_t.

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positive surprise, increasing the returns variance increases $\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))$ so that ρ_t is more variable. Accordingly, increasing the variance affects the response of ρ_t asymmetrically, it induces more underconsumption in good times and less overconsumption in bad times. Although hard to detect with the naked eye, this effect can be seen in figure 1.

Variation in expected returns and predictability. I have shown that the expected risky return, the risk-free return, and the equity premium vary with the consumption-wealth ratio ρ_t. The news-utility implications about the location and shape of the consumption-wealth ratio ρ_t, which are formalized in propositions 2 and 3, directly carry over to the expected return, the risk-free return, and the equity premium. The variation in the expected risky return is generated by the general-equilibrium nature of the model and driven by variation in the agent's willingness to substitute intertemporally as reflected by variation in ρ_t.

In bad states of the world the agent would like to delay adjustments in consumption to let his reference point adjust. To induce the agent to consume his endowment, the price of the Lucas tree must be low and expected returns have to be high. Thus, despite the i.i.d. environment, the expected risky return varies to make the agent willing to hold the entire tree each period. Moreover, the variation in the consumption-wealth ratio generates return predictability. In particular, the realization of ε_t predicts the one-period ahead return R_{t+1}. If ε_t is low then ρ_t the consumption-wealth ratio is high and the one-period ahead return is high; hence, the consumption-wealth ratio positively predicts one-period ahead returns. Moreover, this mechanism generates predictability in excess returns through the consumption-wealth ratio. Bad states predict high future returns, and this implies that the standard deviation of returns is also high and the expected equity premium varies with ε_t. Using the same argument as above, the realization of ε_t then predicts the one-period ahead excess return $R_{t+1} - R^f_{t+1}$.

The following proposition formalizes the model's implications for variation and predictability in returns and the equity premium.

Proposition 4. *If $\theta > 1$ then the realization of the shock ε_t negatively impacts the expected risky return $\frac{\partial E_t[R_{t+1}]}{\partial \varepsilon_t} < 0$, risk-free return $\frac{\partial R^f_t}{\partial \varepsilon_t} < 0$, and equity premium $\frac{\partial (E_t[R_{t+1}] - R^f_{t+1})}{\partial \varepsilon_t} < 0$. This implies predictive power of the period t consumption-wealth ratio ρ_t regarding the period $t+1$ return R_{t+1} and excess return $R_{t+1} - R^f_{t+1}$.*

For illustration, figure 2 in appendix A compares the annualized news-utility return and equity premium with the standard model's ones under the calibration in table 3 with $\lambda = 2$. The expected equity premium amounts to approximately ten percent for low values of ε_t and three percent for high values of ε_t. The high equity premium reflects that the news-utility agent perceives uncertain fluctuations in consumption as being much more painful than the standard agent. The equity premium's variation stems from variation in the quantity of risk as a result of varying intertemporal smoothing incentives. But, the figure also illustrates how the model fails to predict reality: The risk-free return varies considerably, a phenomenon not observed in aggregate data.[21]

[21] Because the consumption-price ratio has a similar shape to the consumption-wealth ratio, the rates of return also correspond. Accordingly, the variation is bounded because gain-loss utility is bounded for a bell-shaped shock distribution. Furthermore, the steepness or responsiveness of the return varies with the amount of economic volatility, which determines the level of gain-loss feelings. Finally, due to the skewness in the variation of the consumption-wealth ratio, expected returns are negatively skewed.

4.2 Basic model: Calibration and moments

In the following, I calibrate the model to gauge its quantitative performance. Before assessing the model's ability to match asset-pricing moments, I illustrate the agent's risk attitudes towards small and large wealth bets. I show that the news-utility model is able to simultaneously match evidence on small-scale and large-scale risk aversion.

4.2.1 Risk attitudes over small and large stakes.

Before moving on to the model's asset-pricing moments I illustrate which news-utility parameter values, i.e., η, λ, and γ, are consistent with existing micro-evidence on risk preferences over small and large stakes and time preferences. I first show that the news-utility model does not generate high equity premia by secretly curving the value function to generate high effective risk aversion. On the contrary, the news-utility model retains a value function with constant curvature, because it is proportional to the power utility of wealth, i.e., $V_t = u(W_t)\Psi_t$ such that $RRA_t = -\frac{W_t V_t''}{V_t'} = \theta$.[22]

In table 1, I illustrate the risk preferences over gambles of various stakes of the standard, news-utility, habit-formation (Campbell and Cochrane (1999)), and long-run risk (Bansal and Yaron (2004)) agents. In particular, I analyze a range of 50-50 win G or lose L gambles at an initial wealth level of 300,000 in the spirit of Rabin (2001) and Chetty and Szeidl (2007). I elicit the agents' risk attitudes by assuming that each of them is presented the gamble after the shock to period t consumption growth has been realized and all consumption C_t in period t has taken place. Thus, the news-utility agent will experience merely prospective gain-loss utility rather than contemporaneous gain-loss utility over the gamble's outcome. In appendix B.6, I show that the news-utility agent is just indifferent to the gamble if

$$(Q+\Omega+\gamma Q\Omega)u(\bar{W}_t) = \gamma(0.5\eta(u(\bar{W}_t+G)-u(\bar{W}_t))Q + \eta\lambda 0.5(u(\bar{W}_t-L)-u(\bar{W}_t))Q) \\ + (Q+\Omega+\gamma Q\Omega)(0.5u(\bar{W}_t+G)+0.5u(\bar{W}_t-L)). \quad (13)$$

The first part on the right hand side of equation (13) represents prospective gain-loss utility, while the second part represents the same value comparison as done by the standard agent, i.e., $u(\bar{W}_t) \lesseqgtr 0.5u(\bar{W}_t+G)+0.5u(\bar{W}_t-L)$. Thus, if γ were zero the news-utility agent's risk attitudes would be the exact same as the standard agent's ones. Moreover, if L and G are small but $G > L$ this second part will certainly be positive as $u(\cdot)$ is almost linear, but the first part will induce prospect-theory risk preferences over future consumption. Although solely λ determines the sign of prospective gain-loss utility, there are restrictions on the other parameters, because the positivity of the second part may dominate the negativity of the first part if γ is small. Empirical estimates for the quasi-hyperbolic parameter β in the $\beta\delta-$model typically range between 0.7 and

[22] The intertemporal elasticity of substitution is disentangled and exhibits variation. The coefficient of relative risk aversion being disentangled from the intertemporal elasticity of substitution is a feature of a broad range of non-time-separable utility functions, such as habit formation.

0.8 (e.g., Laibson, Repetto, and Tobacman (forthcoming)). Thus, the experimental and field evidence on agent's attitudes towards intertemporal consumption trade offs dictates a choice of $\gamma \approx 0.8$ when $\beta \approx 1$.

Simultaneously, the model should match risk attitudes towards bets about immediate consumption, which are determined solely by η and λ, because it can be reasonably assumed that utility over immediate consumption is linear. Thus, $\eta = 1$ and $\lambda \approx 2.5$ is dictated by the laboratory evidence on loss aversion over immediate consumption, i.e., the endowment effect literature Kahneman, Knetsch, and Thaler (1990).[23]

In table 1, I calculate the required G for each value of L to make each agent just indifferent between accepting or rejecting a 50-50 win G or lose L gamble at wealth level $\bar{W}_t = 300,000$. It can be seen that the news-utility agent's risk attitudes take reasonable values for small, medium, and large stakes.[24] In contrast, the standard and long-run risk agents are risk neutral for small stakes and almost risk neutral for medium stakes. The habit-formation agent is risk neutral for small stakes, reasonably risk averse for medium stakes, but that makes him unreasonably risk averse for large stakes. Campbell and Cochrane (1999) also discuss this finding and indicate that the curvature of the habit-formation agent's value function is approximately 80 at the steady-state surplus-consumption ratio; thus, the habit-formation agent behaves similarly to a standard agent with $\theta = 80$. The long-run risk agent behaves similarly to a standard agent with $\theta = 10$, the choice of Bansal and Yaron (2004). Moreover, it can be inferred from this discussion that the disappointment-aversion model (Routledge and Zin (2010)) does not

[23] Let me take a concrete example from Kahneman, Knetsch, and Thaler (1990) assuming that utility over mugs, pens, and small amounts of money is linear. Kahneman, Knetsch, and Thaler (1990) hand out mugs to half the subjects and ask those who did not receive one about their willingness to pay and those who received one about their willingness to accept when selling the mug. The authors observe that the median willingness to pay for the mug is \$2.75 whereas the willingness to accept is \$5.25. Accordingly, I can infer $(1+\eta)u(mug) = (1+\eta\lambda)2.25$ and $(1+\eta\lambda)u(mug) = (1+\eta)5.25$ which implies that $\lambda \approx 3$ when $\eta \approx 1$. For the pen experiment I also obtain $\lambda \approx 3$. Unfortunately, so far I can only jointly identify η and λ. If the news-utility agent exhibits only gain-loss utility I would obtain $\eta\lambda 2.25 \approx 5.25$ and $\eta 2.25 \approx 2.25$, i.e., $\lambda \approx 2.3$ and $\eta \approx 1$ both identified. Alternatively, if I assume that the market price for mugs (or pens), which is \$6 in the experiment (or \$3.75), equals $(1+\eta)u(mug)$ (or $(1+\eta)u(pen)$) I can estimate $\eta = 0.74$ and $\lambda = 2.03$ for the mug experiment and $\eta = 1.09$ and $\lambda = 2.1$ for the pen experiment. These latter assumptions are reasonable given the induced-market experiments of Kahneman, Knetsch, and Thaler (1990). $\eta = 1$ and $\lambda \approx 2.5$ thus seem a reasonable choice and has been typically used in the literature for the static preferences.

[24] While the news-utility agent's risk preferences over contemporaneous consumption exactly match the findings of Kahneman, Knetsch, and Thaler (1990). The news-utility agent's required gain for small gambles about future consumption is somewhat lower than the estimates obtained by Tversky and Kahneman (1992) for instance, even though the authors consider monetary gambles and thus future consumption. But, the news-utility model predicts that people consume entire small gains when being surprised by risk (Koszegi and Rabin (2009)), thus the contemporaneous consumption results might be applicable even for monetary gambles. Moreover, a paper which explicitly considers gambles over future consumption is Andreoni and Sprenger (forthcoming), who find significantly less small-scale risk aversion towards those gambles. In any case, I do not aim to perfectly match experimental evidence here; rather, I want to demonstrate that the model does make a significant step forward in explaining small-stakes risk aversion.

Table 1: **Risk attitudes over small and large wealth bets**

Loss (L)	standard	news-utility contemp.	news-utility prospective	habit-formation	long-run risk
10	10	17	14	10	10
200	200	330	270	200	201
1000	1007	1650	1359	1362	1035
5000	5162	8250	7014	19749	6002
50000	74999	82500	110717	∞	303499
100000	299524	165000	6200303	∞	∞

For each loss L the table's entries show the required gain G to make each agent indifferent between accepting and rejecting a 50-50 gamble win G or lose L at wealth level 300,000.

robustly match risk attitudes towards small and large wealth bets, because the agent is not necessarily "at the kink". The asset-pricing theories based on prospect theory (Barberis, Huang, and Santos (2001); Benartzi and Thaler (1995)) imply plausible attitudes towards small and large wealth bets but not consumption bets and are thus inconsistent with the endowment-effect evidence.

4.2.2 Calibration and asset-pricing moments

Calibration. Table 3 in appendix A displays the calibration and the resulting moments of the news-utility and standard models, a short version of which is table 2. I begin with the model environment and the well-known preference parameters β and θ. I assume a classical Lucas-tree model in which consumption equals dividends, so that the model environment is fully calibrated by μ_c and σ_c. I follow Bansal and Yaron (2004) and choose $\mu_c = 1.8\%$ and $\sigma_c = 2.7\%$ in annualized terms. β and θ are then chosen to roughly match the level of the mean risky return, the mean risk-free return, and the risky return volatility as done by Bansal and Yaron (2004). Following Campbell and Cochrane (1999) and Bansal and Yaron (2004) I simulate the model at a higher frequency to then annualize moments. The news-utility equity premium increases in the model's frequency, which connects to the idea of myopic loss aversion as developed by Benartzi and Thaler (1995). The news-utility agent dislikes fluctuations in beliefs about future consumption. Observing the return realization and readjusting consumption plans at higher frequency, i.e., monthly instead of annually, makes the Lucas tree a less attractive investment opportunity. Therefore, the required compensation for bearing the risk associated with holdings of the Lucas tree increases.

The simulation frequency thus constitutes a calibrational degree of freedom in the news-utility model. At a monthly frequency, θ has to be close to one to match the historical equity premium. To have a bit more scope and aggregate to a quarterly frequency easily, I chose a one-and-a-half month frequency, $\theta = 2$, and $\beta = 0.98$ to match the historical equity premium as well as its volatility. Simulating the model at an annual frequency requires a somewhat higher coefficient of risk aversion θ and comparably high consumption volatility σ_c which are, however, not unusual in the literature. With $\sigma_c = 3.79\%$, as in Barberis, Huang, and Santos (2001), and $\theta = 10$, as in Bansal and Yaron (2004), the annualized news-utility model would roughly match the historical equity premium and its volatility.

The news-utility parameters are calibrated as standard in the prospect-theory literature $\eta = 1$ and $\lambda \in [2; 2.6]$ to match the large array of experimental evidence on loss aversion and to induce reasonable risk attitudes over small and large stakes as can be seen in table 1. These values have also been used in the existing prospect-theory asset-pricing literature; Benartzi and Thaler (1995) assume a coefficient of loss aversion $\lambda = 2.5$ and Barberis, Huang, and Santos (2001) assume a mean coefficient of loss aversion of approximately 2.25. Moreover, to account for the fact that people are present biased, I assume that the agent discounts prospective news utility and set $\gamma = 0.8$. I argue that the existing experimental literature suggests fairly tight ranges for all the news-utility parameters, η, λ, and γ, as well as the standard preference parameters θ and β. Thus, news utility does not allow for large parameter ranges that can be used at one's discretion, as opposed to most preference specifications used in the prospect-theory asset-pricing literature.[25] However, the simulation frequency constitutes a more worrisome degree of freedom, because it has been ignored in static applications of Koszegi and Rabin (2006, 2007) preferences.

Risky and risk-free return moments. As can be seen in table 2, the model matches the historical mean equity premium, its volatility, and the mean risk-free rate elicited from CRSP return data. Quite remarkably, the news-utility model generates the historical equity premium volatility, despite that consumption equals dividends in the basic Lucas-tree model. Thus, the model matches the historical risk-return trade-off with a Sharpe ratio of approximately 0.35. Unfortunately, the news-utility model completely mispredicts the risk-free rate volatility. Moreover, the risk-free rate is countercyclical in the model but procyclical in the data (Fama (1990)).

The model's performance regarding other return moments is mixed, as can be seen in table 3. The model matches the contemporaneous correlation of consumption growth with returns reasonably well but overpredicts the one-period ahead correlation.[26] Predicting too-high correlation between returns and consumption growth is a common failure of leading asset-pricing models as emphasized by Albuquerque, Eichenbaum, and Rebelo (2012) among others. But, because the variation in the consumption-wealth ratio in the news-utility model is a short-run phenomenon, at longer horizons the correlation between consumption growth and asset returns is very low thus matching the data. In contrast, the variation in the consumption-wealth ratio in the long-run risk

[25] Barberis, Huang, and Santos (2001)display results of the parameter k between 3 and 20 and those of b_0 between 0 and 100.

[26] Many asset-pricing models overstate the contemporaneous correlation of consumption and returns, which can be reduced by introducing a separate dividend process. As I roughly match this value I conclude that a separate process for dividends is unnecessary in the basic news-utility model although it will reduce the mispredicted one-period ahead correlation.

Table 2: **Moments of the basic model**

moments	standard and news-utility model				data
	$\eta = 0$	$\lambda = 2$	$\lambda = 2.3$	$\lambda = 2.6$	
$E[r_t - r_t^f]$	0.42	3.02	4.31	5.77	6.33
$\sigma(r_t - r_t^f)$	2.72	14.7	19.2	23.6	19.4
$E[r_t^f]$	5.48	2.15	0.68	-0.92	0.86
$\sigma(r_t^f)$	0.00	12.1	16.6	21.0	0.97
$E[c_t - p_t]$	-5.35	-5.57	-5.61	-5.69	-3.4
$\sigma(c_t - w_t)$	0.00	0.03	0.04	0.053	0.011
$AR(c_t - w_t)$	1.00	0.01	0.01	0.01	0.79
R^2	0.00	0.12	0.15	0.17	0.09

Value-weighted CRSP returns are displayed annualized and in percentage terms. The quarterly moments for the consumption-wealth ratio and predictability regression ($r_{t+1} = \alpha + \beta(c_t - w_t) + \delta r_t^f$) are taken from Lettau and Ludvigson (2001) table II and III.

model is a long-run phenomenon and thus implies counterfactually high correlations between consumption growth and asset returns at longer horizons. Moreover, the autocorrelation of returns is negative in the model as opposed to around zero in the data.[27] Finally, in table 3 I display the moments for a slight increase in θ, γ, and η to give a quantitative idea of the parameters' implications.

The consumption-wealth ratio. The model's simulated consumption-wealth ratio reflects the prior theoretical results. First, the consumption-wealth ratio is lower than in the standard model and exhibits variation. As consumption equals dividends in the classical Lucas-tree model and there is no labor income, the values are difficult to compare with the data. However, the corresponding values in Lettau and Ludvigson (2001) are displayed as an illustration. Both the standard and news-utility model roughly match the level of the consumption-price ratio, but the standard model mispredicts its variation whereas the news-utility model's predicted variation is roughly in line with the data.[28] However, the news-utility consumption-wealth ratio is i.i.d. whereas Lettau and Ludvigson (2001) find relatively high persistence.

The predictability properties compare quite favorably. The model is able to generate predictability in quarterly returns yielding R^2 values of approximately 10% to 17%. Lettau and Ludvigson (2001) emphasize the medium-run predictive power of the aggregate consumption-wealth ratio. The authors obtain R^2 values for quarterly returns of 9% and of 18% for annual excess returns.[29] Lustig, Nieuwerburgh, and Verdelhan (2012)

[27] Although, the aggregation to annualized frequency seems to introduce some spurious correlation as can be seen in the standard model's moments.

[28] Moreover, the consumption-wealth ratio cannot be used to forecast consumption growth, which is in line with the empirical findings in Lettau and Ludvigson (2001).

[29] Lettau and Ludvigson (2001)report small-sample statistics which might be biased up. Therefore, one might argue that the R^2 of the model is somewhat too high, and should be reduced by decreasing

elaborate on the volatility of the consumption-wealth ratio and the return on the consumption claim. As noted by Hirshleifer and Yu (2011), traditional leading asset-pricing models have difficulty matching the volatility of the consumption-wealth ratio and the return on the consumption claim, because they rely on a volatile dividend process, and the only variation in the consumption-wealth ratio stems from heteroskedasticity in consumption growth. I can confirm this finding; using the return on the consumption claim, the R^2 in the habit-formation model of Campbell and Cochrane (1999) is merely 1.6% and the R^2 in the long-run risk model of Bansal and Yaron (2004) is just 2.9%.

Moreover, in figure 3 in appendix A I plot the simulated deviations of the news-utility, standard, habit-formation, and long-run risk consumption-wealth ratio and compare these with the annual $\widehat{cay}$ data provided by Lettau and Ludvigson (2005). For the habit-formation and long-run risk model I use the calibration of Campbell and Cochrane (1999) and Bansal and Yaron (2004) to then aggregate the consumption and wealth time series. Moreover, I feed in the deviations in log consumption growth $\Delta c - 12\mu_c$ of the $\widehat{cay}$ data. As can be seen, news-utility introduces considerably more rapid variation in the consumption-wealth ratio than the standard model or the model augmented with long-run risk, but much less variation than the habit-formation model. While, the long-run risk consumption-wealth ratio appears to be too smooth and habit-formation consumption-wealth ratio too variable, the news-utility variation in the consumption-wealth ratio matches the $\widehat{cay}$ data quite well. Although it is disputable to compare the $\widehat{cay}$ data to the simulated data of a Lucas-tree model, I conclude that the rapid variation is supported by the data.[30]

At first blush, the model's asset pricing implications appear to be mixed. News utility raises the equity premium and its volatility to historical levels even though I omit a separate dividend process. Moreover, the variation in substitution motives generates strong variation in the consumption-wealth ratio and predictability in returns, matching the data better than leading asset-pricing models. However, the model predicts excessive volatility in the risk-free rate, which I address in the following section.

5 Extensions

Motivation. The news-utility model's most important shortcoming is the large predicted variation in the risk-free rate. Nevertheless, I want to take the predictions of

θ, λ, or increasing γ.

[30] Greenwood and Shleifer (2012) compare a variety of survey data on stock market expectations with the predicted expected returns of leading asset-pricing models. The authors show that leading asset-pricing models implied expected returns do not correlate highly with the survey evidence on expected returns. In particular, the $\widehat{cay}$ model of Lettau and Ludvigson (2001) fits the survey data better than the habit-formation and long-run risk models do. I can confirm this finding using the American Association of Individual Investors Sentiment Survey and also find that the news-utility model is more positively correlated with the survey data than the habit-formation, long-run risk, models or the $\widehat{cay}$ data. However, this finding should not be overinterpreted as the annual comparison includes the years 1987 to 2001 only.

the evidence-based utility specification seriously and believe that people are very unwilling to substitute consumption intertemporally in some states of the world. The most important evidence is credit-card borrowing or pay-day loans. However, there may be forces at work that offset the effects on the aggregate risk-free rate. What would a consumption process look like which features an almost constant risk-free rate? An adverse shock to contemporaneous consumption growth has to be associated with an adverse prediction about future consumption growth to keep the risk-free rate low. Thus, the model's risk-free rate process will become more smooth if low values of ε_t are associated with a decrease in μ_c or an increase in σ_c. Variation in the agent's expected consumption growth μ_c has been exploited by Bansal and Yaron (2004) and termed long-run risk. Variation in the agent's expected volatility of consumption growth has been exploited by Campbell and Cochrane (1999) and Bansal and Yaron (2004).[31]

In section 5.1, I reverse-engineer variation in expected consumption growth and its volatility to offset the effect of the variation in the agent's intertemporal smoothing incentives on the risk-free rate. An adverse shock to consumption growth today is then associated with low consumption growth but high volatility in the future. There exists empirical evidence for countercyclical variation in economic uncertainty, or consumption volatility.[32] The empirical evidence on excess sensitivity suggests that there exists positive autocorrelation in consumption growth. However, it turns out that the variation in the agent's smoothing incentives require variation in the agent's expected consumption growth that is too large to be consistent with aggregate consumption data, because the variation in consumption volatility appears to be too weak to significantly affect the strong first-order variation in the agent's risk-free rate. As another alternative, I extend the model to account for time-variant disaster risk to smooth out the risk-free rate in section 5.2. Time-variant disaster risk is a very powerful device under news-utility preferences because they feature left-skewness aversion: The news-utility agent hates the left tail and thus disaster risk. It turns out that time-variant disaster risk is powerful enough to successfully offset the variation in the risk-free rate. Moreover, Barro (2006) provides compelling evidence for the existence of a small probability of economic disaster.

It is important to note that introducing another source of variation does not elim-

[31]Campbell and Cochrane (1999) specify heteroskedasticity in consumption growth to make the risk-free rate exactly constant.

[32]Since French, Schwert, and Stambaugh (1987) it is well known that volatility of stock returns fluctuates considerably over time. Moreover, Black (1976) was one of the first to document that stock returns are negatively correlated with future volatility, an empirical observation which has been referred to as the leverage or volatility-feedback effect. More recently, Lettau and Ludvigson (2004) document that the countercyclical and highly volatile Sharpe ratio is not replicated by leading consumption-based asset pricing models. The Sharpe ratio becomes both more countercyclical and volatile if low returns imply high expected returns and low volatility, as I assume in the extended model. The authors find that the consumption-wealth ratio predicts stock market volatility and provide evidence for variation in aggregate consumption volatility. Furthermore, Tauchen (2011) connects the negative correlation in stock returns and volatility back to the consumption process underlying a standard Lucas-tree model. Finally, robust evidence for heteroskedasticity is provided by Bansal, Khatchatrian, and Yaron (2003).

inate the variation in substitution motives; it merely offsets its effects on the risk-free rate. Moreover, the extended models feature two sources of variation: The news-utility variation in substitution motives and heteroskedasticity in consumption growth or time-variant disaster risk. While the first source of variation concerns intertemporal substitution, the latter work via variation in the price of risk.

5.1 Time-variant consumption growth and volatility

Setup. A decrease in expected consumption growth μ_c or an increase in expected volatility σ_c make the agent consume less and save more. Thus, if an adverse shock is associated with a decrease in expected consumption growth or an increase in expected volatility the agent's intertemporal substitution effects on the risk-free rate will be partially offset. Let consumption growth be given by $log(\frac{C_{t+1}}{C_t}) = \mu_t + \sigma_t\varepsilon_{t+1}$ with $\mu_{t+1} = \mu_c + \nu_\mu(\mu_t - \mu_c) + \tilde{\mu}(\varepsilon_{t+1}) + u_{t+1}$, $u_{t+1} \sim (0, \sigma_u^2)$, and $\tilde{\mu}(\varepsilon_{t+1}) = \bar{\mu}(log(\frac{1-\rho_{t+1}}{\rho_{t+1}}) - E[log(\frac{1-\rho_t}{\rho_t})])$. Moreover, $\sigma_{t+1}^2 = \sigma_c^2 + \tilde{\sigma}(\varepsilon_{t+1}) + \nu_\sigma(\sigma_t^2 - \sigma_c^2) + w_{t+1}$, $w_t \sim (0, \sigma_w^2)$, and $\tilde{\sigma}(\varepsilon_t) = \bar{\sigma}(0.5 - F(\varepsilon_t))$. The variation in $\tilde{\sigma}(\varepsilon_{t+1})$ aims to reflect the variation in the homoskedastic consumption-wealth ratio, because heteroskedasticity is intended to offset the general-equilibrium impact on the risk-free rate. Note that σ_t is a Markovian process, increases in the event of an adverse shock and is characterized by a shape similar to the consumption-wealth ratio determined by the variation in intertemporal substitution motives. Moreover, the conditional expectation of economic volatility is characterized by an AR(1) process with persistence ν_σ. μ_t is chosen to fine-tune the remaining variation in the risk-free rate. The functional form of $\tilde{\mu}(\varepsilon_t)$ is reverse-engineered such that if $\bar{\mu} = 1$ and $\nu_\mu = 0$ the variation in the risk-free rate brought about by the variation in the price-consumption ratio will be exactly offset, as can be seen in equation (11). If $\nu_\mu > 0$ the conditional expectation of consumption growth is characterized by an AR(1) process with persistence ν_μ. The model's simple structure is unaffected by variation in expected consumption growth and derived in appendix C.

Time-variant consumption growth and volatility: Calibration and moments. I slightly modify the calibration presented in table 3 to roughly match the basic asset-pricing moments in the modified model which are displayed in table 4 in appendix A. In particular, I simulate the model at a monthly frequency and decrease β and θ slightly. For illustration, I chose the simplest possible process for expected consumption growth with $\nu_\sigma = \nu_\mu = 0$, $\sigma_w = \sigma_u = 0$ and $\bar{\sigma} = 2$. Moreover, I chose $\bar{\mu} = 0.8$ to smooth out 80% of the variability in the risk-free rate. As can be seen in table 4, the basic asset-pricing moments are matched well in the model with variation in expected consumption growth. Importantly, countercyclical variation in consumption growth does not reduce the variation in the consumption-wealth ratio, such that the model continues to fit the $\widehat{cay}$ data provided by Lettau and Ludvigson (2005) and the model's predictability properties are still present. Thus, the variation in expected consumption growth does not eliminate the variation in intertemporal substitution motives but rather introduces a second channel that offsets the impact on the risk-free rate. If $\nu_\sigma > 0$, a positive shock to economic volatility today implies high volatility in the future because the heteroskedasticity process is autocorrelated. Then, the size of the excess returns will be autocorrelated and the model is able to generate autocorrelation in the returns and long-horizon predictability.[33]

However, μ_c, σ_c, and $\tilde{\mu}(\cdot)$ jointly determine the moments of the annualized consumption growth process, which I also display in table 4 following Bansal and Yaron (2004). Unfortunately, the required variation in $\tilde{\mu}(\cdot)$ significantly changes the moments of the annualized consumption growth process which then fails to match the data even if lower levels for both μ_c and σ_c are chosen. The annualized standard deviation of the simulated consumption process should be at most 3.5%. This value is far exceeded because of the extent of variation in expected consumption growth required to smooth out 80% of the variability of the risk-free rate.

5.2 Time-variant disaster risk

Setup. An increase in the probability of disaster makes the agent value a unit of safe consumption more highly. Thus, if adverse shock realizations of the world are associated with disaster risk the risk-free rate smooths out. Thus, I introduce a small time-variant probability of disaster according to Barro (2006, 2009). In each period t, there is a probability p_t that a disaster occurs in period $t+1$ in which case consumption drops by d percent. Thus, consumption growth is given by $log(\frac{C_{t+1}}{C_t}) = \mu_c + \varepsilon_{t+1} + \upsilon_{t+1}$ with $\varepsilon_{t+1} \sim N(0, \sigma_c^2)$ and $\upsilon_{t+1} = log(1-d)$ with probability p_t and zero otherwise. I assume that ε_{t+1} and υ_{t+1} are independent. The simple process governing the variability in disaster risk is $p_{t+1} = p + \nu(p_t - p) + u_{t+1} + \tilde{g}(\varepsilon_{t+1})$ with $u_{t+1} \sim N(0, \sigma_u^2)$ and $\tilde{g}(\varepsilon_t) = p\bar{p}(0.5 - F(\varepsilon_t))$. Note that p_t is a Markovian process, increases in the event of an adverse shock, and is characterized by a similar shape as the consumption-wealth ratio determined by the variation in intertemporal substitution motives. Moreover, the conditional expectation of disaster risk is characterized by an AR(1) process with persistence ν. The model's simple structure is unaffected by the addition of disaster risk and derived in appendix D.

The news-utility agent is more affected by the probability of disaster than the standard agent, because the news-utility agent dislikes disaster risk more. The utility function's gain-loss component over news is inspired by prospect theory. Classical prospect

[33] Koszegi and Rabin (2007) find that news utility causes variation in risk attitudes. In proposition 1, the authors state that the agent becomes less risk averse when moving from a fixed to a stochastic reference point. With a stochastic reference point, a gamble does not appear as daunting, because some potential losses were previously expected. Thus, the equity premium in period t depends negatively on σ_{t-1}, because it is determined by the price of risk, i.e., $\frac{Cov_t(M_{t+1}, R_{t+1})}{E_t[M_{t+1}]\sigma_t(R_{t+1})}$, which varies with σ_t and σ_{t-1}. If high volatility is expected, ρ_t is less steep and thus less responsive to a shock to consumption growth, which tends to reduce the required equity premium. Hence, news-utility preferences introduce two sources of variation in the price of risk and thus the required equity premium: The price of risk varies with economic volatility σ_t as in the standard model. Furthermore, for any given σ_t, the price of risk varies inversely with the variability of beliefs determined by σ_{t-1}.

theory assumes a value function of the form $v(c-r)$, defined over the actual consumption level c relative to the reference point r. Typically, the value function features a kink at the reference point r, concavity over gains $c > r$, convexity over losses $c < r$, and probability weighting. In contrast, Koszegi and Rabin (2009) specify gain-loss utility as the linear difference in utility values $\mu(u(c) - u(r))$ with $\mu(\cdot)$ being some type of prospect-theory value function. The authors note that diminishing sensitivity or probability weighting may be introduced via $\mu(\cdot)$. However, thus far I have followed the literature and will retain news-utility preferences in their most basic form with $\mu(\cdot)$ being piecewise linear. Interestingly though, using a piecewise linear $\mu(\cdot)$ function results in left-skewness aversion: The news-utility agent hates the left tail. Because the agent assesses gain-loss utility as the linear difference in utility values $u(c) - u(r)$, the left tail, where $u(\cdot)$ becomes steep, is relatively overweighted. In classical prospect theory, left-skewness aversion can only be caused by low-probability overweighting. Thus, the basic form of Koszegi and Rabin (2009) preferences is likely to yield very interesting dynamics with respect to a small disaster probability.

Time-variant disaster risk: Calibration and moments. The extended model yields a realistic set of moments as shown in table 4 in appendix A. The parameters of disaster risk are calibrated according to Barro (2009) with $p = 1.7\%$ and $d = 0.29$. Most importantly, the additional variation does not eliminate the variation in intertemporal substitution motives or the variation in the consumption-wealth ratio. Rather it introduces another channel that offsets the impact of varying intertemporal smoothing incentives on the risk-free rate. A positive autocorrelation in the probability of disaster $\nu > 0$ will generate long-horizon predictability in returns and excess returns. However, for simplicity, I again omit persistence and additional noise in the disaster risk process $\nu = 0$ and $\sigma_u = 0$. The model generates a high equity premium that exhibits considerable variation, although, the model now requires the addition of a separate dividend process to match historical levels of the equity premium's variability. The reason is the low θ I have to choose to not increase the equity premium over and beyond historical levels. But, the variation in the risk-free rate is successfully reduced to approximately 3%, which is reasonable for international data.

6 Welfare and Beliefs-based Present Bias

Last, I illustrate the model's welfare implications. In the spirit of Lucas (1978) and Reis (2009) I show that the news-utility agent would be willing to give up a fraction λ_W of consumption in exchange for a risk-free consumption path, i.e., $E_t[\sum_{\tau=1}^{\infty} \beta^{\tau} u(C_{t+\tau}(1 + \lambda_W))] = \sum_{\tau=1}^{\infty} \beta^{\tau} u(\bar{C}_{t+\tau})$ with $\bar{C}_{t+\tau} = E_t[C_{t+\tau}] = C_t e^{\tau \mu_c + \tau \frac{1}{2} \sigma_c^2}$ for all τ. This fraction determines the costs of business cycle fluctuations and is much higher for the news-utility agent than the standard agent. For the calibration in table 3 with $\lambda = 2$, the news-utility agent would be willing to give up 17.02% of his consumption in exchange for a stable consumption path whereas the standard agent would give up merely 3.37%.

The preferences give rise to a time-inconsistent desire for immediate consumption, which I call beliefs-based present bias. A simplified intuition is that the agent prefers to raise consumption above expectations today instead of increasing consumption and expectations tomorrow. The preferred-personal solution concept requires the agent to choose an equilibrium path that is credible in the sense that beliefs map into the correct behavior and vice versa. However, the agent likes to surprise himself with some extra consumption, taking his beliefs as given in each period. In contrast, on the optimal pre-committed path that maximizes expected utility the agent jointly chooses optimal consumption and beliefs. Hence, the time-consistent equilibrium path does not correspond to the expected-utility maximizing one and the first welfare theorem does not hold. In appendix E.1 I elaborate on the properties of the optimal pre-committed path and how beliefs-based present bias differs from hyperbolic discounting.

7 Conclusion

This paper incorporates expectations-based reference-dependent preferences into the canonical Lucas-tree model. In so doing, I contribute to the prospect-theory asset-pricing literature, pioneered by Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), and Yogo (2008), by assuming a generally-applicable utility function that is based on consumption, does not require a narrow-framing assumption, has a fully developed reference point, and has been shown to be consistent with behavior in a variety of micro domains. News utility generates both desirable and undesirable implications. Most importantly, the preferences shift and introduce strong variation in the consumption-wealth ratio, which is reflected in an increase and variation in the equity premium matching historical levels despite the fact that consumption equals dividends. Intuitively, in bad states of the world, reducing consumption below expectations is particularly painful and the agent becomes unwilling to substitute present for future consumption - as is likely to be true for people engaging in too much credit-card borrowing. However, in a general-equilibrium setup, this translates into large variability in the risk-free rate, a phenomenon not observed in aggregate data. Moreover, I contribute to the asset-pricing literature by making an additional step towards resolving the equity-premium puzzle. In particular, I show that the agent exhibits plausible risk attitudes towards small, medium, and large wealth bets simultaneously; this is not the case for any other preference specification assumed in the literature.

News utility generates a high equity premium, because the agent finds fluctuations in beliefs about future consumption very painful. This relates to the idea of myopic loss aversion and strikes me as deeply true in the sense that people may worry too much about small changes in beliefs about future consumption. Thus, more “newsy” investments should have higher returns to compensate the painful fluctuations in beliefs. In the future, I would like to also analyze such cross-sectional stock market phenomena.

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A More figures and tables

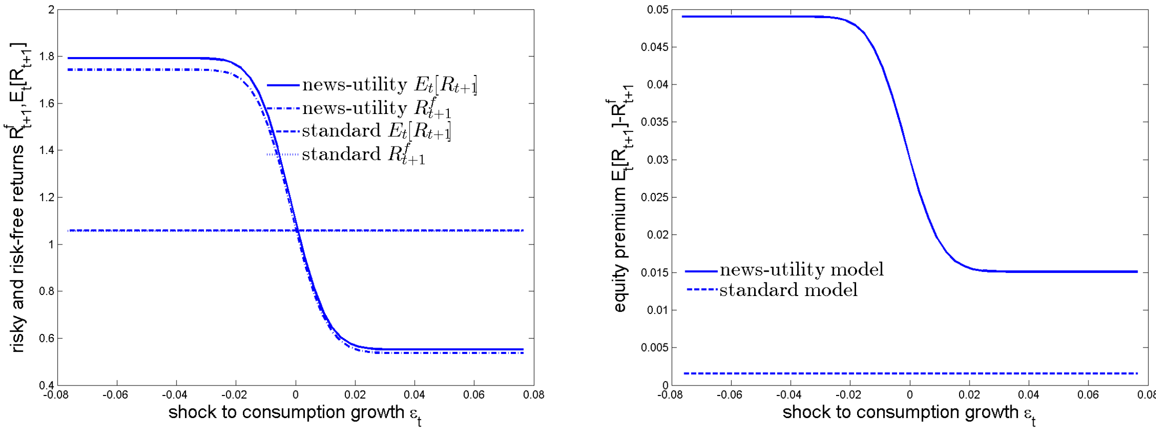

Figure 2: Annualized expected risky $E_t[R_{t+1}]$ and risk-free returns R^f_{t+1} in the news-utility and standard models and annualized equity premium $E_t[R_{t+1}] - R^f_{t+1}$ in the news-utility and standard models

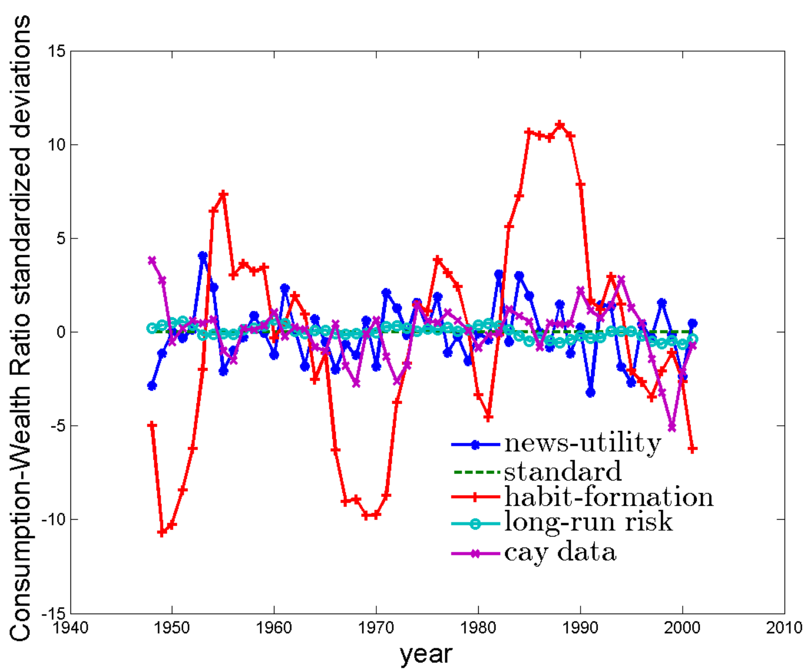

Figure 3: Simulated consumption-wealth ratio and comparison to the $\widehat{cay}$ data as provided by Lettau and Ludvigson (2005).

Table 3: **Calibration and moments of the basic model**

calibration						
μ_c	σ_c	β	θ	η	λ	γ
1.8%	2.7%	.98	2	1	2.3	0.8

moments	standard and news-utility model					$\lambda = 2$		data
	$\eta = 0$	$\lambda = 2$	$\lambda = 2.3$	$\lambda = 2.6$	$\theta = 3$	$\gamma = 1$	$\eta = 1.5$	
$E[r_t - r_t^f]$	0.42	3.02	4.31	5.77	4.25	2.66	3.84	6.33
$\sigma(r_t - r_t^f)$	2.72	14.7	19.2	23.6	16.9	13.3	17.4	19.42
$E[r_t^f]$	5.48	2.15	0.68	-0.92	1.82	2.07	1.23	0.86
$\sigma(r_t^f)$	0.00	12.1	16.6	21.0	14.3	10.6	14.8	0.97
$corr(\Delta c_t, r_t)$	0.71	0.55	0.51	0.48	0.53	0.57	0.52	0.36
$corr(\Delta c_t, r_{t+1})$	0.55	-0.07	-0.12	-0.16	-0.10	-0.04	-0.11	0.09
$AR(r_t)$	0.08	-0.51	-0.54	-0.56	-0.53	-0.48	-0.52	0.011
$E[c_t - p_t]$	-5.35	-5.57	-5.61	-5.69	-5.31	-5.73	-5.61	-3.4
$\sigma(c_t - w_t)$	0.00	0.03	0.04	0.053	0.04	0.03	0.04	0.011
$AR(c_t - w_t)$	1.00	0.004	0.005	0.006	0.004	0.004	0.005	0.79
R^2	0.00	0.12	0.15	0.17	0.14	0.11	0.14	0.09

Return and consumption moments are inferred from value-weighted CRSP return data and BEA data on-real per-capita consumption of nondurables and services for the period 1929-1998 (as in Bansal and Yaron (2004)). All return moments are annualized and in percentage terms. The parameters μ_c, σ_c, and β are annualized. The quarterly moments for the consumption-wealth ratio, the consumption-price ratio, and predictability regression are taken from Lettau and Ludvigson (2001) table II. The R^2 corresponds to a forecasting regression of quarterly stock returns on the quarterly consumption-wealth ratio $r_{t+1} = \alpha + \beta(c_t - w_t) + \delta r_t^f$ (table III in Lettau and Ludvigson (2001)).

Table 4: **Calibration and moments of the extended models**

calibration variation in μ_c and σ_c

μ_c	σ_c	β	θ	η	λ	γ	ν_μ	$\bar{\mu}$	$\bar{\sigma}$
$\frac{1.8}{4}\%$	$\frac{2.7}{\sqrt{4}}\%$	.94	1.3	1	2.3	0.8	0	0.8	2

calibration disaster risk

μ_c	σ_c	β	θ	η	λ	γ	ν	$\bar{p}$
1.8%	2.7%	.95	1.05	1	2	0.8	0	2

moments	extended models		data
	variation in μ_c and σ_c	disaster risk	
$E[r_t - r_t^f]$	5.04	6.17	6.33
$\sigma(r_t - r_t^f)$	19.6	7.31	19.42
$E[r_t^f]$	1.37	1.45	0.86
$\sigma(r_t^f)$	3.72	3.64	0.97
$corr(\Delta c_t, r_t)$	0.75	0.62	0.36
$corr(\Delta c_t, r_{t+1})$	0.60	0.06	0.09
$AR(r_t)$	0.18	-0.44	0.011
$E[c_t - p_t]$	-5.64	-4.91	-3.4
$\sigma(c_t - w_t)$	0.03	0.01	0.011
$AR(c_t - w_t)$	-0.04	0.04	0.79
R^2	0.32	0.05	0.09
$E[\Delta c_{t+1}]$	2.25	1.86	1.89
$\sigma(\Delta c_{t+1})$	13.5	2.24	2.93

Return and consumption moments are inferred from value-weighted CRSP return data and BEA data on-real per-capita consumption of nondurables and services for the period 1929-1998 (as in Bansal and Yaron (2004)). All return moments are annualized and in percentage terms. The parameters μ_c, σ_c, and β are annualized. The quarterly moments for the consumption-wealth ratio, the consumption-price ratio, and predictability regression are taken from Lettau and Ludvigson (2001) table II. The R^2 corresponds to a forecasting regression of quarterly stock returns on the quarterly consumption-wealth ratio $r_{t+1} = \alpha + \beta(c_t - w_t) + \delta r_t^f$ (table III in Lettau and Ludvigson (2001)).

B Derivation and proofs

B.1 Proof of proposition 1

In the following, I quickly guess and verify the model's equilibrium. In section B.2, I derive the model's equilibrium in greater detail and more comprehensively. The exogenous consumption process is $\frac{C_{t+1}}{C_t} = e^{\mu_c + \varepsilon_{t+1}}$ and, in equilibrium, the agent beliefs about consumption are fully determined by it, i.e., $F^t_{C_{t+\tau}} = log{-}N(log(C_t) + \tau\mu_c, \tau^2\sigma_c^2)$. First, I define the following two constants determined by the exogenous parameters only

$$Q = E_t[\sum_{\tau=1}^{\infty} \beta^\tau (\frac{C_{t+\tau}}{C_t})^{1-\theta}] = E_t[\sum_{\tau=1}^{\infty} \beta^\tau (e^{\tau\mu_c + \sum_{j=1}^{\tau} \varepsilon_{t+j}})^{1-\theta}] = \frac{\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}{1 - \beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}$$

and

$$\psi = \beta e^{\mu_c(1-\theta)} E_t[(e^{\varepsilon_{t+1}})^{1-\theta} + (1+\gamma Q)(\eta \int_{-\infty}^{\varepsilon_{t+1}} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta}) dF(\varepsilon) +$$
$$+ \eta\lambda \int_{\varepsilon_{t+1}}^{\infty} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta}) dF(\varepsilon)) + (e^{\varepsilon_{t+1}})^{1-\theta}\psi)].$$

The agent's maximization problem is

$$max_{C_t}\{u(C_t) + n(C_t, F^{t-1}_{C_t}) + \gamma \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) + E_t[\sum_{\tau=1}^{\infty} \beta^\tau U_{t+\tau}]\}.$$

Now, it can be easily noted that $E_t[\sum_{\tau=1}^{\infty} \beta^\tau U_{t+\tau}] = u(C_t)\psi$ and $\gamma \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) = \gamma(\eta \int_{-\infty}^{C_t} (u(C_t)Q - u(c)Q) dF^{t-1}_{C_t}(c)) + \eta\lambda \int_{C_t}^{\infty} (u(C_t)Q - u(c)Q) dF^{t-1}_{C_t}(c))$ in equilibrium.

The agent is a price-taker. In the beginning of each period, the agent observes the realization of his wealth W_t and decides how much to consume C_t and how much to invest into the Lucas tree $P_t = C_t - W_t$. I guess the model's solution as $C_t = W_t\rho_t$ with ρ_t being i.i.d., independent of calender time t, or wealth W_t. Thus, next period's consumption is given by $C_{t+1} = (W_t - C_t)R_{t+1}\rho_{t+1}$ with $R_{t+1} = \frac{P_{t+1}+C_{t+1}}{P_t} = \frac{\rho_t}{1-\rho_t}\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}}$ so that $C_{t+1} = (W_t - C_t)\frac{\rho_t}{1-\rho_t}\frac{C_{t+1}}{C_t}$. From this consideration it can be easily seen that the agent's future value $u(C_t)\psi$ and $u(C_t)Q$ can be rewritten as $u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}\psi$ and $u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}Q$ whereby $\frac{\rho_t}{1-\rho_t}$ stems from the return and is thus taken as exogenous by the agent. In turn, the maximization problem can be rewritten as

$$max_{C_t}\{u(C_t) + \eta \int_{-\infty}^{C_t} (u(C_t) - u(c)) dF^{t-1}_{C_t}(c)) + \eta\lambda \int_{C_t}^{\infty} (u(C_t) - u(c)) dF^{t-1}_{C_t}(c)$$
$$+ \gamma Q(\eta \int_{-\infty}^{C_t} (u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta} - u(c)) dF^{t-1}_{C_t}(c))$$
$$+ \eta\lambda \int_{C_t}^{\infty} (u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta} - u(c)) dF^{t-1}_{C_t}(c)) + u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}\psi\}$$

which yields the following first-order condition

$$C_t^{-\theta}(1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))) = (W_t-C_t)^{-\theta}(\frac{\rho_t}{1-\rho_t})^{1-\theta}(\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))+\psi)$$

as the agent takes his prior beliefs about consumption $F_{C_t}^{t-1}$ as given in the optimization and since $F_{C_t}^{t-1}(C_t) = F(\varepsilon_t)$ with $F \sim N(0, \sigma_c^2)$, because $C_t = C_{t-1}e^{\mu_c+\varepsilon_t}$. Rewriting the first-order condition allows me to verify the solution guess

$$\frac{C_t}{W_t} = \rho_t = \frac{1}{1 + \frac{\psi+\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}}.$$

B.2 Detailed derivation of the model's equilibrium

In the following I derive the model's equilibrium in greater detail. The agent optimally chooses his consumption C_t to maximize his life-time utility

$$max_{C_t}\{u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma\sum_{\tau=1}^{\infty}\beta^\tau \boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1}) + E_t[\sum_{\tau=1}^{\infty}\beta^\tau U_{t+\tau}]\}. \tag{14}$$

The agent's wealth in the beginning of the period W_t is determined by the portfolio return $R_t^p = R_t^f + \alpha_{t-1}(R_t - R_t^f)$, which depends on the risky return realization R_t, the risk-free return R_t^f, and last period's optimal portfolio share α_{t-1}. I impose the equilibrium condition $\alpha_t = 1$ for all t to simplify the maximization problem. Now the agent's problem can be thought of as an infinite-horizon cake-eating problem with a single risky savings device. Thus, the budget constraint is

$$W_t = (W_{t-1} - C_{t-1})R_t \tag{15}$$

which results in the following first-order condition

$$u'(C_t)(1 + \eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1 - F_{C_t}^{t-1}(C_t))) = u'(W_t - C_t)Q_t^0 + u'(W_t - C_t)\psi_t^0 \tag{16}$$

I explain each term in the first-order condition, equation (16), subsequently. The left hand side in equation 16 represents the agent's marginal utility due to consumption utility and gain-loss utility over contemporaneous consumption. Because the agent takes the reference point as given in the optimization and assuming optimal consumption is monotonically increasing in the return realization only the probability masses of states ahead and beneath remain to be considered. As an illustration, consider the following optimization

$$\frac{\partial}{\partial C_t}(\eta\int_{-\infty}^{C_t}(u(C_t) - u(c)))dF_{C_t}^{t-1}(c)) + \eta\lambda\int_{C_t}^{\infty}(u(C_t) - u(c)))dF_{C_t}^{t-1}(c))$$

$$= \eta\int_{-\infty}^{C_t}u'(C_t)dF_{C_t}^{t-1}(c) + \eta\lambda\int_{C_t}^{\infty}u'(C_t)dF_{C_t}^{t-1}(c) = u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1 - F_{C_t}^{t-1}(C_t))$$

$$= u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1 - F_{C_t}^{t-1}(C_t))$$

$$= u'(C_t)(\eta F_{R_t}^{t-1}(R_t) + \eta\lambda(1 - F_{R_t}^{t-1}(R_t)) = u'(C_t)(\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))$$

if C_t is monotonically increasing in the realization of R_t then $F_{R_t}^{t-1}(R_t) = F_{C_t}^{t-1}(C_t)$. In a preferred personal equilibrium the agent would know ex ante if the first-order condition induces him to "jump" realizations of R_t, and expectations over optimal consumption would adjust accordingly such that in equilibrium $F_{C_t}^{t-1}(C_t) = F_{R_t}^{t-1}(R_t)$ for each corresponding realization of C_t and R_t. Moreover, in general equilibrium the agent's beliefs have to match the model environment and hence $F_{R_t}^{t-1}(R_t) = F_{C_t}^{t-1}(C_t) = F(\varepsilon_t)$ for each corresponding realization of C_t, R_t, and ε_t such that both C_t and R_t are necessarily increasing in ε_t.

To explain the right hand side in equation 16 I guess and verify the equilibrium's structure. In each period t, the agent will consume a fraction ρ_t of his wealth W_t, i.e., $C_t = \rho_t W_t$. In the first-order condition, equation 16, the first term on the right hand side represents prospective gain-loss utility over the entire stream of future consumption. Note that, each future optimal consumption as a fraction of wealth can be iterated back to the current savings decision

$$C_{t+\tau} = (W_t - C_t)R_{t+\tau}\rho_{t+\tau}\prod_{j=1}^{\tau-1} R_{t+j}(1-\rho_{t+j}).$$

Then, taking the reference point as given and assuming that optimal savings are monotonically increasing in the return realization results in

$$-(W_t - C_t)^{-\theta}Q_t^0 = \frac{\partial \sum_{\tau=1}^{\infty}\beta^\tau \boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1})}{\partial C_t} = \sum_{\tau=1}^{\infty}\beta^\tau \frac{\partial}{\partial C_t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(u(c)-u(r))dF_{C_{t+\tau}}^{t,t-1}(c,r)$$

$$= -\sum_{\tau=1}^{\infty}\beta^\tau (W_t - C_t)^{-\theta}E_t[R_{t+\tau}^{1-\theta}\rho_{t+\tau}^{1-\theta}\prod_{j=1}^{\tau-1}R_{t+j}^{1-\theta}(1-\rho_{t+j})^{1-\theta}(\eta F_{R_t}^{t-1}(R_t) + \eta\lambda(1-F_{R_t}^{t-1}(R_t)))]$$

$$= -(W_t - C_t)^{-\theta}E_t[\sum_{\tau=1}^{\infty}\beta^\tau R_{t+\tau}^{1-\theta}\rho_{t+\tau}^{1-\theta}\prod_{j=1}^{\tau-1}R_{t+j}^{1-\theta}(1-\rho_{t+j})^{1-\theta}](\eta F_{R_t}^{t-1}(R_t) + \eta\lambda(1-F_{R_t}^{t-1}(R_t))).$$

Moreover,

$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t} = \frac{\rho_t}{1-\rho_t}\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}}$$

such that

$$R_{t+\tau}^{1-\theta}\rho_{t+\tau}^{1-\theta} = (\frac{C_{t+\tau}}{C_{t+\tau-1}}\frac{\rho_{t+\tau-1}}{1-\rho_{t+\tau-1}})^{1-\theta} \text{ and } R_{t+j}^{1-\theta}(1-\rho_{t+j})^{1-\theta} = (\frac{C_{t+j}}{C_{t+j-1}}\frac{\rho_{t+j-1}}{1-\rho_{t+j-1}}\frac{1-\rho_{t+j}}{\rho_{t+j}})^{1-\theta}.$$

Recall that, the model's exogenous consumption process implies $\frac{C_{t+\tau}}{C_t} = e^{\tau\mu_c + \sum_{j=1}^{\tau}\varepsilon_{t+j}}$. Because in a rational-expectations equilibrium, the agent's expectational terms have to match the model's specification $\frac{\partial \sum_{\tau=1}^{\infty} \beta^{\tau} \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}})}{\partial C_t}$ can be rewritten as $-(W_t - C_t)^{-\theta} Q^0_t$

$$-(W_t - C_t)^{-\theta} Q^0_t = -(W_t - C_t)^{-\theta} (\frac{\rho_t}{1-\rho_t})^{1-\theta} \gamma \underbrace{\frac{\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}}_{=Q} (\eta F(\varepsilon_t) + \eta\lambda(1-F(\varepsilon_t)))$$

$$= -(W_t - C_t)^{-\theta} (\frac{\rho_t}{1-\rho_t})^{1-\theta} \gamma Q (\eta F(\varepsilon_t) + \eta\lambda(1-F(\varepsilon_t))).$$

Returning to equation 16, the second term on the right hand side $-(W_t - C_t)^{-\theta} \psi^0_t$ refers to next period's marginal value, which turns out to be linear in the marginal utility of wealth. As above, iterating back next period's marginal utility, i.e., $\frac{\partial u(C_{t+1})}{\partial C_t} = (W_t - C_t)^{-\theta} R^{1-\theta}_{t+1} \rho^{1-\theta}_{t+1}$ and similarly for future consumption, for instance $\frac{\partial u(C_{t+2})}{\partial C_t} = (W_t - C_t)^{-\theta} R^{1-\theta}_{t+1} (1-\rho^{1-\theta}_{t+1}) R^{1-\theta}_{t+2} \rho^{1-\theta}_{t+2}$, yields

$$(W_t - C_t)^{-\theta} \beta E_t[R^{1-\theta}_{t+1} \Psi_{t+1}] = (W_t - C_t)^{-\theta} \beta E_t[R^{1-\theta}_{t+1} \rho^{1-\theta}_{t+1} + \eta \int_{-\infty}^{R_{t+1}\rho_{t+1}} (R^{1-\theta}_{t+1} \rho^{1-\theta}_{t+1} - (R\rho)^{1-\theta}) dF_{R\rho}(R\rho) +$$

$$+ \eta\lambda \int_{R_{t+1}\rho_{t+1}}^{\infty} (R^{1-\theta}_{t+1} \rho^{1-\theta}_{t+1} - (R\rho)^{1-\theta}) ddF_{R\rho}(R\rho) +$$

$$+\gamma(\frac{\rho_{t+1}}{1-\rho_{t+1}})^{1-\theta} Q(\eta \int_{-\infty}^{R_{t+1}(1-\rho_{t+1})} (R^{1-\theta}_{t+1}(1-\rho_{t+1})^{1-\theta} - (R(1-\rho))^{1-\theta}) dF_{R(1-\rho)}(R(1-\rho)) +$$

$$+\eta\lambda \int_{R_{t+1}(1-\rho_{t+1})}^{\infty} (R^{1-\theta}_{t+1}(1-\rho_{t+1})^{1-\theta} - (R(1-\rho))^{1-\theta}) dF_{R(1-\rho)}(R(1-\rho)) + \beta R^{1-\theta}_{t+1}(1-\rho^{1-\theta}_{t+1}) E_{t+1}[R^{1-\theta}_{t+2} \Psi_{t+2}]].$$

Now, let $\psi = \beta E_t[(\frac{C_{t+1}}{C_t} \frac{1}{\rho_{t+1}})^{1-\theta} \Psi_{t+1}] = \beta E_{t+1}[(\frac{C_{t+2}}{C_{t+1}} \frac{1}{\rho_{t+2}})^{1-\theta} \Psi_{t+2}]$ which is constant for any period t because $\frac{C_{t+1}}{C_t}$, ρ_{t+1}, and Ψ_{t+1} are all solely determined by the realization of ε_{t+1} and exogenous parameters. Then, the last term in the equation of $(W_t - C_t)^{-\theta} \beta E_t[R^{1-\theta}_{t+1} \Psi_{t+1}]$ is

$$\beta R^{1-\theta}_{t+1}(1-\rho^{1-\theta}_{t+1}) E_{t+1}[R^{1-\theta}_{t+2} \Psi_{t+2}] = R^{1-\theta}_{t+1}(1-\rho^{1-\theta}_{t+1}) (\frac{\rho_{t+1}}{1-\rho_{t+1}})^{1-\theta} \psi = R^{1-\theta}_{t+1} \rho^{1-\theta}_{t+1} \psi.$$

And, moreover

$$\beta E_t[R^{1-\theta}_{t+1} \Psi_{t+1}] = \beta E_t[(\frac{C_{t+1}}{C_t} \frac{\rho_t}{1-\rho_t} \frac{1}{\rho_{t+1}})^{1-\theta} \Psi_{t+1}] = (\frac{\rho_t}{1-\rho_t})^{1-\theta} \psi$$

such that it follows for the first-order condition, equation 16, that $\psi^0_t = (\frac{\rho_t}{1-\rho_t})^{1-\theta} \psi$.

Plugging in $R_{t+1} = \frac{C_{t+1}}{C_t} \frac{\rho_t}{1-\rho_t} \frac{1}{\rho_{t+1}}$ in the equation for $E_t[R^{1-\theta}_{t+1} \Psi_{t+1}]$ and recalling that $\frac{C_{t+1}}{C_t} = e^{\mu_c + \varepsilon_{t+1}}$ or alternatively simply dividing next period's C_{t+1} terms by C_t allows to express ψ in much simpler terms

$$(\frac{\rho_t}{1-\rho_t})^{1-\theta}\psi = (\frac{\rho_t}{1-\rho_t})^{1-\theta}\beta e^{\mu_c(1-\theta)}E_t[(e^{\varepsilon_{t+1}})^{1-\theta}+(1+\gamma Q)(\eta\int_{-\infty}^{\varepsilon_{t+1}}((e^{\varepsilon_{t+1}})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon)+$$
$$+\eta\lambda\int_{\varepsilon_{t+1}}^{\infty}((e^{\varepsilon_{t+1}})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon))+(e^{\varepsilon_{t+1}})^{1-\theta}\psi)]$$

accordingly $\psi = Q + (1+\gamma Q)\Omega = Q + \Omega + \gamma\Omega Q$ with Ωgiven by

$$\Omega = \frac{\beta e^{\mu_c(1-\theta)}E_t[(\eta\int_{-\infty}^{\varepsilon_{t+1}}((e^{\varepsilon_{t+1}})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon)+\eta\lambda\int_{\varepsilon_{t+1}}^{\infty}((e^{\varepsilon_{t+1}})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon))]}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}$$

$$\Omega = \frac{\beta e^{\mu_c(1-\theta)}\omega(\sigma_c)}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}} \text{ with}$$

$$\omega(\sigma) = \int_{-\infty}^{\infty}(\eta\int_{-\infty}^{z}((e^{z})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon)+\eta\lambda\int_{z}^{\infty}((e^{z})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon))dF(z) \quad z,\varepsilon\sim N(0,\sigma^2)$$
$$= \int_{-\infty}^{\infty}\{\eta F(z)e^{(1-\theta)z}-\eta e^{\frac{1}{2}(1-\theta)^2\sigma^2}(1-F(\frac{(1-\theta)\sigma^2-z}{\sigma}))$$
$$+\eta\lambda(1-F(z))e^{(1-\theta)z}-\eta\lambda e^{\frac{1}{2}(1-\theta)^2\sigma^2}F(\frac{(1-\theta)\sigma^2-z}{\sigma})\}dF(z)$$

In turn, the first-order condition can be rewritten as

$$u'(C_t)(1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))) = u'(W_t-C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}(\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))+\psi).$$

And the general equilibrium consumption-wealth ratio is then given by

$$\frac{C_t}{W_t} = \rho_t = \frac{1}{1+\frac{Q+\Omega+\gamma Q\Omega+\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}}.$$

Now, the solution guess $C_t = \rho_t W_t$ and $W_t - C_t = (1-\rho_t)W_t$ can be verified. The agent's value function is given by $V_t(W_t) = u(W_t)\Psi_t$. Obviously, C_t, $W_t - C_t$, and R_t are all increasing in the realization of ε_t. Finally, note that solving the model using backward induction and taking it to its limit yields this exact same solution.

The stochastic discount factor can be inferred from the first-order condition

$$1 = E_t[M_{t+1}R_{t+1}] = E_t[\frac{\beta u'(W_{t+1})\Psi_{t+1}}{u'(C_t)(1+\eta F(R_t)+\eta\lambda(1-F(R_t)))-E_t[u'(W_t-C_t)Q_t]}R_{t+1}]$$
$$\Rightarrow M_{t+1} = (1+(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))(1-\frac{\rho_t}{1-\rho_t}\gamma Q))^{-1}(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-1}\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi_{t+1}$$
$$= (\frac{\rho_t}{1-\rho_t}\psi)^{-1}(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-1}\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi_{t+1}$$

$$\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi_{t+1} = \beta e^{\mu_c(1-\theta)}\{(e^{\varepsilon_{t+1}})^{1-\theta}+(1+\gamma Q)(\eta\int_{-\infty}^{\varepsilon_{t+1}}((e^{\varepsilon_{t+1}})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon)+$$
$$+\eta\lambda\int_{\varepsilon_{t+1}}^{\infty}((e^{\varepsilon_{t+1}})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon))+(e^{\varepsilon_{t+1}})^{1-\theta}\psi\}$$

If $\eta(\lambda-1)>1$, the stochastic discount factor in the news utility model has a somewhat irritating feature: The existence of gain-loss utility generates negative values of the stochastic discount factor in particularly good states of the world. For any parameter choice, increasing the realization of ε_{t+1} will result in negative values of M_{t+1} at some point. The agent dislikes it if a return pays out in particularly good states of the world because he will experience adverse news-utility in all other states. Therefore, ex ante, the agent would prefer to burn consumption in those particularly pleasurable states. Although a negative stochastic discount factor implies arbitrage opportunities, non-satiated agents would not choose to buy consumption in these states at negative prices because they would experience adverse news utility in all other states. Therefore, the equilibrium is still valid. Moreover, the negativity of the stochastic discount factor in these states is unlikely to matter for the model's implications because, for reasonable parameter combinations, negativity only occurs in the range of four to five standard deviations from the mean. This positive probability of negative state prices is not new to the literature, Chapman (1998) elaborates on the possibility arising in habit-formation endowment economies and Dybvig and Ingersoll (1989) show how it arises in the CAPM.

Note that, for $\eta=0$ the model reduces to non-news or plain power utility in which the consumption-wealth ratio ρ^s is constant:

$$(\frac{\rho^s}{1-\rho^s})^{1-\theta}\psi=\beta E_t[R_{t+1}^{1-\theta}\Psi_{t+1}]\Rightarrow\psi=\beta e^{\mu_c(1-\theta)}E_t[(e^{\varepsilon_{t+1}})^{1-\theta}+(e^{\varepsilon_{t+1}})^{1-\theta}\psi]\Rightarrow\psi=Q$$

$$1=E_t[M_{t+1}R_{t+1}]=E_t[\frac{\beta u'(W_{t+1})((\rho^s)^{1-\theta}+(1-\rho^s)(\frac{\rho^s}{1-\rho^s})^{1-\theta}\psi)}{u'(C_t)}R_{t+1}]$$

$$M_{t+1}=\beta(\frac{1}{\rho^s}\frac{C_{t+1}}{C_t})^{-\theta}(\rho^s)^{1-\theta}(1+\psi)=\beta(\frac{C_{t+1}}{C_t})^{-\theta}$$

B.3 Proof of proposition 2

The marginal value of savings is given by $-\frac{d\beta E_t[u(W_{t+1})Q^0_{t+1}]}{dC_t}=u'(W_t-C_t)(Q+\Omega+\gamma\Omega Q)$ whereas in the standard model $\eta=0\Rightarrow\Omega=0$ and the marginal value of savings is given by $u'(W_t-C_t)Q$. If $\eta>0$, $\lambda>1$ and $\theta>1$ then $\Omega>0$ such that $Q+\Omega+\gamma\Omega Q>Q$ because:

$$\Omega=\frac{\beta e^{\mu_c(1-\theta)}\omega(\sigma_c)}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}\text{ with }\omega(\sigma_c)>0\text{ for }\theta>1$$

$$\text{since }\omega(\sigma)=\int_{-\infty}^{\infty}(\eta\int_{-\infty}^{z}\underbrace{((e^z)^{1-\theta}-(e^\varepsilon)^{1-\theta})}_{<0\text{ for }\theta>1}dF(\varepsilon)+\underbrace{\eta\lambda}_{>\eta}\int_{z}^{\infty}\underbrace{((e^z)^{1-\theta}-(e^\varepsilon)^{1-\theta})}_{>0\text{ for }\theta>1}dF(\varepsilon))dF(z)$$

Therefore, news-utility introduces an additional precautionary savings motive. Moreover, the consumption-wealth ratio is given by

$$\rho_t = \frac{1}{1 + \frac{Q+\Omega+\gamma Q\Omega+\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}} \text{ whereas in the standard model } \rho = \frac{1}{1+Q}.$$

Thus, the consumption-wealth ratio is unambiguously lower than in the standard model for $\gamma = 1$ because $\frac{\frac{Q+\Omega+\gamma Q\Omega}{Q}+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))} > 1$. For $\gamma < 1$, the consumption-wealth ratio is lower if $\gamma > \bar{\gamma}$ with

$$\frac{Q + \Omega + \bar{\gamma} Q\Omega + \bar{\gamma} Q(\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t)))}{1 + \eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))} = Q \Rightarrow \bar{\gamma} = \frac{\eta\lambda - \frac{\Omega}{Q}}{\Omega + \eta\lambda}.$$

As can be easily seen, $\bar{\gamma} < 1$. I chose $F(\varepsilon_t) = 1$ to obtain $\bar{\gamma}$ because $F(\varepsilon_t) = 1$ maximizes ρ_t if $\theta > 1$. Moreover, as can be easily seen $\frac{\partial\Omega}{\partial\eta}, \frac{\partial\Omega}{\partial\lambda} > 0$ if $\theta > 1$. Then

$$\frac{\partial\bar{\gamma}}{\partial\eta} = \frac{\partial\frac{\eta\lambda-\frac{\Omega}{Q}}{\Omega+\eta\lambda}}{\partial\eta} = \frac{(\lambda - \frac{1}{Q}\frac{\partial\Omega}{\partial\eta})(\Omega + \eta\lambda) - (\frac{\partial\Omega}{\partial\eta} + \lambda)(\eta\lambda - \frac{\Omega}{Q})}{(\Omega + \eta\lambda)^2} \leq 0 \text{ if } \Omega \leq \frac{\partial\Omega}{\partial\eta}\eta,$$

$$\frac{\partial\bar{\gamma}}{\partial\lambda} = \frac{\partial\frac{\eta\lambda-\frac{\Omega}{Q}}{\Omega+\eta\lambda}}{\partial\lambda} = \frac{(\eta - \frac{1}{Q}\frac{\partial\Omega}{\partial\lambda})(\Omega + \eta\lambda) - (\frac{\partial\Omega}{\partial\lambda} + \eta)(\eta\lambda - \frac{\Omega}{Q})}{(\Omega + \eta\lambda)^2} < 0 \text{ if } \Omega < \frac{\partial\Omega}{\partial\lambda}\lambda.$$

Additionally, by looking at Ω it is clear that $\frac{\partial\Omega}{\partial\eta} = \frac{\Omega}{\eta}$ and $\frac{\partial\Omega}{\partial\lambda}\lambda > \Omega$ if $\theta > 1$ so that the two conditions always hold.

If $\theta > 0$ then $\Omega > 0$ and $\frac{\partial\Omega}{\partial\eta} > 0$ and $\frac{\partial\Omega}{\partial\lambda} > 0$ such that $\frac{\partial\rho_t}{\partial\eta} < 0$ and $\frac{\partial\rho_t}{\partial\lambda} < 0$ for any ε_t. As $\frac{\partial\rho_t}{\partial\varepsilon_t} < 0$ and if $\frac{\eta-\frac{\Omega}{Q}}{\Omega+\eta} < \gamma < \bar{\gamma}$, such that ρ^s and ρ_t cross at some point $\varepsilon_t = \bar{\varepsilon}_t$ determined by $\rho_t = \rho^s$, then it can be easily inferred that $\bar{\varepsilon}_t$ is decreasing in the news-utility parameters $\frac{\partial\bar{\varepsilon}_t}{\partial\lambda}, \frac{\partial\bar{\varepsilon}_t}{\partial\eta} \leq 0$.

B.4 Proof of proposition 3

The slope of the consumption-wealth ratio is given by

$$\frac{\partial\rho_t}{\partial\varepsilon_t} = -\rho_t^2\frac{(Q + \Omega + \gamma Q\Omega - \gamma Q)\eta f(\varepsilon_t)(\lambda - 1)}{(1 + \eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t)))^2}.$$

Accordingly, $\frac{\partial\rho_t}{\partial\varepsilon_t} \neq 0$ iff $\lambda > 1$ and $Q + \Omega + \gamma\Omega Q \neq \gamma Q$, additionally, $\frac{\partial\rho_t}{\partial\varepsilon_t} < 0$ iff $\lambda > 1$ and $Q + \Omega + \gamma\Omega Q > \gamma Q$ which is necessarily true for $\theta > 1$ or for $\theta < 1$ if $\gamma < \tilde{\gamma}$ with $\tilde{\gamma} = \frac{Q+\Omega}{Q(1-\Omega)}$. Furthermore, if $\theta = 1$ and $\gamma = 1$ then $\frac{\partial\rho_t}{\partial\varepsilon_t} = 0$. If $\theta < 1$ and $\gamma > \tilde{\gamma}$ then $\frac{\partial\rho_t}{\partial\varepsilon_t} > 0$.

B.5 Proof of proposition 4

The news-utility equity premium

$$E_t[R_{t+1}] - R^f_{t+1} = \frac{\rho_t}{1-\rho_t}(\underbrace{E_t[\frac{1}{\rho_{t+1}}\frac{C_{t+1}}{C_t}]}_{\text{constant}} - \underbrace{\psi E_t[\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-\theta}\Psi_{t+1}]^{-1}}_{\text{constant}}).$$

Clearly, $E_t[R_{t+1}]$, R^f_{t+1}, and $E_t[R_{t+1}] - R^f_t$ vary with $\frac{\rho_t}{1-\rho_t}$ whereas the other terms are constant in an i.i.d. world. As $\frac{\partial \frac{\rho_t}{1-\rho_t}}{\partial \varepsilon_t} < 0$ for $\eta > 0$, $\lambda > 1$, and $\theta > 1$ so are $\frac{\partial E_t[R_{t+1}]}{\partial \varepsilon_t} < 0$, $\frac{\partial R^f_t}{\partial \varepsilon_t} < 0$, and $\frac{\partial E_t[R_{t+1}] - R^f_t}{\partial \varepsilon_t} < 0$.

B.6 Risk attitudes towards wealth bets

Recall that $\beta E_t[V_{t+1}(W_{t+1})] = E_t[\sum_{\tau=1}^{\infty}\beta^\tau U_{t+\tau}] = u(C_t)\psi$ and $\gamma\sum_{\tau=1}^{\infty}\beta^\tau \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) = \gamma(\eta\int_{-\infty}^{C_t}(u(C_t)Q - u(c)Q)dF^{t-1}_{C_t}(c)) + \eta\lambda\int_{C_t}^{\infty}(u(C_t)Q - u(c)Q)dF^{t-1}_{C_t}(c))$ such that the news-utility agent will accept the gamble iff

$$\gamma(0.5\eta(u((W_t+G)\rho_t)Q - u((W_t)\rho_t)Q) + \eta\lambda 0.5(u((W_t-L)\rho_t)Q - u((W_t)\rho_t)Q)$$
$$+ 0.5u((W_t+G)\rho_t)\psi + 0.5u((W_t-L)\rho_t)\psi > u(W_t\rho_t)\psi$$
$$\Rightarrow \frac{\gamma(0.5\eta(u(W_t+G) - u(W_t)) + \eta\lambda 0.5(u(W_t-L) - u(W_t)))Q}{Q+\Omega+\gamma Q\Omega} + 0.5u(W_t+G) + 0.5u(W_t-L) > u(W_t)$$

whereas the standard agent will accept the gamble iff

$$0.5u((W_t+G)\rho^s)Q + 0.5u((W_t-L)\rho^s)Q > u(W_t\rho^s)Q \Rightarrow 0.5u(W_t+G) + 0.5u(W_t-L) > u(W_t).$$

C Variation in consumption growth

The model's simple structure is unaffected, $C_t = \rho_t W_t$ and $V_t(W_t) = u(W_t)\Psi_t$ with ρ_t given by

$$\rho_t = \frac{1}{1 + \frac{\psi_t + \gamma Q_t(\eta F_{t-1}(\varepsilon_t) + \eta\lambda(1-F_{t-1}(\varepsilon_t)))}{1+\eta F_{t-1}(\varepsilon_t) + \eta\lambda(1-F_{t-1}(\varepsilon_t))}}.$$

Fluctuations in beliefs about economic volatility make the exogenous parameters $\psi_t = f^\psi(\mu_t, \sigma_t)$ and $Q_t = f^Q(\mu_t, \sigma_t)$ variant, and the calculation of ψ_t and Q_t thus becomes somewhat more complicated, by the same argument as above

$$Q_t = E_t[\beta(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}] + E_t[\beta(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}Q_{t+1}] = e^{(1-\theta)\mu_t}(E_t[\beta(e^{\sigma_t\varepsilon_{t+1}})^{1-\theta}] + E_t[\beta(e^{\sigma_t\varepsilon_{t+1}})^{1-\theta}Q_{t+1}]).$$

And ψ_t is

$$\psi_t = \beta E_t[(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta} + \eta(\lambda-1)\int_{-\infty}^{\varepsilon_{t+1}}((e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta} - (e^{\mu_t+\sigma_t x})^{1-\theta})dF(x) +$$
$$+\eta(\lambda-1)\int_{-\infty}^{\varepsilon_{t+1}}((e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}Q_{t+1} - (e^{\mu_t+\sigma_t x})^{1-\theta}f^Q(\tilde{\mu}(\mu_t, x), \tilde{\sigma}(\sigma_t, x)))dF(x) + \beta(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}\psi_{t+1}].$$

Note that in the standard model $\rho_t = \frac{1}{1+\psi_t^s}$ with

$$\psi_t^s = f^{\psi^s}(\mu_t, \sigma_t) = \beta E_t[(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}] + \beta E_t[(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{(1-\theta)}\psi_{t+1}^s]$$

Unfortunately, the heteroskedasticity model can no longer be solved analytically. But, thanks to the geometric-sum nature of Q_t and ψ_t they can be computed numerically using a simple interpolation procedure that iterates until convergence. The numerical solution procedure appears to be very robust and pricing errors in $1 = E_t[M_{t+1}R_{t+1}]$ are very small.

D Disaster risk

The model's simple structure is unaffected by disaster risk in the consumption process, $C_t = \rho_t W_t$ and $V_t(W_t) = u(W_t)\Psi_t$, but ρ_t now depends on the probability of disaster p_t and if disaster happened v_t and is given by

$$\rho_t = \frac{1}{1 + \frac{\psi_t + \gamma Q_t(\eta F_{t-1}(\varepsilon_t, v_t) + \eta\lambda(1-F_{t-1}(\varepsilon_t, v_t)))}{1+\eta F_{t-1}(\varepsilon_t, v_t)+\eta\lambda(1-F_{t-1}(\varepsilon_t, v_t))}}.$$

Note that $F_{t-1}(\varepsilon_t, 0) = p_{t-1}F(\varepsilon_t - log(1-d)) + (1-p_{t-1})F(\varepsilon_t)$ if a disaster does not occur with probability $1-p_{t-1}$ and $F_{t-1}(\varepsilon_t, log(1-d)) = p_{t-1}F(\varepsilon_t) + (1-p_{t-1})F(\varepsilon_t + log(1-d))$ if a disaster occurs with probability p_{t-1}.

If disaster risk is invariant, Q and ψ are constant, from the same arguments as above

$$Q = E_t[\sum_{\tau=1}^{\infty} \beta^\tau (e^{\tau\mu_c + \sum_{j=1}^{\tau}\varepsilon_{t+\tau} + \sum_{j=1}^{\tau} v_{t+\tau}})^{1-\theta}]$$

$$\text{with } E_t[e^{(1-\theta)\sum_{j=1}^{\tau} v_{t+j}}] = E_t[e^{(1-\theta)v_{t+1}}]^\tau = (1-p+p(1-d)^{1-\theta})^\tau$$

$$\text{such that } Q = \frac{\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}(1-p+p(1-d)^{1-\theta})}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}(1-p+p(1-d)^{1-\theta})}$$

$$\psi = Q + \Omega + \gamma Q\Omega \text{ and } \Omega = \frac{\beta e^{\mu_c(1-\theta)}((1-p)\omega(p) + p\omega_p(p))}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}(1-p+p(1-d)^{1-\theta})} \text{ with}$$

$$\omega(p) = \int_{-\infty}^{\infty} \eta(\lambda-1) \int_{z}^{\infty} (1-p)((e^z)^{1-\theta} - (e^\varepsilon)^{1-\theta}) + p((e^z)^{1-\theta} - (e^\varepsilon(1-d))^{1-\theta})dF(\varepsilon)dF(z)$$

$$\omega_p(p) = \int_{-\infty}^{\infty} +\eta(\lambda-1) \int_{e^z(1-d)}^{\infty} (1-p)((e^z(1-d))^{1-\theta} - (e^\varepsilon)^{1-\theta}) + p((e^z(1-d))^{1-\theta} - (e^\varepsilon(1-d))^{1-\theta})dF(z)$$

For time-variation in disaster risk, $v_{t+1} \sim (p_t, d)$, $Q_t = f^Q(p_t)$ and $\psi_t = f^\psi(p_t)$ become variant with p_t

$$Q_t = \beta E_t[(e^{\mu_c+\sigma_c\varepsilon_{t+1}+v_{t+1}})^{1-\theta}]+\beta E_t[(e^{\mu_c+\sigma_c\varepsilon_{t+1}+v_{t+1}})^{1-\theta}Q_{t+1}] = E_t[\sum_{\tau=1}^{\infty}\beta^{\tau}(e^{\tau\mu_c+\sum_{j=1}^{\tau}\varepsilon_{t+j}+\sum_{j=1}^{\tau}v_{t+\tau}})^{1-\theta}]$$

$$= \beta(1-p_t+p_t(1-d)^{1-\theta})(E_t[(e^{\mu_c+\sigma_c\varepsilon_{t+1}})^{1-\theta}] + E_t[(e^{\mu_c+\sigma_c\varepsilon_{t+1}})^{1-\theta}Q_{t+1}])$$

$$\psi_t = \beta e^{(1-\theta)\mu_c}E_t[\{(1-p_t)(e^{\sigma_t\varepsilon_{t+1}})^{1-\theta} + p_t(e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}$$

$$+(1-p_t)\eta(\lambda-1)\int_{\varepsilon_{t+1}}^{\infty}((1-p_t)((e^{\sigma_t\varepsilon_{t+1}})^{1-\theta}-(e^{\sigma_t x})^{1-\theta})+p_t((e^{\sigma_t\varepsilon_{t+1}})^{1-\theta}-(e^{\sigma_t x}(1-d))^{1-\theta}))dF(x)$$

$$+p_t\eta(\lambda-1)\int_{log(e^{\varepsilon_{t+1}}(1-d))}^{\infty}((1-p_t)((e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}-(e^{\sigma_t x})^{1-\theta})+p_t((e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}-(e^{\sigma_t x}(1-d))^{1-\theta}))dF(x)$$

$$+\gamma(1-p_t)\eta(\lambda-1)\int_{\varepsilon_{t+1}}^{\infty}(1-p_t)((e^{\sigma_c\varepsilon_{t+1}})^{1-\theta}Q_{t+1}-(e^{\sigma_c x})^{1-\theta}f^Q(\tilde{p}(x,p_t)))+p_t((e^{\sigma_c\varepsilon_{t+1}})^{1-\theta}Q_{t+1}$$

$$-(e^{\sigma_c x}(1-d))^{1-\theta}f^Q(\tilde{p}(x,p_t)))dF(x)+\gamma p_t\eta(\lambda-1)\int_{e^{\sigma_t\varepsilon_{t+1}}(1-d)}^{\infty}((1-p_t)((e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}Q_{t+1}-(e^{\sigma_t x})^{1-\theta}$$

$$f^Q(\tilde{p}(x,p_t)))+p_t((e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}Q_{t+1}-(e^{\sigma_t x}(1-d))^{1-\theta})f^Q(\tilde{p}(x,p_t)))dF(x)+(1-p_t+p_t(1-d)^{1-\theta})e^{(1-\theta)\sigma_c\varepsilon_{t+1}}\psi_{t-}$$

E Online Appendix

E.1 Beliefs-based present-bias

As can be seen in the first-order condition 8, the news discounting parameter γ is unambiguously positively related to the consumption-wealth ratio. For lower values of the news discounting parameter, the agent consumes more of his wealth because positive news about the present is overweighted. Therefore, the model induces overconsumption if the agent discounts news about future consumption. But, the preferences feature a more conceptual desire for time-inconsistent overconsumption. In equilibrium, the agent takes his beliefs as given and optimizes over consumption. In contrast, on some optimal pre-committed path the agent jointly optimizes over consumption and beliefs. The following proposition summarizes how the pre-committed consumption path differs from the time-consistent one.

Proposition 5. *If there is uncertainty $\sigma_c > 0$, and $\theta \neq 1$ then the expected-utility-maximizing consumption path does not correspond to the Markovian rational-expectations equilibrium consumption path. In particular, for $\theta > 1$ the agent chooses a suboptimal overconsumption equilibrium path. The pre-committed consumption-wealth ratio is generally lower and the gap increases in good states:*

$$\rho_t < \rho_t^c \text{ and } \frac{\partial(\rho_t - \rho_t^c)}{\partial\varepsilon_t} > 0$$

Proof of proposition 5 The optimal pre-committed and non-pre-committed consumption-wealth ratios are given by

$$\rho_t^c = \frac{1}{1 + \frac{Q+\Omega+\gamma Q\Omega+\gamma Q\eta(\lambda-1)(1-2F(\varepsilon_t)))}{1+\eta(\lambda-1)(1-2F(\varepsilon_t))}} \text{ and } \rho_t = \frac{1}{1 + \frac{Q+\Omega+\gamma Q\Omega+\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}}.$$

For $\sigma_c = 0$ if $\gamma > \frac{1}{\lambda}$ then $\rho_t^c = \rho_t$, if $\gamma < \frac{1}{\lambda}$ then $\rho_t^c < \rho_t$.

For $\sigma_c > 0$, $\rho_t^c < \rho_t$ iff $\theta > 1$ as $\eta(\lambda-1)(1-2F(\varepsilon_t))) < \eta F(\varepsilon_t) + \eta\lambda(1-F(\varepsilon_t))$ for all $\varepsilon_t \sim N(0, \sigma_c^2)$ and $Q + \Omega + \gamma\Omega Q > \gamma Q$. Moreover, $\rho_t - \rho_t^c$ is increasing in ε_t because $\eta(1-F(\varepsilon_t)) + \eta\lambda F(\varepsilon_t)$ is increasing in ε_t, i.e., $\frac{\partial(\rho_t-\rho_t^c)}{\partial\varepsilon_t} > 0$. ∎

Suppose the agent can pre-commit to an optimal history-dependent consumption path for each possible future contingency. When choosing the optimal pre-committed consumption in each state, the marginal gain-loss utility is no longer solely composed of the sensation of increasing consumption in that state $u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1 - F_{C_t}^{t-1}(C_t)))$. Additionally, the agent considers that in all other states of the world he experiences fewer feelings of gain and more feelings of loss due to increasing consumption in that contingency $-u'(C_t)(\eta(1 - F_{C_t}^{t-1}(C_t)) + \eta\lambda F_{C_t}^{t-1}(C_t))$. Marginal gain-loss utility is then given by $\eta(\lambda-1)(1-2F(\varepsilon_t)) \in [-\eta(\lambda-1), \eta(\lambda-1)]$. Let me illustrate this derivation in greater depth.

Suppose the agent has the ability to pick an optimal history-dependent consumption path for each possible future contingency in period zero when he does not experience any gain-loss utility. The maximization problem can be represented in recursive format as above

$$max\{u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma\sum_{\tau=1}^{\infty}\beta^\tau \boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1}) + \beta V(W_{t+1})\}$$

The crucial difference is that in period zero the agent chooses optimal consumption in period t in each possible contingency jointly with his beliefs, which of course coincide with the agent's optimal state-contingent plan. For instance, consider the joint optimization over consumption and beliefs for $C(W^*)$ when wealth W^* has been realized:

$$\frac{\partial}{\partial C(W^*)}\{\int\int \mu(u(C(W)) - u(C(W')))dF(W')dF(W)\}$$

$$= \frac{\partial}{\partial C(W^*)}\int \eta\int_{-\infty}^{W}\{(u(C(W)){-}u(C(W')))dF(W'){+}\eta\lambda\int_{W}^{\infty}(u(C(W)){-}u(C(W')))dF(W')\}dF(W)$$

$$= u'(C(W^*))(\eta F(W^*) + \eta\lambda(1-F(W^*))) - u'(C(W^*))(\eta(1-F(W^*)) + \eta\lambda F(W^*))$$

$$= u'(C(W^*))\eta(\lambda-1)(1-2F(W^*)) \text{ with } \eta(\lambda-1)(1-2F(W^*)) > 0 \text{ for } F(W^*) < 0.5$$

Consider the difference from the term in the initial first-order condition $u'(C_t)(\eta F(\varepsilon_t) + \eta\lambda(1-F(\varepsilon_t))$: When choosing the pre-committed plan, the additional utility of increasing consumption a little bit is no longer only composed of the additional step in the probability distribution. Instead, the two additional negative terms account for the fact that in all other states of the world, the agent experiences less feelings of gain and more feelings of loss due to increasing consumption in that contingency. The equation indicates that the marginal utility of state W^* will be increased by news utility if the realization is below the median. For realizations above the median, marginal utility will be decreased and the agent will consume relatively less. In general equilibrium, again the agent's expectational terms have to match the model's setup and the above expression becomes:

$$= C_t^{-\theta}(1+\eta(\lambda-1)(1-2F(\varepsilon_t))) \text{ with } \eta(\lambda-1)(1-2F(\varepsilon_t)) \in [-\eta(\lambda-1), \eta(\lambda-1)]$$

Accordingly, by the same reasoning as above the first-order condition for the pre-committed consumption path is given by:

$$C_t^{-\theta}(1+\eta(\lambda-1)(1-2F(\varepsilon_t))) = (W_t - C_t)^{-\theta}(\frac{\rho_t}{1-\rho_t})^{1-\theta}(\gamma Q\eta(\lambda-1)(1-2F(\varepsilon_t))+\psi)$$

$$\rho_t^c = \frac{1}{1+\frac{\psi+\gamma Q\eta(\lambda-1)(1-2F(\varepsilon_t)))}{1+\eta(\lambda-1)(1-2F(\varepsilon_t))}} \text{ and } \frac{\partial \rho_t^c}{\partial \varepsilon_t} = -(\rho_t^c)^2 \frac{(\psi-\gamma Q)\eta(\lambda-1)2f(\varepsilon_t)}{(1+\eta(\lambda-1)(1-2F(\varepsilon_t)))^2}$$

Not surprisingly, the agent's first-order condition has only changed with respect to present gain-loss utility over current and future consumption. In the non-pre-committed optimization, the agent took the beliefs he had as given, now he considers the true costs of increasing consumption on his gain-loss feelings in all other states of the world.[34]

Marginal pre-committed gain-loss utility is generally lower and thus the pre-committed agent consumes less in all states. Moreover, pre-committed marginal utility will only be increased by news utility if the realization is below the median. For realizations above the median marginal utility will be decreased. In contrast, on the non-pre-committed path $\eta F_{C_t}^{t-1}(C_t)+\eta\lambda(1-F_{C_t}^{t-1}(C_t)) \in [\eta, \eta\lambda]$ and marginal gain-loss utility is always positive, as the agent enjoys the sensation of increasing consumption in any state. Thus, in good states, the conceptual problem of beliefs-based present bias is more powerful: Pre-committed marginal gain-loss utility is negative, which never happens on the non-pre-committed path. Therefore, the degree of present bias is reference-dependent and increasing in good states.[35]

[34]Unfortunately, there is a problem that arises in the pre-commitment optimization problem that was absent in the non-pre-committed one: When beliefs are taken as given, the agent optimizes over two concave functions, consumption utility and the first part of gain-loss utility. Accordingly, the first-order condition specifies a maximum. In contrast, when the agent simultaneously chooses his beliefs and his consumption, he also optimizes over the second, convex part of gain-loss utility. The additional part determining marginal utility $-u'(C_t)(\eta(1-F(\varepsilon_t))+\eta\lambda F(\varepsilon_t))$ is largest in particularly good states of the world, as increasing consumption in these states implies additional feelings of loss in almost all other states of the world. It can be easily shown that the sufficient condition for the optimization problem holds if the parameters satisfy the following simple condition: $\eta(\lambda-1)(2F(\varepsilon_t)-1) < 1$. Accordingly, for $\eta(\lambda-1) > 1$, which is true for a range of commonly used parameter combinations, the first-order condition no longer specifies the optimum for favorable states $F(\varepsilon_t) = 1$. For the purposes of this paper, the pre-commitment case was merely meant to illustrate the agent's present bias. Hence, at this point, I am not going to pursue the issue of convexity in the pre-committed optimization further.

[35]The news-utility induced beliefs-based present-bias is not only conceptually very different from

E.2 Prospective gain-loss using the ordered comparison

Koszegi and Rabin (2009) assume that the decision-maker experiences prospective gain-loss utility by means of an ordered comparison of her prior and updated beliefs about the stream of future consumption. The ordered comparison is slightly different from the static comparison assumed in Koszegi and Rabin (2009). Rigorously applying the static comparison to prospective gain-loss utility would imply that the agent experiences gain-loss utility over risk, which has been priorly expected, but not resolved. I circumvent this problem by excluding future uncertainty from the static comparison. This captures a similar intuition but is not exactly the same as the ordered comparison. In the following, I outline the model solution under the assumptions of the ordered comparison. Prospective gain-loss about consumption in each future period $C_{t+\tau}$ is then given by:

$$\sum_{\tau=1}^{\infty}\beta^{\tau}N(F^{t}_{C_{t+\tau}},F^{t-1}_{C_{t+\tau}})=\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}\mu(u(C_{F^{t}_{C_{t+\tau}}}(p))-u(C_{F^{t-1}_{C_{t+\tau}}}(p)))dp$$

As above $C_{t+\tau}$ can be expressed as:

$$C_{t+\tau}=C_{t}e^{\tau\mu_{c}+\sum_{j=1}^{\tau}\varepsilon_{t+j}}=(W_{t-1}-C_{t-1})R_{t}\rho_{t}e^{\tau\mu_{c}+\sum_{j=1}^{\tau}\varepsilon_{t+j}}$$
$$=(W_{t-1}-C_{t-1})\frac{\rho_{t-1}}{1-\rho_{t-1}}\frac{C_{t}}{C_{t-1}}e^{\tau\mu_{c}+\sum_{j=1}^{\tau}\varepsilon_{t+j}}=(W_{t-1}-C_{t-1})\frac{\rho_{t-1}}{1-\rho_{t-1}}e^{(\tau+1)\mu_{c}+\sum_{j=0}^{\tau}\varepsilon_{t+j}}$$

Thus, I can write

$$\frac{\partial\sum_{\tau=1}^{\infty}\beta^{\tau}n(F^{t}_{C_{t+\tau}},F^{t-1}_{C_{t+\tau}})}{\partial C_{t}}=$$
$$=-(W_{t}-C_{t})^{-\theta}(\frac{\rho_{t}}{1-\rho_{t}})^{1-\theta}\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}(e^{\tau\mu_{c}+\sum_{j=1}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta}\mu_{I}(u(C_{F^{t}_{C_{t+\tau}}}(p))-u(C_{F^{t-1}_{C_{t+\tau}}}(p)))dp$$

with $\mu_{I}(x)=\eta$ if $x\geq 0$ and$\mu_{I}(x)=\eta\lambda$ if $x<0$. Moreover, the sum of expected consumption and gain-loss utility, ψ, looks slightly different. Recall, the agent's value

$\beta\delta-$preferences, but as well observationally distinguishable. In the Lucas-tree model $\beta\delta-$preferences, with a hyperbolic-discounting factor denoted by $b<1$, would merely lead to an upward shift of the consumption-wealth ratio $\rho^{b}=\frac{1}{1+bQ}$ whereas in the standard model $\rho^{s}=\frac{1}{1+Q}$ and the $\beta\delta-$agent would like to pre-commit to the standard agent's path. Thus, there are three main differences between $\beta\delta-$preferences and news utility: First, news utility introduces an additional precautionary savings effect which is absent in the $\beta\delta-$model. Rather, uncertainty increases the future marginal propensity to consume, which increases the effective discount rate, so that the agent tends to consume more (Laibson (1998)). Secondly, the optimal pre-committed consumption path is time-variant. In contrast to $\beta\delta-$preferences, the agent does not have a universal desire to pre-commit himself and consume at his liquidity constraint each period (Laibson (1997)). With illiquid and liquid savings the news-utility agent would trade-off the benefits of smoothing consumption and news utility with his present-bias. Last but not least, news-utility preferences predict a state-dependent b, the agent's degree of present-bias varies. In particular, the agent is better behaved in bad times. In my opinion $\beta\delta-$preferences could be a reduced form of a more fundamental source of present-bias as introduced by news utility for instance.

$V_{t+1}(W_{t+1}) = u(W_{t+1})\Psi_{t+1}$:

$$\beta E_t[V_{t+1}] = \beta E_t[u(W_{t+1})\Psi^{OC}_{t+1}] = u(W_t - C_t)\beta E_t[R^{1-\theta}_{t+1}\Psi^{OC}_{t+1}] = u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}\psi$$

$$= u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}\beta e^{\mu_c(1-\theta)}E_t[(e^{\varepsilon_{t+1}})^{1-\theta} + (\eta\int_{-\infty}^{\varepsilon_{t+1}}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon) +$$

$$+ \eta\lambda\int_{\varepsilon_{t+1}}^{\infty}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon))] +$$

$$\gamma\beta(((W_t - C_t)\frac{\rho_t}{1-\rho_t})^{1-\theta})^{-1}\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}\mu(u(C_{F^t_{C_{t+\tau}}}(p)) - u(C_{F^{t-1}_{C_{t+\tau}}}(p)))dp$$

$$+ (\frac{\rho_t}{1-\rho_t})^{1-\theta}\beta e^{\mu_c(1-\theta)}E_t[(e^{\varepsilon_{t+1}})^{1-\theta}\psi)]$$

since $C_{t+\tau} = (W_t - C_t)\frac{\rho_t}{1-\rho_t}e^{\tau\mu_c + \sum_{j=1}^{\tau}\varepsilon_{t+\tau}}$ with $(W_t - C_t)\frac{\rho_t}{1-\rho_t}$ known in period t

accordingly $\psi = Q + \Omega + \gamma\Omega^{OC}$ with Ω^{OC}given by

$$\Omega^{OC} = \frac{\beta\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu((e^{\tau\mu_c + \varepsilon_{t+1} + \sum_{j=2}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta} - (e^{\tau\mu_c + \sum_{j=1}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta})dpdF(\varepsilon_{t+1})}{1 - \beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2\sigma_c^2}}$$

The stochastic discount factor is then given by:

$$1 = E_t[M_{t+1}R_{t+1}] = E_t[\frac{\beta u'(W_{t+1})\Psi^{OC}_{t+1}}{u'(C_t)(1 + \eta F(R_t) + \eta\lambda(1 - F(R_t))) - u'(W_t - C_t)Q^{OC}_t}R_{t+1}]$$

$$Q^{OC}_t = \gamma(\frac{\rho_t}{1-\rho_t})^{1-\theta}\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}(e^{\tau\mu_c + \sum_{j=1}^{\tau}\varepsilon_F(p)})^{1-\theta}\mu_I(u(C_{F^t_{C_{t+\tau}}}(p)) - u(C_{F^{t-1}_{C_{t+\tau}}}(p)))dp$$

$$\Rightarrow M_{t+1} = ((1 + \eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))) - (\frac{1-\rho_t}{\rho_t})^{-\theta}Q^{OC}_t))^{-1}(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-1}\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi^{OC}_{t+1}$$

$$\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi^{OC}_{t+1} = \beta e^{\mu_c(1-\theta)}\{(e^{\varepsilon_{t+1}})^{1-\theta} + \eta\int_{-\infty}^{\varepsilon_{t+1}}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon) +$$

$$+ \eta\lambda\int_{\varepsilon_{t+1}}^{\infty}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon)) +$$

$$+ \sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}\mu((e^{(\tau-1)\mu_c + \varepsilon_{t+1} + \sum_{j=2}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta} - (e^{(\tau-1)\mu_c + \sum_{j=1}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta})dp + (e^{\varepsilon_{t+1}})^{1-\theta}\psi\}$$

The term $\mu_I(u(C_{F^t_{C_{t+\tau}}}(p)) - u(C_{F^{t-1}_{C_{t+\tau}}}(p)))$ in the agent's first-order condition prevents an analytical solution. Instead I have to obtain the function for ρ_t by numerically finding a fixed point. The numerical procedures are very robust and pricing errors are very small. The results when using the ordered comparison are qualitatively and quantitatively very similar to the results under the static comparison excluding future uncertainty.

E.3 Comparison to the partial-equilibrium model

The Lucas-tree general-equilibrium setup simplifies the analysis considerably. In a partial-equilibrium model, in which R_t is i.i.d. and exogenous, the consumption-wealth ratio appears to be slightly more complicated

$$\rho_t = \frac{1}{1 + \left(\frac{\psi + \gamma Q(\eta F_{R_t}(R_t) + \eta\lambda(1 - F_{R_t}(R_t)))}{1 + \eta F_{R_t}(R_t) + \eta\lambda(1 - F_{R_t}(R_t))}\right)^{\frac{1}{\theta}}}.$$

Q and ψ are constant but need to be solved simultaneously with ρ_t. Thus, the model needs to be solved with a simple fixed-point numerical procedure in an infinite-horizon model. In contrast, in general equilibrium Q and ψ depend on exogenous parameters, which gives rise to an analytical solution for ρ_t. Moreover, in the partial-equilibrium model, it has to be verified that consumption C_t and savings $W_t - C_t$ are increasing in the return realization R_t. In the Lucas-tree model, this is necessarily the case as consumption C_t, savings $W_t - C_t$, and returns are all increasing in ε_t.