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Abstract: We present a hidden Markov model of discrete strategic heterogeneity and learning in first price independent private values auctions. The model includes three latent bidding rules: constant absolute mark-up, constant percentage mark-up, and strategic best response. Rule switching probabilities depend upon a bidder's past auction outcomes. We apply this model to a new experiment that varies the number of bidders, the auction frame between forward and reverse, and includes the collection of saliva samples - used to measure subjects' sex hormone levels. We find the proportion of bidders following constant absolute mark-up increases with experience, particularly when the number of bidders is large. The primary driver here is subjects' increased propensity to switch strategies when they experience a loss (win) reinforcement when following a strategic (heuristic) rule. This affect is stronger for women and leads them spend more time following boundedly rational rules. We also find women in the Luteal and Menstrual phases of their menstrual cycle bid less aggressively, in terms of surplus demanded, when following the best response rule. This combined with spending more time following simple rules of thumbs explains gender differences in earnings.

Keywords: private value auction; discrete heterogeneity; learning; gender difference; hidden Markov model; laboratory experiment

JEL Classification Numbers: D44; C72; C92; D87; C15

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1 Introduction

Economists frequently study auctions because of their common usage and their amenability to various modes of inquiry. There is a particularly extensive literature documenting and modeling the deviations of behavior from the predictions of standard theory in first price sealed bid auctions experiments.[1] A recent strand in this literature explores mixture models in which a bidder may follow one of several alternative bidding rules of varying strategic sophistication. We introduce a dynamic mixture model that allows a bidder to change his rule in response to past auction outcomes. We estimate our model with the data from a new experiment. This exercise generates insights into when and why bidders increase their use of simple bidding heuristics and identifies behavioral mechanisms behind recently noted gender earnings differences in auction experiments.

Studies of symmetric independent private value (IPV, hereafter) first price sealed bid auctions by Crawford and Iriberri (2007), Kirchkamp and Reiss (2008), and Shachat and Wei (2012) introduced mixture bidding models with distributions of strategic and non-strategic bidders.[2] Crawford and Iriberri (2007) formulated a Level-k model, where k indicates the number of steps of iterated best response a bidder performs when selecting a strategy. They considered two non-strategic $k = 0$ rules: bid one's value or bid randomly according to a uniform distribution over the interval from the minimum allowable bid to value. A $k = 1$ type believes all other bidders follow a particular $k = 0$ strategy and best responds. Correspondingly, a $k = 2$ type believes all other bidders are $k = 1$ and best responds, and so on. Applying this model to the first five rounds of bidding in the IPV first price auction experiments of Goeree and Holt (2002), they found approximately 4%, 76%, and 20% of the subjects followed the level $k = 0, 1$ and 2 rules, respectively. Kirchkamp and Reiss (2008) introduced a mixture model with two types. One type simply bids a fixed markdown of

[1]See Kagel (1995) and Kagel and Levin (2011) for authoritative reviews of the literature.

[2]Isaac et al. (2012) also estimated a mixture model of reduced form linear bidding strategies for data from an IPV first price sealed bid auction experiment with an unknown number of bidders.

his valuation; the other is rational and best responds taking account of the proportions of bidders following the markdown rule and those also best responding. Kirchkamp and Reiss tested their model in an experiment that allowed bids less than the lowest possible valuation and found roughly 30% of the subjects followed the markdown rule.

Shachat and Wei (2012) extended the previous static approach by allowing rule switching according to a first order Markov process with exogenous transition probabilities. We estimated the model using data from an experiment on first price sealed bid reverse auctions (bidders are trying to sell, rather than purchase, an object). The mixture model consists of two simple pricing rules of thumb suggested by Baumol and Quandt (1964): bidding a constant absolute mark-up of one's cost and bidding a constant percentage mark-up on one's cost. The third bidding rule is to best respond to the mixture probabilities and mark-up parameters. Like previous studies, we found initial high frequencies of strategic bidding in early auctions with approximately 75% of the subject following the best response rule. Surprisingly this percentage quickly fell to a steady state of approximately 62% and the percentage of absolute mark-up bidding rises to over 30%. Skepticism of this result is natural; the model lacks a behavioral mechanism explaining this learning to bid irrationally.

This paper extends this dynamic discrete heterogeneity approach by modeling rule switching as a function of how bidders react to ex post auction outcomes. We do this through myopic rule specific reinforcement learning dynamics similar to those introduced by Erev and Roth (1998) into the behavioral economic literature. Specifically, the relative attractiveness of the currently adopted rules adjusts when (1) the bidders wins the auction or (2) the bidder loses the auction and could have profitably won with an alternative bid. We call the current model the Hidden Markov Bidding Model, or simply the HMBM.

Only a limited number of studies have addressed how individuals learn to bid over time in first price auctions. The most developed of these literature strands uses the directional learning framework of Selten and Buchta (1998). Directional learning is a behavioral principle where individuals adjust their strategies toward those offering ex-post higher payoffs conditional upon available information. Studies such as (Selten and Buchta, 1998; Guth et al., 2003; Neugebauer and Selten, 2006) estimate how bidding rules adjust according to the ex post information provided on auction outcomes. These models, unlike ours, have individuals calculating the counterfactual payoffs for strategies not played.

Using the HMBM to analyze the new experiment, we offer insights into the recent efforts of Chen et al. (2013), Pearson and Schipper (2013), and Schipper (2012) to document and explain gender differences in earnings in IPV first price sealed bid forward auction experiments.[3] These three studies examined bidding behavior through reduced form regression models with socioeconomic covariates, controls for risk attitudes collected in a lottery choice task, information on the use of oral hormonal contraceptives, and the current phase of a female subject's menstrual cycle.[4] These studies found that women bid higher and earn less than their male counterparts when they are in the Luteal and Menstrual phases of their Menstrual cycle - this is the second half of the cycle when there is a lower likelihood of conception; we call these subjects LP Females. Meanwhile, these gender differences are absent or reduced for women in the Follicular or Ovulatory phases of their cycle; we call these subjects HP Females. Interestingly the effects are stronger for women who use oral contraceptives versus those who don't. In our new experiment, the subject pool is mainland Chinese university students, none of whom reported using hormonal contraceptives. Nonetheless, we find similar earnings differences.

We estimate the HMBM with data from a new experiment that involves two within-subject treatments and the collection of saliva samples to measure levels of three different sex hormones. Each experimental session consists of 18 subjects participating in a sequence of 100 n-bidder first price IPV auctions. The first treatment variable is the auction frame. 50 of the auctions are forward auctions where the bidders attempt to purchase an object and

[3] Note, Casari et al. (2007) also document gender performance differences in first price common value auction experiments.

[4] Chen et al. (2013) and Pearson and Schipper (2013) collect the relevant use of contraceptive and menstrual cycle information from surveys, and Schipper (2012) augments survey information with the measurements of various sex hormones from saliva samples.

the other 50 are reverse auctions. This treatment identifies differences in the bidding rules of thumb used when in the role of a buyer versus when in the role of the seller. While the forward and reverse framing are analogous in standard game theoretic models, the percentage mark-up rule will generate quite different outcomes and earnings in the two frames. The second treatment variable is the number of bidders n. Half of the time $n = 3$, and the other half of the time $n = 6$. This variation creates identification between the best response rule and the percentage mark-up rule in the forward auction frame. We also collect a saliva sample from each subject before and after the 100 auctions. These two samples are combined and then tested for levels of Testosterone, Estradiol, and Progesterone.

The estimated HMBM provides the following insights.

1. We find substantial variation in the bidding rule parameters according to gender types. One of the most relevant variations is that LP Females have greater implied risk aversion than HP Females or Males when following the rational bidding rule.
2. For LP and HP Females, the probability of switching from a rational bidding rule to a boundedly rational one spikes after losing an auction that could have been won profitably.
3. The probability that a subject switches from the constant absolute mark-up rule to the best response rule spikes after a bidder wins the auction.
4. The gender differences in these reinforcement effects leads both LP and HP Females to spend more time using boundedly rational rules than Males.
5. The nature of these feedbacks leads to decreasing use of rational rules over time in 6 bidder auctions.
6. We show that about 75% of the earnings difference between Males and LP Females in forward auctions - there is no such earnings difference in reverse auctions - arises from differences in parameter variations within rules and 25% from the differences in the time using alternative rules.

We present the HMBM in the next section. Then we describe the experimental design. This is followed by an analysis of gender earnings differences in the data. In the penultimate section, we present the estimated bidding model and the bulk of the empirical results. We offer discussion of the scope of our results and extensions in the conclusion. There are two appendices: one presenting some of the detailed theoretical analysis and another detailing the Gibbs Sampler and Markov Chain Monte Carlo method used to conduct the Bayesian estimation.

2 The Hidden Markov Bidding Model

We consider the setting in which a bidder participates in a sequence of single object auctions, indexed $t = 1, \ldots, T$. In each auction there are n_t bidders, indexed by i. Each bidder is characterized by a vector of time invariant socioeconomic variables z_i. The auction frame, f_{it}, denotes whether the auction is a forward (F) or reverse (R) one. While the number of bidders and the frame may vary within the auction sequence, these values are always common knowledge. A bidder's type in an auction period, denoted v_{it}, is private information. In a forward action v_{it} is bidder i's value for the object, and in a reverse auction it is his cost. Each v_{it} is an independent draw from the uniform distribution on $[L, H]$. Bidders simultaneously submit bids in a forward (reverse) auction; the one submitting the highest (lowest) bid purchases (sells) the object and pays (receives) the amount of his bid. The winning bidder's payoff is the amount of realized consumer (producer) surplus, and losing bidders' payoffs are zero. Bidders are myopic - only concerning themselves with the current auction payoff - and types are drawn anew each auction.

The HMBM consists of three components. First, there is a finite set of latent linear bid rules mapping from the auction frame and bidder type to bid amount. This set can consist of rules derived strategically and those representing simple heuristics. The strategies are latent because a bidder follows his strategy subject to some random perturbation. The second component is an exogenous multinomial distribution governing the initial assignment of bidders to bidding rules. The third component is a first order Markov matrix of transition probabilities governing the switching of rules. These transition probabilities are functions of a bidder's previous auction participation outcome.

2.1 The set of latent bidding rules

We assume the set of latent bidding rules contains three elements, $\{AM, PM, BR\}$ with generic element s, each reflecting a distinct behavioral heuristic. The constant absolute mark-up (AM) bidder always demands a fixed surplus independent of his value. The AM bidding rule is the affine function with slope of one,

$$b_{AM}(v_{it}|f_{it}, z_i) = \kappa(f_{it}, z_i) + v_{it} + \epsilon_{AMit}.$$

We specify that the markup parameter $\kappa(f_{it}, z_i)$, or expressed more compactly as κ, should be negative for forward frames and positive for reverse frames. It is a linear combination of the socioeconomic effects conditional on the frame. We don't impose that κ, conditional of the value of z_i, has the same magnitude in the two auction frames. Also notice that the AM bidding rule does not depend up the number of bidders nor the distribution of private types. Finally, ϵ_{AMit} is a heteroscedastic independent random perturbation following the normal distribution with mean of zero and variance of $\sigma^2_{AM}(z_i)$.

The constant percentage mark-up (PM) bidder always demands surplus that is a fixed percentage of his realized type. Thus, the PM bidding rule, conditional on the frame is,

$$b_{PM}(v_{it}|f_{it}, z_i) = \big(1 + \rho(f_{it}, z_i)\big)v_{it} + \epsilon_{PMit}$$

Again we allow the possibility that percentage mark-up $\rho(f_{it}, z_i)$, compactly denoted ρ, may differ in sign and magnitude in forward and reverse auctions. The PM rule, like the AM rule, does not depend up the number of bidders nor the distribution of private types. Finally note that this rule is adopted imperfectly with the heteroscedastic independent normally distributed perturbation ϵ_{PMit} with a mean of zero and variance of $\sigma^2_{PM}(z_i)$.

The final latent bidding rule is the strategic best response or BR. A bidder adopting the BR rule maximizes his expected utility conditional upon his realized type, the mark-up parameters κ and ρ, the number of bidders n, and his beliefs regarding the rules each of the bidders is currently adopting. Regarding these beliefs, let π_{AM} be the probability any bidder is a AM bidder, π_{PM} be the probability any bidder is a PM bidder, and that

$(1-\pi_{AM}-\pi_{PM})$ is the probability any bidder is a BR bidder.[5] We assume each bidder has the von Neumann-Morgenstern expected utility function $U(y)=\eta y^{\frac{1}{\eta}}$, where y is a non-negative change in wealth and η is his constant coefficient of relative risk aversion and can be a linear function of z_i, denoted $\eta(z_i)$. In Appendix A, we show the BR bidding rule is:

$$b_{BR}(v_{it}|f_{it},z_i)=\begin{cases}\dfrac{(L+\pi_{AM}\kappa)(1+\rho)}{\big[1+\rho(1-\pi_{PM})\big]M}+\dfrac{M-1}{M}v_{it}+\epsilon_{BRit} & \text{if } f_{it}=F\\[2ex] \dfrac{(H+\pi_{AM}\kappa)(1+\rho)}{\big[1+\rho(1-\pi_{PM})\big]M}+\dfrac{M-1}{M}v_{it}+\epsilon_{BRit} & \text{if } f_{it}=R\end{cases},$$

where $M=\eta(z_i)(n-1)+1$. The heteroscedastic random perturbation ϵ_{BRit} is independently normally distributed with a mean of zero and variance of $\sigma^2_{BR}(z_i)$.[6]

The BR rule has two notable features. First, the mark-up rule parameters κ and ρ, and the beliefs about the distribution of bidding types only affect the intercept term. Second, when the number of bidders and the risk attitude are consistent between the forward and reverse frames, the slope terms are the same.

At this point, we have a fully formulated static model, from which we can generate several alternative models with appropriate restrictions on parameters and beliefs. The Bayes-Nash equilibrium model (Vickrey, 1961) occurs when beliefs are $1-\pi_{AM}-\pi_{PM}=1$ and we restrict bidders to be risk neutral, i.e. $\eta(z_i)=1$. If we instead restrict the constant coefficient of risk aversion to be the same for all bidders and in the open unit interval, we obtain the risk averse Bayes-Nash equilibrium model of Holt (1980). We recover the model of Kirchkamp and Reiss (2008) by setting $\pi_{PM}=0$. Finally, we can obtain a version of the Level-k model of Crawford and Iriberri (2007) by setting $\pi_{AM}=1$ and the absolute mark-up $\kappa=0$.

[5] In the repeated auction context we can think of these beliefs as a state variable. While the myopia assumption allows us think of a bidder as only concerned about his current auction payoff, the formation of these beliefs can involve a complicated inference problem depending upon the informational feedback provided in the auction. In our experiment, subjects are randomly rematched into new bidding cohorts every period which eliminates this conditional inference problem.

[6] It is important to note that we assume the BR bidder does not consider the noise terms ϵ_{rit} when calculating his optimal bid as he would in statistical equilibrium concepts like the Quantal Response Equilibrium (McKelvey and Palfrey, 1995).

2.2 Markovian rule switching and auction feedback

In the HMBM, the evolution of strategy adoption begins with an initial assignment bidding rules according to multinomial distribution Π_1. The variable s_{it} indicates the rule bidder i uses in auction t. From the second auction onward, we assume s_{it} depends upon s_{it-1}, the feedback from i's participation in auction $t-1$, and the socioeconomic variable z_i. We summarize the transition probabilities with the Markov matrix P,

$$P = \begin{pmatrix} \Pr_{BR,BR}(o_{it-1}, z_i) & \Pr_{BR,AM}(o_{it-1}, z_i) & \Pr_{BR,PM}(o_{it-1}, z_i) \\ \Pr_{AM,BR}(o_{it-1}, z_i) & \Pr_{AM,AM}(o_{it-1}, z_i) & \Pr_{AM,PM}(o_{it-1}, z_i) \\ \Pr_{PM,BR}(o_{it-1}, z_i) & \Pr_{PM,AM}(o_{it-1}, z_i) & \Pr_{PM,PM}(o_{it-1}, z_i) \end{pmatrix},$$

where $\Pr_{jk}(o_{it-1}, z_i) = \Pr(s_{it} = j | s_{it-1} = k, o_{it-1}, z_i)$ is the transition probability of moving from bidding rule j to bidding rule k, and o_{it-1} is the feedback bidder i receives from his participation in auction $t-1$.

We classify the outcome o_{it}, as one of three possible types; NR, LR, and WR. The NR outcome is *neutral reinforcement*; the bidder loses the auction, but there was no other bid at which he could have won and earned positive surplus. The second outcome is *loss reinforcement* (LR) in which the bidder loses the auction but there was an alternative bid at which he could have won and earned positive surplus. The final potential outcome is *win reinforcement* (WR); the bidder wins the auction.

We quantify these auction outcome effects through a state dependent index indicating the attractiveness of each rule. We assume the auction outcome adjusts the index of a bidder's adopted rule prior to the determination his subsequent rule. Bidder i's rule indices for period t, conditional on $s_{it} = j$, are

$$\Psi_{it}^{jk} = \begin{cases} \gamma_{jk} z_i + \xi_{it}^{jk} & \text{if } j \neq k \\ \gamma_{jk} z_i + D_{LR}(\gamma_{j1} z_i + \gamma_{j2} z_i L_{it}) + D_{WR}(\gamma_{j3} z_i + \gamma_{j4} z_i W_{it}) + \xi_{it}^{jk} & \text{if } j = k \end{cases}, \qquad (1)$$

where ξ_{it}^{jk} is an independent standard normal innovation.[7] The dummy variable D_x takes the value of one when $o_{it} = x$ and zero otherwise. Loss or win reinforcements have affine impacts on the index of the adopted strategy. The variables L_{it} and W_{it} are the surplus amounts associated with the respective reinforcements calculated as follows:

$$L_{it} = \begin{cases} v_{it} - p_{it} & \text{if } f_{it} = F \\ p_{it} - v_{it} & \text{if } f_{it} = R \end{cases},$$

where p_{it} is the winning bidding in auction t, and

$$W_{it} = \begin{cases} v_{it} - b_{it} & \text{if } f_{it} = F \\ b_{it} - v_{it} & \text{if } f_{it} = R \end{cases}.$$

We assume bidder i transitions to the bidding rule with the largest index Ψ_{it}^{jk}. This implies that each row of the matrix P is a multinomial probit choice model.

3 Experiment design

We ran all experiments at the Finance and Economics Experimental Laboratory (FEEL) of Xiamen University. We used the ORSEE system (Greiner, 2004) to recruit subjects. All subjects were either undergraduate or Master level students from a cross section of schools in the university. The study consisted of 10 sessions each with 18 subjects. Each session took approximately 2 hours to complete a common sequence of four tasks:

Task 1: collection of a saliva sample which includes reading task specific instructions,
Task 2: reading instructions for and participation in 100 auction periods,
Task 3: completion of a survey, and
Task 4: collection of a second saliva sample.

After completing the four tasks, we paid subjects privately as they exited one-by-one.

[7] Setting the variance term σ_ξ^2 equal to one is without loss of generality, it allows identification of the transition probability parameters.

3.1 The auction task

Within the 100 auction periods of each session there are two within subject treatments: the auction frame (forward and reverse), and the number of bidders (3 and 6). After period 50, we switch the auction frame and make a public announcement to remind subjects of this. In one-half of the sessions subjects participate in the forward frame first, and the other sessions start with the reverse frame. Within the first 50 auction periods, we vary the numbers of bidders between the first and second blocks of 25 periods. This order is switched in the second session half. Table 1 presents the sequences of these treatments for each session. Subjects are randomly matched in new groups each period to limit repeated game effects.

Table 1: The assigned sequence of within subject treatments by experimental session

Session	Periods 1-25	Periods 26-50	Periods 51-75	Periods 76-100
1	Forward; n =6	Forward; n =3	Reverse; n =3	Reverse; n =6
2	Forward; n =3	Forward; n =6	Reverse; n =6	Reverse; n =3
3	Forward; n =3	Forward; n =6	Reverse; n =6	Reverse; n =3
4	Forward; n =6	Forward; n =3	Reverse; n =3	Reverse; n =6
5	Forward; n =3	Forward; n =6	Reverse; n =6	Reverse; n =3
6	Reverse; n =6	Reverse; n =3	Forward; n =3	Forward; n =6
7	Reverse; n =6	Reverse; n =3	Forward; n =3	Forward; n =6
8	Reverse; n =3	Reverse; n =6	Forward; n =6	Forward; n =3
9	Reverse; n =3	Reverse; n =6	Forward; n =6	Forward; n =3
10	Reverse; n =6	Reverse; n =3	Forward; n =3	Forward; n =6

Each auction is a first price sealed bid auction. In each period, the computer program[8] informs a subject of the auction frame, the number of bidders, and his value/cost. Each subject is asked to submit a bid in the range of 0 to 60. The subject who bids the highest (lowest) price in each group of bidders wins the forward (reverse) auction and pays (receives) the amount of his bid.[9] After the auction concludes, the computer program informs each subject whether or not he won, the winning price and his payoff in the period. Note during

[8]Developed with the Z-tree programming language (Fischbacher, 2007).

[9]In the case of multiple subjects submitting the winning bid amount within a group, one of them is randomly selected to be the auction winner.

the auction periods, a subject can view his entire past auction experience.

To directly compare the impact of auction frame, we use the symmetrical setting between the forward and reverse formats. For each of the 50 forward auctions, subject i's values are independently drawn from a uniform distribution on the range 20 to 40. Suppose V_i is the vector of these realized draws. For the 50 reverse auctions, subject i's cost vector is generated by the formula $C_i = 40-(V_i-20) = 60-V_i$. So there are 50 pairs of value and cost for each subject. The orders of first 25 elements of V_i and C_i are randomly sorted to form the actual sequence of types in the 3 bidder auctions, and the last 25 are randomly sorted to form the actual sequence of types in the 6 bidder auctions.

3.2 Salivary hormone sampling, socioeconomic survey, and the construction of gender variables

We collect a saliva sample from each subject, which we have tested for levels of progesterone, estradiol, and testosterone.[10] In the invitation to participate, subjects are informed that they will provide two saliva samples in the session. We collect a sample from each subject at the beginning and at the end of the session, then we combine these two samples. On average, the two samples are taken one hour and forty-five minutes apart. Then, all subjects' samples are analyzed at the Xiamen University School of Medicine.[11] As food consumption can result in erroneous salivary hormone measurements, we take two precautions. First, all the sessions start at either 3:00 or 7:30pm (2.5 hours after the standard lunch time or 1.5 hours after the standard dinner time of Xiamen University). Second, before subjects entered the lab, we sequentially ask each to gargle three times to remove possible food residues.

In the third task, subjects complete a computerized survey. The survey contains questions about individual characteristics for all subjects and, for female subjects, additional questions

[10]We do not use the measurements of estradiol and testosterone in the reported data analysis. In unreported results, we find the levels of these two hormones uncorrelated with earnings or bidding behavior. This surprising non-significant results, especially with respect to testosterone, is also consistent with Schipper (2012) which studies the correlation between sex hormone levels and the levels auction earnings and bids.

[11]The three testing kits we used to analyze samples come from the DRG International, Incorporated. Their website provides technical descriptions of the kit, testing procedure, and other specifications for Progesterone, Estradiol, and Testosterone.

about their menstrual cycle.

We combine survey responses regarding the timing of a subject's last menstrual cycle and her measured level of progesterone to divide female participants into two categories: LP Female and HP Female. In the LP Female category, a female subject is either in the Luteal Phase (salivary progesterone is higher than 99.1 pg/ml) or in the first 5 days of her menstrual cycle (determined by the response to the survey question, "How many days ago was the start of your last menstruation?"). Otherwise, the female subject is included in the HP Female category; i.e., in the Follicular or Ovulatory phases of the Menstrual cycle. Hence, the LP Female and HP Female categories correspond to relatively low and high probabilities of conception. In total, our study includes 81 Males (M), 44 LP Females, and 55 HP Females. Finally, note the use of hormonal contraceptives is extremely low amongst Chinese college students[12], and no subject affirmatively answered the survey question about the use of oral contraceptives. Thus female subjects in our study are all naturally cycling women.

4 Earnings treatment effects

In this section we examine whether our study replicates previously documented differences in earnings by gender types. We also probe deeper to look at how the interaction of auction format, the number of bidders, and gender impact earnings. Table 2 presents the average auction earnings, in Chinese renminbi, and the standard deviation. The columns are organized by gender categories, and the rows by the auction framing and the number of bidders. First, we can see our experiment replicates the payoff earnings differences recorded by Pearson and Schipper (2013) and Schipper (2012), LP Females earn less than HP Females and Males in the 3 bidder forward auctions.[13] However, payoffs are flat across gender classifications in 6 bidder auctions. This is not surprising as, by the sheer force of greater competition, 6 bidder auctions yield low earning levels by construction. With respect to the

[12] A survey of over 74,000 students across 8 mainland Chinese universities by Zhou et al. (2009) reported that only 10.1% of the female respondents reported to previously had sex, and of these sexually active females only 28.8% report using oral contraceptives.

[13] Our results are not an exact replication as other studies only look at two-bidder auctions.

auction frame effect, pooled earnings of subjects are slightly lower in the reverse than the forward frame, and differences across gender classifications are smaller as well.

Table 2: Auction task earnings: average and *standard deviation*

Format	Bidders	Total	Male	Female	HP Female	LP Female
Forward	$n = 3$	19.44	20.57	18.52	21.18	15.20
		8.66	*9.15*	*8.18*	*8.65*	*6.18*
	$n = 6$	7.54	7.84	7.29	7.04	7.60
		4.35	*4.28*	*3.93*	*3.51*	*4.42*
	$n = 3$ & 6	26.97	28.41	25.81	28.22	22.80
		10.05	*10.94*	*9.16*	*9.54*	*7.75*
Reverse	$n = 3$	18.98	18.99	18.79	19.97	17.72
		9.21	*9.20*	*9.26*	*8.47*	*10.12*
	$n = 6$	7.22	7.79	6.76	6.78	6.74
		4.91	*5.43*	*4.41*	*4.58*	*4.24*
	$n = 3$ & 6	26.20	26.78	25.73	26.75	24.46
		11.29	*11.68*	*11.00*	*10.17*	*11.95*
For & Rev	$n = 3$	38.42	39.56	37.49	41.15	32.92
		14.07	*15.76*	*14.50*	*13.62*	*14.42*
	$n = 6$	14.76	15.63	14.06	13.83	14.34
		8.02	*8.91*	*7.19*	*7.07*	*7.41*
	$n = 3$ & 6	53.18	55.19	51.55	54.98	47.26
		17.86	*19.31*	*16.51*	*15.47*	*16.95*

We test the differences in earnings within the different categories of the levels of gender classification, number of bidders, and auction frame using two sided t-tests and non-parametric Wilcoxon rank-sum tests. We report the p-values of these tests in Table 3. The only statistical rejections we find involve Males vs. LP Females and HP vs LP Females in the forward auctions. However, we see these rejections arise because of the differences in the 3 bidder auctions. Thus we find the same results as Pearson and Schipper (2013) and Schipper (2012); men outperform women only when they are in the high fertility phase of their menstrual cycles.[14] However, we find these results only hold in the 3 bidder forward auction, failing to extend to the reverse auction and 6 bidder cases.

[14] Our results are partially consistent with Chen et al. (2013), who find the payoff ordering Male, HP Female, and LP Female.

Table 3: Reported p-values for two sided hypothesis tests for differences in average earnings

	Null Hypothesis	$n = 3\&6$ t-test	$n = 3\&6$ Wilcoxon	$n = 3$ t-test	$n = 3$ Wilcoxon	$n = 6$ t-test	$n = 6$ Wilcoxon
	For = Rev	0.49	0.67	0.63	0.65	0.52	0.32
For & Rev	M = F	0.18	0.16	0.37	0.58	0.20	0.28
	M = HP	0.94	0.88	0.53	0.41	0.19	0.31
	M = LP	0.02	0.02	0.02	0.05	0.39	0.44
	LP = HP	0.02	0.04	0.00	0.01	0.73	0.85
Forward	M = F	0.09	0.11	0.14	0.09	0.43	0.58
	M = HP	0.91	0.88	0.75	0.79	0.27	0.51
	M = LP	0.00	0.00	0.00	0.00	0.77	0.90
	HP = LP	0.00	0.00	0.00	0.00	0.48	0.70
Reverse	M = F	0.54	0.43	0.94	0.74	0.17	0.08
	M = HP	0.99	0.88	0.61	0.65	0.30	0.14
	M = LP	0.30	0.22	0.49	0.98	0.20	0.21
	HP = LP	0.31	0.28	0.30	0.63	0.87	0.88

5 Estimation Results

In this section we present the estimates of the HMBM. We find gender differences in the parameters of the latent bidding rules. We also find gender differences in the response to auction feedback. Specifically we find, when following the BR bidding rule, both LP and HP Females react to loss reinforcement causing a sharp increase in the probability of switching to a non-strategic bidding rule. All subjects are sensitive to win reinforcement when following the AM bidding rule. In this case, the probability of abandoning the AM rule and switching to the rational BR rule rises sharply. These reactions lead to different dynamic paths of rule adoption, with Females spending more time following simple bidding heuristics. We finally show how both differences in the bidding rules and the amount of time spent following non-strategic rules contribute to the gender differences in earnings.

We adopt a Bayesian statistical approach to estimate the unknown parameters of the HMBM. The exercise starts by specifying independent and diffuse marginal priors on the parameters. Then we use an iterative Monte Carlo Markov Chain (MCMC) with a Gibbs sampler to generate estimates of the marginal posterior distributions of the parameters. A key property of this procedure is that the empirical distribution of random parameter draws converges to the true marginal posteriors. Thus, we conduct 3000 iterations and establish that the empirical distributions of the drawn parameters has converged (Geweke, 1991). Then we run the procedure an additional 2000 iterations, from which we construct empirical density functions. These empirical density functions are used to calculate the posterior means and confidence intervals reported in this section. In an appendix, we provide a full description of this statistical procedure.

The are a couple of items to consider before proceeding. First, the vector of socioeconomic variables, z_i we consider contains only indicator variables for the three gender classifications M, HP, and LP. The dummy variables D_F and D_R are used to indicate a Forward and Reverse auction frame respectively.

5.1 Estimated bidding rules

We first consider the estimated posterior means of the AM and PM bidding rule parameters, presented in Table 4 along with respective 95% confidence intervals. Columns 3-5 report the posterior means straddled by the upper and lower bound of the 95% confidence intervals. In the forward auction we see the absolute consumer surplus demanded is quite low, roughly sixty cents below value. Also there is a small, both in terms of economic and statical significance, gender difference with the absolute mark-up of Males lower than those of the LP and HP Females. In the reverse auction, the absolute mark-ups are significantly smaller for all the gender types. With respect to the percentage mark-up demanded by PM bidders, there is also a gender effect. But in this case, Male bidders are more aggressive in the mark-up demands than both Female types. Unlike the AM bidding rule, there is no significant effect on the auction frame and the size of the mark-up.

Next we consider the estimated BR bidding rule parameters. First we examine the slope terms, reported for the 3 bidder auctions. There is a statistically significant ordering of

Table 4: Mark-up bidding rule parameters: posterior means with 95% confidence intervals

Rule	Variable	Male			HP Female			LP Female		
	$D_F \cdot \kappa$	-0.59	-0.56	-0.53	-0.65	-0.61	-0.58	-0.64	-0.60	-0.56
AM	$D_R \cdot \kappa$	0.45	0.48	0.50	0.47	0.51	0.55	0.40	0.44	0.48
	σ^2_{AM}	0.14	0.15	0.17	0.15	0.16	0.18	0.14	0.15	0.17
	$D_F \cdot \rho$	-0.72	-0.58	-0.45	-0.45	-0.40	-0.36	-0.51	-0.39	-0.30
PM	$D_R \cdot \rho$	0.35	0.48	0.60	0.38	0.43	0.48	0.24	0.32	0.40
	σ^2_{PM}	115.18	131.64	150.68	18.55	20.96	23.88	31.39	38.50	47.82

the slope terms that is increasing for Male, HP Females, and LP Females. This ordering is consistent with increasing coefficients of constant relative risk aversion η across the gender classifications. Also this bid responsiveness to type is the same across auction frames - thus when following a strategic rule - bidders do treat the forward and reverse auctions isometrically. The change in the slope going from 3 to 6 bidders is in the correct direction; however, the magnitude does not reflect a constant η.[15] One surprising result, is that gender differences in the slope are not found in the six bidder auctions.

5.2 Estimated endogenous rule switching probabilities

We report the estimates the parameters of attractiveness indices underlying the multinomial probit models of rule transition probabilities in Table 6. Examination of these results reveals a interesting asymmetry. The only impact of loss reinforcement is a reduction in the attractiveness of the BR strategy, and the only impact of win reinforcement is a *reduction* in the attractiveness of the AM strategy. In other words, when a subject follows the strategic rule and loses the auction when he could have won with a lower surplus demanding bid, this increases the probability of switching to one of the simple bidding heuristics. It's as though one regrets following such aggressive strategies. On the other hand, when a subject follows

[15] Not finding constant coefficients of relative risk aversion when the number of bidders changes is a leading critique against risk aversion based explanations of overbidding in first price auctions; for example, see Kagel and Levin (1993).

Table 5: Strategic Best Response (BR) bidding rule parameters: posterior means with 95% confidence intervals

Variable	Male			HP Female			LP Female		
D_F	3.59	3.89	4.21	2.84	3.21	3.58	2.16	2.53	2.87
$D_F \cdot D_6$	-2.01	-1.59	-1.18	-2.48	-1.92	-1.36	-1.28	-0.77	-0.23
D_R	7.38	7.70	8.01	6.64	7.02	7.42	5.73	6.14	6.57
$D_R \cdot D_6$	-2.30	-1.85	-1.40	-2.72	-2.17	-2.72	-2.60	-2.03	-1.48
$D_F \cdot v_{it}$	0.79	0.80	0.81	0.82	0.83	0.84	0.85	0.86	0.87
$D_F \cdot D_6 \cdot v_{it}$	0.06	0.07	0.08	0.06	0.07	0.09	0.02	0.04	0.05
$D_R \cdot v_{it}$	0.79	0.80	0.81	-0.81	0.83	0.84	0.84	0.85	0.86
$D_R \cdot D_6 \cdot v_{it}$	0.04	0.05	0.06	0.04	0.06	0.07	0.04	0.06	0.08
σ^2_{BR}	0.91	0.96	1.02	0.78	0.83	0.90	0.61	0.68	0.74

the simple absolute mark-up rule and wins the auction, then this increases the probability of switching away from this strategy and, as we shall see shortly, predominantly towards the BR strategy. It appears, in this case, subjects regret following the conservative non-strategic rule.

There are some clear gender differences in these reinforcements. Let's first focus on the loss reinforcement effects when following the BR rule. For LP Females, their is a significant negative impact from simply experiencing the loss reinforcement state, captured by D_{LR} coefficient, that does not vary with the size of the loss, captured by the L_{it} coefficient. HP females also experience a significant impact, but in this case the impact proportional to the size of loss. Males do not exhibit a significant loss reinforcement effect.

Now turning our attention to the AM bidding rule and the impact of win reinforcement. First note by definition, the size of W_{it} conditional upon experiencing a win reinforcement state should have little variation. Thus, there should be some co-linearity between the variables W_{it} and D_{WR}. Inspection of the estimates of the coefficients reveals significant negative impact of D_{WR} for HP Females, and W_{it} for LP Females and Males. When combining the

Table 6: Estimated reinforcement effects on rule attractiveness indices: posterior means with 95% confidence intervals

Rule	Variable	Male			HP Female			LP Female		
BR	D_{WR}	-0.12	0.02	0.15	-0.18	0.00	0.17	-0.16	0.04	0.24
	W_{it}	-0.14	-0.07	0.00	-0.14	-0.05	0.07	-0.22	-0.11	0.02
	D_{LR}	-0.44	-0.18	0.10	-0.54	-0.24	0.07	-0.83	-0.50	-0.19
	L_{it}	-0.14	-0.03	0.09	-0.31	-0.16	-0.02	-0.19	-0.02	0.16
AM	D_{WR}	-0.23	0.06	0.35	-0.64	-0.32	-0.01	-0.45	-0.13	0.19
	W_{it}	-0.80	-0.44	-0.09	-0.51	-0.13	0.24	-0.90	-0.52	-0.11
	D_{LR}	-0.41	-0.01	0.41	-0.42	0.06	0.57	-0.40	0.09	0.60
	L_{it}	-0.44	0.08	0.65	-0.60	-0.04	0.50	-0.61	-0.03	0.58
PM	D_{WR}	-0.15	0.30	0.80	-0.49	0.07	0.63	-0.08	0.44	0.95
	W_{it}	-0.17	-0.08	0.02	-0.10	0.06	0.38	-0.21	-0.04	0.20
	D_{LR}	-0.70	-0.26	0.19	-0.65	-0.24	0.18	-0.65	-0.18	0.33
	L_{it}	-0.15	-0.04	0.07	-0.09	0.02	0.13	-0.26	-0.09	0.08

estimates of these coefficients with their respective estimated values of the absolute mark-ups κ in Table 4, we find the impact is the similar across gender classifications.

The estimated values of the reinforcement coefficients alone doesn't adequately convey how these outcomes affect the transition probability matrix P. To better convey how the auction outcome impacts P, we calculate the estimated matrix P for each of the three auction outcomes and the three gender classification. We report these the estimated P's in Table 7.

When inspecting the posterior mean transition probabilities there are three consistent regularities. First, the continuation probabilities of following the same rule, given in the grey filled cells on the main diagonals, are usually larger than 0.60. This suggests inertia in subjects' rule adoptions. The small number of cases where this doesn't hold we discuss shortly. Second, when subjects transition away from one of the non-strategic rules, AM and PM, it will almost always be to the strategic PM rule. In other words, the probability of switching from one non-strategic rule to another is very low. See the underlined transition probabilities in Table 7. Third, when subjects switch away from the BR rule, they are much more likely to adopt the AM rather than the PM rule. These general characteristic are similar to those of Shachat and Wei (2012); indicating they are robust to auction frames, subject pools, and the number of bidders.

The impact of the different reinforcements, and gender differences in these impacts, are also evident in Table 7. Consider the differences in P between neutral versus loss reinforcement outcomes. For Males, the only difference is the reduction in the continuation probability of the PM rule. However, for both HP and LP females loss reinforcement leads to a reduction in the BR continuation probability from 0.89 to 0.72 and 0.76 respectively. Loss Reinforcement also negatively impacts the LP Female's continuation probability of PM as well.

Now consider the differences in P between neutral and win reinforcements. There is no discernable impact in the BR continuation probability for any gender type. However, win reinforcement reduces the AM continuation probability for Males and LP Females respectively from 0.82 and 0.86 to 0.45. For HP females there is a reduction from 0.86 to 0.61. With respect to the PM rule, win reinforcement leads to increases in the PM continuation probability for all three gender classifications.

5.3 Evolution of bidding rule adoption

We show the estimated HMBM leads to different dynamic patterns of rule adoption for different gender classifications. One of the key latent variables in the HMBM is the sequence of bidding rule states a subject occupies over the 100 auction periods. The Gibbs sampler generates a realization of this random latent sequence for each subject in every iteration of the MCMC estimation. We calculate the mean posterior estimate of this sequence for each subject as follows. For auction period t, we calculate the proportion of the 2000 realized bidding rule sequences the subject adopts each bidding rule. The we average element-by-element the appropriate sets of individual sequences to form different aggregate series of

Table 7: Transition matrices conditional upon auction outcome and gender classification

		Neutral Reinforcement			**Loss Reinforcement**			**Win Reinforcement**		
		BR_t	AM_t	PM_t	BR_t	AM_t	PM_t	BR_t	AM_t	PM_t
Male[A]	BR_{t-1}	0.89	0.08	0.03	0.84	0.12	0.04	0.86	0.11	0.03
	AM_{t-1}	0.20	0.80	$\underline{0.01}$	0.18	0.82	$\underline{0.00}$	0.55	0.45	$\underline{0.00}$
	PM_{t-1}	0.34	$\underline{0.02}$	0.63	0.48	$\underline{0.03}$	0.49	0.29	$\underline{0.02}$	0.69
HP Female[B]	BR_{t-1}	0.89	0.09	0.02	0.72	0.22	0.06	0.87	0.11	0.03
	AM_{t-1}	0.17	0.82	$\underline{0.00}$	0.14	0.86	$\underline{0.00}$	0.38	0.61	$\underline{0.01}$
	PM_{t-1}	0.20	$\underline{0.02}$	0.78	0.25	$\underline{0.02}$	0.74	0.18	$\underline{0.02}$	0.81
LP Female[C]	BR_{t-1}	0.89	0.09	0.02	0.76	0.19	0.05	0.85	0.12	0.03
	AM_{t-1}	0.16	0.84	$\underline{0.00}$	0.14	0.86	$\underline{0.00}$	0.54	0.45	$\underline{0.01}$
	PM_{t-1}	0.30	$\underline{0.03}$	0.67	0.41	$\underline{0.04}$	0.56	0.21	$\underline{0.02}$	0.77

[A] Evaluated at the following average Win and Loss reinforcement levels: BR, $\overline{W} = 2.69$ and $\overline{L} = 1.83$; AM, $\overline{W} = 0.65$ and $\overline{L} = 0.42$; and PM, $\overline{W} = 5.31$ and $\overline{L} = 3.68$.
[B] Evaluated at the following average Win and Loss reinforcement levels: BR, $\overline{W} = 2.48$ and $\overline{L} = 1.53$; AM, $\overline{W} = 0.73$ and $\overline{L} = 0.46$; and PM, $\overline{W} = 4.55$ and $\overline{L} = 3.34$.
[C] Evaluated at the following average Win and Loss reinforcement levels: BR, $\overline{W} = 2.25$ and $\overline{L} = 1.32$; AM, $\overline{W} = 0.65$ and $\overline{L} = 0.40$; and PM, $\overline{W} = 4.07$ and $\overline{L} = 3.05$.

interest regarding gender classification, auction frame, and the number of bidders.

Figure 1 presents the estimated 100 period sequences of auction rule adoption by gender classifications.[16] At the start of the experiment, we can see Male subjects have a higher initially probability of following the BR rule than either Female type. We also see all three types initially use the PM rule in greater proportion than the AM rule. However, we see for all three gender classifications the proportion adopting the PM rule drops to 10% or below within ten periods. We also observe a steady rise of the AM rule to the 35-40% range. We

[16] For these series subjects are randomly assigned the sequence they face the auction frame and the number of bidder treatments.

also see, after an initial 10 to 20 period adjustment, the average proportional use of the BR bidding strategy Males, HP Females, and LP Females respectively are 65%, 60%, and 55%. These initial conditions are quite different than those reported by Shachat and Wei (2012); their U.S. and Singaporean subject pools show much higher initial strategic behavior. But surprisingly the convergence of rule adoption proportions is quite similar. This suggests we have identified robust learning.

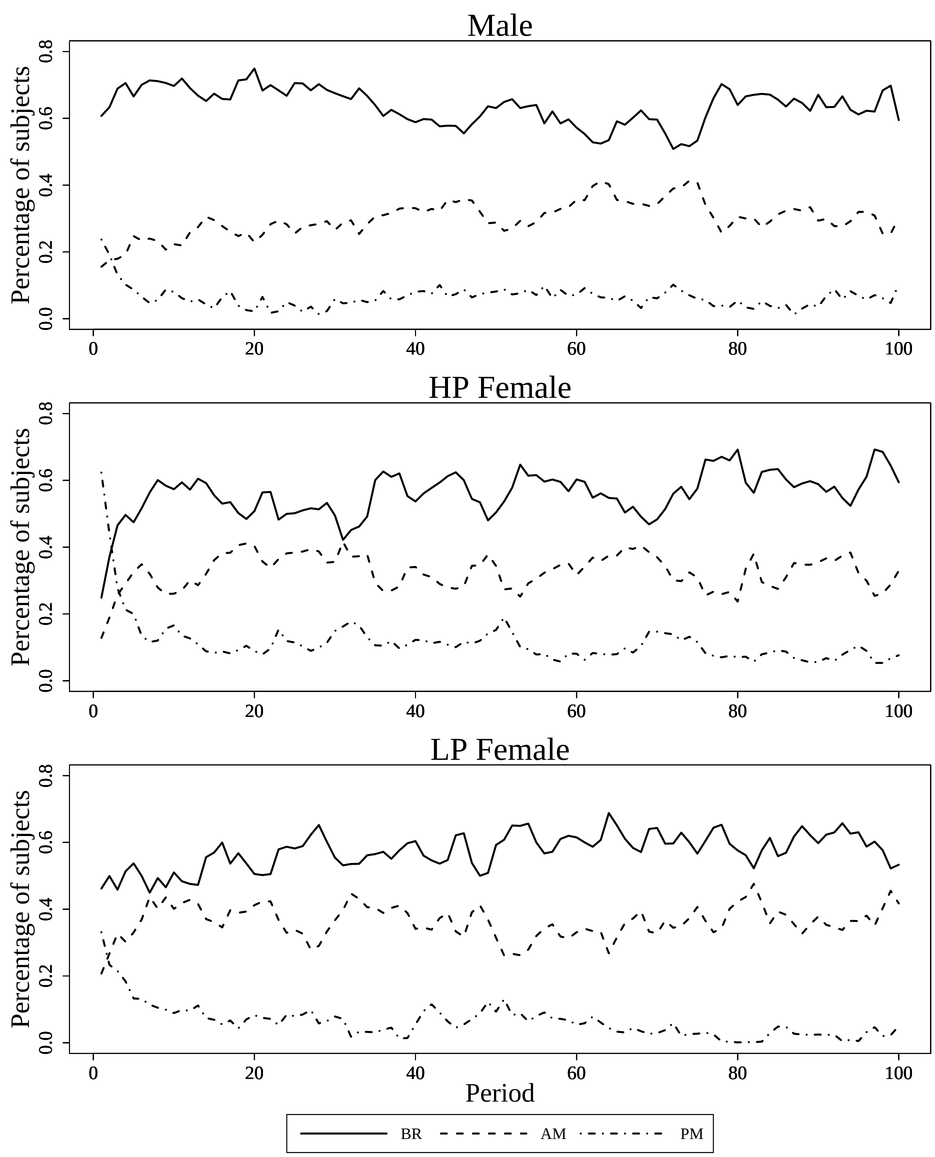

Figure 1: Estimated sequences by gender of subject proportions using the AM, PM, and BR rules

One of this paper's innovations is introducing endogenous transition probabilities conditional upon the reinforcements of auction outcomes. Differences in the bidding rule parameter values and varying auction conditions such as the number of bidders can generate varying patterns of learning and rule adoption. Differing patterns are not possible when the matrix P is exogenous as in Shachat and Wei (2012).

Figure 2 is a 4×3 array of sequence plots that exhibits how the rule learning dynamics vary according to the number of bidders, auction frame, and gender.[17] The four rows in the array correspond to each of the 25 period blocks of the within crossing of the auction frame and the number of bidder treatment variables. The columns correspond to the three gender classifications. In the 3 bidder auctions we observe fairly stable patterns previously noted when looking at the 100 period sequences. In contrast, the 6 bidder auctions all exhibit diminishing adoption of the BR and a corresponding increasing the use of the AM. A final general feature of this array is that gender differences in how a bidder responds to auction outcomes and differences in bidding rule parameters leads to Males behaving more strategic in most settings.

5.4 The sources of gender differences in earnings

We conclude or analysis by identifying the sources of gender earning differences according to the estimated HMBM. The estimated HMBM exhibits gender differences both in the propensities to follow alternative rules and in the amount of surplus demanded in each bidding rule. We report the results of Oaxaca-Blinder decompositions (Oaxaca, 1973; Blinder, 1973) that allow us to attribute how much of the observed gender differences in earnings result from each of these two sources.

We construct statistics regarding the proportion of periods that subjects follow each rule and their earnings when using each rule. We start by tracking the random draw of the rule state for each subject and match it with his actual period earnings in each of the final 2000 MCMC iterations. We group these matched sequences by gender classification. For each

[17]For these series, subjects are randomly assigned to which 25 period block they face the depicted treatment.

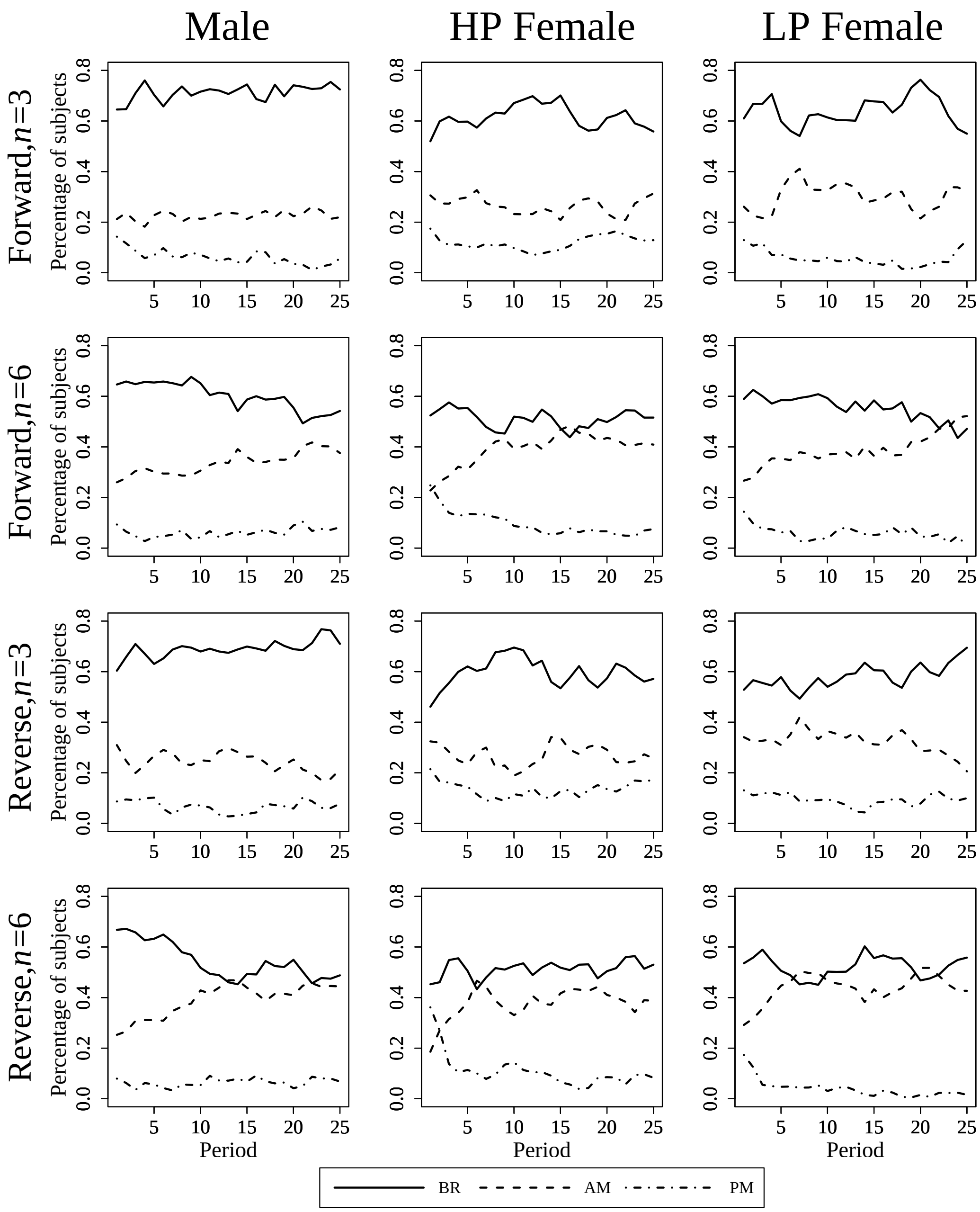

Figure 2: Estimated sequences by gender of subject proportions using the AM, PM, and BR rules

auction frame, we calculate the proportion of periods that subjects follow each of the three rules, as well as the average realized period earnings conditional upon rule. The product of these two numbers and 50 (the number of periods of the auction frame) is our estimate of the total earnings when following a particular rule. We present these statistics in Table 8.

Table 8 shows that Males follow the strategic best response bidding rule with greater frequency than both HP and LP Females do. Further, when following the BR rule Males and HP Females earn about the same earnings per period and more than the LP Females. This partially explains why HP female and Males don't have the significant earning differences in the forward auction. The Male earnings per bid in the reverse auction is lower than in the forward auction. Therefore, the difference of total earnings among male and female become smaller in reverse auction.

Table 8: Posterior average frequency and earning by rule for each auction frame

		Forward				Reverse			
	Rule	% use of rule	Earnings per bid	Rule Earnings	Earnings Share	% use of rule	Earnings per bid	Rule Earnings	Earnings Share
Male	BR	0.66	0.76	25.08	0.88	0.62	0.74	22.93	0.86
	AM	0.28	0.13	1.80	0.06	0.32	0.11	1.67	0.06
	PM	0.06	0.50	1.53	0.05	0.07	0.66	2.18	0.08
	Total	1.00	0.57	28.41	1.00	1.00	0.54	26.78	1.00
HP	BR	0.56	0.74	21.01	0.74	0.55	0.71	19.64	0.73
	AM	0.33	0.17	2.81	0.10	0.32	0.16	2.52	0.09
	PM	0.11	0.82	4.41	0.16	0.12	0.75	4.59	0.17
	Total	1.00	0.56	28.22	1.00	1.00	0.54	26.75	1.00
LP	BR	0.60	0.63	18.94	0.83	0.55	0.65	17.83	0.73
	AM	0.34	0.15	2.58	0.11	0.38	0.11	2.04	0.08
	PM	0.06	0.43	1.27	0.06	0.07	1.37	4.59	0.19
	Total	1.00	0.46	22.80	1.00	1.00	0.49	24.46	1.00

The Oaxaca-Blinder decomposition allow us to ask what part of the gender differences in earnings comes from females using rules that demand less surplus and what part comes from females spending a larger proportion of time following non-strategic rules. Consider the following way of expressing the average earnings per bid for a gender classification g, across all rules:

$$y_g = \hat{\pi}_g' \cdot x_g,$$

where y_g is the average earnings per bid, $\hat{\pi}_g$ is the 3×1 vector of proportions in which rules are followed, and x_g is the 3×1 vector of earnings per bid averages of the rules. We can express the difference in average earnings per bid between gender types g and h as

$$y_g - y_h = \hat{\pi}_g' \cdot (x_g - x_h) + {x_h}' \cdot (\hat{\pi}_g - \hat{\pi}_h). \tag{2}$$

The first term of Equation 2 consists of the differences in average earnings for each rule weighted by the proportions by which g uses the rules. We call this the within rule earnings difference. The second term of equation 2 consists of the differences in the proportions with which rules are used weighted by the average rule earnings of h. We call this the between rule earnings difference.

Table 9 reports, by auction frame, the within rule and between rule earnings differences for the Male type and each of the HP and LP Female types. We can see the second term is always positive implying males always spend a larger proportion of time following with higher earnings. However, the differences in within rule earnings are more varied and explain why we observe gender difference in earnings. Notice for the forward auction, about 75% of the earnings difference between Male and LP Female subjects comes from within rule differences. While in this auction frame, HP Female subjects actually earn more than Male subjects within rules, cancelling out the loss from spending more time following simple bidding heuristics. In the reverse auction, the within rule earnings difference is much smaller for Male and LP Female subjects and, despite a slightly larger between rule difference, leads the lower overall earnings difference. We further see the within and between rule earnings differences are essentially zero for the Male and HP Female subjects in the reverse auction.

Table 9: Oaxaca-Blinder decomposition of gender differences in earnings

	Forward auction			Reverse auction		
Earnings difference	Within rule	Between rule	Total	Within rule	Between rule	Total
Male - LP Female	4.13	1.48	5.62	0.64	1.68	2.31
Male - HP Female	−0.99	1.19	0.19	−0.03	0.05	0.02

6 Conclusion

We introduced the HMBM, which provides a dynamic generalization of mixture models of bidding in auctions. Within this flexible framework, we considered the case where auction outcomes impact the transition probabilities between strategic best response rules and simple rules of thumb. We applied the HMBM to a new experiment and found an interesting rule switching dynamic. After experiencing a loss reinforcement, the probability a female subject following the rational BR rule switches to a rule of thumb spikes. In contrast, when a subject - both male and female - follows AM, the predominantly used rule of thumb, the probability of switching to the rational BR rule spikes after experiencing a win reinforcement.

We don't provide, nor are aware of, a model of optimizing behavior that generates such dynamic responses. However, we conjecture the following. Individuals appreciate there is a potential benefit to choosing a strategic rule. But ascertaining which strategic rule offers the highest reward is a difficult cognitive task, especially in our auction setting. This generates uncertainty in the value of the calculated strategic rule. On the other hand, the value of following a rule of thumb is easy to perceive and cognitively simple to execute. Consequently, a negative reinforcement undermines the bidder's confidence in his ability to successfully bid strategically. This leads to abandoning the pursuit of the less certain-higher reward strategy in favor of the easier to value rule of thumb. In the other direction, a win reinforcement when following the rule of thumb stirs a sense of lost opportunity from not trying to identify and pursue a more profitable strategic rule.

We find that women are sensitive to this loss reinforcement, while men are not. This, along with parameter variations within rules, provides a behavioral principle for observed gender differences in auction performance. This differential response to loss reinforcement could be an important phenomenon across a wide variety of settings. For example, Gill and Prowse (2012) study gender differences in productivity in real effort tournament competition. They find losing a competition negatively impacts females' effort levels in subsequent competitions. Interestingly, much like our result for LP Females, the reduction of subsequent effort is sensitive to experiencing the loss reinforcement state but not to the size of the lost prize. Male productivity is not sensitive to this loss reinforcement unless the prize is sufficiently large. Further we conjecture this same dynamic is a plausible explanation of women learning more rapidly than men to avoid the winner's curse in common value auctions (Casari et al., 2007).

A Deriving the strategic BR bidding rule

We first consider the case of the forward auction. Suppose $b_{BR}(v_{it})$ is a strictly increasing continuous bounded function, and thus has a continuous inverse. Now consider bidder j who follows the BR rule; he chooses the bid p_{jt} to maximize his expected utility conditional upon his value v_{jt}. The probability p_{jt} is greater than another bidder i's bid is

$$\Pr\big(p_{jt} \geq p_{it}\big) = \pi_{AM} \Pr\big(p_{jt} \geq v_{it} + \kappa\big) + \pi_{PM} \Pr\big(p_{jt} \geq (1+\rho)v_{it}\big) + \pi_{BR} \Pr\big(p_{jt} \geq b_{BR}(v_{it})\big),$$

where $\pi_{BR} = 1 - \pi_{AM} - \pi_{PM}$. Since each of the three bidding rules has an inverse we can restate this probability, with F denoting the uniform distribution on $[L, H]$, as

$$\Pr(p_{jt} > p_{it}) = \pi_{AM} F\big(p_{jt} - \kappa\big) + \pi_{PM} F\left(\frac{p_{jt}}{1+\rho}\right) + \pi_{BR} F\big(b_{BR}^{-1}(p_{jt})\big).$$

The probability bidder j wins an n bidder auction is the probability p_{jt} is the first order statistic of the n realized bids, or

$$\Pr(p_{jt} > p_{it}; i = 1, \ldots, n-1) = \big[Pr(p_{jt} > p_{it})\big]^{n-1}.$$

Thus, bidder j's expected utility from bidding p_{jt} conditional on realized value v_{jt} is,

$$E[U(p_{jt}|v_{jt})] = \eta(v_{jt} - p_{jt})^{\frac{1}{\eta}} \left[\pi_{AM} \frac{p_{jt} - \kappa - L}{H - L} + \pi_{PM} \frac{\frac{p_{jt}}{1+\rho} - L}{H - L} + \pi_{BR} \frac{b_{BR}^{-1}(p_{jt}) - L}{H - L}\right]^{n-1}$$

The first order condition of maximizing expected utility with respect to the bid price is:

$$\begin{aligned} \frac{1}{\eta}\left[\pi_{BR}\big(b_{BR}^{-1}(p_{jt}) - L\big) + \pi_{AM}(p_{jt} - \kappa - L) + \pi_{PM}\left(\frac{p_{jt}}{1+\rho} - L\right)\right] \\ = (n-1)(v_{jt} - p_{jt}) \left[\frac{\pi_{PM}}{b'_{BR}(v_{jt})} + \pi_{AM} + \frac{\pi_{PM}}{1+\rho}\right]. \end{aligned}$$

Now if one assumes that all BR bidders use the same bid function, the above expression reduces to the following differential equation:

$$\frac{v_{it} - L - \kappa\pi_{AM} - \frac{\rho}{1+\rho}\pi_{PM}}{v_{jt} - p_{jt}} = \frac{\eta(n-1)\pi_{BR}}{b'_{BR}(v_{jt})} + \big(\eta(n-1) + 1\big)\left(\pi_{AM} + \frac{\pi_{PM}}{1+\rho}\right)$$

The solution this differential equation and the BR bidding rule is

$$b_{BR}(v_{it}) = \frac{(L + \pi_{AM}\kappa)(1+\rho)}{(1 + \rho(1 - \pi_{PM}))M} + \frac{M}{(M-1)} v_{it},$$

where $M = \eta(n-1)+1$.

The derivation of the BR rule for the reverse auction frame is found in Shachat and Wei (2012).

B The MCMC estimation of the HMBM

This paper adopts a Bayesian methodology to make inferences regarding the parameters and unobserved components of the HMBM. The HMBM is a statistical process with three components. First, we can write the state space of linear bid functions as,

$$\begin{pmatrix} b_{AM}(v_{it}, z_i, f_{it}) \\ b_{PM}(v_{it}, z_i, f_{it}) \\ b_{BR}(v_{it}, z_i, f_{it}, n_{it}) \end{pmatrix} = \begin{pmatrix} \kappa(z_i, f_{it)} & 1 \\ 0 & (1+\rho(z_i, f_{it})) \\ \alpha(z_i, f_{it}, n_{it}) & \beta(z_i, f_{it}, n_{it}) \end{pmatrix} \begin{pmatrix} 1 \\ v_{it} \end{pmatrix} + \begin{pmatrix} \epsilon_{AMit} \\ \epsilon_{PMit} \\ \epsilon_{BRit} \end{pmatrix}.$$

Let Φ denote the matrix of bidding rule parameters and $\Sigma = (\sigma_r^2(z_i))$ denote the vector of heteroscedastic variances of the bidding rule perturbations. Second, Γ indicates the matrix of the parameters of the transition matrix P, where each row of Γ consists of the multinomial probit model parameters of the corresponding bidding rule. Third, Π_1 is the multinomial distribution governing the initial assignment of bidding rules and has the parameters π_{AM} and π_{PM}. The output of the HMBM consists of the observable the sequences of bids and auctions outcomes, B and O respectively, for each bidder, and the unobservable sequence of bidding rules, S, adopted by each bidder. Also, for notational convenience, let V be the collection of all V_i's.

Consider the joint posterior density function of the HMBM parameters and the unobserved realized state sequences, $h(S, \Phi, \Sigma, \Gamma, \Pi_1|B, V, O)$. We first assume that parameters of the bidding rules are independent from the auction outcomes. Then we express this joint density as the product of the marginal density of HMBM parameters conditional on the observed bids, values, and outcomes; and the unobserved states with the marginal density of the states conditional upon action choices and outcomes.

$$h(S, \Phi, \Sigma, \Gamma, \Pi_1|B, V, O) = h(\Phi, \Sigma, \Gamma, \Pi_1|S, B, V, O)h(S|B, V, O).$$

Next, we assume the distribution of the bidding rule parameters are independent of the distribution of the rule transition probability parameters when both are condition upon the observable elements (B, V, O). This allows us to state

$$h(S, \Phi, \Sigma, \Gamma, \Pi_1|B, V, O) = h(\Phi, \Sigma|S, B, V)h(\Gamma, \Pi_1|S, O)h(S|B, V, O). \quad (3)$$

This product of three conditional posteriors permits a simple Markov Chain procedure of sequentially sampling from these distributions. The MCMC approach relies on augmenting the parameter space with filtered values of the unobserved states S, and using Gibbs sampling procedures generate sequential draws from the marginal distributions of equation 3. After a large number of iterations, indexed by l, the empirical density of these draws converges in probability to the true posterior density functions (Geman and Geman, 1987). The first iteration of MCMC algorithm, $l = 0$, starts with the construction of S^0 by random draws from the uniform multinomial distribution, Π_1^0 from the uniform Dirichlet distribution, and draws for all values parameter values Φ^0, Σ^0, and Γ^0 from the standard uniform distribution. Iterations $l > 0$, consists of the following three steps.

Step 1: Sample $\Phi^{(l)}$ and $\Sigma^{(l)}$ by using $S^{(l-1)}$, B, V;
Step 2: Sample $S^{(1)}$ by using $\Gamma^{(l-1)}$, O, and $S^{(l-1)}$; and
Step 3: Sample $\Gamma^{(l)}$ by using $S^{(l)}$, $\Psi^{(l)}$ and O.

The Gibbs sampler is run for a large number of iterations until the empirical distribution of all the parameters has converged according to the convergence test of Geweke (1991). Then the sampling procedure is allowed to continue to run for another number of iterations to build up an empirical distribution that corresponds to the posterior distribution of the HMBM parameters. It is from this empirical distribution that we conduct statistical inferences. We now describe the three steps of an iteration of the Gibbs sampler in detail.

Step 1: Sampling the rule parameters $\{\mathbf{\Phi},\ \mathbf{\Sigma}\}^{(l)}$

Given the values of S, We can summarize all the gender specific rules summarized as linear models:

$$b_s(v_{it}|g) = \phi_{gs0} + \phi_{gs1}v_{it} + \epsilon_{gsit}.$$

Define B_{gs} to be the vector of bids when subjects of gender type g adopt rule s, and V_{gs} to be the matrix of right-hand side variables when subjects adopt rule s. We start by assuming the prior joint distribution of $(\phi_{gs}^{(l)}, \sigma_{gs}^{(l)})$ follows the Normal-Inverse Gamma distribution, N-IG$(\tilde{\phi}_s, \tilde{A}_s, \tilde{\sigma^2}_s)$. Note the prior parameters are generally set to zero, except for the slope term in the AM rule where we use the point prior $\phi_{gAM1} = 1$ and the intercept term in the PM rule where we use the point prior $\phi_{gPM0} = 0$.

The posterior distribution of $(\phi_{gs},\ \sigma_{gs}^2)$ has the Normal-Inverse Gamma form:

$$\phi_{gs} \sim \mathbf{N}\left(\bar{\phi}_{gs}, \bar{A}_{gs} \cdot \sigma_{gs}^2\right)$$
$$\sigma_{gs}^2 \sim \mathbf{IG}\left(\frac{\bar{\nu}_{gs}}{2}, \frac{\bar{\nu}_{gs} \cdot \bar{\sigma}_{gs}^2}{2}\right)$$

where $\bar{\nu}_{gs}$ is the degrees of freedom of the linear model, the number of time gender type g uses rule s in the sequence of states S.

We draw values for $(\sigma^2_{gs})^{(l)}$ from the posterior inverse-gamma distribution with $\bar{\nu}_s$ and $\bar{\sigma}^2_s$, which can be calculate by following formulas.

$$
\begin{aligned}
\bar{A}_{gs} &= \left(\tilde{A}_{gs} + V'_{gs} \cdot V_{gs}\right) \\
\bar{\phi}_{gs} &= \bar{A}^{-1}_{gs} \cdot \left(\tilde{A}_{gs} \cdot \tilde{\phi}_{gs} + V'_{gs} \cdot B_{gs}\right) \\
\bar{\sigma}^2_{gs} &= \bar{\nu}^{-1}_{gs} \left(\bar{\nu}_{gs}\tilde{\sigma}^2_{gs} + \left(B_{gs} - \bar{\phi}_{gs}V_{gs}\right)' \left(B_{gs} - \bar{\phi}_{gs}V_{gs}\right) + \left(\bar{\phi}_{gs} - \tilde{\phi}_{gs}\right)' \tilde{A}_{gs} \left(\bar{\phi}_{gs} - \tilde{\phi}_{gs}\right)\right)
\end{aligned}
$$

Where $\tilde{\phi}_{gs}$, $\tilde{A}_{gs}$ and $\tilde{\sigma}^2_{gs}$ are the prior parameters. Value of $\phi^{(l)}_{gs}$ can be drawn from the normal distribution with the variance $(\sigma^2_{gs})^{(l)}$. The prior parameters $\tilde{\phi}_{gs}$, $\tilde{A}_{gs}$ and $\tilde{\sigma^2}_{gs}$ are chosen to be non informative. To avoid the switching of estimate rules, we restrict $\sigma^2_{PM} > \sigma^2_{BR} > \sigma^2_{AM}$. The restriction is consistent with the E-MLE results of Shachat and Wei (2012).

Step 2: Sampling the state sequences $\mathbf{S}^{(l)}$

To generate $s^{(l)}_{it}$, we start at $t = 100$ and recursively calculate state probabilities. Then we determine state $s^{(l)}_{it}$ by taking a realization from the standard uniform distribution and comparing it to the calculated state probabilities. The formula for the state probabilities are

$$
\Pr(s^{(l)}_{it} = r) = \frac{\Theta(r)}{\sum_{j\in\{AM,PM,BR\}} \Theta(j)},
$$

where

$$
\Theta(r) = \begin{cases}
\Pr\left(b_{it}|s^{(l)}_{it} = r, v_{it}, \phi^{(l)}_{gr}, \sigma^{(l)}_{gr}\right) \Pr\left(s^{(l)}_{it} = r|s^{(l-1)}_{it-1}, o_{it-1}, \Gamma^{(l-1)}\right) & \text{if } t = 100 \\
\Pr\left(b_{it}|s^{(l)}_{it} = r, v_{it}, \phi^{(l)}_{gr}, \sigma^{(l)}_{gr}\right) \Pr\left(s^{(l)}_{it} = r|s^{(l-1)}_{it-1}, o_{it-1}, \Gamma^{(l-1)}\right) & \\
\quad \times \Pr\left(s^{(l)}_{it+1}|s^{(l)}_{it} = r, o_{it}, \Gamma^{(l-1)}\right) & \text{if } 2 < t < 100 \\
\Pr\left(b_{it}|s^{(l)}_{it} = r, v_{it}, \phi^{(l)}_{gr}, \sigma^{(l)}_{gr}\right) \Pr\left(s^{(l)}_{it} = r|\Pi^{(l-1)}_{g1}\right) & \\
\quad \times \Pr\left(s^{(l)}_{it+1}|s^{(l)}_{it} = r, o_{it}, \Gamma^{(l-1)}\right) & \text{if } t = 1
\end{cases}.
$$

Step 3: Sampling $\Pi^{(l)}_{g1}$ and $\Gamma^{(l)}$

For each gender specific Π_{g1} we assume a uniform Dirichlet prior, $D(\Pi_{g1,s}; d_{AM}, d_{PM}, d_{BR})$ by setting the shape parameters, d_s to one. The Dirichlet distribution is the conjugate prior for the multinomial distribution, and the shape parameters of the conditional posterior are simply the number of occurrences of each state in the first element of the sequences of gender type g bidders in $S^{(l)}$. Let $n^0_{g,j}$ be the number incidences of $s^{(l)}_{i,1} = j$ for g type bidders. Thus, the conditional posterior is

$$h(\Pi_{g1,s}|S^{(l)}) = h(\Pi_{g1,s}; d_{AM} + n^0_{g,AM}, d_{PM} + n^0_{g,PM}, d_{BR} + n^0_{g,BR}).$$

We generate $\Pi^{(l)}_{g1}$ by sampling from this distribution.

To characterize the marginal conditional posterior distribution for the rule transition probability matrix parameters Γ and to generate a sample $\Gamma^{(l)}$, we use the methods introduced by Albert and Chib (1993) and Filardo and Gordon (1998). First, reduce the three indices of equation 1 to two by normalize on the BR rule as follows[18];

$$\Psi^{j1}_{it} = \Psi^{jAM}_{it} - \Psi^{jBR}_{it} \text{ and } \Psi^{j2}_{it} = \Psi^{jPM}_{it} - \Psi^{jBR}_{it}.$$

Now we can restate the transition probabilities; Given S, Γ and the following inequality constraint:

$$\begin{aligned}
\Pr(s_{it} = BR|s_{it-1}, o_{it-1}) &= \Pr(\Psi^{s_{it-1}1}_{it-1} \leq 0, \Psi^{s_{it-1}2}_{it-1} \leq 0)\\
\Pr(s_{it} = PM|s_{it-1}, o_{it-1}) &= \Pr(\Psi^{s_{it-1}1}_{it-1} \geq \Psi^{s_{it-1}2}_{it-1}, \Psi^{s_{it-1}1}_{it-1} \geq 0).\\
\Pr(s_{it} = AM|s_{it-1}, o_{it-1}) &= \Pr(\Psi^{s_{it-1}2}_{it-1} \geq \Psi^{s_{it-1}1}_{it-1}, \Psi^{s_{it-1}2}_{it-1} \geq 0)
\end{aligned}$$

We construct realized values of the Ψ^{j1}_{it} and Ψ^{j2}_{it} by using $\Gamma^{(l-1)}$, $s^{(l)}_{it}$, and o_{it} with perturbations randomly generated from the appropriate truncated bivariate normal distributions. These realized normalized indices are now simple linear regression models with unit variance. With the known variance and assumed normal distributed prior with zero mean, the conditional posterior distribution is also normal and we can generate draws similarly to how we do in Step 1.

[18]Note that this normalization highlights that $\gamma_{jk} z_i$ is not identified, only the differences between rules indices under neutral reinforcement are.

References

Albert, James H and Siddhartha Chib (1993), "Bayesian analysis of binary and polychotomous response data." *Journal of the American Statistical Association*, 88, 669–679.

Baumol, William J. and Richard E. Quandt (1964), "Rules of thumb and optimally imperfect decisions." *The American Economic Review*, 54, 23–46.

Blinder, Alan S (1973), "Wage discrimination: reduced form and structural estimates." *Journal of Human resources*, 436–455.

Casari, Marco, John C. Ham, and John H. Kagel (2007), "Selection bias, demographic effects, and ability effects in common value auction experiments." *American Economic Review*, 97, 1278–1304.

Chen, Yan, Peter Katuk, and Emre Ozdenoren (2013), "Why can't a woman bid more like a man?" *Games and Economic Behavior*, 77, 181 – 213.

Crawford, Vincent P. and Nagore Iriberri (2007), "Level-k auctions: can a nonequilibrium model of strategic thinking explain the winner's curse and overbidding in private-value auctions?" *Econometrica*, 75, 1721–1770.

Erev, Ido and Alvin E. Roth (1998), "Predicting how people play games: reinforcement learning in experimental games with unique, mixed strategy equilibria." *American Economic Review*, 88, 848–81.

Filardo, Andrew I. and Stephen F. Gordon (1998), "Business cycle durations." *Journal of Econometrics*, 85, 99–123.

Fischbacher, Urs (2007), "z-tree: Zurich toolbox for ready-made economic experiments." *Experimental Economics*, 10, 171–178.

Geman, Stuart and Donald Geman (1987), "Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images." In *Readings in computer vision: issues, problems, principles, and paradigms* (Martin A. Fischler and Oscar Firschein, eds.), 564–584, Morgan Kaufmann Publishers Inc., San Francisco, CA, USA.

Geweke, John (1991), "Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments." Staff Report 148, Federal Reserve Bank of Minneapolis.

Gill, David and Victoria Liza Prowse (2012), "Gender differences and dynamics in competition: the role of luck." MPRA Paper, University Library of Munich, Germany.

Goeree, Jacob and Charles A. Holt (2002), "Quantal response equilibrium and overbidding in private-value auctions." *Journal of Economic Theory*, 104, 247–272.

Greiner, Ben (2004), "An online recruitment system for economic experiments." In *Forschung und wissenschaftliches Rechnen* (Kurt Kremer and Volker Macho, eds.), volume 63 of *Ges. fur Wiss. Datenverarbeitung*, 79–93, GWDG Bericht.

Guth, Werner, Radosveta Ivanova-Stenzel, Manfred Konigstein, and Martin Strobel (2003), "Learning to bid - an experimental study of bid function adjustments in auctions and fair division games." *Economic Journal*, 113, 477–494.

Holt, Charles A (1980), "Competitive bidding for contracts under alternative auction procedures." *The Journal of Political Economy*, 433–445.

Isaac, Mark, Svetlana Pevnitskaya, and Kurt S. Schnier (2012), "Individual behavior and bidding heterogeneity in sealed bid auctions where the number of bidders is unknown." *Economic Inquiry*, 50, 516–533.

Kagel, John H (1995), "Auctions: a survey of experimental research." In *The Handbook of Experimental Economics* (Alvin E Roth and John H Kagel, eds.), first edition, chapter 7, 501 – 585, Princeton University Press, Princeton, NJ.

Kagel, John H and Dan Levin (1993), "Independent private value auctions: bidder behaviour in first-, second-and third-price auctions with varying numbers of bidders." *The Economic Journal*, 868–879.

Kagel, John H and Dan Levin (2011), "Auctions: a survey of experimental research, 1995–2010." Manuscript, Ohio State University.

Kirchkamp, Oliver and J. Philipp Reiss (2008), "Heterogeneous bids in auctions with rational and markdown bidders - theory and experiment." Jena Economic Research Papers 2008-066, Friedrich-Schiller-University Jena, Max-Planck-Institute of Economics.

McKelvey, Richard D and Thomas R Palfrey (1995), "Quantal response equilibria for normal form games." *Games and economic behavior*, 10, 6–38.

Neugebauer, Tibor and Reinhard Selten (2006), "Individual behavior of first-price auctions: The importance of information feedback in computerized experimental markets." *Games and Economic Behavior*, 54, 183–204.

Oaxaca, Ronald (1973), "Male-female wage differentials in urban labor markets." *International Economic Review*, 14, 693–709.

Pearson, Matthew and Burkhard C. Schipper (2013), "Menstrual cycle and competitive bidding." *Games and Economic Behavior*, 78, 1 – 20.

Schipper, Burkhard (2012), "Sex hormones and competitive bidding." Working Papers 128, University of California, Davis, Department of Economics.

Selten, Reinhard and Joachim Buchta (1998), "Experimental sealed bid first price auctions with directly observed bid functions." In *Games and Human Behavior: Essays in Honor of Amnon Rapoport* (David V. Budescu, Ido Erev, and Rami Zwick, eds.), chapter 5, 79–102, Lawrence Erlbaum Ass., Mahwah (NJ).

Shachat, Jason and Lijia Wei (2012), "Procuring commodities: first-price sealed-bid or english auctions?" *Marketing Science*, 31, 317–333.

Vickrey, William (1961), "Counterpeculation, auctions, and competitive sealed tenders." *Journal of Finance*, 16, 8–37.

Zhou, Yuanzhong, Chengliang Xiong, Ping Yin, Xuejun Shang, Sheng Wei, Hong Liu, Guohui Liu, Xueyi Wang, Meimei Zhang, Xinwei Li, Huangtao Guan, Dunzheng Xiao, Xuebing Pang, Ning Ren, and Min Wang (2009), "Survey of status and requirement about sexual behavior and contraception among unmarried college/university students in china." *Journal of Huazhong University of Science and Technology (Medical Sciences)*, 38, 561–580.