Analyzing Time-Frequency Relationship between Oil Price and Exchange Rate in Pakistan through Wavelets

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Abstract:

This study analyzes the time-frequency relationship between oil price and exchange rate for Pakistan by using measures of continuous wavelet such as wavelet power, cross-wavelet power, and cross-wavelet coherency. The results of cross-wavelet analysis indicate that covariance between oil price and exchange rate are unable to give clear-cut results but both variables have been in phase and out phase (i.e. they are anti-cyclical and cyclical in nature) in some or other durations. However, results of squared wavelet coherence disclose that both variables are out of phase and real exchange rate was leading during the entire period studied, corresponding to the 10~15 months scale. These results are the unique contribution of the present study, which would have not been drawn if one would have utilized any other time series or frequency domain based approach. This finding provides evidence of anti-cyclical relationship between oil price and real effective exchange rate. However; in most of the period studied, real exchange rate was leading and passing anti-cycle effects on oil price shocks which is the major contribution of the study.

Keywords: Oil prices, exchange rate, Pakistan

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I. Introduction

Oil prices affect real effective exchange rate via supply-side and demand side mechanisms. Explaining supply-side effects, crude oil is basic input used in production process. So, rise in oil prices is linked with an increase in the cost of production of non-tradable products and in resulting, this rise in prices of non-tradable goods appreciates real effective exchange rate. Indirectly, an increase in disposable income also leads an appreciation in real effective exchange rate. Consumer spending power is highly affected by a rise in oil prices. This reduces the demand for non-tradable products and in turn, prices of non-tradable goods have fallen which deprecates the real effective exchange rate. Hamilton, [1] started a debated on the relationship between oil prices and macroeconomic variables. For example; Hamilton, [1] explored the association between oil prices and US business cycle. Various researchers contributed in existing literature by investigating relationship between oil prices and stock market returns (Sadorsky, [2, 3,4]; Papapetrou, [5]); between economic growth and stock market development (Shahbaz et al. [6]) and relationship between exchange rate and oil prices is also investigated by Bénassy-Quéré et al. [7]; Chen and Chen, [8]; Huang and Guo, [9]; Olomola and Adejumo, [10]; Kutan and Wyzan, [11] and many more.

This paper deals with empirical investigation between oil prices and real effective exchange rate in case of Pakistan over the period of 1986M_2-2009M_3. Few studies are also available examining Pakistan's macroeconomic fundamentals. For example, nominal and real effective exchange rates (Shahbaz, [12]; saving-investment and capital outflows (Shahbaz et al. [13]); capital inflows and economic growth (Shahbaz and Rahman, [14]); real effective exchange rate and trade balance (Shahbaz et al. [15]); terms of trade and trade balance i.e. J-curve (Shahbaz et al. [16]); devaluation and economic growth (Shahbaz et al. [17]); financial development, foreign direct investment and economic growth (Shahbaz and Rahman, [18]); trade openness and economic growth (Shahbaz, [19])[1] and, money supply and interest rate (Khattak et al. [21]). What is in remainder to investigate the relationship between oil prices and real effective exchange rate in case of Pakistan. Pakistan is lower middle income country heavily depends on oil imports to boost economic activity and hence economic growth. The empirical investigation of impact of oil prices on exchange rate is very important for policy making point of view.

Our study makes important contributing in the existing literature by three ways. In the first place, the much debated question of whether or not a causal relationship exists between the oil price and exchange rate, calling upon the notion of causality based on the pioneering work of Granger, [22]. Secondly, within this causality debate, the relevance of frequency domain concepts is introduced as Granger and Lin, [23] documented that the extent and direction of causality can differ between frequency bands. Thirdly, we introduced time series concept with frequency domain, hence, we analyzed time-frequency relationship as in the frequency domain framework time information is lost. So, in our contribution, we used continuous wavelets tools such as the wavelet power spectrum, wavelet coherency, and wavelet phase difference to analyze the impact of oil price changes on exchange rate and vice-versa. The wavelet power spectrum illustrates the evolution of the variance of a time-series at the different frequencies; the wavelet coherency demonstrates the correlation coefficient in the time-frequency space; and the information on the delay between the oscillations of two time-series i.e. lead-lag relationship s provided by phase difference. Our major contribution lies in providing evidence of anti-cyclical relationship between oil price and real effective exchange rate however, in most of the period studied we found that real exchange rate was leading and passing anti-cycle effects on oil price shocks.

The rest of paper is organized as following: section-II provides review of literature; section-III explains methodology and data collection. Results are interpreted in section-IV and conclusions and future research are drawn in section-V.

II. Literature Review

The relationship between oil prices and exchange rate nexus has been discussed by Krugman, [24]; Golub, [25]; Corden, [26]; Rogoff, [27] and many more. These studies argument that in oil exporting countries, a rise in oil prices appreciates local currency against US dollar and US dollar exchange rate depreciates due to rise in oil prices in oil importing countries. Moreover, Amano and van Norden, [28] examined the cointegration and causality relationship between oil prices and exchange rate in case of Germany, Japan and the United States. Their results showed cointegration relationship between the variables and oil price change leads to appreciate the US dollar in long span of time. The causality analysis also supported their view by providing unidirectional causality running from oil prices to exchange rate in Germany, Japan and US. In case of OECD countries, Chauhuri and Daniel, [29] discussed the issue of oil prices and exchange rate by applying Engle-Granger, [30] cointegration approach and reported that both variables cointegrated in 13 out of 16 countries. Furthermore, exchange rate is Granger caused by oil price changes.

Rautava, [31, 32] examined the role of oil prices and exchange rate changes in Russian economy. The results showed cointegration between the variables and oil prices inversely affect output and Russian exchange rate[2]. Bénassy-Quéré et al. [7] unveiled the relationship between oil prices and real effective exchange rate in case of China. They reported that both variables are cointegrated for long run relationship and the VECM Granger causality analysis showed positive causal relationship runs from oil prices to exchange rate in short run while in long run causality is negative running from oil prices to exchange rate. Similarly, for Dominican Republic, Dawson, [34] investigated the impact of oil prices on exchange rate using VAR approach. The findings indicated that oil prices rise is inversely linked with exchange rate. Dawson documented that a 2.9 percent depreciation in real exchange rate is Granger caused by 1 percent increase in oil price and this phenomenon is more relevant for short run. Chen and Chen, [8] borrowed model developed by Chauhuri and Daniel, [29] to analyse the relationship between oil prices and exchange rate using data of G-7 countries. Their results indicated that both variables are cointegrated for long run using panel cointegration approach.

Huang and Guo, [9] collected data on oil prices and exchange rate in case of China to examine the nature of relationship between them. By applying SVAR model, they found that real oil price shocks appreciate the US dollar minimally in long run due to lesser dependence on oil imports. Issa et al. [35] revisited the association between energy prices and Canadian dollar by applying the model advanced by Amano and van Norden, [28][3]. Their results reported that energy prices depreciate Canadian dollar. Coudert et al. [37] reconsidered the relationship between oil prices and exchange rate in case US. They found cointegration between the variables and oil price Granger cause exchange rate[4]. Rickne, [39] found that co-movements between oil prices and US dollar exchange rate depends on political and legal institutions. Currencies are less effective by oil price changes in countries where bureaucracies and legal systems are strong. In case of US, Huang and Tseng, [40] investigated the relationship between crude oil prices and US dollar exchange rate using auxiliary regression. They found cointegration between both variables while causality results indicated that feedback effect is found between oil prices and exchange rate[5]. Leili, [42] unveiled the relationship between real oil prices and real effective exchange rate using the data of OPEC. The finding indicates that dominant source of movements in exchange rate is due to real oil prices shocks in long run. In case of US, Lizardo and Mollick, [43] found long run relationship between the series. They reported that rise in oil prices depreciate the US dollar against net exporter countries such as Canada, Mexico, and Russia and vise versa for oil importer countries[6].

In case of India; Ghosh, [45] probed the relationship between crude oil prices and exchange rate using generalized autoregressive conditional heteroskedasticity (GARCH) and exponential GARCH (EGARCH) models. Ghosh reported unidirectional causality running from oil prices to exchange rate (depreciating Indian currency against US dollar) and changes in oil prices affect exchange rate permanently. In case UAE; Al-mulali and Sab, [46] examined impact of oil prices shocks on exchange rate of UAE Dirham in fixed exchange regime. They found that oil prices shocks do not seem to lead exchange rate but stimulated gross domestic product and liquidity which in resulting caused domestic prices to rise and hence inflation. Treviño, [47] disclosed the relationship between oil price and exchange rate in oil-rich countries of the Central African Economic and Monetary Community (CEMAC) by applying procedure developed by Ismail [48][7]. The results showed that oil price appreciates real exchange rate which confirms the presence of Dutch disease that restrict these economies to attain high economic growth in long run. In addition; Kanturk, [49] examined the effect of oil prices on exchange rate volatility in Turkey and concluded that rise in oil prices has significant impact on exchange rate volatility but impact is restrained during financial crisis.

Wu et al. [50] examined co-movements between oil prices and exchange rate by applying copula-based GARCH model in case of US economy. They reported that copula GARCH model label the volatility and dependence structure of oil price and US dollar exchange rate returns, CAMP model reveals that feedback trading activities are found significant in oil market but inference is not drawn in USDX market, GARCH model indicates that short run volatility is persistence and less than long run volatility for oil prices features but it is significant for USDX features. Similarly; Reboredo, [51] conducted a study to model the co-movements between oil prices and exchange rate using correlations and copulas approaches[8]. The results indicated that oil price rise is weakly linked with US dollar depreciation and vice versa, the strength of co-movements is found different across currencies. For example, intensity is high for oil exporting countries such as Canada, Norway and Mexico and low intensity is found in oil importing courtiers, especially in Japan where there is no interdependence between oil prices and exchange rate movements. The interdependence between the variables is increased after financial crisis, once, the coefficient of linear correlation raises to maximum value of 0.45. Basher et al. [52] unveiled the relationship between oil price and exchange rate in emerging markets[9]. Their results reported that a positive shock to oil prices Granger cause US dollar exchange rate to decline in short run in all emerging economies. In case of Malaysia; Hussein et al. [53] investigated the causality between oil price and US dollar exchange rate. They found cointegration between the variables and unidirectional causal relationship is found running from oil price to US dollar exchange rate. In US economy; Benhmad, [54] disclosed unidirectional causality running from real oil prices to real effective exchange rate applying linear and non-linear Granger causality approaches.

Beckmann and Czudaj, [55] examined the dynamics between oil prices and US dollar exchange rate and reported bidirectional causality between the both variables. Coleman et al. [56] disclosed the nexus between oil price and exchange rate in African countries[10]. They found cointegration between the variables and oil price changes paly vital role in determining real exchange rate but this effect is different across the countries may be due to difference in economic structure. Turhan et al. [57] uncovered the role of oil price in determining the exchange rate in emerging economies[11]. Their results indicated that oil price rise is leading indicator to appreciate the currencies of these market against US dollar. Moreover, analysis of generalized impulse response function reveals that impact of oil price on exchange rate is significant after 2008 financial crisis. Adeniyi et al. [58] analyzed the relationship between oil prices and exchange rate using GARCH and exponential GARCH (EGARCH) in case of Nigeria. They found that oil price rise appreciates Nigerian currency against US dollar and same inference was drawn by Olomola and Adejumo, [10] and latter on by Oriavwote and Eriemo, [59][12]. Apart from that Englama et al. [60] reported that a 1 per cent increase in permanent shock in oil prices adds in 0.54 per cent shock in exchange rate unpredictability in long run while foreign exchange demand leads exchange rate volatility dominate. Latter on, Hassan and Zahid, [61] and, Ozsoz and Akinkunmi, [62] also found positive impact of oil prices on Nigerian real effective exchange rate. Recently, Tiwari et al. [63] applied the wavelet approach to probe the relationship between oil prices and real effective exchange rate using the data of Indian economy. They found neutral effect between both variables at the lower time scale but at higher time scales, real effective exchange rate Granger cause oil prices.

III. Data and methodology

For empirical purpose, we have collected monthly frequency data on oil prices and real effective exchange rate over the period of 1986-2009. The data span is large and sufficient for reliable and consistent results. Exchange rate is proxied by real effective exchange rate and collected from international financial statistics (CD-ROM, 2010). The crude oil price variable is expressed in real terms, i.e. deflated by U.S. consumer price index following Faria et al. [64]. The data on crude oil prices are the spot prices and collected from Pakistan Energy Year Book (Government of Pakistan).

III.I Motivation and Introduction to Methodology

Existing economic literature provides various studies applying various approaches to investigate the relationship between oil prices and exchange rate changes. The researchers have not paid their attention to apply the time domain and frequency domain approaches in examining the relationship between both series. There may be a relationship between the series at different frequencies such as oil price may act like a supply shock at lower and medium frequencies (Naccache, [41]) and in resulting it affects real effective exchange rate. In short span of time i.e. at the higher frequencies, real effective exchange rate affects oil prices following demand-effect. It is a general practice in existing economic literature to unveil relationship between the series at different frequencies using Fourier analysis. The demerits of Fourier analysis are also discussed in the existing literature. For example, Fourier transform does not seem to capture the time information which makes difficult to get information about short-lived relationship or structural break stemming in the series. These structural breaks are very important for policy making point of view. Furthermore, results provided by Fourier transform are less reliable. This approach works well when series do not have unit root problem at level but macroeconomic variables are found non-stationary at level usually. This implies that error term of non-stationary series is not normally distributed and provides biased results. This issue has been resolved by Gabor, [65] who advanced a specific transformation of Fourier transform which is also called a short time Fourier transformation. The short time Fourier transformation breaks into smaller sub-samples to apply Fourier approach on each sub-sample. Although, this approach is also criticized on the basis of its efficiency as it takes equal frequency resolution across all dissimilar frequencies (see Raihan et al. [66] for detail). This issue has provided a space for wavelet transform approach. This approach is advantageous over Fourier transformation from various aspects. For example, wavelet transform approach performs "natural local analysis of a time-series in the sense that the length of wavelets varies endogenously: it stretches into a long wavelet function to measure the low-frequency movements; and it compresses into a short wavelet function to measure the high-frequency movements" Aguiar-Conraria and Soares [67]. Wavelet transform works within the spectral framework for analysis of the time series and it is a function of time. This implies that wavelet approach illustrates the changes stemming in the series with the passage of time and at various periodic components or frequency bands.

It is noted that discrete wavelet transformation is not applied extensively in economics and finance. There is a question, up to what extent we should decompose while employing discrete wavelet approach. Moreover, wavelet discrete analysis is not much helpful for economists and policy makers in formulating a comprehensive economic policy. This continues transformation of discrete wavelet analysis may provide more reliable and understandable results at each scale following the variations in the time series data. For example, looking at Figure-1 one can immediately conclude the evolution of the variance of the return series of oil price and exchange rate at the several time scales along the 20 year observation and extract the conclusions with just a single diagram. Aguiar-Conraria et al. [68] pointed out the wavelets due to its two interesting features. For example, Aguiar-Conraria et al. ([68], p. 2865) pointed out that "first, in most economic applications the (discrete) wavelet transform has mainly been used as a low and high pass filter, it being hard to convince an economist that the same could not be learned from the data using the more traditional, in economics, band pass-filtering methods. The second reason is related to the difficulty of analyzing simultaneously two (or more) time series. In economics, these techniques have either been applied to analyze individual time series or used to individually analyze several time series (one each time), whose decompositions are then studied using traditional time-domain methods, such as correlation analysis or Granger causality".

The above issued have been solved by Hudgins et al. [69] and Torrence and Compo, [70] developing the cross-wavelet power, the cross-wavelet coherency, and the phase difference to accommodate the analysis of time frequency dependencies between two time series. Using cross-wavelet tools, we can analyze relationship between two series at different frequencies. The single wavelet power spectrum is helpful to recognize the development of variations in the series at different frequencies as well as with periods of large variance associated with periods of large power at the different scales. Furthermore, cross-wavelet power also shows the curbed covariance between the variables. The wavelet coherency can be interpreted as correlation coefficient in the time-frequency space. This phase term indicates the position of pseudo-cycle of the time which is function of occurrence. Similarly, the phase difference gives us information "on the delay, or synchronization, between oscillations of the two time series" (Aguiar-Conraria et al. [68], p. 2867).

III.II The Continuous Wavelet Transform (CWT)[13]

The continuous wavelet transform approach is contained to both frequency and time having zero mean. The major advantage of continuous wavelet transform is that it can be characterized by localizing continuous wavelet transform in time (Δt) and frequency ($\Delta\omega$) or in both. It is exposed by Heisenberg uncertainty principle that tradeoff exists between localization in time and frequency. We have to properly define Δt and $\Delta\omega$ because there is a minimum limit for the uncertainty product (yellow I have edited) $\Delta t \cdot \Delta\omega$. The Morlet wavelet, most often used in research, is defined as following:

$$\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0\eta} e^{-\frac{1}{2}\eta^2}. \qquad (1)$$

where dimensionless frequency and time is indicated by ω_0 and η respectively. The Morlet wavelet (with ω_0=6) approach is an appropriate option for feature extraction because it provides a good balance between time and frequency localization. This approach is applied to the wavelet as band pass filter to the time series. The wavelet is stretched in time by varying its scale (s), so that $\eta = s \cdot t$ and normalizing it to have unit energy. For the Morlet wavelet (with ω_0=6) the Fourier period (λ_{wt}) is almost equal to the scale ($\lambda_{wt} = 1.03\,s$). The CWT of a time series ($x_n, n = 1,...,N$) with uniform time steps δt, is defined as the convolution of x_n with the scaled and normalized wavelet. We write as:

$$W_n^X(s) = \sqrt{\frac{\delta t}{s}} \sum_{n'=1}^{N} x_{n'} \psi_0\left[(n'-n)\frac{\delta t}{s}\right] \qquad (2)$$

The power of wavelet is defined as $\left|W_n^X(s)\right|^2$ and local phase is simple interpretation of the complex argument of $W_n^X(s)$. The CWT has edge artifacts because the wavelet is not completely localized in time. The introduction of Cone of Influence (COI) is useful to a point where edge effects are accepted. We observe the COI as the area in which the wavelet power caused by a discontinuity at the edge has dropped to e^{-2} of the value at the edge. The null hypothesis is used to asses the statistical significance of wavelet power while background power spectrum (P_k) is used to generate stationary process[14]. Torrence and Compo, [70] estimated the white noise as well ass red noise wavelet power spectra. Both estimates have been derived from corresponding distribution following wavelet power spectrum at every point of time n as well as scale s. The corresponding distribution is as following:

$$D\left(\frac{\left|W_n^X(s)\right|^2}{\sigma_X^2} < p\right) = \frac{1}{2} P_k \chi_v^2(p), \tag{3}$$

where v is equal to 1 for real and 2 for complex wavelets.

III.III The Cross Wavelet Transform (CWT)

$W^{XY} = W^X W^{Y*}$ is definition of the cross wavelet transform (CWT) of the two variables such as x_n and y_n. The W^X and W^Y are transformation of the wavelet transforms for x and y time series respectively and * is complex conjugation. $\left|W^{XY}\right|$ is the definition of cross wavelet power. The i complex argument such as W^{xy} interprets the local relative phase for x_n and y_n time series using time frequency space. Torrence and Compo, [70] generated the distribution of the cross wavelet power for two time series with background power spectra P_k^X and P_k^Y which is given as following:

$$D\left(\frac{\left|W_n^X(s)W_n^{Y*}(s)\right|}{\sigma_X \sigma_Y} < p\right) = \frac{Z_v(p)}{v}\sqrt{P_k^X P_k^Y}, \tag{4}$$

where $Z_v(p)$ is level of confidence linked with the probability p for a pdf which is defined by χ^2 distributions.

III.IV Wavelet Coherency (WTC)

Following Fourier spectral approach, we can define Wavelet Coherency (WTC). The Wavelet Coherency is a ratio of the cross-spectrum to the product of the spectrum of each series. This indicates local correlation between two time series within time and frequency. So, the Wavelet Coherency presents a high resemblance if coherence is near to 1 otherwise no relationship is found between the time series. The Wavelet power spectrum shows the variance of the series. The larger variance in Wavelet power spectrum shows large power. The covariance between the time series is represented by the Cross Wavelet power following all frequencies or scales. Aguiar-Conraria et al. ([68], p. 2872) defines Wavelet Coherency as "the ratio of the cross-spectrum to the product of the spectrum of each series, and can be thought of as the local (both in time and frequency) correlation between two time-series".

So, Torrence and Webster, [71] define the Wavelet Coherence of two time series as following:

$$R_n^2(s) = \frac{\left|S\left(s^{-1}W_n^{XY}(s)\right)\right|^2}{S\left(s^{-1}\left|W_n^X(s)\right|^2\right)\cdot S\left(s^{-1}\left|W_n^Y(s)\right|^2\right)}, \tag{5}$$

where, smoothing operator is shown by S. It is a traditional correlation coefficient definition which is helpful in thinking of the Wavelet Coherence as a localized correlation coefficient following time frequency space. We rewrite the equation-5 once smoothing operator is equalant to 1 and smoothing operator S as a convolution in time and scale:

$$S(W) = S_{scale}(S_{time}(W_n(s))) \tag{6}$$

where S_{scale} denotes smoothing along the wavelet scale axis and S_{time} indicates smoothing in time. The time of convolution is determined using Gaussian and scale convolution is done by regular window (Torrence and Compo, [70]). The functional form of smoothing power following Morlet wavelet is articulated as:

$$S_{time}(W)|_s = \left(W_n(s) * c_1^{-t^2/2s^2}\right)|_s \tag{7}$$

$$S_{scale}(W)|_n = \left(W_n(s) * c_2\Pi(0.6s)\right)|_n \tag{8}$$

where, normalized constants are c_1 and c_2 while Π denotes is the rectangle function. The scale de-correlation length for Morlet wavelet is empirical determined by 0.6. Practically, we determine co-evolutions directly but normalized coefficients indirectly. The Monte Carlo simulation approach is used to asses the theoretical distributors of wavelet coherency. To test the wavelet coherency, we follow Aguiar-Conraria and Soares, [67] rather than Wavelet Cross Spectrum. Aguiar-Conraria and Soares, ([67], p. 649) gives two arguments for this: "(1) the wavelet coherency has the advantage of being normalized by the power spectrum of the two time-series, and (2) that the wavelets cross spectrum can show strong peaks even for the realization of independent processes suggesting the possibility of spurious significance tests".

III.V Cross Wavelet Phase Angle

The phase difference between the components is estimated by using mean and confidence interval of the phase difference of two time series. The phase relation is measured by the circular mean of the phase, which is if, over the regions higher than 5 per cent level of significance and outside the COI to measures the phase relation. The useful and general method to investigate mean phase using a set of angles ($a_i, i = 1,...,n$) can be defied as following:

$$a_m = \arg(X, Y) \text{ with } X = \sum_{i=1}^{n} \cos(a_i) \text{ and } Y = \sum_{i=1}^{n} \sin(a_i) \qquad (9)$$

The independence of phase angles is helpful in calculating reliable confidence interval of the mean angle. The number of angles used in the calculation can be set arbitrarily high simply by increasing the scale resolution. Nevertheless, the scatter of angles around the mean is very interesting. The spherical standard deviation can be defined as following:

$$s = \sqrt{-2\ln(R/n)}, \tag{10}$$

where $R = \sqrt{(X^2 + Y^2)}$. The spherical standard deviation is similar to linear standard deviation and its value ranges from zero to infinity. The close distribution of angles around the mean angle does not make difference in results and results are similar with linear standard deviation. In some cases there might be reasons for calculating the mean phase angle for each scale, and then the phase angle can be quantified as a number of years. The Monte Carlo methods are used to find statistical level of significance of the wavelet coherence. We generate a large ensemble of surrogate data set pairs with the same AR(1) coefficients as the input datasets. We investigate the wavelet coherence for each pair. The level of significance for every scale is estimated using the values outside the COI. The number of lags in phase for wavelets is defined as following:

$$\phi_{x,y} = \tan^{-1}\frac{I\{W_n^{xy}\}}{R\{W_n^{xy}\}}, \phi_{x,y} \in [-\pi, \pi] \tag{11}$$

The real and imaginary parts are indicated by I and R respectively, of the smooth power spectrum. The phase relationship between the two times series is characterized using path difference which is considered useful. The time series moves together with specified frequency if value of phase difference ranges to zero. The series move in phase if $\phi_{x,y} \in [0, \pi/2]$ when series x is lead by y series. On contrary, if $\phi_{x,y} \in [-\pi/2,0]$ then x is leading. We have an anti-phase relation (analogous to negative covariance) if we have a phase difference of π (or $-\pi$) meaning $\phi_{x,y} \in [-\pi/2,\pi] \cup [-\pi,\pi/2]$. If $\phi_{x,y} \in [\pi/2,\pi]$ then x is leading, and the time series y is leading if $\phi_{x,y} \in [-\pi,-\pi/2]$.

INSERT FIGURE 1 ABOUT HERE

IV. Results and their Discussions

We have converted both series into logarithm to obtain unbiased and efficient results. The graphs of both variables are shorn in Figure-1. The results of descriptive statistics and correlation matrices are reported in Table-1 containing log levels as well as in returns. The sample mean of log of real effective exchange rate and for returns data of oil prices is positive. The sample mean of oil prices is negative and higher as compared to returns data for real effective exchange rate. The degree skewness shows that real effective exchange rate and oil prices in level are positively skewed and negatively skewed in return data set for both variables. The data is more skewed of level oil prices and returns data set for real effective exchange rate. The measure of Kurtosis indicates that oil prices in level as well as in return data set has more leptokurtic distribution compared to normal distribution. The correlation coefficient reports that real effective exchange rate and oil prices are negatively correlated in level form and same inference is drawn for return data set of both series.

INSERT TABLE 1 ABOUT HERE

To test the stationery properties of running series, we applied ADF (Dickey and Fuller, [72]) and PP (Philips and Perron, [73]). The results of both tests are shown in Table-2. The Table-2 indicates that both variables show unit root behavior at level and found stationery at 1st difference with intercept and trend. The findings of ADF and PP unit root tests may be biased due not having information about structural breaks stemming in the series. To solve this issue, we have applied de-trended Zivot and Andrews, [74] structural break unit test to examine the integrating orders of the variables in the presence of structural breaks. The results are reported in lower segment of Table-2. We found that the series are found non-stationery at level showing structural breaks in 2002M_1 and 1998M_{12} in real effective exchange rate and oil prices respectively[15]. Real effective exchange rate and oil prices are found to be integrated at I(1). This implies that variables have same level of stationarity i.e. I(1).

INSERT TABLE 2 ABOUT HERE

The next step is to apply the ARDL bounds testing approach in the presence of structural breaks due to major economic events affecting these series. The appropriate leg order is prerequisite to apply bounds testing to examine long run relationship between the variables. In doing, we chose Akaike information criteria (AIC) to select lag order of the variables. It is suggested by (Lütkepohl, [75]) that we should apply AIC to choose leg length because it provides better results as compared to sequential modified LR test statistic (LR), final prediction error (FPE); Schwarz information criterion (SIC) and Hannan-Quinn information criterion (HQ). The lag order 5 is chosen following the minimum value of AIC and results of shown in second column of Table-3.

The next step is to compute F-statistic following the ARDL bounds testing approach to cointegration to examine long run relationship between real effective exchange rate and oil prices. We use critical bounds generated by Pesaran et al. [76] for large sample (279 observations). The empirical evidence shows that our computed F-statistic exceeds upper critical bound (UCB) i.e. 7.269 > 4.68 once real effective exchange rate is used as forcing variable. This implies that there is one cointegrating found confirming long run relationship exists between real effective exchange rate and oil prices in case of Pakistan over the period of 1986M_1-2009M_3.

INSERT TABLE 3 ABOUT HERE

The next step is to analyze continuous wavelet power spectrum for both variables. It is evident from Figure-2 that there are some common islands. In particular, the common features in the wavelet power of the two time series are evident in 1990s and 2006 and then 2007. The important point here is less evidence of Common Island for common frequencies and for same year. However, the similarity between the portrayed patterns in these periods is not very much clear and it is therefore hard to tell if it is merely a coincidence. The cross wavelet transform helps in this regard. We further, analyzed the nature of data through cross wavelet and presented results in Figure-3.

INSERT FIGURE 2 ABOUT HERE

INSERT FIGURE 3 ABOUT HERE

It is very interesting to see that in Figure-3, the direction of arrows at different periods (i.e. frequency bands) over the time period studied is not same. We observe that the variables during 1990s are out of phase (if we focus on significance region) and up to 1998 they are out of phase, if we consider high-power region. However, after 1998 we observe that the variables are in phase (if we consider high power region) and significant region is not observed after 1998. The most critical point is that direction of the arrows i.e. whether they are right-up or left-up (or right-down or left-down) is not very clear. So, it is very difficult to tell which variable is leading and which one is lagging in different frequency bands and periods. In other words, outside the areas with significant power, the phase relationship and also lead-lag relationship is also not very clear. Even if, now, we do not have very clear results but this type of results one analyst would have not got if he/she would have utilized either time series or spectral or frequency analysis based methods. Overall we, therefore, speculate that there is a stronger link between returns series of oil price and exchange rate than that implied by the cross wavelet power.

Further, it is worthy to mention that wavelet cross-spectrum (i.e. cross wavelet) describes the common power of two processes without normalization to the single wavelet power spectrum. This can produce misleading results, because one essentially multiplies the continuous wavelet transform of two time series. For example, if one of the spectra is locally and the other exhibits strong peaks, peaks in the cross spectrum can be produced that may have nothing to do with any relation of the two series. This leads us to conclude that wavelet cross spectrum is not suitable to test the significance of relationship between two time series. Therefore, in our conclusion we relied on the wavelet coherency (as it is able to detect a significant interrelation between two time series, to know more about refer to the section-III.II). However, one can still use wavelet cross-spectrum to estimate the phase spectrum. The wavelet coherency is used to identify both frequency bands and time intervals within which pairs of indices are co-varying. Finally, we presented results of cross-wavelet coherency in Figure-4.

INSERT FIGURE 4 ABOUT HERE

The squared WTC of return series of oil prices and real effective exchange rate are shown in Figure-4. If we compare results of WTC and XWT i.e. if we compare Figure-3 and Figure-4, we find very clear results of phase difference of lead-lag relationship between returns series of oil prices and real effective exchange rate in Figure-4. It is hard to tell the exact lead-lag relationship at very short scales of less than 5 months. However, arrows are left-up, in general, throughout the period corresponding to the 10~15 months scale. This finding indicates that the variables are out of phase throughout the period i.e. anti-cyclical effects are observed. Since arrows are pointing up this indicate that real exchange rate is leading. This is the most interesting part which comes now in existence (which did not appear in XWT analysis). Now with the application of WTC analysis we have very clear evidence on lead-lag relationship between return series of oil price and exchange rate. Further, we also come to know whether one variable influence or influenced by the other through anti-cyclical or cyclical shocks. Definitely these results would have not been drawn through the application of time series or fourier transformation analysis if one could have attempted.

V. Conclusions and Future Research

The study analyzed Granger-causality between oil prices and real effective exchange rate for Pakistan by using monthly data covering the period of $1986M_2$-$2009M_3$. We have decomposed the time-frequency relationship between oil price and real effective exchange rate by applying continuous wavelet approach. We have also used structural break unit test to examine the order of integration of both series and the ARDL bounds testing approach is applied to investigate long run relationship between oil price and real effective exchange rate in case of Pakistan.

Our results indicate that the variables are integrated at I(1) and are cointegrated for long run relationship. Furthermore, our results found from the continuous power spectrum that the common features in the wavelet power of the two time series are evident in 1990s and 2006 and latter on in 2007. The results of XWT, which indicate the covariance between oil price and exchange rate, are unable to give clear-cut results but indicate that both variables have been in phase and out phase (i.e. they are anti-cyclical and cyclical in nature) in some or other durations. However, our results of cross-wavelet coherency or squared wavelet coherence (WTC), which can be interpreted as correlation, reveal that both variables are out of phase and real exchange rate was leading during the entire period studied, corresponding to the 10~15 months scale. These results are the unique contribution of the present study, which would have not been drawn if one would have utilized any other time series or frequency domain based approach. Our results indicate that causal and reverse causal relations between oil price and real effective exchange rate vary across scale in case of Pakistan. There are evidence of anti-cyclical relationship between oil price and real effective exchange rate however, in most of the period studied real exchange rate was leading and passing anti-cycle effects on oil price shocks.

Our results found that exchange rate is leading to pass anti-cyclical effects on oil prices shocks. To avoid these shocks, Pakistan government should control the exchange rate movement as it affects are moving backwards. This affects production process i.e. increases the cost of production where oil is used are input. The government should also focus in exploring news sources of energy as alternate of oil energy. This not only lowers the heavy dependence of Pakistan on oil imports but it would also be helpful in controlling the rapid movements in exchange rates due to sustainable deficit in trade as well ass in balance of payments. For future research, current study can be extended by analyzing the trivariate wavelet based approach which might include different interest rates and/or stock market return as a third variable as theoretically all the three variables are expected to be highly correlated with each other following Basher et al. [52]. The copula-based GARCH models can also be used to analyze co-movements between real effective exchange rate and oil prices following Wu et al. [50].

Footnotes

1. Nasreen, [20] also investigated the directional of causal relationship between exports and economic growth using data of Pakistan, India and Bangladesh.
2. Hamilton, [33] found that oil prices affects economic growth negatively.
3. Narayan et al. [36] also reported that oil prices lead exchange rate appreciation for Fiji Islands.
4. Aziz [38] reported no significant association between oil prices and real effective exchange rate.
5. Naccache, [41] examined co-movements between oil price and world macroeconomy proxies by Morgan Stanley Capital International (MSCI) and reported reverse causality theory.
6. Zhang et al. [44] noted that crude oil prices Granger cause US dollar to depreciate
7. Chad, Cameroon, Equatorial Guinea, Gabon, Republic of Congo, Benin, Burkina Faso, Cote d'Ivoire, Guinea Bissau, Mali, Niger, Senegal, Togo, Central African Republic
8. Germany, France, Italy, Netherlands, Belgium/Luxembourg, Ireland, Spain, Austria, Finland, Portugal, Greece, Slovenia, Cyprus, Slovakia, Malta, Australia, Canada, United Kingdom, Japan, Norway and Mexico.
9. Qatar, China, Kazakhstan, Algeria, Kuwait, Saudi Arabia, United Arab Emirates, Ecuador, Singapore, India, USA
10. Burkina Faso, Cameroon, Ivory Coast, Kenya, Madagascar, Mauritius, Morocco, Nigeria, Rwanda, Senegal, Seychelles, South Africa and Togo
11. Argentina, Brazil, Columbia, Indonesia, Mexico, Nigeria, Peru, Philippines, Poland, Russia, South Africa, South Korea and Turkey
12. Oriavwote and Eriemo [59] also reported unidirectional causality running from crude oil to real exchange rate.

13. We are grateful to Grinsted and co-authors for making codes available at: http://www.pol.ac.uk/home/research/waveletcoherence/, which was utilized in the present study.

14. It is pointed out by Torrence and Compo, [70] that how the statistical significance of wavelet power can be assessed against the null hypothesis that the data generating process is given by an *AR (0)* or *AR (1)* stationary process with a certain background power spectrum (P_k), for more general processes one has to rely on Monte-Carlo simulations.

15. The structural break in real effective exchange rate series is dealt with complete shift of Pakistan economy to floating exchange rate system. The structural break in oil prices deals with negative supply shock which raised real price of oil in the country.

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Figure1: Plot of the real effective rupee exchange returns and oil returns

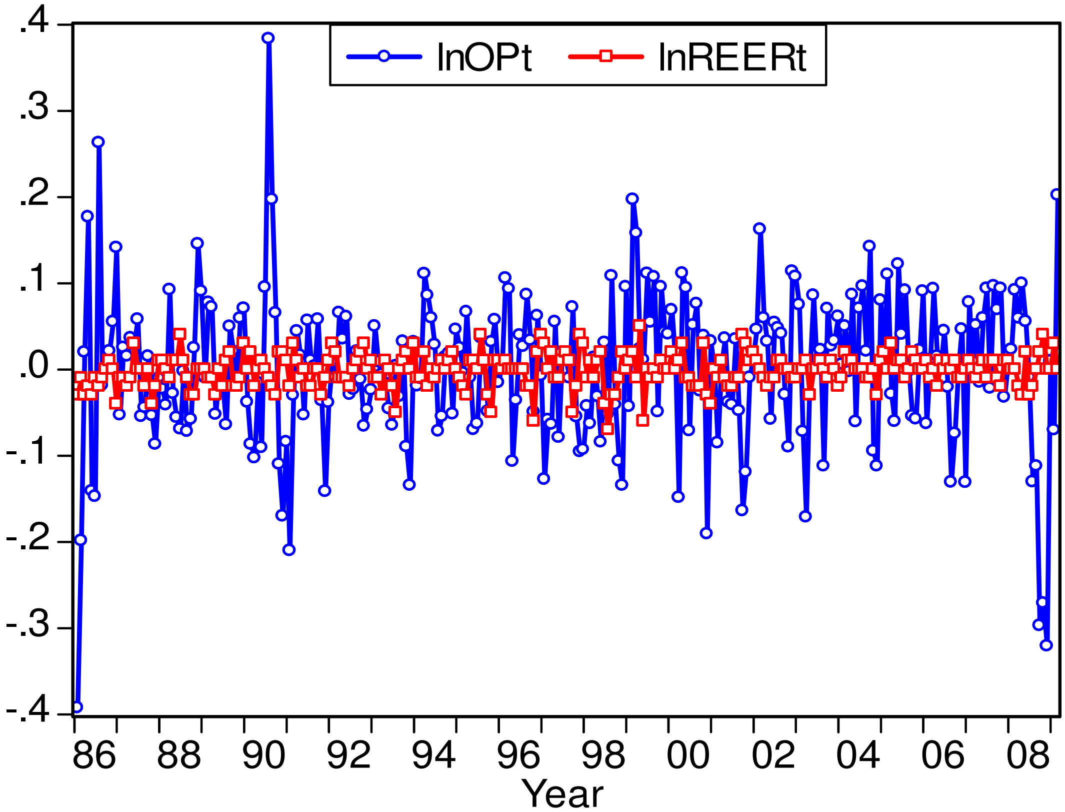

Figure-2: Continuous Wavelet Power Spectrum of $\ln REER_t$ and $\ln OP_t$

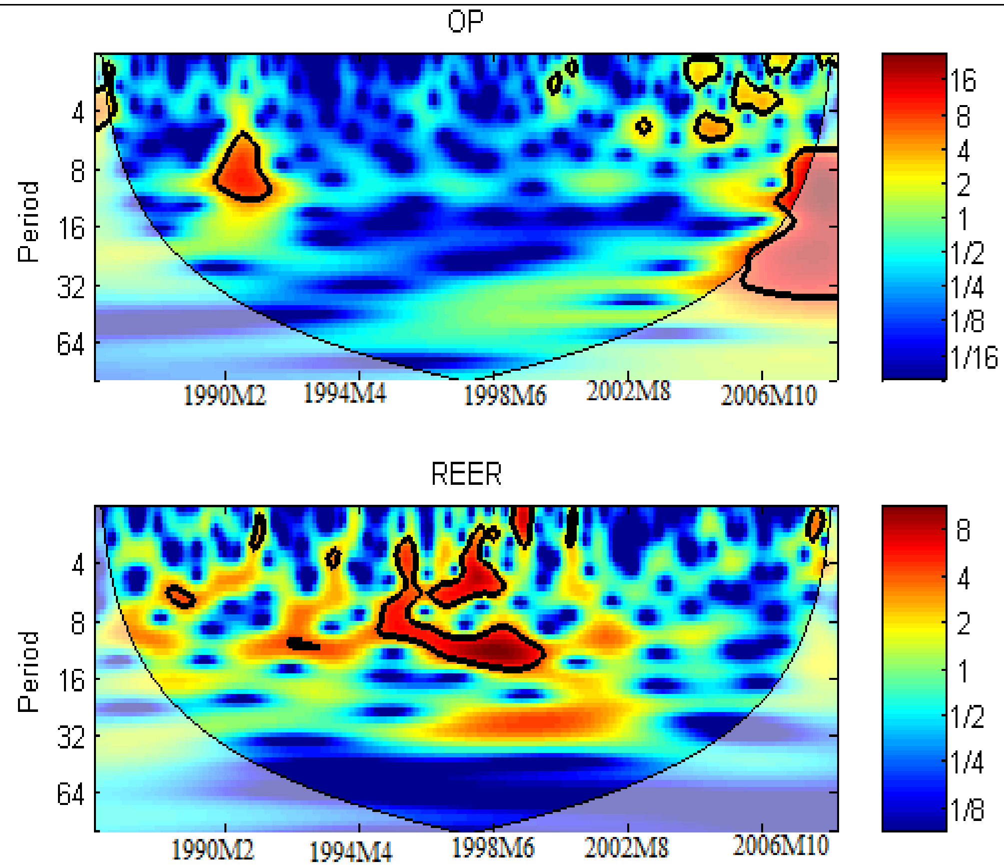

Figure-2: The continuous wavelet power spectrum of both *OP* (oil prices in the top) and *REER* (real effective exchange rate in the bottom) series are shown here. The thick black contour designates the 5% significance level against red noise and the cone of influence (COI) where edge effects might distort the picture is shown as a lighter shade. The color code for power ranges from blue (low power) to red (high power). Y-axis measures frequencies or scale and X-axis represent the time period studied.

Figure-3: Cross Wavelet Transform of $\ln REER_t$ and $\ln OP_t$

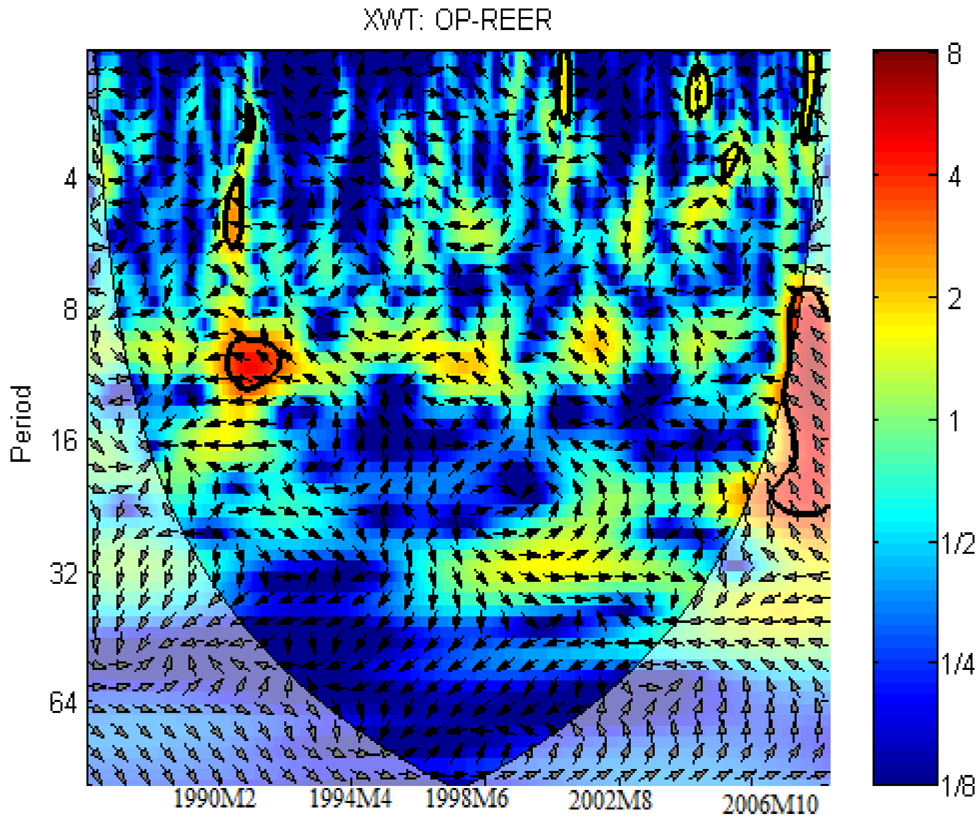

Figure-3: Cross wavelet transform of the OP and $REER$ time series. The thick black contour designates the 5% significance level against red noise which estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is shown with a lighter shade black line. The color code for power ranges from blue (low power) to red (high power). The phase difference between the two series is indicated by arrows. Arrows pointing to the right mean that the variables are in phase. To the right and up, with $REER$ is lagging. To the right and down, with $REER$ is leading. Arrows pointing to the left mean that the variables are out of phase. To the left and up, with $REER$ is leading. To the left and down, with

REER is lagging. In phase indicate that variables will be having cyclical effect on each other and out of phase or anti-phase shows that variable will be having ant-cyclical effect on each other.

Figure-4: Cross-wavelet Coherency of $\ln REER_t$ and $\ln OP_t$

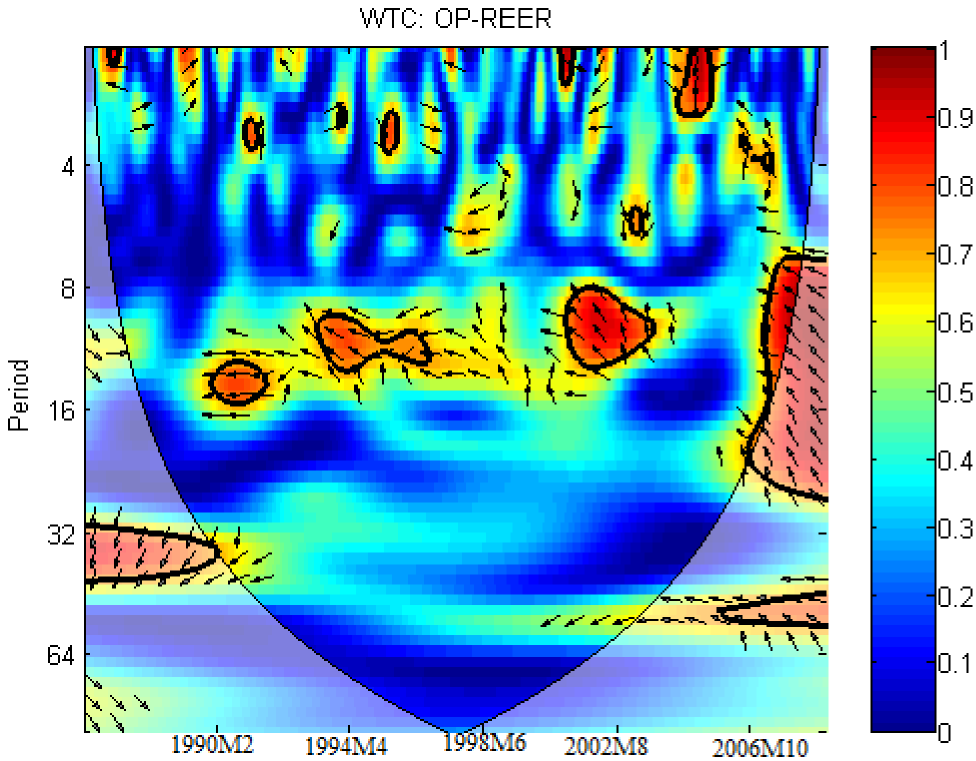

Figure-4: Cross-wavelet coherency or squared wavelet coherence. The thick black contour designates the 5% significance level against red noise which is estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is also shown with a light black line. The color code for coherency ranges from blue (low coherency-close to zero) to red (high coherency-close to one). The phase difference between the two series is indicated by arrows. Arrows pointing to the right mean that the variables are in phase. To the right and up, with *REER* is lagging. To the right and down, with *REER* is leading. Arrows pointing to the left mean that the variables are out of phase. To the left and up, with *REER* is leading.

To the left and down, with $REER$ is lagging. In phase indicate that variables will be having cyclical effect on each other and out of phase or anti-phase shows that variable will be having ant-cyclical effect on each other.

Table-1: Descriptive Statistics of Level and Returns Series

Variables	$\ln REER_t$	$\ln OP_t$	$\Delta \ln REER_t$	$\Delta \ln OP_t$
Mean	4.7407	-1.1218	-0.0019	0.0002
Median	4.7500	-1.2071	0.0000	0.0078
Maximum	5.1800	0.1784	0.0500	0.3832
Minimum	4.5300	-2.0004	-0.0700	-0.3924
Std. Dev.	0.1338	0.4076	0.0187	0.0878
Skewness	0.6621	0.9770	-0.3023	-0.3849
Kurtosis	3.0349	3.8340	3.8172	6.0161
$\ln REER_t$	1.0000	….	….	….
$\ln OP_t$	-0.4045	1.0000	….	….
$\Delta \ln REER_t$	….	….	1.0000	….
$\Delta \ln OP_t$	….	….	-0.1027	1.0000

Table-2: Unit Root Analysis

Variable	ADF Unit Root Test		PP Unit Root Test	
	T-statistic	Prob. Values	T-statistic	Prob. values
$\ln REER_t$	-2.8608 (7)	0.1771	-2.9321 (15)	0.2191
$\ln OP_t$	-2.7259 (5)	0.2270	-2.9636 (6)	0.1445
$\Delta \ln REER_t$	-8.1984 (5)*	0.0000	-13.9484 (15)*	0.0000
$\Delta \ln OP_t$	8.2266 (5)*	0.0000	12.7440 (6)*	0.0000
Variable	**ZA at Level**		**ZA at 1st Difference**	
	T-statistic	Time Break	T-statistic	Time Break
$\ln REER_t$	-3.147 (1)	$2002M_1$	-10.844 (2)*	$1997M_8$
$\ln OP_t$	-4.942 (1)	$1998M_{12}$	-8.432 (3)*	$2000M_2$

Note: * represent significant at 1 per cent level of significance. Lag order is shown in parenthesis.

Table-3: The ARDL Bounds Testing Cointegration Analysis

Estimated Model	$F_{REER}(\ln REER_t / \ln OP_t)$	$F_{OP}(\ln OP_t / \ln REER_t)$
Optimal Lag Length	(5, 3)	(5, 5)
Structural Break	$2002M_1$	$1998M_{12}$
F-statistics	2.518	7.269*
Critical values#	Lower Critical Bound	Upper Critical Bound
1 per cent level	3.41	4.68
5 per cent level	2.62	3.79
10 per cent level	2.26	3.35
R^2	0.1114	0.2062
Adjusted- R^2	0..0702	0.1597
F-statistics	2.7065*	4.4354*
Durbin-Watson	2.0074	1.9040

Note: * denotes the significance at 1% level respectively. The optimal lag structure is determined by AIC. Critical values bounds are from Narayan, [77].