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A Mean Probability Event for a Set of Events

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In this paper, we present an eventological model of a mean probability event for a set of events. This model is analogous to the notion of a mean measure set [3].

Keywords: eventology, probability, universal probability space, universal elementary outcome, universal event, set of universal events, mean measure set, mean probability event, mean probability terrace partition.

Introduction

The concept of a mean measure set was introduced by Oleg Vorobyev in 1973. It was presented (the first time in 1975, the last — in 1984) in the following publications: [1–3], [4, p. 644]. A mean measure set is a mean set characteristic of a random set, whose values are subsets of a measurable space with a measure; this characteristic is to a random set as an expected value or a mean is to a random element with values from linear space (for a random value: a vector, a matrix, a function, etc.).

There is a concept of a mean set of events in eventology [5]. The mean set of events is the result of literal application of definition of a mean measure set to a random set of events. In this work, definition of a mean measure set was used as an example for definition of a new concept of a mean probability event. What does this analogy tell us about a mean probability event?

As a measurable space with a measure corresponds to the universal probability space, so do its subsets — to events, a measure — to a probability measure, a set of events acts as an analogy of a random set. A set of events is given the role, which according to analogy with a random set can be called — a "random" event.

However, the direct definition of a "random" event according to the usual scheme of definition of a random element demands causeless introduction of an additional probability measure, which is responsible for the nature of "randomness" of this event.

Therefore, instead of a "random" event as an analogy to a random set, we propose a set of homogeneous events, in which all events belong to one type. It is possible to consider such set of

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similar events as a source of the additional "homogeneous" measure, which attributes the similar meaning to each event from the set.

Assuming such uniformity of events, what matters the most now is not that a certain subset of events from a set comes into being, but that a portion of events from this set comes into being. The given assumption is natural in a number of applications, for example, in those fields that consider eventological models of a total subject and a total object [6].

Hereafter we will use the following notations:

- $(\Omega, \mathcal{A}, \mathbf{P})$ — the universal probability space,
- Ω — the space of universal elementary outcomes $\omega \in \Omega$,
- $\mathcal{A}$ — algebra of universal events[‡] $\mu \subseteq \Omega$,
- $\mathbf{P}$ — probability measure at A,
- $\mathfrak{M} \subset \mathcal{A}$ — finite set of universal events $\mu \in \mathfrak{M}$.

Mean Measure Set

Traditionally in eventology each *set of events* $\mathfrak{M} \subset \mathcal{A}$ connects with the equivalent concept of *a random set of events* which is defined on universal probability space $(\Omega, \mathcal{A}, \mathbf{P})$ as a random element

$$K_{\mathfrak{M}} : (\Omega, \mathcal{A}, \mathbf{P}) \to \left(2^{\mathfrak{M}}, 2^{2^{\mathfrak{M}}}\right)$$

with values

$$K_{\mathfrak{M}}(\omega) = \{\mu \in \mathfrak{M} : \omega \in \mu\} \subseteq \mathfrak{M}$$

— subsets of events from $\mathfrak{M}$, occuring upon occurence of a universal elementary outcome $\omega \in \Omega$.

Both concepts, the set of events $\mathfrak{M}$ and the random set of events $K_{\mathfrak{M}}$, are equivalent. They are defined by the same eventological distribution (E-distribution) — a set of

$$\{p(Y /\!\!/ \mathfrak{M}), Y \subseteq \mathfrak{M}\}$$

probabilities

$$p(Y /\!\!/ \mathfrak{M}) = \mathbf{P}(\mathsf{ter}(Y /\!\!/ \mathfrak{M}))$$

of terrace events

$$\mathsf{ter}(Y /\!\!/ \mathfrak{M}) = \bigcap_{\mu \in Y} \mu \bigcap_{\mu \in Y^c} \mu^c,$$

which create a partitioning Ω:

$$\Omega = \sum_{Y \subseteq \mathfrak{M}} \mathsf{ter}(Y /\!\!/ \mathfrak{M}),$$

generated by $\mathfrak{M}$.

For the random set of events $K_{\mathfrak{M}}$, in literal case as well as in general case of a random set of any elements [3,4], the mean measure set of events is defined as the set of events $\mathcal{E}K_{\mathfrak{M}} \subseteq \mathfrak{M}$ which satisfies the inclusion relation:

$$\{\mu : \mathbf{P}(\mu) > h\} \subseteq \mathcal{E}K_{\mathfrak{M}} \subseteq \{\mu : \mathbf{P}(\mu) \geqslant h\},$$

[‡]Further events are always considered as universal events; for brevity, the term "event" will be used instead of a "universal event".

in which the level $h \in [0,1]$ is such that approximate equation

$$|\mathcal{E}K_{\mathfrak{M}}| \overset{\min}{\approx} \mathbf{E}|K_{\mathfrak{M}}|. \tag{1}$$

is done with the least error, in which $\mathcal{E}|K_{\mathfrak{M}}| = \sum_{\mu\in\mathfrak{M}} \mathbf{P}(\mu)$ according to Robbins' theorem [7].

Lemma 1 (extreme properties of a mean measure set of events). *A mean measure set of events $\mathcal{E}K_{\mathfrak{M}}$ minimizes a mean distance (mean capacity of a symmetric difference)*

$$\mathbf{E}|K_{\mathfrak{M}} \Delta\, \mathcal{E}K_{\mathfrak{M}}| = \min_{|Y| \overset{\min}{\approx} \mathbf{E}|K_{\mathfrak{M}}|} \mathbf{E}|K_{\mathfrak{M}} \Delta\, Y|$$

Mean Probability Event

Next, the new concept of a mean probability event for the given set of events $\mathfrak{M} \subset \mathcal{A}$, which is designated as

$$\widehat{\mu}_{\mathfrak{M}} \subseteq \Omega,$$

occurs with the probability equal to the mean probability of events $\mu \in \mathfrak{M}$

$$\mathbf{P}\left(\widehat{\mu}_{\mathfrak{M}}\right) = \frac{1}{|\mathfrak{M}|} \sum_{\mu\in\mathfrak{M}} \mathbf{P}(\mu),$$

where $\sum\limits_{\mu\in\mathfrak{M}} \mathbf{P}(\mu) = \sum\limits_{Y\subseteq\mathfrak{M}} |Y| p(Y)$ according to Robbins's theorem [7].

As much as the mean measure set [3] plays a role of the mean set characteristic of random subsets of a set, the mean probability event plays a role of the mean set characteristic of events from $\mathfrak{M}$ as subsets of Ω. Let's give a definition of a mean probability event.

Definition (a mean probability event). *The mean probability event* of the set of events $\mathfrak{M} \subset \mathcal{A}$ is a universal event $\mu \in \mathfrak{M}$, which satisfies inclusions

$$\sum_{|Y|>m} \mathsf{ter}(Y /\!\!/ \mathfrak{M}) \subseteq \widehat{\mu}_{\mathfrak{M}} \subseteq \sum_{|Y|\geqslant m} \mathsf{ter}(Y /\!\!/ \mathfrak{M}), \tag{2}$$

and comes with probability

$$\mathbf{P}(\widehat{\mu}_{\mathfrak{M}}) = \frac{1}{|\mathfrak{M}|} \sum_{\mu\in\mathfrak{M}} \mathbf{P}(\mu) \tag{3}$$

every time when among events from $\mathfrak{M}$ occur not less than m of events, where $m \in \{0, 1, \ldots, |\mathfrak{M}|\}$ satisfies inequalities

$$\sum_{|Y|>m} p(Y /\!\!/ \mathfrak{M}) < \mathbf{P}(\widehat{\mu}_{\mathfrak{M}}) \leqslant \sum_{|Y|\geqslant m} p(Y /\!\!/ \mathfrak{M}). \tag{4}$$

Here, if in the right inequality equality is reached:

$$\mathbf{P}(\widehat{\mu}_{\mathfrak{M}}) = \sum_{|Y|\geqslant m} p(Y /\!\!/ \mathfrak{M}), \tag{4$'$}$$

then the mean probability event for $\mathfrak{M}$ is unique:

$$\widehat{\mu}_{\mathfrak{M}} = \sum_{|Y|\geqslant m} \mathsf{ter}(Y /\!\!/ \mathfrak{M}). \tag{2$'$}$$

Definition (probabilistic distance of an event to a set). *The probabilistic distance of an event $\widehat{\mu}_{\mathfrak{M}} \in \mathcal{A}$ to a set $\mathfrak{M} \subset \mathcal{A}$* is defined by a formula:

$$\rho(\widehat{\mu}_{\mathfrak{M}}, \mathfrak{M}) = \sum_{\mu \in \mathfrak{M}} \mathbf{P}(\mu \Delta \, \widehat{\mu}_{\mathfrak{M}}) = \sum_{\mu \in \mathfrak{M}} \mathbf{P}(\widehat{\mu}_{\mathfrak{M}} \cap \mu^c) + \sum_{\mu \in \mathfrak{M}} \mathbf{P}(\widehat{\mu}^c_{\mathfrak{M}} \cap \mu). \tag{5}$$

Theorem 1 (an extremal property of the mean probability event). *A mean probability event $\widehat{\mu}_{\mathfrak{M}}$ for a set of events $\mathfrak{M}$ is minimized probabilistic distance to $\mathfrak{M}$:*

$$\rho(\widehat{\mu}_{\mathfrak{M}}, \mathfrak{M}) = \min_{\substack{\alpha \in \mathcal{A} \\ \mathbf{P}(\alpha) = \widehat{\mathbf{P}}_{\mathfrak{M}}}} \rho(\alpha, \mathfrak{M}) \tag{6}$$

among such events from algebra $\mathcal{A}$, which occur with the probability equal to the mean probability

$$\widehat{\mathbf{P}}_{\mathfrak{M}} = \frac{1}{|\mathfrak{M}|} \sum_{\mu \in \mathfrak{M}} \mathbf{P}(\mu)$$

of events from $\mathfrak{M}$.

The proof of Theorem is found in the paper [8].

On a Commutativity of Set-Theoretical Operations by Minkowski and Operations of a Capture of a Mean Probability Event

For designation of *a mean probability event* we will use a bulky designation

$$\widehat{\mu}_{\mathfrak{M}} = \mathcal{E}(\mu/\mathfrak{M}) \subset \Omega$$

instead of a quite convenient abbreviation $\widehat{\mu}_{\mathfrak{M}}$. The new designation has a more traditional form and is more suitable for use in this paragraph, where the standard designations of mean probability events will also be used for sets of events, acting as results of set-theoretical operations by Minkowski performed over several sets of events. Equivalent designations of a mean probability event for a set $\mathfrak{M}$ as an event

$$\mathcal{E}(\cdot/\mathfrak{M}) = \widehat{(\cdot)}_{\mathfrak{M}} \subset \Omega$$

may also be deemed more suitable for use in some contexts, rather than $\mathcal{E}(\mu/\mathfrak{M}) = \widehat{\mu}_{\mathfrak{M}} \subset \Omega$.

Set-theoretical operations by Minkowski (M-operations) performed over sets of events are such operations over sets of events, the result of which is defined as the set of events containing corresponding results of set-theoretical operations over events of these sets.

For example, the set

$$\mathfrak{M}^{(c)} = \{\mu^c : \mu \in \mathfrak{M}\} \subset \mathcal{A}$$

is called the complement of the set of events $\mathfrak{M}$ by Minkowski, or M-complement;

$$\mathfrak{M}(\cup)\mathfrak{L} = \{\mu \cup \lambda : \mu \in \mathfrak{M}, \lambda \in \mathfrak{L}\} \subset \mathcal{A}$$

is called the M-union of sets of events $\mathfrak{M}$ and $\mathfrak{L}$;

$$\mathfrak{M}(\cap)\mathfrak{L} = \{\mu \cap \lambda : \mu \in \mathfrak{M}, \lambda \in \mathfrak{L}\} \subset \mathcal{A}$$

is called the M-intersection of sets of events $\mathfrak{M}$ and $\mathfrak{L}$;

$$\mathfrak{M}(\Delta)\mathfrak{L} = \{\mu\Delta\lambda : \mu \in \mathfrak{M}, \lambda \in \mathfrak{L}\} \subset \mathcal{A}$$

is called the M-symmetric difference of sets of events $\mathfrak{M}$ and $\mathfrak{L}$, etc.

We are interested in the connection between mean probability events

$$\mathcal{E}(\mu/\mathfrak{M}), \quad \mathcal{E}(\lambda/\mathfrak{L})$$

and mean probability events

$$\mathcal{E}(\mu^c/\mathfrak{M}^{(c)}), \quad \mathcal{E}(\mu \cup \lambda \,/\, \mathfrak{M}(\cup)\mathfrak{L}), \quad \mathcal{E}(\mu \cap \lambda \,/\, \mathfrak{M}(\cap)\mathfrak{L}), \quad \mathcal{E}(\mu\Delta\lambda \,/\, \mathfrak{M}(\Delta)\mathfrak{L})$$

for results of M-operations performed over them, listed above, and also other M-operations.

Lemma 2 (a mean probability for an M-complement). *A mean probability event for a set of complements $\mathfrak{M}^{(c)}$ events from $\mathfrak{M}$ coincides with the complement of the mean probability event for the set of events $\mathfrak{M}$:*[§]

$$\mathcal{E}\left(\mu^c/\mathfrak{M}^{(c)}\right) = \Big(\mathcal{E}(\mu/\mathfrak{M})\Big)^c = \Omega - \mathcal{E}(\mu/\mathfrak{M}).$$

Lemma 3 (a mean probability event for a binary M-operation). *A mean probability event for a binary M-operation $\mathfrak{M}(*)\mathfrak{L}$ performed over sets of events $\mathfrak{M}$ and $\mathfrak{L}$ coincides with the binary set-theoretical operation over mean probability events for these two sets of events:*

$$\mathcal{E}\left(\mu(*)\lambda \,/\, \mathfrak{M}(*)\mathfrak{L}\right) = \mathcal{E}(\mu/\mathfrak{M}) * \mathcal{E}(\lambda/\mathfrak{L}).$$

Before proceeding to the next statement, we shall consider the following set

$$\ddot{\mathfrak{M}} = \{\ddot{\mu} : \ddot{\mu} = \{\mu_1, \ldots, \mu_n\}, \mu \in \mathfrak{M}\},$$

consisting of n-sets of events $\ddot{\mu}$, when $\mu \in \mathfrak{M}$. Also we shall define the terrace M-operation over a set $\ddot{\mathfrak{M}}$ as follows for $Y \subseteq \mathfrak{M}$:

$$\mathsf{ter}(\{\ddot{\mu}, \mu \in Y\} /\!\!/ \ddot{\mathfrak{M}}) = \Big(\bigcap_{\mu \in Y}\Big)\, \ddot{\mu} \,\Big(\bigcap_{\mu \in Y^c}\Big)\, \ddot{\mu}^{(c)} =$$

$$= \Big\{\mathsf{ter}(\{\mu_i, \mu \in Y\} /\!\!/ \{\mu_i, \mu \in \mathfrak{M}\}), i = 1, \ldots, n\Big\} = \left\{\bigcap_{\mu \in Y} \mu_i \bigcap_{\mu \in Y^c} \mu_i^c, i = 1, \ldots, n\right\}.$$

Lemma 4 (a mean probability event for a terrace M-operation). *A mean probability event for a terrace M-operation (3) over a set $\ddot{\mathfrak{M}}$ coincides with the corresponding terrace operation performed over a set of mean probability events* :

$$\mathcal{E}\left(\mathsf{ter}(\{\ddot{\mu}, \mu \in Y\} /\!\!/ \ddot{\mathfrak{M}}) \,/\, \ddot{\mathfrak{M}}\right) = \bigcap_{\mu \in Y} \mathcal{E}(\cdot/\ddot{\mu}) \bigcap_{\mu \in Y^c} \Big(\mathcal{E}(\cdot/\ddot{\mu})\Big)^c =$$

$$= \mathsf{ter}\Big(\{\mathcal{E}(\cdot/\ddot{\mu}), \mu \in Y\} /\!\!/ \{\mathcal{E}(\cdot/\ddot{\mu}), \mu \in \mathfrak{M}\}\Big).$$

[§]For comparison — the same relation recorded using the abbreviated designation $\widehat{\mu^c}_{\mathfrak{M}^{(c)}} = \widehat{\mu}^c_{\mathfrak{M}} = \Omega - \widehat{\mu}_{\mathfrak{M}}$.

Mean Probability Terrace Partition

The concept of a mean probability event $\widehat{\mu}_{\mathfrak{M}}$ for the set of events $\mathfrak{M}$, which can be considered as partitioning of the space of universal elementary outcomes into two terrace events

$$\Omega = \widehat{\mu}_{\mathfrak{M}} + (\widehat{\mu}_{\mathfrak{M}})^c,$$

each of which is a mean probability event¶, can be generalized to the concept of a mean probability terrace partition $\mu^*_{\mathfrak{M}^*}$ for the set $\mathfrak{M}^* = \{\mu^* : \mu \in \mathfrak{M}\}$ of homogeneous terrace partitions $\mu^* \in \mathfrak{M}^*$, each fragment of which is a mean probability event.

Let's examine this statement in more detail. Let $\mathfrak{M}$ be a set of events $\mu \in \mathfrak{M}$. In each of these events the subsets forming an n-set of events are allocated

$$\ddot{\mu} = \{\mu_i \subseteq \mu : i = 1, \dots, n\} \subset \mathcal{A},$$

which generates a terrace partition μ for $2n$ terrace events:

$$\mu = \sum_{Y \subseteq \ddot{\mu}} \mu(Y /\!\!/ \ddot{\mu}),$$

they are abbreviated as

$$\mu(Y /\!\!/ \ddot{\mu}) = \mu \cap \mathsf{ter}(Y /\!\!/ \ddot{\mu}).$$

They are fragments of a 2n-terrace partition of an event $\mu \in \mathfrak{M}$. Let's designate the 2n-terrace partition and a set of all such 2n-terrace partitions

$$\mu^* = \{\mu(Y /\!\!/ \ddot{\mu}), Y \subseteq \ddot{\mu}\},$$

and a set of all such 2^n-terrace partitions

$$\mathfrak{M}^* = \{\mu^* : \mu \in \mathfrak{M}\}.$$

Definition (a mean probability terrace partition). A mean probability 2n-terrace partition of an event‖

$$\mathsf{ter}(\mathfrak{M} /\!\!/ \mathfrak{M}) = \bigcap_{\mu \in \mathfrak{M}} \mu,$$

which defines a totality of $2n$-terrace partitions μ^* of all events $\mu \in \mathfrak{M}$ in the same mean sense, as the mean probability event $\widehat{\mu}_{\mathfrak{M}}$ defines the totality of events $\mathfrak{M}$, is called the set of mean probability terrace

$$\mu^*_{\mathfrak{M}^*} = \{\mu(Y /\!\!/ \ddot{\mu})_{\mathfrak{M}(Y)}, Y \subseteq \ddot{\mu}\},$$

forming the partition of event

$$\mathsf{ter}(\mathfrak{M} /\!\!/ \mathfrak{M}) = \sum_{Y \subseteq \ddot{\mu}} \mu(Y /\!\!/ \ddot{\mu})_{\mathfrak{M}(Y)},$$

in which each fragment of the partition

$$\mu(Y /\!\!/ \ddot{\mu})_{\mathfrak{M}(Y)}$$

¶As: $(\widehat{\mu}_{\mathfrak{M}})^c = \widehat{\mu}_{\mathfrak{M}(c)}$.

‖In particular when $\mu = \Omega$ for all $\mu \in \mathfrak{M}$, this event coincides with space of universal elementary outcomes: $\mathsf{ter}(\mathfrak{M} /\!\!/ \mathfrak{M}) = \Omega$.

is a mean probability terrace event for a set of terrace events

$$\mathfrak{M}(Y) = \{\mu \cap \mathsf{ter}(Y /\!\!/ \ddot{\mu}), \mu \in \mathfrak{M}\}.$$

In order to formulate extreme properties of a mean probability terrace partition it is necessary to define probabilistic distance between terrace partitions, as the sum of probabilistic distances between their corresponding fragments (terrace events), and probabilistic distance from terrace partition to a set of terrace partitions, as the sum of probabilistic distances before each terrace partition from this set.

Definition (a probabilistic distance between terrace partitions). Let $\mu, \lambda \in \mathfrak{M}$ be two events, which are characterized by terrace partitions μ^* and λ^*. *The probabilistic distance between terrace partitions* μ^* and λ^* is defined as

$$\rho(\mu^*, \lambda^*) = \sum_Y \mathbf{P}\Big(\mu(Y_\mu /\!\!/ \ddot{\mu}) \Delta\, \lambda(Y_\lambda /\!\!/ \ddot{\lambda})\Big)$$

which is the sum of probabilistic distances between the corresponding fragments of these terrace partitions, where the correspondence is one-to-one and, in particular, for doublets $\ddot{\mu} = \{\mu_1, \mu_2\}$ and $\ddot{\lambda} = \{\lambda_1, \lambda_2\}$ can be recorded as:

$$\begin{aligned} \emptyset_\mu &\leftrightarrow \emptyset_\lambda, \\ \{\mu_1\}_\mu &\leftrightarrow \{\lambda_1\}_\lambda, \\ \{\mu_2\}_\mu &\leftrightarrow \{\lambda_2\}_\lambda, \\ \{\mu_1, \mu_2\}_\mu &\leftrightarrow \{\lambda_1, \lambda_2\}_\lambda, \end{aligned}$$

which makes the structure of such correspondence clear in general situations.

Definition (a probabilistic distance from terrace partitions to a set of terrace partitions). Let $\mathfrak{M}^*$ be a set of homogeneous terrace partitions $\mu^* \in \mathfrak{M}^*$, and $\alpha^* = \{Y \subseteq \ddot{\alpha}\}$ is terrace partition of the same kind. *The probabilistic distance of terrace partition* α^* to a set of terrace partitions $\mathfrak{M}^*$ is defined as

$$\rho(\alpha^*, \mathfrak{M}^*) = \sum_{\mu \in \mathfrak{M}} \rho(\alpha^*, \mu^*)$$

— the sum of probabilistic distances from α^* before each terrace partition $\mu^* \in \mathfrak{M}^*$.

Besides, it is required to introduce designation

$$\mathcal{P}(\alpha^*) = \{\mathbf{P}(\alpha(Y)) : Y \subseteq \ddot{\alpha}\}$$

for eventological distribution of a terrace partition

$$\alpha^* = \{\alpha(Y) : Y \subseteq \ddot{\alpha}\},$$

connected with an event $\alpha \in \mathcal{A}$ and its $2n$-terrace partition α^*, the generated n-set of events $\ddot{\alpha}$. And also — a designation for a mean eventological distribution of terrace partition from a set $\mathfrak{M}^*$:

$$\widehat{\mathcal{P}}_{\mathfrak{M}^*} = \frac{1}{|\mathfrak{M}|} \sum_{\mu \in \mathfrak{M}} \mathcal{P}(\mu^*).$$

Let's also introduce designation $\mathcal{A}^*$ for a set of all the $2n$-partitions generated by n-sets $\ddot{\alpha} = \{\alpha_i \in \mathcal{A} : i = 1, \ldots, n\}$ of events from algebra $\mathcal{A}$.

Theorem 2 (an extremal property of the mean probability terrace partition). *A mean probability terrace partition $\mu^*_{\mathfrak{M}^*}$ for a set of homogeneous terrace partitions $\mathfrak{M}^*$ minimizes probabilistic distance to $\mathfrak{M}^*$:*

$$\rho(\mu^*_{\mathfrak{M}^*}, \mathfrak{M}^*) = \min_{\substack{\alpha^* \in \mathcal{A}^* \\ \mathcal{P}(\alpha^*) = \widehat{\mathcal{P}}_{\mathfrak{M}^*}}} \rho(\alpha^*, \mathfrak{M}^*)$$

among such terrace partitions which have the mean eventological distribution

$$\widehat{\mathcal{P}}_{\mathfrak{M}^*} = \frac{1}{|\mathfrak{M}|} \sum_{\mu \in \mathfrak{M}} \mathcal{P}(\mu^*)$$

among all eventological distributions of terrace partitions from $\mathfrak{M}^*$.

Conclusion

The concept of a mean probability event for the given set of events has been introduced recently out of the growing need to examine in detail the accurate definition of a total system of events and its differences from a totality of systems of events [6]. A new concept of the total system of events was required in eventological system analysis [9,10] for the eventological description of a set of subjects, which in applications is often regarded as a total subject possessing "personal" eventological characteristics, similar to personal eventological characteristics of isolated subjects.

A mean probability event, as well as a mean probability terrace partition, can be used in any eventological research of a set of homogeneous events or sets of homogeneous systems of events, in case a mean characteristic of an event or system of events, the characteristic of the same kind, as to which isolated events and systems of events of the given totality belong to, is necessary.

For example, in the eventological theory of fuzzy events [5], which defines a fuzzy event as a set of Kolmogorov's events, the mean probability event acts as a mean event characteristic of a fuzzy opinion of multiple subjects (sources of the fuzziness). As the mean probability event has all the properties of Kolmogorov's event and occurs with the probability, it can serve as an eventologically accurate justification of defuzzification.

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Средневероятное событие для множества событий

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Предлагается эвентологическая модель средневероятного события для множества событий, имеющая аналогии с понятием среднемерного множества [3].

Ключевые слова: эвентология, вероятность, всеобщее вероятностное пространство, всеобщий элементарный исход, всеобщее событие, множество всеобщих событий, среднемерное множество, средневероятное событие, средневероятное террасное разбиение.