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Harashima, Taiji

Kanazawa Seiryō University

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# Escaping a Liquidity Trap: Keynes' Prescription Is Right But His Reasoning Is Wrong

Taiji Harashima\*

## Abstract

Keynes' original intention in introducing the concept of a liquidity trap was to explain the reason why persistent large amounts of unutilized resources were generated during the Great Depression. This paper shows that this type of phenomenon cannot be explained in the framework of a traditional competitive market equilibrium. Instead, it can be understood in terms of a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path because a Nash equilibrium can conceptually coexist with Pareto inefficiency. Such a Nash equilibrium will be selected when an upwards time preference shock occurs. At this Nash equilibrium, monetary policies are useless but fiscal policies are very effective as Keynes argued, but for different reasons.

JEL Classification code: E32, E52, E62

Keywords: Liquidity trap; Monetary policy; Fiscal policy; Pareto inefficiency; Time preference

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\*Correspondence: Taiji HARASHIMA, Kanazawa Seiryō University, 10-1 Goshomachi-Ushi, Kanazawa-shi, Ishikawa, 920-8620, Japan.

Email: [harashim@seiryō-u.ac.jp](mailto:harashim@seiryō-u.ac.jp) or [t-harashima@mve.biglobe.ne.jp](mailto:t-harashima@mve.biglobe.ne.jp).

# 1 INTRODUCTION

The term “liquidity trap” has recently been used differently than Keynes (1936) originally intended. It seems to express an economic situation that has at least one of the following features: (1) nominal interest rates are nearly zero, (2) investments do not respond to very low nominal interest rates, and (3) monetary policies are significantly ineffective. Some authors emphasize feature (1) and others feature (3) (see, e.g., Krugman, 1998; Benhabib et al., 2001a, b, 2002; Eggertsson and Woodford, 2003; Jeanne and Svensson, 2007; Eggertsson and Krugman, 2012). Keynes’ (1936) original intention when introducing the concept of the liquidity trap was to explain the reason why persistent large amounts of unused resources (e.g., persistent high unemployment rates and large amounts of idle capital) were observed during the Great Depression in the 1930s. In this sense, merely stressing feature (1) would not be a correct usage of the term liquidity trap. An essential element is not very low nominal interest rates but the existence of large and persistent amounts of unused resources, a situation that was observed not only during the Great Depression but also in Japan’s lost decades in the 1990s and 2000s as well as the Great Recession beginning in 2008. If a large amount of resources is persistently not utilized, investments will not increase even though nominal interest rates are very low; thus, the economy will not respond to monetary policies. If very low nominal interest rates are the main cause of persistent large amounts of unused resources, feature (1) is important, but understanding why persistent large amounts of unused resources are generated and what counter measures are the most effective in fixing this problem are more important.

Keynes’ and his early followers’ explanation of persistent large amounts of unused resources is now viewed as basically unacceptable because it has no micro-foundation. New-Keynesians’ explanations are based on micro-founded mechanisms of some kinds of price rigidity, but they have not been regarded as sufficiently successful because price rigidity has been criticized for its fragile theoretical (micro-) foundation and its inability to explain the persistent nature of inflation. Mankiw (2001) argued that the so-called New-Keynesian Phillips curve is ultimately a failure and is not consistent with the standard stylized facts about the dynamic effects of monetary policy (see also, e.g., Fuhrer and Moore, 1995; Galí and Gertler, 1999). In multi-equilibria, indeterminate, or sunspot models, a situation that satisfies at least one of the above three features can be generated (e.g., Benhabib and Farmer, 2000). For example, a *zero interest rate equilibrium* can possibly exist (Benhabib et al., 2001a., 2001b, 2002), and some models exhibit the existence of Pareto inferior and superior equilibria. Nevertheless, these multi-equilibria, indeterminate, or sunspot models have the common feature that markets are basically cleared in equilibria. Hence, they cannot demonstrate a mechanism by which persistent large amounts of unused resources are generated. So, if the focus is only on feature (1), these models may be useful, but they are not suitable for analyzing the economic situation Keynes originally intended to explain by using the liquidity trap concept.

This paper examines a mechanism by which persistent large amounts of unused resources can be generated and evaluates appropriate counter measures by taking a fundamentally different approach from New-Keynesian and multi-equilibria, indeterminate, or sunspot models. The mechanism is explained based on the model developed in Harashima (2004a, 2012, 2013b). The essential point of the model is that persistent large amounts of unused resources exist at a special Nash equilibrium that consists of strategies that generate Pareto inefficient payoffs. Moreover, they probably exist only at such an equilibrium if all agents are rational. In the framework of a traditional competitive market equilibrium, it is very difficult to show a rational mechanism that generates persistent large amounts of unused resources, that is, a persistent substantial Pareto inefficiency. One of a few ways to show such a mechanism in this framework is to assume some kinds of rigidity, particularly in prices. This approach was originally explored by Keynes, and since then, numerous studies have been devoted to this line of research. However, as Mankiw (2001) argued, this approach is not regarded as sufficiently successful. Humans are considered to be so clever and rational that they

cannot be cheated persistently; for example, they soon exploit the opportunities that price rigidities provide and price rigidity will thereby soon disappear. Unlike traditional competitive market equilibrium, however, a Nash equilibrium can conceptually coexist with Pareto inefficiency, and such a mechanism can exist without the need for rigidity.

A Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path of consumption to the steady state (hereafter called a “Nash equilibrium of a Pareto inefficient path”) is generated even in a frictionless economy if—and probably only if—the rate of time preference shifts. An essential reason for the generation of this path is that households are intrinsically risk averse and not cooperative. In a strategic environment, this generates the possibility that, if consumption needs to be substantially and discontinuously increased to keep Pareto optimality, a non-cooperative household’s strategy to deviate from the Pareto optimal path gives a higher expected utility than the strategy of choosing the Pareto optimal path.

The Nash equilibrium of a Pareto inefficient path should not be confused with a Pareto inferior Nash equilibrium or a Nash equilibrium that is Pareto inefficient. They are conceptually quite different, although the Nash equilibrium of a Pareto inefficient path discussed in this paper is both a Pareto inferior Nash equilibrium and a Nash equilibrium that is Pareto inefficient. Multiple equilibria resulting from, for example, increasing returns, an externality, or a complementarity in a macro-economic framework are usually Pareto ranked equilibria and include a Pareto inferior equilibrium (e.g., Morris and Shin, 2001). Such a Pareto inferior equilibrium usually indicates lower production and consumption than in a Pareto superior equilibrium, suggesting a recession. However, if consumption is immediately adjusted completely when the economy is switched from a Pareto superior equilibrium to the inferior one, unutilized resources will not be generated as a result of the switch; therefore, merely showing the possibility of multiple Pareto ranked equilibria is not sufficient to explain the generation mechanism of persistent Pareto inefficiency. A mechanism that generates huge and persistent unutilized resources during the transition path to the new equilibrium should also be presented, and the Nash equilibrium of a Pareto inefficient path fully explains this mechanism.

If households are cooperative, they will always proceed on Pareto efficient paths because they will coordinate with each other to perfectly utilize all resources. Conversely, if they do not coordinate with each other, they may strategically not utilize all resources; that is, they may select a Nash equilibrium of a Pareto inefficient path. Such a possibility cannot be denied *a priori*, because a Nash equilibrium can coexist with Pareto inefficiency. In fact, households are intrinsically not cooperative—they act independently of one another. Suppose that an upward shift of the time preference rate occurs. All households will be knocked off the Pareto efficient path on which they have proceeded prior to the shift. At that moment, each household must decide on a direction in which to proceed. Because they are no longer on a Pareto efficient path, households strategically choose a path on the basis of the expected utility calculated considering other households’ choices; that is, each household behaves non-cooperatively in its own interest considering other households’ strategies. This situation can be described by a non-cooperative mixed strategy game, and there is a Nash equilibrium of a Pareto inefficient path in this game.

This paper argues that the situation labeled as a liquidity trap is a Nash equilibrium of a Pareto inefficient path and, based on the nature of the Nash equilibrium of a Pareto inefficient path, Keynes’ prescription for counter measures was right although his explanation of why they work was wrong. Although Keynes’ original arguments have been severely criticized, his prescription has been widely used by policymakers. This gap between theory and practice was still evident during the recent Great Recession. This paper shows that, as Keynes argued, monetary policies are useless, but fiscal policies are effective to counter a liquidity trap.

As a tool to finance fiscal policies, households are indifferent in the choice between tax increases and increased borrowing if the Barro–Ricardo equivalence theorem holds. However, this paper shows that the government may not be indifferent when choosing between the two if it is a Leviathan government.

The paper is organized as follows. Section 2 shows that a Nash equilibrium of a Pareto

inefficient path is rationally generated when the time preference rates of risk-averse and non-cooperative households shift. In Section 3, the effects of monetary policies when an economy is on a Nash equilibrium of a Pareto inefficient path are examined and evaluated. In Section 4, the effects of fiscal policies are examined and evaluated, and I also show that the types of fiscal policies to be selected will depend on the shape of the government's utility function. Finally, I offer concluding remarks in Section 5.

## 2 THE NASH EQUILIBRIUM OF A PARETO INEFFICIENT PATH

### 2.1 *Model with non-cooperative households*<sup>1</sup>

#### 2.1.1 The shock

The model describes the utility maximization of households after an upward time preference shock. This shock was chosen because it is one of the few shocks that result in a Nash equilibrium of a Pareto inefficient path (other possible shocks are discussed in Section 2.5). Another important reason for selecting an upward time preference shock is that it shifts the steady state to lower levels of production and consumption than before the shock, which is consistent with the phenomena actually observed in a recession.

Although the rate of time preference is a deep parameter, it has not been regarded as a source of shocks for economic fluctuations, possibly because the rate of time preference is thought to be constant and not to shift suddenly. There is also a practical reason, however. Models with a permanently constant rate of time preference exhibit excellent tractability (see Samuelson, 1937). However, the rate of time preference has been naturally assumed and actually observed to be time-variable. The concept of a time-varying rate of time preference has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930). More recently, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant rates of time preference by nature and that economic and social factors affect the formation of time preference rates. Their arguments indicate that many incidents can affect and change the rate of time preference throughout a person's life. For example, Parkin (1988) examined business cycles in the United States, explicitly considering the time-variability of the time preference rate, and showed that the rate of time preference was as volatile as technology and leisure preference.

#### 2.1.2 Households

Households are not intrinsically cooperative. Except in a strict communist economy, households do not coordinate themselves to behave as a single entity when consuming goods and services. The model in this paper assumes non-cooperative, identical, and infinitely long living households and that the number of households is sufficiently large. Each of them equally maximizes the expected utility

$$E_0 \int_0^{\infty} \exp(-\theta t) u(c_t) dt \quad , \quad (1)$$

subject to

$$\frac{dk_t}{dt} = f(A, k_t) - \delta k_t - c_t \quad , \quad (2)$$

where  $y_t$ ,  $c_t$ , and  $k_t$  are production, consumption, and capital per capita in period  $t$ , respectively;

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<sup>1</sup> The model in Section 2 is based on the model by Harashima (2012). See also Harashima (2004a, 2013b).

$A$  is technology and constant;  $u$  is the utility function;  $y_t = f(A, k_t)$  is the production function;  $\theta (> 0)$  is the rate of time preference;  $\delta$  is the rate of depreciation; and  $E_0$  is the expectations operator conditioned on the agents' period 0 information set.  $y_t$ ,  $c_t$ , and  $k_t$  are monotonously continuous and differentiable in  $t$ , and  $u$  and  $f$  are monotonously continuous functions of  $c_t$  and  $k_t$ , respectively. All households initially have an identical amount of financial assets equal to  $k_t$ , and all households gain the identical amount of income  $y_t = f(A, k_t)$  in each period. It is assumed that  $\frac{du(c_t)}{dc_t} > 0$  and  $\frac{d^2u(c_t)}{dc_t^2} < 0$ ; thus, households are risk averse. For simplicity, the utility function is specified to be the constant relative risk aversion utility function

$$\begin{aligned} u(c_t) &= \frac{c_t^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ u(c_t) &= \ln(c_t) & \text{if } \gamma = 1 \end{aligned} ,$$

where  $\gamma$  is a constant and  $0 < \gamma < \infty$ . In addition,  $\frac{\partial f(A, k_t)}{\partial k_t} > 0$  and  $\frac{\partial^2 f(k_t)}{\partial k_t^2} < 0$ . Both technology ( $A$ ) and labor supply are assumed to be constant.

The effects of an upward shift in time preference are shown in Figure 1. Suppose first that the economy is at steady state before the shock. After the upward time preference shock, the vertical line  $\frac{dc_t}{dt} = 0$  moves to the left (from the solid vertical line to the dashed vertical line in

Fig. 1). To keep Pareto efficiency, consumption needs to jump immediately from the steady state before the shock (the prior steady state) to point  $Z$ . After the jump, consumption proceeds on the Pareto efficient saddle path after the shock (the posterior Pareto efficient saddle path) from point  $Z$  to the lower steady state after the shock (the posterior steady state). Nevertheless, this discontinuous jump to  $Z$  may be uncomfortable for risk-averse households that wish to smooth consumption and not to experience substantial fluctuations. Households may instead take a shortcut and, for example, proceed on a path on which consumption is reduced continuously from the prior steady state to the posterior steady state (the bold dashed line in Fig. 1), but this shortcut is not Pareto efficient.

Choosing a Pareto inefficient consumption path must be consistent with each household's maximization of its expected utility. To examine the possibility of the rational choice of a Pareto inefficient path, the expected utilities between the two options need be compared. For this comparison, I assume that there are two options for each non-cooperative household with regard to consumption just after an upward shift in time preference. The first is a jump option,  $J$ , in which a household's consumption jumps to  $Z$  and then proceeds on the posterior Pareto efficient saddle path to the posterior steady state. The second is a non-jump option,  $NJ$ , in which a household's consumption does not jump but instead gradually decreases from the prior steady state to the posterior steady state, as shown by the bold dashed line in Figure 1. The household that chooses the  $NJ$  option reaches the posterior steady state in period  $s (\geq 0)$ . The difference in consumption between the two options in each period  $t$  is  $b_t (\geq 0)$ . Thus,  $b_0$  indicates the difference between  $Z$  and the prior steady state.  $b_t$  diminishes continuously and becomes zero in period  $s$ . The  $NJ$  path of consumption ( $c_t$ ) after the shock is monotonously continuous and differentiable in  $t$  and  $\frac{dc_t}{dt} < 0$  if  $0 \leq t < s$ . In addition,

$$\bar{c} < c_t < \hat{c}_t \quad \text{if } 0 \leq t < s$$

$$c_t = \bar{c} \quad \text{if } 0 \leq s \leq t \text{ ,}$$

where  $\hat{c}_t$  is consumption when proceeding on the posterior Pareto efficient saddle path and  $\bar{c}$  is consumption in the posterior steady state. Therefore,

$$\begin{aligned} b_t &= \hat{c}_t - c_t > 0 & \text{if } 0 \leq t < s \\ b_t &= 0 & \text{if } 0 \leq s \leq t \text{ .} \end{aligned}$$

It is also assumed that, when a household chooses a different option from the one the other households choose, the difference in the accumulation of financial assets resulting from the difference in consumption ( $b_t$ ) before period  $s$  between that household and the other households is reflected in consumption after period  $s$ . That is, the difference in the return on financial assets is added to (or subtracted from) the household's consumption in each period after period  $s$ . The exact functional form of the addition (or subtraction) is shown in Section 2.1.4.

### 2.1.3 Firms

Unutilized products ( $b_t$ ) are eliminated quickly in each period by firms because holding  $b_t$  for a long period is a cost to firms. Elimination of  $b_t$  is accomplished by discarding the goods or preemptively suspending production, thereby leaving some capital and labor inputs idle. However, in the next period, unutilized products are generated again because the economy is not proceeding on the Pareto efficient saddle path. Unutilized products are therefore successively generated and eliminated. Faced with these unutilized products, firms dispose of the excess capital used to generate  $b_t$ . Disposing of the excess capital is rational for firms because the excess capital is an unnecessary cost, but this means that parts of the firms are liquidated, which takes time and thus disposing of the excess capital will also take time. If the economy proceeds on the  $NJ$  path (that is, if all households choose the  $NJ$  option), firms dispose of all of the remaining excess capital that generates  $b_t$  and adjust their capital to the posterior steady-state level in period  $s$ , which also corresponds to households reaching the posterior steady state. Thus, if the economy proceeds on the  $NJ$  path, capital  $k_t$  is

$$\begin{aligned} \bar{k} < k_t \leq \hat{k}_t & \quad \text{if } 0 \leq t < s \\ k_t &= \bar{k} & \quad \text{if } 0 \leq s \leq t \text{ ,} \end{aligned}$$

where  $\hat{k}_t$  is capital per capita when proceeding on the posterior Pareto efficient saddle path and  $\bar{k}$  is capital per capita in the posterior steady state.

The real interest rate  $i_t$  is

$$i_t = \frac{\partial f(A, k_t)}{\partial k_t} \text{ .} \quad (3)$$

Because the real interest rate equals the rate of time preference at steady state, if the economy proceeds on the  $NJ$  path,

$$\begin{aligned} \tilde{\theta} \leq i_t < \theta & \quad \text{if } 0 \leq t < s \\ i_t &= \theta & \quad \text{if } 0 \leq s \leq t \text{ ,} \end{aligned} \quad (4)$$

where  $\tilde{\theta}$  is the rate of time preference before the shock and  $\theta$  is the rate of time preference

after the shock.  $i_t$  is monotonously continuous and differentiable in  $t$  if  $0 \leq t < s$ .

### 2.1.4 Expected utility after the shock

The expected utility of a household after the shock depends on its choice of the  $J$  or  $NJ$  path. Let  $Jalone$  indicate that the household chooses option  $J$ , but the other households choose option  $NJ$ ;  $NJalone$  indicate that the household chooses option  $NJ$ , but the other households choose option  $J$ ;  $Jtogether$  indicate that all households choose option  $J$ ; and  $NJtogether$  indicate that all households choose option  $NJ$ . Let  $p$  ( $0 \leq p \leq 1$ ) be the subjective probability of a household that the other households choose the  $J$  option (e.g.,  $p = 0$  indicates that all the other households choose option  $NJ$ ). With  $p$ , the expected utility of a household when it chooses option  $J$  is

$$E_0(J) = pE_0(Jtogether) + (1-p)E_0(Jalone) , \quad (5)$$

and when it chooses option  $NJ$  is

$$E_0(NJ) = pE_0(NJalone) + (1-p)E_0(NJtogether) , \quad (6)$$

where  $E_0(Jalone)$ ,  $E_0(NJalone)$ ,  $E_0(Jtogether)$ , and  $E_0(NJtogether)$  are the expected utilities of the household when choosing  $Jalone$ ,  $NJalone$ ,  $Jtogether$ , and  $NJtogether$ , respectively. Given the properties of  $J$  and  $NJ$  shown in Sections 2.1.2 and 2.1.3,

$$E_0(J) = pE_0 \left[ \int_0^s \exp(-\theta t) u(c_t + b_t) dt + \int_s^\infty \exp(-\theta t) u(\hat{c}_t) dt \right] \\ + (1-p)E_0 \left[ \int_0^s \exp(-\theta t) u(c_t + b_t) dt + \int_s^\infty \exp(-\theta t) u(\bar{c} - \bar{a}) dt \right] , \quad (7)$$

and

$$E_0(NJ) = pE_0 \left[ \int_0^s \exp(-\theta t) u(c_t) dt + \int_s^\infty \exp(-\theta t) u(\hat{c}_t + a_t) dt \right] \\ + (1-p)E_0 \left[ \int_0^s \exp(-\theta t) u(c_t) dt + \int_s^\infty \exp(-\theta t) u(\bar{c}) dt \right] , \quad (8)$$

where

$$\bar{a} = \theta \int_0^s b_r \exp \int_r^s i_q dq dr , \quad (9)$$

and

$$a_t = i_t \int_0^s b_r \exp \int_r^s i_q dq dr , \quad (10)$$

and the shock occurred in period  $t = 0$ . Figure 2 shows the paths of  $Jalone$  and  $NJalone$ . Because there is a sufficiently large number of households and the effect of an individual household on the whole economy is negligible, in the case of  $Jalone$ , the economy almost proceeds on the  $NJ$  path. Similarly, in the case of  $NJalone$ , it almost proceeds on the  $J$  path. If the other households choose the  $NJ$  option ( $Jalone$  or  $NJtogether$ ), consumption after  $s$  is constant as  $\bar{c}$  and capital is adjusted to  $\bar{k}$  by firms in period  $s$ . In addition,  $a_t$  and  $i_t$  are constant after  $s$  such that  $a_t$  equals  $\bar{a}$  and  $i_s$  equals  $\theta$ , because the economy is at the posterior



steady state. Nevertheless, during the transition period before  $s$ , the value of  $i_t$  changes from the value of the prior time preference rate to that of the posterior rate. If the other households choose option  $J$  ( $NJalone$  or  $Jtogether$ ), however, consumption after  $s$  is  $\hat{c}_t$  and capital is not adjusted to  $\bar{k}$  by firms in period  $s$  and remains at  $\hat{k}_t$ .

As mentioned in Section 2.1.2, the difference in the returns on financial assets for the household from the returns for each of the other households is added to (or subtracted from) its consumption in each period after period  $s$ . This is described by  $a_t$  and  $\bar{a}$  in equations (7) and (8), and equations (9) and (10) indicate that the accumulated difference in financial assets resulting from  $b_t$  increases by compound interest between the period  $r$  to  $s$ . That is, if the household takes the  $NJalone$  path, it accumulates more financial assets than each of the other  $J$  households, and instead of immediately consuming these extra accumulated financial assets after period  $s$ , the household consumes the returns on them in every subsequent period. If the household takes the  $Jalone$  path, however, its consumption after  $s$  is  $\bar{c} - \bar{a}$ , as shown in equation (7).  $\bar{a}$  is subtracted because the income of each household,  $y_t = f(A, k_t)$ , including the  $Jalone$  household, decreases equally by  $b_t$ . Each of the other  $NJ$  households decreases consumption by  $b_t$  at the same time, which compensates for the decrease in income; thus, its financial assets (i.e., capital per capita;  $k_t$ ) are kept equal to  $\hat{k}_t$ . The  $Jalone$  household, however, does not decrease its consumption, and its financial assets become smaller than those of each of the other  $NJ$  households, which results in the subtraction of  $\bar{a}$  after period  $s$ .

## 2.2 Pareto inefficient transition path <sup>2</sup>

### 2.2.1 Rational Pareto inefficient path

#### 2.2.1.1 Rational choice of a Pareto inefficient path

Before examining the economy with non-cooperative households, I first show that, if households are cooperative, only option  $J$  is chosen as the path after the shock because it gives a higher expected utility than option  $NJ$ . Because there is no possibility of  $Jalone$  and  $NJalone$  if households are cooperative, then  $E_0(J) = E_0(Jtogether)$  and  $E_0(NJ) = E_0(NJtogether)$ . Therefore,

$$\begin{aligned} & E_0(J) - E_0(NJ) \\ &= E_0 \left[ \int_0^s \exp(-\theta t) u(c_t + b_t) dt + \int_s^\infty \exp(-\theta t) u(\hat{c}_t) dt \right] - E_0 \left[ \int_0^s \exp(-\theta t) u(c_t) dt + \int_s^\infty \exp(-\theta t) u(\bar{c}) dt \right] \\ &= E_0 \left\{ \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t) [u(\hat{c}_t) - u(\bar{c})] dt \right\} > 0 \end{aligned}$$

because  $c_t < c_t + b_t$  and  $\bar{c} < \hat{c}_t$ .

Next, I examine the economy with non-cooperative households. First, the special case with a utility function with a sufficiently small  $\gamma$  is examined.

**Lemma 1:** If  $\gamma (0 < \gamma < \infty)$  is sufficiently small, then  $E_0(Jalone) - E_0(NJtogether) > 0$ .

**Proof:**  $\lim_{\gamma \rightarrow 0} [E_0(Jalone) - E_0(NJtogether)]$

$$\begin{aligned} &= E_0 \int_0^s \exp(-\theta t) \lim_{\gamma \rightarrow 0} [u(c_t + b_t) - u(c_t)] dt + E_0 \int_s^\infty \exp(-\theta t) \lim_{\gamma \rightarrow 0} [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\ &= E_0 \int_0^s \exp(-\theta t) b_t dt - E_0 \int_s^\infty \exp(-\theta t) \bar{a} dt \end{aligned}$$

<sup>2</sup> The idea of a rationally chosen Pareto inefficient path was originally presented by Harashima (2004b).

$$\begin{aligned}
&= E_0 \int_0^s \exp(-\theta t) b_t dt - E_0 \theta \left[ \int_0^s \left( b_r \exp \int_r^s i_q dq \right) dr \right] \int_s^\infty \exp(-\theta t) dt \\
&= E_0 \int_0^s \exp(-\theta t) b_t dt - E_0 \exp(-\theta s) \int_0^s \left( b_r \exp \int_r^s i_q dq \right) dr \\
&= E_0 \exp(-\theta s) \int_0^s b_t \left\{ \exp[\theta(s-t)] - \exp \int_t^s i_q dq \right\} dt > 0,
\end{aligned}$$

because, if  $0 \leq t < s$ , then  $i_t < \theta$  and  $\exp[\theta(s-t)] > \exp \int_t^s i_q dq$ . Hence, because  $\exp[\theta(s-t)] > \exp \int_t^s i_q dq$ ,  $E_0(\text{Jalone}) - E_0(\text{NJtogether}) > 0$  for sufficiently small  $\gamma$ . ■

Second, the opposite special case (i.e., a utility function with a sufficiently large  $\gamma$ ) is examined.

**Lemma 2:** If  $\gamma (0 < \gamma < \infty)$  is sufficiently large and if  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ , then  $E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0$ .

**Proof:** Because  $0 < b_t$ , then

$$\lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} [u(c_t + b_t) - u(c_t)] = \lim_{\gamma \rightarrow \infty} \left[ \left( \frac{c_t + b_t}{\bar{c}} \right)^{1-\gamma} - \left( \frac{c_t}{\bar{c}} \right)^{1-\gamma} \right] = 0$$

for any period  $t (< s)$ . On the other hand, because  $0 < \bar{a}$ , then for any period  $t (< s)$ , if  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ ,

$$\lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} [u(\bar{c} - \bar{a}) - u(\bar{c})] = \lim_{\gamma \rightarrow \infty} \left[ \left( 1 - \frac{\bar{a}}{\bar{c}} \right)^{1-\gamma} - 1 \right] = \infty.$$

Thus,

$$\begin{aligned}
&\lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} [E_0(\text{Jalone}) - E_0(\text{NJtogether})] \\
&= \lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} \int_0^s \exp(-\theta t) \lim_{\gamma \rightarrow \infty} [u(c_t + b_t) - u(c_t)] dt \\
&+ \lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} \int_s^\infty \exp(-\theta t) \lim_{\gamma \rightarrow \infty} [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\
&= 0 + \infty > 0.
\end{aligned}$$

Because  $\frac{1-\gamma}{\bar{c}^{1-\gamma}} < 0$  for any  $\gamma (1 < \gamma < \infty)$ , then if  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ ,  $E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0$  for sufficiently large  $\gamma (< \infty)$ . ■

The condition  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$  indicates that path *NJ* from  $c_0$  to  $\bar{c}$  deviates sufficiently from the posterior Pareto efficient saddle path and reaches the posterior steady state  $\bar{c}$  not taking

much time. Because steady states are irrelevant to the degree of risk aversion ( $\gamma$ ), both  $c_0$  and  $\bar{c}$  are irrelevant to  $\gamma$ .

By Lemmas 1 and 2, it can be proved that  $E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0$  is possible.

**Lemma 3:** If  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ , then there is a  $\gamma^* (0 < \gamma^* < \infty)$  such that if  $\gamma^* < \gamma < \infty$ ,  $E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0$ .

**Proof:** If  $\gamma (> 0)$  is sufficiently small, then  $E_0(\text{Jalone}) - E_0(\text{NJtogether}) > 0$  by Lemma 1, and if  $\gamma (< \infty)$  is sufficiently large and if  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ , then  $E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0$  by Lemma 2. Hence, if  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ , there is a certain  $\gamma^* (0 < \gamma^* < \infty)$  such that, if  $\gamma^* < \gamma < \infty$ , then  $E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0$ . ■

However,  $E_0(\text{Jtogether}) - E_0(\text{NJalone}) > 0$  because both *Jtogether* and *NJalone* indicate that all the other households choose option *J*; thus, the values of  $i_t$  and  $k_t$  are the same as those when all households proceed on the posterior Pareto efficient saddle path. Faced with these  $i_t$  and  $k_t$ , deviating alone from the Pareto efficient path (*NJalone*) gives a lower expected utility than *Jtogether* to the *NJ* household. Both *Jalone* and *NJtogether* indicate that all the other households choose option *NJ* and  $i_t$  and  $k_t$  are not those of the Pareto efficient path. Hence, the sign of  $E_0(\text{Jalone}) - E_0(\text{NJtogether})$  varies depending on the conditions, as Lemma 3 indicates.

By Lemma 3 and the property  $E_0(\text{Jtogether}) - E_0(\text{NJalone}) > 0$ , the possibility of the choice of a Pareto inefficient transition path, that is,  $E_0(\text{J}) - E_0(\text{NJ}) < 0$ , is shown.

**Proposition 1:** If  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$  and  $\gamma^* < \gamma < \infty$ , then there is a  $p^* (0 \leq p^* \leq 1)$  such that if  $p = p^*$ ,  $E_0(\text{J}) - E_0(\text{NJ}) = 0$ , and if  $p < p^*$ ,  $E_0(\text{J}) - E_0(\text{NJ}) < 0$ .

**Proof:** By Lemma 3, if  $\gamma^* < \gamma < \infty$ , then  $E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0$  and  $E_0(\text{Jtogether}) - E_0(\text{NJalone}) > 0$ . By equations (5) and (6),

$$E_0(\text{J}) - E_0(\text{NJ}) = p[E_0(\text{Jtogether}) - E_0(\text{NJalone})] + (1-p)[E_0(\text{Jalone}) - E_0(\text{NJtogether})] .$$

Thus, if  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$  and  $\gamma^* < \gamma < \infty$ ,  $\lim_{p \rightarrow 0} [E_0(\text{J}) - E_0(\text{NJ})] = E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0$  and  $\lim_{p \rightarrow 1} [E_0(\text{J}) - E_0(\text{NJ})] = E_0(\text{Jtogether}) - E_0(\text{NJalone}) > 0$ . Hence, by the intermediate value theorem, there is  $p^* (0 \leq p^* \leq 1)$  such that if  $p = p^*$ ,  $E_0(\text{J}) - E_0(\text{NJ}) = 0$  and if  $p < p^*$ ,  $E_0(\text{J}) - E_0(\text{NJ}) < 0$ . ■

Proposition 1 indicates that, if  $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ ,  $\gamma^* < \gamma < \infty$ , and  $p < p^*$ , then the choice of option *NJ* gives the higher expected utility than that of option *J* to a household; that is, a household may make the rational choice of taking a Pareto inefficient transition path. The lemmas and proposition require no friction, so a Pareto inefficient transition path can be chosen even in a frictionless economy. This result is very important because it offers counter-evidence against

the conjecture that households never rationally choose a Pareto inefficient transition path in a frictionless economy.

### 2.2.1.2 Conditions for a rational Pareto inefficient path

The proposition requires several conditions. Among them,  $\gamma^* < \gamma < \infty$  may appear rather strict. If  $\gamma^*$  is very large, path  $NJ$  will rarely be chosen. However, if path  $NJ$  is such that consumption is reduced sharply after the shock, the  $NJ$  option yields a higher expected utility than the  $J$  option even though  $\gamma$  is very small. For example, for any  $\gamma (0 < \gamma < \infty)$ ,

$$\begin{aligned} & \lim_{s \rightarrow 0} \frac{1}{s} [E_0(Jalone) - E_0(NJtogether)] \\ &= \lim_{s \rightarrow 0} \frac{1}{s} \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + \lim_{s \rightarrow 0} \frac{1}{s} \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\ &= u(c_0 + b_0) - u(c_0) - \frac{1}{\theta} \lim_{s \rightarrow 0} \frac{u(\bar{c}) - u(\bar{c} - s\theta b_0)}{s} = u(c_0 + b_0) - u(c_0) - b_0 \frac{du(\bar{c})}{d\bar{c}} \\ &= \frac{(c_0 + b_0)^{1-\gamma} - c_0^{1-\gamma}}{1-\gamma} - b_0 \bar{c}^{-\gamma} = \bar{c}^{-\gamma} \left\{ \bar{c}^\gamma \left[ \frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] - b_0 \right\} < 0, \end{aligned}$$

because  $\lim_{\gamma \rightarrow 1} \bar{c}^\gamma \left[ \frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] = \bar{c} [\ln(c_0 + b_0) - \ln(c_0)] = \bar{c} \ln \left( 1 + \frac{b_0}{c_0} \right) < b_0$  and

$$\lim_{\gamma \rightarrow \infty} \bar{c}^\gamma \left[ \frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] = \lim_{\gamma \rightarrow \infty} \bar{c}^\gamma c_0^{1-\gamma} \left[ \frac{\left( 1 + \frac{b_0}{c_0} \right)^{1-\gamma} - 1}{1-\gamma} \right] = 0 \quad \text{because } \bar{c} < c_0. \quad \text{That is, for}$$

each combination of path  $NJ$  and  $\gamma$ , there is  $s^* (> 0)$  such that, if  $s < s^*$ , then  $E_0(Jalone) - E_0(NJtogether) < 0$ .

Consider an example in which path  $NJ$  is such that  $b_t$  is constant and  $b_t = \bar{b}$  before  $s$  (Figure 3); thus,  $E_0 \int_0^s b_t = s\bar{b}$ . In this  $NJ$  path, consumption is reduced more sharply than it is in the case shown in Figure 2. In this case, because  $\bar{a} > E_0 \theta \int_0^s b_t = \theta s \bar{b}$ ,  $0 < \gamma$ , and  $c_s < c_t$  for  $t < s$ , then  $E_0 \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt < E_0 \int_0^s \exp(-\theta t) dt [u(c_s + \bar{b}) - u(c_s)] = E_0 \frac{1 - \exp(-\theta s)}{\theta} [u(c_s + \bar{b}) - u(c_s)]$ , and in addition,  $E_0 \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\bar{c})] dt = E_0 \int_s^\infty \exp(-\theta t) dt [u(\bar{c} - \bar{a}) - u(\bar{c})] = E_0 \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \bar{a}) - u(\bar{c})] < E_0 \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \theta s \bar{b}) - u(\bar{c})]$ . Hence,

$$\begin{aligned} & E_0(Jalone) - E_0(NJtogether) \\ &= E_0 \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + E_0 \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\ &< E_0 \frac{1 - \exp(-\theta s)}{\theta} [u(c_s + \bar{b}) - u(c_s)] + E_0 \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \theta s \bar{b}) - u(\bar{c})] \end{aligned}$$

$$= E_0 \frac{1 - \exp(-\theta s)}{\theta} \left\{ \left[ u(c_s + \bar{b}) - u(c_s) \right] - \frac{\exp(-\theta s)}{1 - \exp(-\theta s)} \left[ u(\bar{c}) - u(\bar{c} - \theta s \bar{b}) \right] \right\}.$$

As  $\gamma$  increases, the ratio  $\frac{u(c_s + \bar{b}) - u(c_s)}{u(\bar{c}) - u(\bar{c} - \theta s \bar{b})}$  decreases; thus, larger values of  $s$  can satisfy

$E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0$ . For example, suppose that  $\bar{c} = 10$ ,  $c_s = 10.2$ ,  $\bar{b} = 0.3$ , and  $\theta = 0.05$ . If  $\gamma = 1$ , then  $s^* = 1.5$  at the minimum, and if  $\gamma = 5$ , then  $s^* = 6.8$  at the minimum. This result implies that, if option *NJ* is such that consumption is reduced relatively sharply after the shock (e.g.,  $b_t = \bar{b}$ ) and  $p < p^*$ , option *NJ* will usually be chosen. Choosing option *NJ* is not a special case observed only if  $\gamma$  is very large, but option *NJ* can normally be chosen when the value of  $\gamma$  is within usually observed values. Conditions for generating a rational Pareto inefficient transition path therefore are not strict. In a recession, consumption usually declines sharply after the shock, which suggests that households have chosen the *NJ* option.

## 2.3 Nash equilibrium

### 2.3.1 A Nash equilibrium consisting of *NJ* strategies

A household strategically determines whether to choose the *J* or *NJ* option, considering other households' choices. All households know that each of them forms expectations about the future values of its utility and makes a decision in the same manner. Since all households are identical, the best response of each household is identical. Suppose that there are  $H$  ( $\in N$ ) identical households in the economy where  $H$  is sufficiently large (as assumed in Section 2.1). Let  $q_\eta$  ( $0 \leq q_\eta \leq 1$ ) be the probability that a household  $\eta$  ( $\in H$ ) chooses option *J*. The average utility of the other households almost equals that of all households because  $H$  is sufficiently large. Hence, the average expected utilities of the other households that choose the *J* and *NJ* options are  $E_0(\text{Jtogether})$  and  $E_0(\text{NJtogether})$ , respectively. Hence, the payoff matrix of the  $H$ -dimensional symmetric mixed strategy game can be described as shown in Table 1. Each identical household determines its behavior on the basis of this payoff matrix.

In this mixed strategy game, the strategy profiles

$$(q_1, q_2, \dots, q_H) = \{(1, 1, \dots, 1), (p^*, p^*, \dots, p^*), (0, 0, \dots, 0)\} \quad (12)$$

are Nash equilibria for the following reason. By Proposition 1, the best response of household  $\eta$  is *J* (i.e.,  $q_\eta = 1$ ) if  $p > p^*$ , indifferent between *J* and *NJ* (i.e., any  $q_\eta \in [0, 1]$ ) if  $p = p^*$ , and *NJ* (i.e.,  $q_\eta = 0$ ) if  $p < p^*$ . Because all households are identical, the best-response correspondence of each household is identical such that  $q_\eta = 1$  if  $p > p^*$ ,  $[0, 1]$  if  $p = p^*$ , and 0 if  $p < p^*$  for any household  $\eta \in H$ . Hence, the mixed strategy profiles  $(1, 1, \dots, 1)$ ,  $(p^*, p^*, \dots, p^*)$ , and  $(0, 0, \dots, 0)$  are the intersections of the graph of the best-response correspondences of all households. The Pareto efficient saddle path solution  $(1, 1, \dots, 1)$  (i.e., *Jtogether*) is a pure strategy Nash equilibrium, but a Pareto inefficient transition path  $(0, 0, \dots, 0)$  (i.e., *NJtogether*) is also a pure strategy Nash equilibrium. In addition, there is a mixed strategy Nash equilibrium  $(p^*, p^*, \dots, p^*)$ .

### 2.3.2 Selection of equilibrium

Determining which Nash equilibrium, either *NJtogether*  $(0, 0, \dots, 0)$  or *Jtogether*  $(1, 1, \dots, 1)$ , is dominant requires refinements of the Nash equilibrium, which necessitate additional criteria. Here, if households have a risk-averse preference in the sense that they avert the worst scenario when its probability is not known, households suppose a very low  $p$  and select the *NJtogether*

$(0,0,\dots,0)$  equilibrium. Because

$$\begin{aligned}
& E_0(Jalone) - E_0(NJalone) \\
&= E_0 \left\{ \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\hat{c}_t + a_t)] dt \right\} \\
&< E_0 \left\{ \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \right\} \\
&= E_0(Jalone) - E_0(NJtogether) < 0,
\end{aligned} \tag{13}$$

by Lemma 3, *Jalone* is the worst choice in terms of the amount of payoff, followed by *NJtogether*, and *NJalone*, and *Jtogether* is the best. The outcomes of choosing option *J* are more dispersed than those of option *NJ*. If households have a risk-averse preference in the above-mentioned sense and avert the worst scenario when they have no information on its probability, a household will prefer the less dispersed option (*NJ*), fearing the worst situation that the household alone substantially increases consumption while the other households substantially decrease consumption after the shock. This behavior is rational because it is consistent with preferences. Because all households are identical and know inequality (13), all households will equally suppose that they all prefer the less dispersed *NJ* option; therefore, all of them will suppose a very low  $p$ , particularly  $p=0$ , and select the *NJtogether*  $(0,0,\dots,0)$  equilibrium, which is the Nash equilibrium of a Pareto inefficient path. Thereby, unlike most multiple equilibria models, the problem of indeterminacy does not arise, and “animal spirits” (e.g., pessimism or optimism) are unnecessary to explain the selection.

## 2.4 Amplified generation of unutilized resources

A Nash equilibrium of a Pareto inefficient path successively generates unutilized products ( $b_t$ ). They are left unused, discarded, or preemptively not produced during the path. Unused or discarded goods and services indicate a decline in sales and an increase in inventory for firms. Preemptively suspended production results in an increase in unemployment and idle capital. As a result, profits decline and some parts of firms need to be liquidated, which is unnecessary if the economy proceeds on the *J* path (i.e., the posterior Pareto efficient path). If the liquidation is implemented immediately after the shock,  $b_t$  will no longer be generated, but such a liquidation would generate a tremendous shock. The process of the liquidation, however, will take time because of various frictions, and excess capital that generates  $b_t$  will remain for a long period. During the period when capital is not reduced to the posterior steady-state level, unutilized products are successively generated. In a period,  $b_t$  is generated and eliminated, but in the next period, another, new,  $b_t$  is generated and eliminated. This cycle is repeated in every period throughout the transition path, and it implies that demand is lower than supply in every period. This phenomenon may be interpreted as a general glut or a persisting disequilibrium by some definitions of equilibrium.

## 2.5 Time preference shock as the exceptional shock

Not all shocks result in a Nash equilibrium of a Pareto inefficient path. If anything, this type of shock is limited because such a shock needs to force consumption to fluctuate very jaggedly to maintain Pareto efficiency. A Pareto inefficient path is preferred, because these substantially jagged fluctuations can be averted. An upward time preference shock is one shock that necessitates a substantially jagged fluctuation as shown in Figure 1. Other examples are rare because shocks that do not change the steady state (e.g., monetary shocks) are not relevant. One other example is technology regression, which would move the vertical line  $\frac{dc_t}{dt} = 0$  to the left

in Figure 1 and necessitate a jagged consumption path to keep Pareto efficiency. In this sense, technology and time preference shocks have similar effects on economic fluctuations. However,

a technology regression also simultaneously moves the curve  $\frac{dk_t}{dt} = 0$  downwards, and accordingly, the Pareto efficient saddle path also moves downwards. Therefore, the jagged consumption is smoothed out to some extent. As a result, the substantially jagged consumption that can generate a recession would require a large-scale, sudden, and sharp regression in technology, which does not seem very likely. An upward time preference shock, however, only moves the vertical line  $\frac{dc_t}{dt} = 0$  to the left.

In some macro-economic models with multiple equilibria, changing equilibria may necessitate substantially jagged consumption to keep Pareto optimality. There are many types of multiple equilibria models that depend on various types of increasing returns, externalities, or complementarities, but they are vulnerable to a number of criticisms (e.g., insufficient explanation of the switching mechanism; see, e.g., Morris and Shin, 2001). Examining the properties, validity, and plausibility of each of these many and diverse models is beyond the scope of this paper.

### 3. MONETARY POLICY

#### 3.1 *Irresponsive monetary policy*

As argued in Section 1, a feature of a liquidity trap is that households and firms are unresponsive to monetary policies. Here, monetary policy is defined as the policy of the monetary authority to manipulate nominal interest rates by intervening in financial markets. On a Nash equilibrium of a Pareto inefficient path, monetary policies are naturally unresponsive because, unless  $\bar{a}$  is made substantially small, households' choice of the  $NJ$  path does not change. Through the use of monetary policies, it is difficult to make  $\bar{a}$  substantially small. Households and firms rationally determine their behaviors on the basis of their expectations of a future economic path from now until an indefinite future. If monetary policies cannot affect expectations about the  $NJ$  path, they will be useless.

Suppose that the real interest rate  $i_t$  is lowered by monetary policies, and thus  $\bar{a}$  in equation (9) becomes smaller. If

$$E(\bar{a}) = E\left(\theta \int_0^s b_r \exp \int_r^s i_q dq dr\right)$$

becomes substantially small or negative, households may switch from the  $NJ$  path to the  $J$  path based on the same strategic calculations shown in Section 2. However, there is a critical point  $E(\bar{a}^*)$  such that, for a given  $\tilde{\gamma}$  ( $\gamma^* < \tilde{\gamma} < \infty$ ), if  $E(\bar{a}) > E(\bar{a}^*)$ , then households will not switch to the  $J$  path. The proof is almost same as that given in Proposition 1. In essence, whatever the monetary policies are, the paths of consumption, investments, and production will be unchanged unless  $E(\bar{a})$  is made less than  $E(\bar{a}^*)$ .

It is, however, very difficult to make  $E(\bar{a})$  less than  $E(\bar{a}^*)$  because  $\theta \int_0^s b_r dr$  (i.e., the main component of  $\bar{a}$ ) cannot be affected by monetary policies and  $\exp \int_r^s i_q dq$  cannot be made substantially small even if  $i$  is lowered by monetary policies. For example, if  $i$  is lowered from 0.03 to 0.02 by monetary policies for 10 years,  $\exp \int_0^{10} i_q dq$  is reduced from 1.34 to 1.22 (i.e., only by 9.3%), and if from  $i = 0.03$  to 0.01,  $\exp \int_0^{10} i_q dq$  is from 1.34 to 1.10 (by 18%). Hence, even if monetary authorities can successfully lower  $i$  for a

long period,  $E(\bar{a})$  cannot be made substantially small. An exception is the case in which monetary authorities can make long-term real interest rates substantially negative for a long period. Nevertheless, this case is generally very unlikely and actually has not been historically observed in the modern era.

In addition, it is debatable whether the monetary authority can artificially hold the real interest rate substantially below the marginal product of capital for a long period. Many economists agree that monetary policies can affect the real interest rate in the short-run by manipulating nominal interest rates, but many of them do not agree that they can do so in the long-run. The real interest rate is basically determined to be equal to the marginal product of capital in the long-run.

In sum, monetary policies usually do not have enough power to shift households from the  $NJ$  path to the  $J$  path. Even if aggressive monetary policies are taken, in the sense of almost zero nominal interest rates, the paths of consumption, investments, and production will remain unchanged, except in the unlikely case that the real interest rate is substantially negative for a long period. Because investments are unchanged even if nominal interest rate are very low, banks have difficulty in lending money corresponding to  $b_t$  to firms. This therefore is not a credit crunch, but rather a “debt crunch.” This phenomenon may also be called “excess savings.” As a last resort, banks have to increase their purchases of government bonds.

## **3.2 Unconventional monetary policies**

### **3.2.1 Effects on consumption and investments**

As shown in Section 3.1, an important feature of the so-called liquidity trap is that traditional monetary policy (i.e., manipulating nominal interest rates) is useless. However, some economists argue that an unconventional monetary policy (i.e., manipulating the quantity of money directly by the monetary authority) is still useful. In particular, they argue that by continuous injections of large amounts of money, the economy can eventually get out of a liquidity trap. However, how the quantity of money affects consumption, investments, and production is theoretically unclear. The Friedman rule (Friedman, 1969) indicates that money should be provided until demand for it is saturated and the quantity of money reaches its optimal level. Therefore, in this case, the nominal interest rate should be zero and the inflation rate should be negative.

The model in Section 2 indicates that unless households switch from the  $NJ$  path to the  $J$  path, the paths of consumption, investment, and production are unchanged. Therefore, unless unconventional monetary policies can make real interest rates substantially negative for a long period, households will not switch to the  $J$  path. However, it is highly unlikely that real interest rates can be kept substantially negative for a long period through the manipulation of the quantity of money. As a result, the model in Section 2 indicates that the unconventional monetary policy is also not useful.

### **3.2.2 Excess money**

If the government is not benevolent but instead is economically Leviathan, the optimal quantity of money is not determined at the point where the nominal interest rate is zero. Harashima (2004b, 2008, 2013a) shows that inflation acceleration is not generated by an increasing money supply but instead by the gap of time preference rates between the Leviathan government and households. The optimal quantity of money is determined at the point where both the government and households can simultaneously satisfy their all optimality conditions. If a quantity of money over this optimal quantity is supplied, it is merely excess money and useless for households in the sense that they cannot satisfy all their optimality conditions.

### **3.2.3 Equivalence to tax**

Because the  $NJ$  paths of consumption, investment, and production are unchanged by



unconventional monetary policy, the excess money that it injects into the economy is not spent on consumption and investment. Therefore, households and firms that possess the excess money are forced to buy government bonds that finance the money for buying  $b_t$ , directly or indirectly via banks. As a result, the demand for government bonds will increase and the real rate of return on government bonds will decrease. Furthermore, if there is an insufficient amount of government bonds in financial markets, some households may not be able to purchase enough bonds and will have to unwillingly hold onto part of the excess money.

The unwillingly held excess money is practically equivalent to seigniorage. Because seigniorage is a type of tax, this excess money represents a tax increase. The lowered real rate of return on government bonds resulting from this excess money can also be interpreted as a kind of tax imposed on the return. The revenue from these taxes is used to finance the money used to buy  $b_t$ . Hence, the question of what effects unconventional monetary policies have resolves itself into the question of whether taxes or borrowing should be used for utilizing  $b_t$ . Adopting unconventional monetary policy in essence means that taxes have been chosen.

### 3.2.4 Possible divergence of the bonds' interest rate from marginal productivity

Usually, the real rate of return on government bonds is kept equal to the marginal product of capital by arbitrage if other factors (e.g., transaction costs, risk premium, depreciation rate, etc.) are neglected. However, as shown above, the excess money lowers the real rate of return on government bonds, and it may become lower than the marginal product of capital. This divergence will not be corrected by arbitrage because the economy is on the  $NJ$  path. The paths of consumption and investments in capital are unchanged by the excess money; thus, firms do not increase investments in capital. Hence, there is no room for firms to exploit opportunities provided by the excess money by investing more in capital. As a result, households and firms have to buy government bonds for utilizing  $b_t$ , even though their rate of return is lower than the marginal product of capital. This divergence is rational. Conversely, the real rate of return on government bonds is not necessarily a good proxy variable for the marginal product of capital if an economy is on the  $NJ$  path.

## 4. FISCAL POLICY

### 4.1 *Pareto efficiency*

#### 4.1.1 Pareto improvement

The fiscal policy examined in this paper is one in which  $b_t$  is bought and utilized by the government, and the government's expenditure is financed by increases in taxes or borrowing. For simplicity, it is assumed that the tax is a lump-sum tax and the borrowing is accomplished by issuing government bonds.

As was the case with monetary policies, this fiscal policy generally does not have the power to force a switch from the  $NJ$  path to the  $J$  path because the expected utilities of *Jalone*, *NJtogether*, *Jtogether*, and *NJtogether* are basically not affected by the policy. An increase in the lump-sum tax will cause consumption to decrease equally across households regardless of the paths taken. Therefore, the expected utilities for the four paths will decrease similarly by the amount of the tax increase; thus, Proposition 1 is basically held. In addition, an increase in borrowing will not make the real interest rate substantially negative for a long period. Rather, it may raise nominal interest rates and then, in the short run, the real interest rate may also rise. Hence, an increase in borrowing will not make  $\bar{a}$  substantially small; thus, the path will not switch from  $NJ$  to  $J$  as a result of fiscal policy.

Nevertheless, fiscal policy is very effective in the sense that unused resources are utilized. That is, the use of fiscal policy will improve Pareto efficiency, but the argument that Pareto efficiency is improved is not simple. Keynes (1936) argued that it was better for the

government to pay people to dig holes in the ground and then fill them up to decrease unemployment than doing nothing. But does this policy improve Pareto efficiency? To answer this question, we first need to examine how the government's behaviors are determined.

### 4.1.2 Two different views on government behavior

There are two extremely different views regarding government behavior in the literature on political economy: the Leviathan view and the benevolent view (e.g., Downs 1957; Brennan and Buchanan 1980; Alesina and Cukierman 1990). From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household, but a Leviathan government does not. Whereas the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government's own policy objectives.<sup>3</sup> For example, if a Leviathan government considers national security to be the most important political issue, defense spending will increase greatly, but if improving social welfare is the top political priority, spending on social welfare will increase dramatically, even though the increased expenditures may not necessarily increase the economic utility of the representative household.

The Leviathan view generally requires the explicit inclusion of government expenditures, tax revenues, and related activities in the government's political utility function (e.g., Edwards and Keen 1996). Because an economically Leviathan government derives political utility from expenditure for its political purposes, the government will be happier as expenditures increase. But raising tax rates will provoke people's antipathy, which increases the probability of being replaced by an opposing party that also nearly represents the median household. Thus, an economically Leviathan government regards taxes as necessary costs to obtain the freedom of expenditure for its own purposes. The government therefore will derive utility from expenditure and disutility from taxation. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the representative household. Consumption and labor hours are both control variables, and as such, the government's expenditure and tax revenues are also control variables. As a whole, the political utility function of an economically Leviathan government can be expressed as  $u_G(g_t, x_t)$  where  $g_t$  and  $x_t$  are the government's expenditure and tax revenues in period  $t$ , respectively. In addition, it can be assumed on the basis of the previously mentioned arguments that  $\frac{\partial u_G}{\partial g_t} > 0$  and  $\frac{\partial^2 u_G}{\partial g_t^2} < 0$ , and therefore that  $\frac{\partial u_G}{\partial x_t} < 0$  and  $\frac{\partial^2 u_G}{\partial x_t^2} > 0$ . An economically Leviathan government therefore maximizes the expected sum of these utilities discounted by its time preference rate under the constraint of deficit financing.

### 4.1.3 The benevolent view and $b_t$

A benevolent government behaves to maximize the utility of households. Because the economy is on the  $NJ$  path, private consumption of households does not change as a result of fiscal policy. If the expenditure to utilize  $b_t$  increases the provision of public goods, however, households' consumption of public goods increases and their utilities also increase. Hence, the government's use of  $b_t$  improves Pareto efficiency.

However, there is a problem with this benevolent view. It justifies the fiscal policy because it increases public goods, but the most important motive of a government in adopting

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<sup>3</sup> The government behavior assumed in the fiscal theory of the price level reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.

fiscal policies is not to increase the consumption of public goods but to decrease unemployment during a recession. Increasing the consumption of public goods is only a tool that is being used to decrease unemployment. Actually, a government will adopt the fiscal policy to make unemployment decrease regardless of an increase in public goods. However, the benevolent view justifies an increase in public goods even though employment does not increase. The benevolent view therefore is not necessarily consistent with the motive of government.

It may be argued that, if an originally unemployed household member becomes employed as a result of the fiscal policy, the household's welfare will improve; thus, the fiscal policy can be directly linked to a reduction in unemployment, but this is not true. Suppose that the stochastic process of unemployment is described as a Markov chain, and its stationary distribution is equal to all identical households. As a result of fiscal policies, originally unemployed household members become employed, and the average income of all households increases. However, this increased income is taken away through increased taxes or borrowing by the government to finance the increased expenditures. As a result, the path of private consumption is unchanged regardless of unemployment rate. Hence, lowering the unemployment rate does not directly improve Pareto efficiency. Pareto efficiency is only improved if the provision of public goods increases. Therefore, the theoretical fiscal policy of employing people to dig holes and fill them up, which Keynes supported, is meaningless from the benevolent point of view.

#### **4.1.4 The Leviathan view and $b_t$**

The Leviathan view is consistent with the motive of government. The expenditure to reduce unemployment directly increases a Leviathan government's utility regardless of households' utilities or the usefulness of increased public goods. Because people's political desires are directly reflected in the government's utility, materialization of their desires, for example, a significant reduction in unemployment, greatly increases the government's utility. An increase in the government's utility improves Pareto efficiency. Hence, unlike the benevolent view, the policy of employing people to dig holes and fill them up is justified from the Leviathan point of view.

Not only is the Leviathan view consistent with the motive of government, it is also highly likely that the view actually prevails because a government is generally chosen by median households under a proportional representation system (e.g., Downs, 1957), whereas the representative household usually presumed in the economics literature is the mean household. The economically representative household is not usually identical to the politically representative household, and a majority of people could support a Leviathan government even if they know that the government does not necessarily pursue only the economic objectives of the economically representative household. In other words, the government presented here is an economically Leviathan government that maximizes the political utility of people, whereas the conventional economically benevolent government maximizes the economic utility of people. In addition, because the politically and economically representative households are different (the median and mean households, respectively), the preferences of future governments will also be similarly different from those of the mean representative household. In this sense, the current and future governments presented in the model can be seen as a combined government that goes on indefinitely; that is, the economically Leviathan government always represents the median representative household.

## **4.2 Artificial jump to the new saddle path**

If the government completely utilizes  $b_t$ , the total demand in the economy, that is, the sum of private consumption, investments, and government expenditure, is exactly equal to production (the total supply). There is no excess demand or supply. Because  $b_t$  is completely utilized by the government, the economy jumps to the new saddle path (the  $J$  path) from the  $NJ$  path. Households, however, still proceed on the  $NJ$  path. The path households choose is, in essence,

artificially and compulsorily corrected by the government's fiscal policy.

Another important feature is that even though the government's expenditure is financed by borrowing, the fiscal policies have no negative effect on capital formation. Capital is adjusted to the level of the new steady state regardless of government borrowing. The path of investment is predetermined before the fiscal policy is implemented. Conversely, the fiscal policy also has no "positive" effect on capital formation in the sense that it does not increase investments above the level on the *NJ* path.

Note that the above argument is based on the assumption that the government has a firm will to tax households after period  $s$  to pay for the borrowing used to finance the policy to utilize  $b_t$ . If the government behaves in this manner, the extra debt accumulated does not matter, even though the ratio of the government's debts to GDP can become very high before period  $s$ .

### 4.3 Increased taxes or borrowing

Increases in tax revenues and borrowing are the choices available to finance the money needed to utilize  $b_t$ . If the Barro–Ricardo equivalence theorem holds, households are indifferent to both forms of financing. The government, however, may not necessarily be indifferent. The expected utility of government will vary depending on the functional form of its utility function.

#### 4.3.1 The Leviathan government's alternatives

Suppose that the government is Leviathan and its expenditure is  $g_t$ , the lump-sum tax revenue is  $x_t$ , and its borrowing is  $\Delta B_t$  in period  $t$ . The Leviathan government's utility function is

$$u_G(g_t, x_t),$$

and

$$\frac{\partial u_G}{\partial g_t} > 0 \quad \text{and} \quad \frac{\partial^2 u_G}{\partial g_t^2} < 0, \quad \text{and} \quad \frac{\partial u_G}{\partial x_t} < 0 \quad \text{and} \quad \frac{\partial^2 u_G}{\partial x_t^2} > 0 .$$

In addition, it is assumed that for simplicity

$$\begin{aligned} u_G(g_t, x_t) &= \frac{g_t^{1-\gamma_g}}{1-\gamma_g} - \frac{x_t^{1-\gamma_x}}{1-\gamma_x} && \text{if } \gamma_g \neq 1 \text{ and } \gamma_x \neq 1 , \\ u_G(g_t, x_t) &= \ln(g_t) - \frac{x_t^{1-\gamma_x}}{1-\gamma_x} && \text{if } \gamma_g = 1 , \\ u_G(g_t, x_t) &= \frac{g_t^{1-\gamma_g}}{1-\gamma_g} - \ln(x_t) && \text{if } \gamma_x = 1 , \end{aligned}$$

where  $\gamma_g$  and  $\gamma_x$  are constant. It is also assumed that, before the shock,  $x_t$ ,  $\Delta B_t$ , and  $g_t$  are constant for any period; that is, they are steady state or, so to speak, permanent values. The permanent expenditure  $g_t$  is financed by  $x_t$  and  $\Delta B_t$  in each period such that

$$g_t = x_t + \Delta B_t .$$

Because of the inclusion of government in the model, households' private consumption is smaller by  $g_t$  than that in models without government (Fig. 4). Suppose that, after the shock, the *NJ* path is selected and a fiscal policy to completely utilize  $b_t$  is implemented. Let the government's extra expenditure to utilize  $b_t$ , its extra lump-sum tax, and its extra borrowing to finance the expenditure in the period  $t$  be  $g_{b,t}$ ,  $x_{b,t}$  and  $\Delta B_{b,t}$  respectively. Therefore, after the

shock, the total expenditure of the government, the total tax revenue, and the total borrowing in period  $t$  before period  $s$  are  $g_t + g_{b,t}$ ,  $x_t + x_{b,t}$  and  $\Delta B_t + \Delta B_{b,t}$ , respectively. Hence, after the shock,

$$g_t + g_{b,t} = x_t + x_{b,t} + \Delta B_t + \Delta B_{b,t} \quad (14)$$

in period  $t$  before  $s$ ; thus,  $g_{b,t} = x_{b,t} + \Delta B_{b,t}$ .

If only a tax increase is used to finance  $g_{b,t}$ , then equation (14) degenerates to

$$g_t + g_{b,t} = x_t + x_{b,t} + \Delta B_t,$$

before period  $s$ , and after  $s$ , to

$$g_t = x_t + \Delta B_t.$$

If only an increase in borrowing is used, then equation (14) degenerates to

$$g_t + g_{b,t} = x_t + \Delta B_t + \Delta B_{b,t}$$

before period  $s$ , and after  $s$ , to

$$g_t + g_{a,t} = x_t + \bar{a} + \Delta B_t,$$

where  $g_{a,t}$  is the extra expenditure to pay for interest of the accumulated extra government bonds  $\Delta B_{b,t}$ .  $\bar{a}$ , which is shown in equation (9), here indicates the extra tax revenue. Because  $b_t$  is completely utilized, the accumulated extra government bonds can be expressed as

$$\int_0^s b_r \exp \int_r^s i_q dq dr,$$

and the real interest rate after period  $s$  is equal to  $\theta$ ; thus,  $\bar{a}$  is equal to  $g_{a,t}$ . That is, the government imposes an extra tax to pay for  $g_{a,t}$ .

In addition, tax cuts can be combined with an increase in borrowing. Particularly, the following case is examined:

$$g_t + g_{b,t} - \Delta g_t = g_t = x_t - \Delta x_t + \Delta B_t + \Delta B_{b,t}$$

before the period  $s$ , and after  $s$ ,

$$g_t + g_{a,t} = x_t + \bar{a} + \Delta B_t,$$

where  $\Delta g_t$  is a reduction in permanent expenditure and  $\Delta g_t = g_{b,t}$ , and  $\Delta x_t$  is the amount of the tax cuts and  $\Delta g_t = \Delta x_t$ . In this case, the government increases expenditure by  $g_{b,t}$  as the fiscal policy, but at the same time, it reduces permanent expenditure by  $\Delta g_t$ . As a result, the total expenditure is unchanged even though the fiscal policy has been adopted, and the reduced expenditure  $\Delta g_t$  is used to finance tax cuts  $\Delta x_t$ .

### 4.3.2 The expected utility of a Leviathan government

If only an increase in tax is used to finance the utilization of  $b_t$ , the government's expected utility is

$$U_1 = \int_0^s \exp(-\theta_G t) u_G(g_t + G_{b,p} x_t + G_{b,t}) dt + \int_s^\infty \exp(-\theta_G t) u_G(g_t, x_t) dt ;$$

if only an increase in borrowing is used, it is

$$U_2 = \int_0^s \exp(-\theta_G t) u_G(g_t + G_{b,p} x_t) dt + \int_s^\infty \exp(-\theta_G t) u_G(g_t, x_t + \bar{a}) dt \quad ;$$

and if tax cuts and borrowing are combined as shown above, it is

$$U_3 = \int_0^s \exp(-\theta_G t) u_G(g_t, x_t - G_{b,t}) dt + \int_s^\infty \exp(-\theta_G t) u_G(g_t, x_t + \bar{a}) dt \quad .$$

If  $U_2 > U_1$ , then financing by increased borrowing is preferred to that by increased taxes. Here,

$$\begin{aligned} U_2 - U_1 &= \int_0^s \exp(-\theta_G t) [u_G(g_t + G_{b,p} x_t) - u_G(g_t + G_{b,p} x_t + G_{b,t})] dt \\ &\quad + \int_s^\infty \exp(-\theta_G t) [u_G(g_t, x_t + \bar{a}) - u_G(g_t, x_t)] dt \quad . \end{aligned}$$

In addition,

$$\int_s^\infty \exp(-\theta_G t) [u_G(g_t, x_t + \bar{a}) - u_G(g_t, x_t)] dt < 0$$

and

$$\int_0^s \exp(-\theta_G t) [u_G(g_t + G_{b,p} x_t) - u_G(g_t + G_{b,p} x_t + G_{b,t})] dt > 0 \quad .$$

By the same procedure as used in Proposition 1 and Lemmas 1, 2, and 3, there is  $\gamma_x^* (0 < \gamma_x^* < \infty)$  such that if  $\gamma_x^* < \gamma_x < \infty$ , then

$$\int_0^s \exp(-\theta_G t) [u_G(g_t + G_{b,p} x_t) - u_G(g_t + G_{b,p} x_t + G_{b,t})] dt + \int_s^\infty \exp(-\theta_G t) [u_G(g_t, x_t + \bar{a}) - u_G(g_t, x_t)] dt < 0 \quad ;$$

that is,  $U_2 < U_1$ . Conversely, if  $\gamma_x$  is sufficiently small, then  $U_2 > U_1$  and an increase in borrowing is preferred. A small  $\gamma_x$  indicates that the disutility per unit of tax does not largely diminish as the amount of tax decreases.

Next, if  $U_3 > U_1$ , then a combination of tax cuts and borrowing is preferred to the tax increase. Here,

$$\begin{aligned} U_3 - U_1 &= \int_0^s \exp(-\theta_G t) [u_G(g_t, x_t - G_{b,t}) - u_G(g_t + G_{b,p} x_t + G_{b,t})] dt \\ &\quad + \int_s^\infty \exp(-\theta_G t) [u_G(g_t, x_t + \bar{a}) - u_G(g_t, x_t)] dt \quad , \end{aligned}$$

and

$$\int_s^\infty \exp(-\theta_G t) [u_G(g_t, x_t + \bar{a}) - u_G(g_t, x_t)] dt < 0 \quad .$$

In addition, if  $\frac{\partial u_G}{\partial g_t} < \left| \frac{\partial u_G}{\partial x_t} \right|$ , then

$$\int_0^s \exp(-\theta_G t) [u_G(g_t, x_t - G_{b,t}) - u_G(g_t + G_{b,p} x_t + G_{b,t})] dt > 0 . \quad (15)$$

Even though  $\frac{\partial u_G}{\partial g_t} > \left| \frac{\partial u_G}{\partial x_t} \right|$ , if  $\frac{\partial u_G}{\partial g_t} < \left| \frac{\partial u_G}{\partial x_t} \right| + \alpha$  for a certain positive constant  $\alpha$ , then inequality (15) holds. That is, if the values of  $\frac{\partial u_G}{\partial g_t}$  and  $\left| \frac{\partial u_G}{\partial x_t} \right|$  are not so different, even though  $\frac{\partial u_G}{\partial g_t} > \left| \frac{\partial u_G}{\partial x_t} \right|$ , inequality (15) holds. Thus, if  $\frac{\partial u_G}{\partial g_t} < \left| \frac{\partial u_G}{\partial x_t} \right|$  or the values of  $\frac{\partial u_G}{\partial g_t}$  and  $\left| \frac{\partial u_G}{\partial x_t} \right|$  are not so different, then inequality (15) holds. Conversely, if  $\frac{\partial u_G}{\partial g_t} > \left| \frac{\partial u_G}{\partial x_t} \right|$  and the values of  $\frac{\partial u_G}{\partial g_t}$  and  $\left| \frac{\partial u_G}{\partial x_t} \right|$  are sufficiently different,  $U_3 - U_1 < 0$ .

If  $\frac{\partial u_G}{\partial g_t} < \left| \frac{\partial u_G}{\partial x_t} \right|$  or the values of  $\frac{\partial u_G}{\partial g_t}$  and  $\left| \frac{\partial u_G}{\partial x_t} \right|$  are not so different, by the same procedure as used in Proposition 1 and Lemmas 1, 2, and 3, there is  $\gamma_x^{**}$  ( $0 < \gamma_x^{**} < \infty$ ) such that if  $\gamma_x^{**} < \gamma_x < \infty$ , then

$$U_3 - U_1 = \int_0^s \exp(-\theta_G t) [u_G(g_t, x_t - G_{b,t}) - u_G(g_t + G_{b,p} x_t + G_{b,t})] dt + \int_s^\infty \exp(-\theta_G t) [u_G(g_t, x_t + \bar{a}) - u_G(g_t, x_t)] dt < 0 .$$

In addition, similarly, there is  $\gamma_g^{**}$  ( $0 < \gamma_g^{**} < \infty$ ) such that if  $\gamma_g^{**} < \gamma_g < \infty$ , then  $U_3 - U_1 < 0$ . Conversely, if  $\gamma_x$  and/or  $\gamma_g$  is sufficiently small, then  $U_3 - U_1 > 0$ . In these cases, the combination of tax cuts and borrowing is preferred.

Finally, if  $U_3 > U_2$ , then the combination of tax cuts and increased borrowing is preferred to only an increase in borrowing. Here,

$$U_3 - U_2 = \int_0^s \exp(-\theta_G t) [u_G(g_t, x_t - G_{b,t}) - u_G(g_t + G_{b,p} x_t)] dt .$$

The sign of  $\int_0^s \exp(-\theta_G t) [u_G(g_t, x_t - G_{b,t}) - u_G(g_t + G_{b,p} x_t)] dt$  depends on the property of  $u_G$ . If  $\frac{\partial u_G}{\partial g_t} > \left| \frac{\partial u_G}{\partial x_t} \right|$ , then  $U_3 - U_2 < 0$ ; that is, the fiscal policy consisting only of an increase in borrowing is preferred to that consisting only of a combination of tax cuts and increased borrowing. Conversely, if  $\frac{\partial u_G}{\partial g_t} < \left| \frac{\partial u_G}{\partial x_t} \right|$ , then  $U_3 - U_2 > 0$ .

### 4.3.3 A shift of government's preferences

The values of  $\gamma_g$  and  $\gamma_x$  are empirical questions. However, they may decrease when  $b_t$  is generated. A shock that generates  $b_t$  will cause the preferences of a Leviathan government (i.e., political desires of the median household) to shift because the existence of extra unemployment is significantly unacceptable for the government. The representative household will not perceive

economic disutility from extra unemployment as a result of  $b_t$  because its consumption (i.e., the average consumption of all households) does not change. However, the increased possibility of unemployment resulting from  $b_t$  will significantly affect the political perception of individual households. Although fears of being unemployed will not affect the economic utility derived from consumption, it will have physiological and political effects. Being unemployed may substantially change the course of a person's life. An unemployed person may have to unwillingly and greatly change his or her plans. Uncertainty about various elements of life will substantially increase. Although these kinds of physiological and political disutilities are not reflected in economic disutility, they will have a large impact on people's political behaviors, and the political desires of the median household will be substantially affected. Hence, the values of  $\gamma_g$  and  $\gamma_x$  may shift as a result of a shock that generates  $b_t$ .

In response to the increased and very strong fear of unemployment, a Leviathan government's political desires to increase government expenditures to finance the utilization of  $b_t$  to decrease extra unemployment will be very strong. Therefore, far more expenditures will be preferred by the government when  $b_t$  is generated. In addition, an increase in taxes when  $b_t$  is generated will be seen as a measure that worsens unemployment so a tax increase will be far less preferred when  $b_t$  is generated as compared to when it is not. Such changes in the government's preferences induced by the shock suggest that the values of  $\gamma_g$  and  $\gamma_x$  will become significantly small if  $b_t$  is generated. As stated in Section 4.3.2, if such shifts in preferences actually occur, the government will prefer to increase borrowing over tax increases to finance the expenditures for utilizing  $b_t$ .

## 5 CONCLUDING REMARKS

Keynes' and his early followers' explanations of persistent large amounts of unused resources have come to be viewed as basically unacceptable because they have no micro-foundation. In addition, New-Keynesian explanations based on micro-founded mechanisms of some kinds of price rigidity have not been regarded as sufficiently successful because price rigidity has been criticized for its fragile theoretical (micro-) foundation and its inability to explain the persistent nature of inflation. In this paper, I present a mechanism based on the model in Harashima (2004a, 2012, 2013b). An essential part of this mechanism is that a Nash equilibrium can conceptually coexist with Pareto inefficiency.

A Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path of consumption to the steady state exists because households are intrinsically risk averse and not cooperative. In a strategic environment, this generates the possibility that, if consumption needs to be substantially and discontinuously increased to keep Pareto optimality, a non-cooperative household's strategy to deviate from the Pareto optimal path gives a higher expected utility than the strategy of choosing the Pareto optimal path. Such a strategic situation is generated when the rate of time preference shifts upwards.

The effects of monetary and fiscal policies when an economy is on a Nash equilibrium of a Pareto inefficient path were also explored. The results indicate that, although Keynes' prescription for escaping a liquidity trap (i.e., the use of fiscal policy) was right, his explanation for it was wrong. Monetary policies are useless during periods known as liquidity traps because they do not have enough power to shift a Nash equilibrium of a Pareto inefficient path to that of a Pareto efficient path. Fiscal policy can fill the gap between the Pareto efficient and inefficient paths and thus improve Pareto efficiency without affecting capital formation. The economy essentially jumps to the new Pareto efficient saddle path as a result of the fiscal policy. As a tool to finance fiscal policies, households are indifferent to the choice of tax increases and borrowing as the method of financing the additional expenditures if the Barro–Ricardo equivalence theorem holds. However, if the government is economically Leviathan, it will generally prefer to increase borrowing over raising taxes.

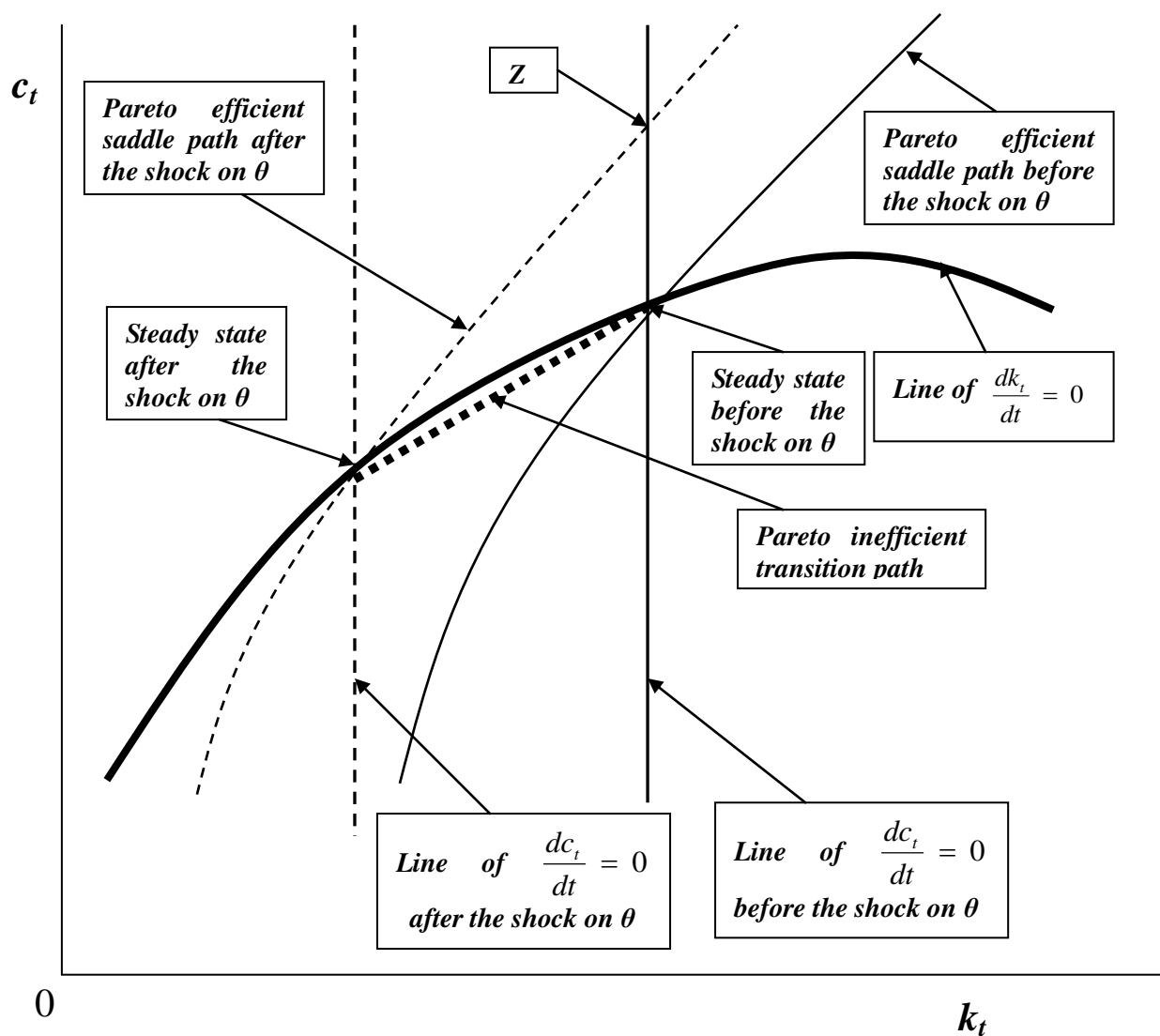


## References

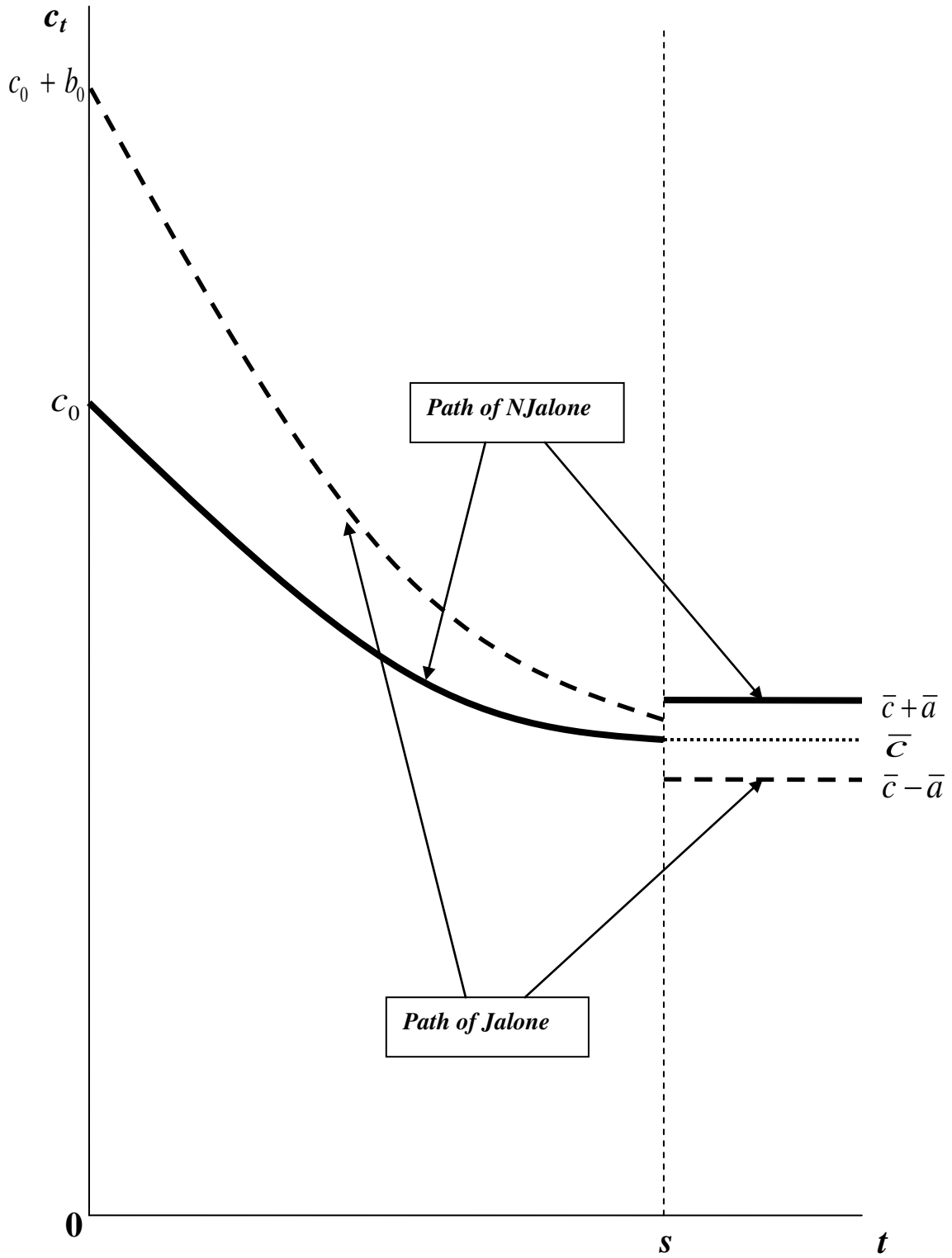
- Alesina, Alberto and Alex Cukierman. (1990) "The Politics of Ambiguity," *The Quarterly Journal of Economics*, Vol. 105, No. 4, pp. 829–850.
- Becker, Gary S. and Casey Mulligan (1997) "The Endogenous Determination of Time Preference," *Quarterly Journal of Economics*, Vol. 112, No. 3, pp. 729–758.
- Benhabib, Jess, and Roger Farmer (2000) "Indeterminacy and Sunspots in Macroeconomics" , in John Taylor and Michael Woodford (eds.): *Handbook of Macroeconomics*, North Holland.
- Benhabib, Jess, Stephanie Schmitt-Grohé and Martín Uribe (2001a) "The Perils of Taylor Rules", *Journal of Economic Theory*, Vol. 96, pp. 40–96.
- Benhabib, Jess, Stephanie Schmitt-Grohé and Martín Uribe (2001b) "Monetary Policy and Multiple Equilibria", *American Economic Review*, Vol. 91, pp. 167–186.
- Benhabib, Jess, Stephanie Schmitt-Grohé and Martín Uribe (2002) "Avoiding Liquidity Traps", *Journal of Political Economy*, Vol. 110, No. 3, pp. 535–563.
- Böhm-Bawerk, Eugen von (1889) *Capital and Interest*, Reprinted by Libertarian Press, South Holland, IL, 1970.
- Brennan, Geoffrey and James M. Buchanan. (1980) *The power to tax: analytical foundations of a fiscal constitution*, Cambridge University Press, Cambridge MA.
- Christiano, Lawrence and Terry J. Fitzgerald. (2000) "Understanding the Fiscal Theory of the Price Level," *Federal Reserve Bank of Cleveland Economic Review*, Q II, pp. 2-38.
- Downs, Anthony. (1957) *An economic theory of democracy*, Harper, New York.
- Edwards, Jeremy and Michael Keen. (1996) "Tax Competition and Leviathan," *European Economic Review*, Vol. 40, No. 1, pp. 113–134.
- Eggertsson, Gauti B. and Paul R. Krugman. (2012) "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *The Quarterly Journal of Economics*, Vol. 127, No. 3, pp. 1469-1513.
- Eggertsson, Gauti B. and Michael Woodford. (2003) "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, Vol. 34, pp. 139-233.
- Fisher, Irving (1930) *The Theory of Interest*, Macmillan, New York
- Friedman, Milton. (1969) *The Optimal Quantity of Money and Other Essays*, Aldine, Chicago.
- Fuhrer, Jeff and George Moore. (1995) "Inflation Persistence," *Quarterly Journal of Economics*, Vol. 110, No. 1, pp. 127–159.
- Galí, Jordi and Mark L. Gertler (1999) "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, Vol. 44, No. 2, pp. 195–222.
- Harashima, Taiji (2004a) "A More Realistic Endogenous Time Preference Model and the Slump in Japan," *EconWPA Working Papers*, ewp-mac0402015.
- Harashima, Taiji (2004b) "The Ultimate Source of Inflation: A Microfoundation of the Fiscal Theory of the Price Level," *EconWPA Working Papers*, ewp-mac/ 0409018.
- Harashima, Taiji (2008) "The Cause of the Great Inflation: Interactions between the Government and the Monetary Policymakers," *The Journal of World Economic Review*, Vol. 3, No. 1, pp. 69-89.
- Harashima, Taiji (2012) "A Mechanism of Cyclical Volatility in the Vacancy-Unemployment Ratio: What Is the Source of Rigidity?" *The Munich Personal RePEc Archive Paper No. 36895*.
- Harashima, Taiji (2013a) "The Optimal Quantity of Money Consistent with Positive Nominal Interest Rates," *Journal of Kanazawa Seiryō University*, Vol. 46, No. 2, pp. 27-36 (in Japanese).
- Harashima, Taiji (2013b) *The Economics of Fiscal Deficits (Zaisei Akaji No Keizaigaku)*, Buitsu-Soryushon, Nagoya, Japan (in Japanese).
- Jeanne, Olivier, and Lars E. O. Svensson (2007) "Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank," *The*

- American Economic Review*, Vol. 97, pp. 474-490.
- Keynes, John Maynard (1936) *The General Theory of Employment, Interest and Money*, Macmillan Cambridge University Press, London.
- Krugman, Paul R. (1998) "Its Baaack: Japans Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, Vol. 29, pp. 137-187.
- Lawrance, Emily C. (1991) "Poverty and the Rate of Time Preference: Evidence from Panel Data," *Journal of Political Economy*, Vol. 99, No. 1, pp. 54-77.
- Mankiw, N. Gregory (2001) "The Inexorable and Mysterious Tradeoff between Inflation and Unemployment," *Economic Journal*, Vol. 111, Issue 471, pp. C45-61
- Morris, Stephen and Hyun Song Shin (2001) "Rethinking Multiple Equilibria in Macroeconomic Modeling," *NBER Macroeconomics Annual 2000*, Vol. 15, pp. 139-182.
- Parkin, Michael (1988) "A Method for Determining Whether Parameters in Aggregative Models Are Structural," *Carnegie-Rochester Conference Series on Public Policy*, Vol. 29, No. 1, pp. 215-252.
- Samuelson, Paul (1937) "A Note on Measurement of Utility," *Review of Economic Studies*, Vol. 4, No. 2, pp. 155-161.

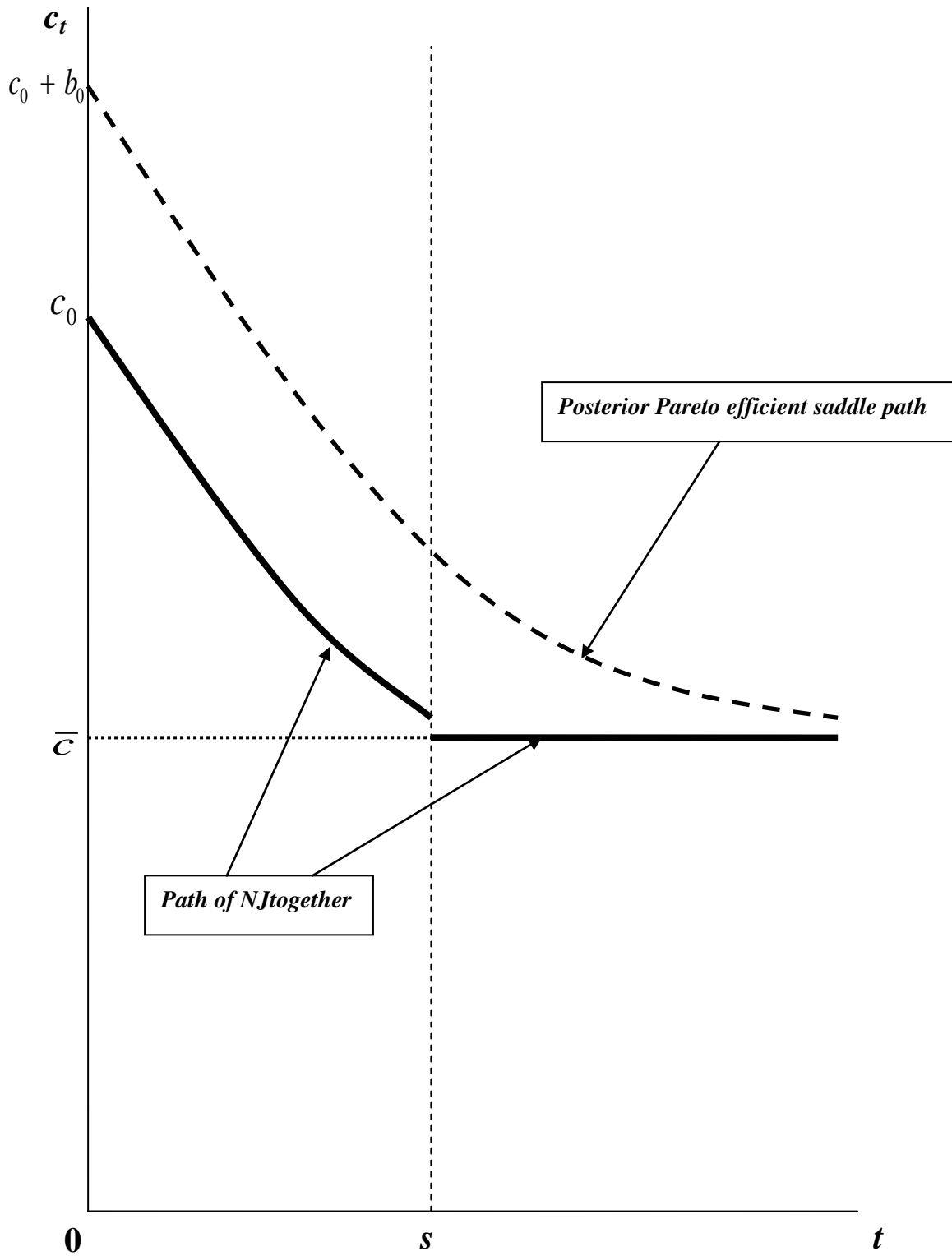
**Figure 1: A time preference shock**



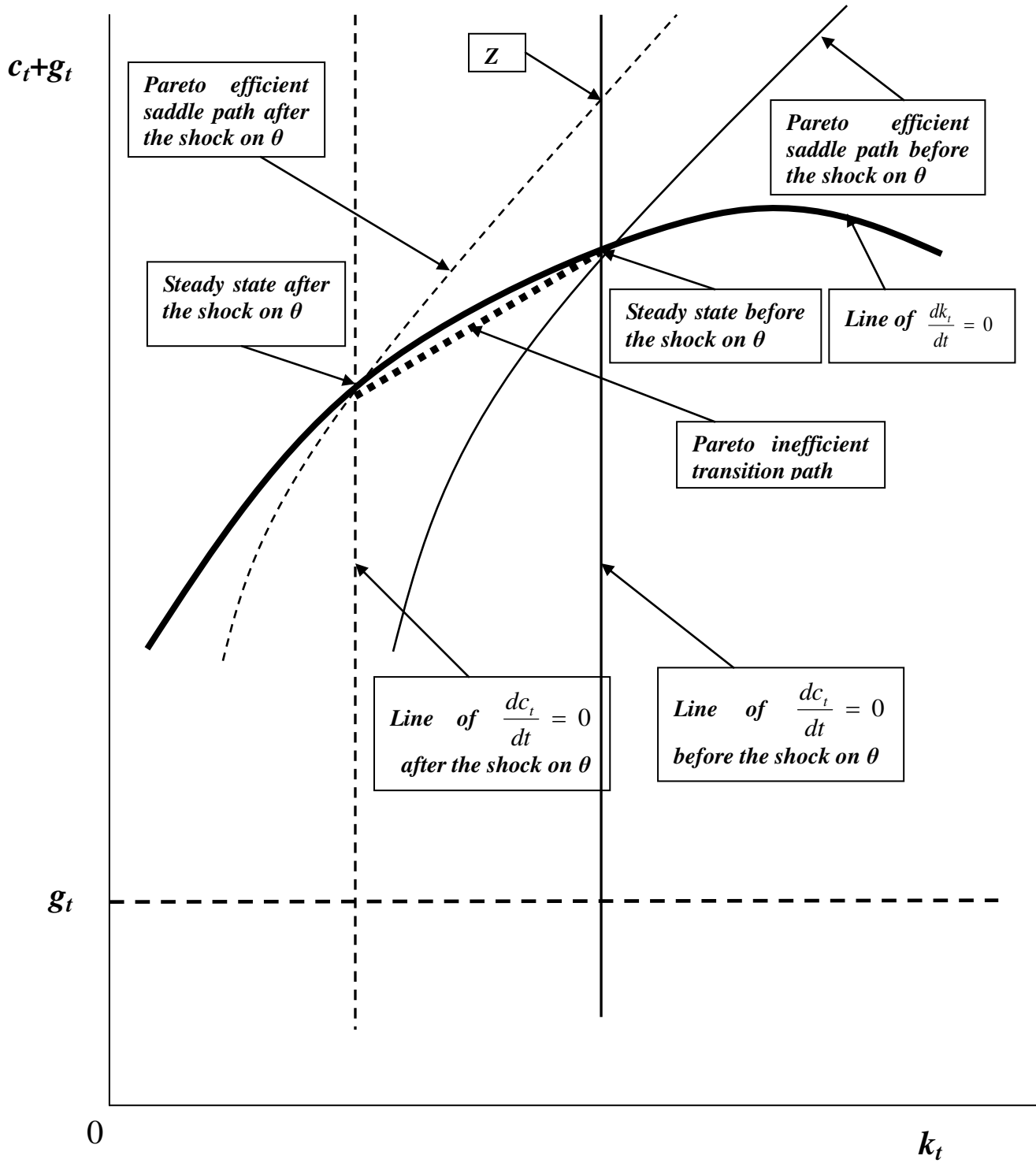
**Figure 2: The paths of *Jalone* and *NJalone***



**Figure 3: A Pareto inefficient transition path**



**Figure 4: A time preference shock in an economy with a government**



**Table 1 The payoff matrix**

		Any other household	
		<i>J</i>	<i>NJ</i>
Household A	<i>J</i>	$E_0(Jtogether), E_0(Jtogether)$	$E_0(Jalone), E_0(NJtogether)$
	<i>NJ</i>	$E_0(NJalone), E_0(Jtogether)$	$E_0(NJtogether), E_0(NJtogether)$