



Munich Personal RePEc Archive

Did Greenspan Open Pandora's Box? Testing the Taylor Hypothesis and Beyond

Nuno Palma

Nova School of Business and Economics

21. January 2013

Online at <http://mpa.ub.uni-muenchen.de/48197/>

MPRA Paper No. 48197, posted 10. July 2013 12:19 UTC

DID GREENSPAN OPEN PANDORA'S BOX? TESTING THE TAYLOR HYPOTHESIS AND BEYOND

Nuno Palma

Nova School of Business and Economics

Keywords: Business Cycle, Financial Crisis, Great Moderation, Monetary Shocks, Persistence

JEL codes: E32, E37, E43, E52, E58, E65

First version: September 19th, 2009. Current draft: January 21th, 2013 *

Word Count: < 2.000

ABSTRACT

The Taylor hypothesis is the conjecture that the 2007-2009 financial crisis and the 2008-present downturn have been caused by loose monetary policy during 2002-2006. According to the Taylor hypothesis the Fed deviated from well-know rules of monetary policy-making over this period, and this deviation caused an inefficient boom and subsequent bust. I use a well know economic model of the US aggregate economy (Christiano, Eichenbaum and Evans 2005) to test this hypothesis. I interpret shocks as deviations from Taylor-type rules. I conclude that the Taylor hypothesis for the Taylor rule fails to reproduce observed fluctuations in the data. Output increases only 0.3% at maximum which occurs at 2004:Q2. In the data, the output gap was at it's maximum in 2006:Q3. However, the Taylor hypothesis modified to incorporate persistence in the policy rule can partly explain the boom of the economy after 2001.

*I thank Luis Bryce, Larry Christiano and Carlos Madeira for helpful discussions. I am especially grateful to Daisuke Ikeda. The usual disclaimer applies.

1 Introduction

Recent house price increases are attributable mainly to economic fundamentals
Bernanke (2005)

John Taylor (2007, 2009 and elsewhere) has argued that the origins of the 2007-2009 financial crisis lie in deviations from the Federal Reserve System away from Taylor-style rules of monetary policy making during 2002-2006. In this paper I test for this hypothesis using the benchmark model in Christiano, Eichenbaum and Evans (2005), henceforth CEE (2005), imposing all functional forms, and two alternative Taylor-style policy rules. This is an appropriate choice because it is a consensual framework for empirically accurate simulated quantitative effects of monetary policy shocks. I calculate the magnitude of deviations from two Taylor-style rules for the 2002-2006 period. If these were the relevant shocks of the period, it is necessary to calculate variance decompositions, which is in line with the original Taylor hypothesis. Thus the model's counterfactual unshocked growth path constitutes a reasonable approximation to what the economy would have evolved like if the deviations had not existed. By feeding these shocks into the model, I observe whether they are sufficient to reproduce observed fluctuations in the data.

2 The Taylor Hypothesis

Much of the discussion about the recent crisis and downturn has focused on contemporary policy responses. The actual mechanism which lead to the crisis has been given less attention. For example, in order to study optimal policy response to the crisis, Gertler and Kardi (2011) interpret the origins of the crisis as a capital quality shock. While this shock-type interpretation can be useful from the static perspective of an immediate policy response, it clearly leaves the actual origins of the crisis as an open question.

Understanding the underlying cause of the crisis is important because it may suggest, first, how to prevent future occurrences; and second, how to best respond at present as well – especially in a way that doesn't create the "seeds" to a possible future crisis – a reasonable possibility in a dynamic context. Optimal policy responses derived from unmeasurable and exogenous shock-based interpretations to the source of the crisis ignore such possibility.

While many have blamed the crisis to inefficiencies in the market sector and insufficient regulation, John Taylor has been a prominent supporter of a different explanation:

During the period from 2003 to 2006 the federal funds rate was well below what experience during the previous two decades [would have predicted] ... with low money market rates, housing finance was very cheap and attractive ... the surge in housing demand led to a surge in housing price inflation ... this jump in housing price inflation then accelerated the demand for housing in a upward spiral (Taylor 2007)

Taylor's dating is that the departure initially occurred in the 2nd quarter of 2002 and ended in the 3rd quarter of 2006. The quantitatively stronger departure period was 2003-2005. A reasonable dating for the start of the crisis is August 2007. This was when the spread between the term LIBOR rates and the overnight Fed funds rare jumped and soon the Fed was taking unprecedented monetary policy measures which have to be considered subsequent deviations from rules-based monetary policy. Thus in order to be internally consistent our analysis ends in the 2nd quarter of 2007.

3 Model

The CEE (2005) model consists of five departures to the one sector DSGE model. These are: sticky wages and prices (Calvo-style nominal contracts, with lagged inflation indexation, i.e. dynamic price updating); adjustment costs in investment; habit dependent preferences; variable capital utilization; and a working capital channel (firms must borrow the wage bill). I also impose all of CEE's benchmark functional forms: separable preferences, logarithmic in consumption $u(\cdot) = \log(\cdot)$, quadratic in leisure $z(\cdot) = \psi_0(\cdot)$ and CES in real balances $v(\cdot) = \psi_q \frac{(\cdot)^{1-\sigma_q}}{1-\sigma_q}$, plus investment adjustment costs S of the form $F(i_t, i_{t-1}) = \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t$, $S(1) = S'(1) = 0$, $S''(1) > 0$, and two restrictions on the capital utilization function: $a(u_t)$ s.t. $u_t = 1$ in steady state, and $a(1) = 0$. The price updating rule I use is dynamic updating $\pi_t = \pi_{t-1}$. The equilibrium dynamics of the system (except for the description of monetary policy, which I discuss below) is described in the appendix. I close the model with our only deviation from the benchmark CEE model, the description of monetary policy.

3.0.1 Monetary Policy

Taylor rule (benchmark):

$$\hat{R}_t = 1.5E_{t-1}\hat{\pi}_t + 0.5\hat{y}_t + \varepsilon$$

Below I consider an alternative to this policy, but the above equation corresponds to the benchmark of the model, and arguably the closest description of the original Taylor hypothesis.

3.1 Solution Strategy

I adopt the standard sequence of markets equilibrium concept and the solution method in Christiano (2002). Briefly, this involves the following steps. First, I find the nonstochastic steady state, which is the steady state where all shocks are set to their means, and I show it is unique. It is to be noted that to a first order approximation, all Tak-Yun distortions are 1 (see Yun 1996).

Second, I take a linear approximation of the economy around the nonstochastic steady state. Third, I find a solution. A solution is a linear feedback rule relating the current period endogenous variables to a set of state variables. It consists of two matrices, the feedback part, with consists of endogenous state variables (controls of the previous period), and the feedforward part, generated by some exogenous stochastic process (shocks). Together these two determine the current period endogenous variables. The only shock I consider here is the shock to monetary policy, ε . Define a vector z_t which contains the 13 variables of the system which are endogenous at period t :

$$z_t \equiv [\hat{\pi}_t \quad \hat{q}_t \quad \hat{w}_t \quad \hat{c}_t \quad \hat{K}_{t+1} \quad \hat{m}_t \quad \hat{L}_t \quad \hat{R}_t \quad \hat{H}_t \quad \hat{u}_{c,t} \quad \hat{P}_{k',t} \quad \hat{I}_t \quad \hat{K}_t]$$

where $\bar{q}_t \equiv Q_t/P_{t-1}$, $\bar{w}_t = W_t/P_{t-1}$ and a hat denotes a deviation from the non-stochastic steady-state, i.e. $\hat{x}_t \equiv (x_t - x)/x$, with x corresponding to the nonstochastic steady-state of x_t .

I search for a solution of the form:

$$z_t = Az_{t-1} + B\varepsilon$$

where A is the feedback part and B the feedforward part.

3.2 Econometric Methodology

Again I follow the methodology of CEE (2005) to estimate and evaluate our model. The only exception is again regarding the treatment of monetary policy. I also use their benchmark estimations for the group of model parameters $(\lambda_f, \zeta_w, \zeta_p, \sigma_q, \kappa, b, \sigma_a)$.

3.3 Parameterization

A first group of parameters $(\beta, \phi, \alpha, \delta, \psi_0, \psi_q, \lambda_w)$ is set by calibration. $\beta = 1.03^{-0.25}$, $\alpha = 0.36$, $\delta = 0.025$, $\mu = 1.017$, $\lambda_w = 1.05$. The economic rationale of such parameterization is described in CEE (2005). I also calculate the fixed cost parameter ϕ such that steady-state profits are zero, ψ_0 such that the steady-state value of $L = 1$ and ψ_q such that the steady state value of the ratio Q/M is 0.44. See CEE (2005) for discussion and details. For monetary policy, I use the Taylor rule as described above. Finally, the third group of parameters $(\lambda_f, \zeta_w, \zeta_p, \sigma_q, \kappa, b, \sigma_a)$ should minimize some measure of the distance between the model and empirical impulse response functions. For the present purpose I will take as a first approximation CEE's estimates from their own model (which differs in the monetary policy specification). These are $\lambda_f = 1.20$, $\zeta_w = .64$, $\zeta_p = .6$, $\sigma_q = 10.62$, $\kappa = 2.48$, $b = .65$, $\sigma_a = .01$.

4 Results

In this section, I interpret shocks as deviations from the Taylor rule. I feed these shocks on the model and compare the counterfactual response with actual fluctuations in the data. Thus I estimate the magnitude of those shocks for the 2002-2006 period. I use Taylor (2007)'s own estimation: $R_t = 1 + 1.5\pi_t + 0.5\hat{y}_t$ where π_t is CPI inflation rate and \hat{y}_t is the deviation from a Hodrick-Prescott filter.

4.1 Pure Taylor Rule

Consistent with Taylor's hypothesis, I assume the relevant shocks to the economy in the 2002-2006 period where shocks to monetary policy in the form of deviations from the Taylor rule. For this reason I don't calculate variance decompositions. I calculate the magnitude of these shocks for the period, using Taylor's own approach, i.e. backing it out of the Taylor rule. I use official Federal Reserve Economic Data (FRED) data for the actual federal funds rate and consumer price index. For the output gap I use Congressional Budget office (CBO) data.

In the first column of table 1 I present the shocks that the data suggests existed to the rule of the form:

$$\hat{R}_t = 1.5E_{t-1}\hat{\pi}_{t+1} + 0.5\hat{y}_t + \varepsilon$$

I feed these shocks into the CEE model described above. The result of this simulation is presented on figure 1, which shows, for each variable of interest, the percentage deviation from steady state values. The impulse response functions (IRFs) are on figure 2. The simulation is defined relative to the counterfactual unshocked growth path, or what the economy would have looked like without the deviations.

Figure 1 here

Figure 2 here

The policy shock does not generate a boom in output. Output increases only 0.3% at maximum. The timing is also off. In the data, the output gap was at its maximum in 2006:Q3. In this simulation, the output was its maximum after 10 quarters, thus 2004:Q2. Output response is not persistent and strong under the pure Taylor rule. This is consistent with the IRFs in figure 2, which show little persistence and amplification.

4.2 CGG Taylor-type rule

I now consider an alternative treatment of monetary policy. I still interpret deviations as a sequence of shocks to a static characterization of monetary policy. However now I allow these shocks to be correlated. I adopt the Clarida et al (1999), post-1979 monetary policy characterization, which is a Taylor-style rule of the form,

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho)[a_\pi E_{t-1} \hat{\pi}_{t+1} + a_y \hat{y}_t] + \varepsilon_t$$

where ε_t is a shock. I parameterize as Clarida et al (1999): $\rho = 0.80$, $a_\pi = 1.5$, and $a_y = 0.5$. The simulation is shown on figure 3, with the corresponding IRFs on figure 4.

Figure 3 here

Figure 4 here

This time there is persistence and amplification. The magnitude is also much larger than before: now in the peak the output is more than 5% larger than its steady state level. Second, the peak of output is now delayed to 16th period, 2005:Q4. The Taylor hypothesis modified to incorporate persistence can explain to a large degree the boom of the economy after 2001.

5 Conclusion

The Fed deviated from well-know rules of monetary policy making during 2002-2006. These deviations started in early 2002 and peaked early to mid 2004, when the Fed funds rate was about 1% while the Taylor rule suggested 4.2%. The deviations lasted until 2006 and their cumulative magnitude was substantial.

Evidence from the CEE model combined with these shocks suggests the following. For a pure Taylor rule, the deviations are too small to generate the observed fluctuations and timing of the data. For this simulation, the policy shock does not generate a boom in output. Output increases only 0.3% at maximum. The timing is also wrong. In the data, the output gap was at its maximum in 2006:Q3. In this simulation, the output was its maximum after 10 quarters, thus about 2004:Q2. Output response is not persistent and strong under the pure Taylor rule. The impulse response functions lack persistence and amplification.

For a persistence-augmented Taylor rule of the Clarida-Gali-Gertler type, the simulated response explains better the magnitude of the fluctuations. Following the deviations output booms to more than 5% its steady state level, and the the peak is delayed to 2005:Q4. This version can help explaining the post-2001 boom and bubble (i.e the above steady-state growth) build-up.

So what remains to be explained? In the simulation output it is already decreasing during 2006, but in the data it is was still rising until 2006:Q3, and then it crashed about a year later. Further, the crash is more sudden in 2007 and again in 2008 in the data than in the model where the downturn is smoother. One possibility is that market participants interpreted successive deviations as a regime shift and hence adjusted their expectations about the Fed's behavior. Agents may have believed the Fed had come to care more about stabilization and market growth and less about inflation relative to what the CGG rule suggests. Indeed the "Greenspan put" was an expression widely well-know in the financial markets which suggests that agents started perceiving that each time the stock market went down, the Fed would react (Ravn 2012). This must have had an effect on asset pricing, narrower credit spreads, and more risk taking. If so this can explain why the rise in the Federal Funds rate starting in 2004 did not lead to an increase in the long-term rates (the 2005 "Greenspan conundrum"), why output kept rising until late 2006, and the suddenness of the crash in 2007 – as agents go back to the original expectations about the behavior of the Fed. Under the possibility that what may look like several shocks to the rule actually reflected a change in preferences of the central bank, and that this change was eventually internalized by the agents (or at least that a change was perceived by the agents even if in fact the shocks were additive), the correct treatment would be a change in the parameters of the rule itself, as opposed to an additive shock. This is a relevant possibility, but one which is beyond the more humble scope of this paper.

References

- [1] Bernanke, B. (2009). The Crisis and the Policy Response. Stamp Lecture, LSE. Available at <http://www.federalreserve.gov/newsevents/speech/bernanke20090113a.htm>
- [2] Christiano, L. (2002). Solving Dynamic Equilibrium Models by a Method of Undetermined Coefficients. *Computational Economics*, 20 (October): 21-55.
- [3] Christiano, L., M. Eichenbaum and C. Evans (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113:1.
- [4] Clarida, R., J. Galí, M. Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature*, Vol. 37, No. 4: 1661-1707.
- [5] Gertler and Kardi (2011). A Model of Unconventional Monetary Policy. *Journal of Monetary Economics*. 58(1): 17–34
- [6] Ravn, S. (2012). Has the Fed Reacted Asymmetrically to Stock Prices? *B.E. Journal of Macroeconomics*, vol. 12:1 (Topics), article 14.
- [7] Taylor, J. (2007). Housing and Monetary Policy: Proceedings of Jackson Hole Symposium. Federal Reserve Bank of Kansas City.
- [8] Taylor, J. (2009b). Getting Off Track: How Government Actions and Interventions Caused, Prolonged, and Worsened the Financial Crisis. Hoover Institution Press Publication.
- [9] Yun, T. (1996). Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles. *Journal of Monetary Economics*, 37(2): 345-370.

Figures

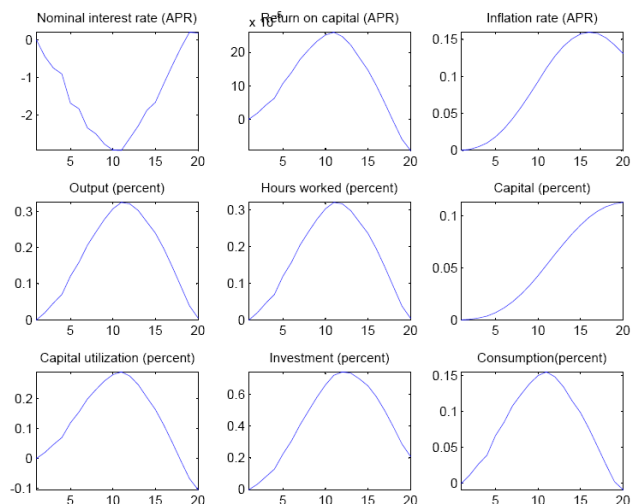


Figure 1. Percentage deviations from steady state (pure Taylor rule case)

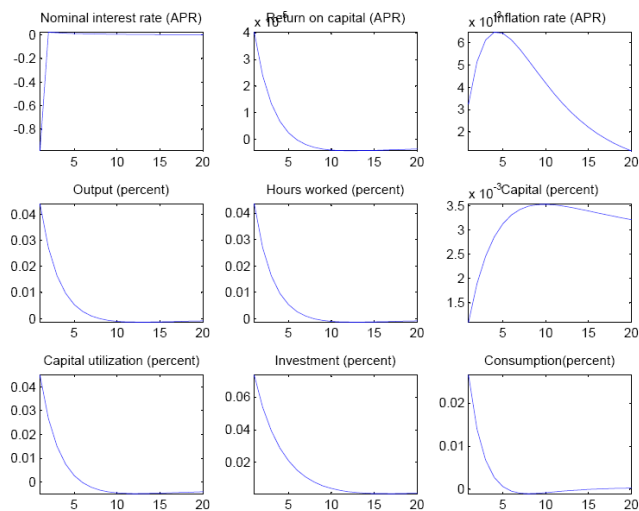


Figure 2. Impulse response functions to an additive shock (pure Taylor rule case)

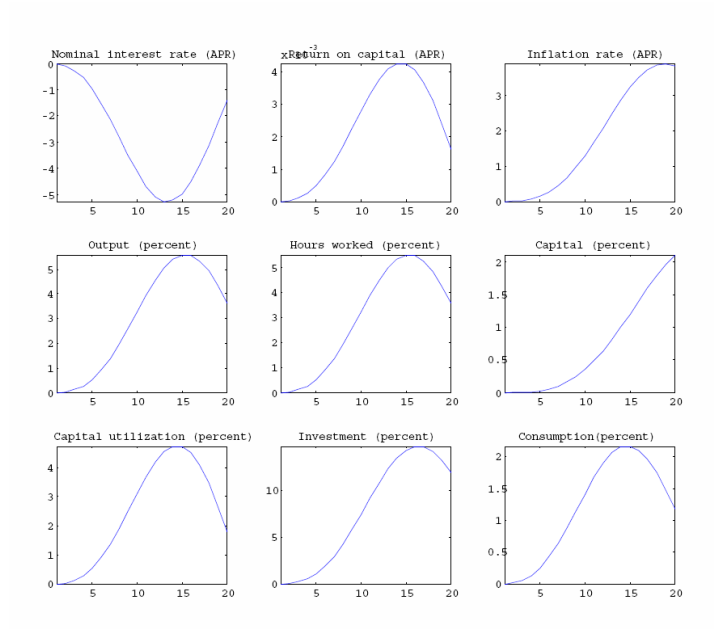


Figure 3. Percentage deviations from steady state (Clarida-Gali-Gertler Taylor-type rule)

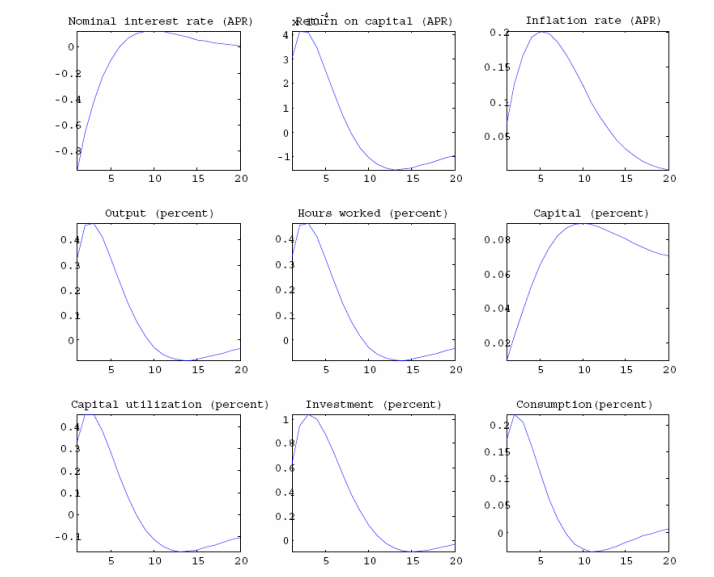


Figure 4. Impulse response functions to an additive shock (Clarida-Gali-Gertler Taylor-type rule)

Quarterly deviations from the Taylor rule

	Taylor	CGG
02:Q1	0.00	0
02:Q2	-0.45	-0.09
02:Q3	-0.77	-0.23
02:Q4	-0.95	-0.37
03:Q1	-1.76	-0.65
03:Q2	-1.94	-0.91
03:Q3	-2.49	-1.22
03:Q4	-2.69	-1.52
04:Q1	-3.00	-1.81
04:Q2	-3.20	-2.09
04:Q3	-3.24	-2.32
04:Q4	-2.94	-2.44
05:Q1	-2.62	-2.48
05:Q2	-2.21	-2.43
05:Q3	-1.99	-2.34
05:Q4	-1.47	-2.16
06:Q1	-0.96	-1.92
06:Q2	-0.46	-1.63
06:Q3	-0.01	-1.31
06:Q4	0.00	-1.05

Table 1. Quarterly deviations from the Taylor rule

Model description

Households

Consumption Euler equation (c_t FONC):

$$E_{t-1}u_{c,t} = E_{t-1}\psi_t \quad (1)$$

where marginal utility is,

$$u_{c,t} = \frac{1}{c_t - H_t} - \beta E_t b \frac{1}{c_{t+1} - bH_{t-1}} - v \left(\frac{\pi_t}{m_t} \right)^{1-\sigma_q} c_t^{-\sigma_q} \quad (2)$$

Deposits/Bonds/M Euler equation (Fisher) (M_{t+1} FONC or "Fisher equation"):

$$\psi_t = \beta E_t \psi_{t+1} \frac{R_{t+1}}{\pi_{t+1}} \quad (3)$$

Euler equation for real/cash balances (i.e. Q_t FONC or "money demand", after imposing the $v(\cdot) = \psi_q \frac{(\cdot)^{1-\sigma_q}}{1-\sigma_q}$ functional form):

$$\psi_q q_t^{-\sigma_q} + \psi_t = \psi_t R_t \quad (4)$$

Euler equation for \bar{k}_{t+1} :

$$E_{t-1}\psi_t = \beta E_{t-1}\psi_{t+1} \left[\frac{u_{t+1} r_{t+1}^k - a(u_{t+1}) + P_{k',t+1}(1-\delta)}{P_{k',t}} \right] \quad (5)$$

Euler equation for investment (i_t FONC) after imposing the $F(i_t, i_{t-1}) = \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t$, $S(1) = S'(1) = 0$, $S''(1) > 0$ functional form:

$$E_{t-1}\psi_t = E_{t-1} [\psi_t P_{k',t} F_{1,t} + \beta \psi_{t+1} P_{k',t+1} F_{2,t+1}] \quad (6)$$

Note that the functional form implies that in a nonstochastic steady state, $F_1 = 1$, $F_2 = 0$ which leads to the condition $P_{k',t} = P_{k',t+1} = 1$.

Euler equation for capital utilization (u_t FONC):

$$E_{t-1}\psi_t [r_t^k - a'(u_t)] = 0 \quad (7)$$

The functional form restriction that $u_t = 1$ in steady state then implies $a' = r^k$.
Habit evolution equation:

$$H_t = \psi H_{t-1} + bc_{t-1} \quad (8)$$

Calvo wages (imposing the demand curve for $h_{j,t}$ into the \tilde{W}_t FONC, and substituting):

$$\left[\frac{1 - \zeta_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1 - \zeta_w} \right]^{1-\lambda_w} = \psi_L \frac{K_{w,t}}{F_{w,t}} \quad (9)$$

where,

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}} = \frac{\tilde{w}_t}{\tilde{w}_{t-1}} \pi_t \quad (10)$$

and \tilde{w}_t relates to w_t :

$$w_t = \left[\frac{\psi_L F_{w,t}}{\tilde{w}_t K_{w,t}} \right]^{\frac{1-\lambda_w}{1-(1+\lambda_w)\sigma_L}} \quad (11)$$

and the laws of motion,

$$F_{w,t} = l_t \frac{\psi_t}{\lambda_w} + \beta \zeta_w \left(\frac{1}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\tilde{\pi}_{w,t}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} \quad (12)$$

$$K_{w,t} = l_t^{1+\sigma_L} + \beta \zeta_w \left[\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right]^{\frac{\lambda_w}{1-\lambda_w} (1-\sigma_L)} K_{w,t+1} \quad (13)$$

Firms

Calvo pricing (\tilde{P}_t FONC, and dividing by P_{t-1} and rearranging):

$$\frac{1 - \zeta_p \left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_p}}}{(1 - \zeta_p)} = \left(\frac{K_{p,t}}{F_{p,t}}\right)^{\frac{1}{1-\lambda_p}} \quad (14)$$

where,

$$E_t \left[\psi_t y_t + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_p}} \beta \zeta_p F_{p,t+1} - F_{p,t} \right] = 0 \quad (15)$$

and

$$E_t \left[\lambda_f \psi_t y_t s_t + \beta \zeta_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} - K_{p,t} \right] = 0 \quad (16)$$

where output is \bar{k}_{t-1} instead of k_t reflects adjustment costs,

$$y_t = (p_t^*)^{\frac{\lambda_f}{\lambda_f-1}} \left[\bar{k}_{t-1}^\alpha \left((w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t \right)^{1-\alpha} - \phi \right] \quad (17)$$

where capital services k_t are related to physical capital \bar{k}_t through a utilization rate:

$$k_t = u_t \bar{k}_t \quad (18)$$

Updating Rules

$$\tilde{\pi}_t = \pi_{t-1} \quad (19)$$

$$\tilde{\pi}_{w,t} = \pi_{w,t-1} \quad (20)$$

Loan Market Clearing

$$W_t L_t = \mu_t M_t - Q_t$$

After dividing by P_{t-1} :

$$\bar{w}_t L_t = \mu_t m_t - q_t$$

where (noting $m_t = \frac{M_t}{P_{t-1}}$):

$$\mu_t = \frac{M_t}{M_{t-1}} = \frac{m_t P_{t-1}}{m_{t-1} P_{t-2}} = \frac{m_t}{m_{t-1}} \pi_{t-1}$$

so,

$$\bar{w}_t L_t = \left(\frac{m_t}{m_{t-1}} \pi_{t-1} \right) m_t - q_t \quad (21)$$

Aggregate Resource Constraint

$$a(u_t)\bar{k}_t + c_t + i_t \leq y_t \quad (22)$$

where

$$h_t = l_t(w_t^*)^{\frac{\lambda_w}{1-\lambda_w}} \quad (23)$$

Capital transition law:

$$\bar{k}_{t+1} = (1 - \delta)\bar{k}_t + F(i_t, i_{t-1}) \quad (24)$$

Other conditions

The rental rate of capital

$$r_t^k = s_t \alpha \left(\frac{l_t}{k_t} \right)^{1-\alpha} \quad (25)$$

and marginal cost

$$s_t = \frac{\tilde{w}_t}{(1 - \alpha)} \left(\frac{(w_t^*)^{\frac{\lambda_w}{1-\lambda_w}} h_t}{k_{t-1}} \right)^\alpha \quad (26)$$

Two Distortions

$$\left[(1 - \zeta_p) \left(\frac{1 - \zeta_p \left(\frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \zeta_p} \right)^{\lambda_f} + \zeta_p \left(\frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]^{\frac{1-\lambda_f}{\lambda_f}} = p_t^* \quad (27)$$

$$\left[(1 - \zeta_w) \left(\frac{1 - \zeta_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \zeta_w} \right)^{\lambda_w} + \zeta_p \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}} = w_t^* \quad (28)$$

This is a under-determined system of 29 variables in 28 equations. The variables are $u_{c,t}, \psi_t, c_t, H_t, \pi_t, m_t, R_{t+1}, q_t, u_{t+1}, r_{t+1}^k, P_{k,t+1}, \pi_{w,t}, k_{w,t}, F_{w,t}, \tilde{w}_t, K_{p,t}, F_{p,t}, Y_t, \bar{k}_t, \bar{w}_t, i_t, l_t, k_t, s_t, r_t, \tilde{\pi}_t, \tilde{\pi}_{w,t}, p_t^*, w_t^*$. The description of monetary policy closes the system.