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Algorithm of construction of Optimum Portfolio of stocks using Genetic Algorithm

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Abstract:

The objective of this paper is to develop an algorithm to create an Optimum Portfolio from a large pool of stocks listed in a single market index SPX 500 Index: USA (for example) using Genetic Algorithm. The algorithm selects stocks on the basis of a priority index function designed on company fundamentals, and then genetically assigns optimum weights to the selected stocks by finding a genetically suitable combination of return and risk on the basis of historical data. The effect of genetic evolution on portfolio optimization has been demonstrated by developing a MATLAB code to implement the genetic application of reproduction, crossover and mutation operators. The effectiveness of the obtained portfolio has been successfully tested by running its performance over a six month holding period. It is found that genetic algorithm is successful in providing the optimum weights to stocks which were initially screened through a predetermined priority index function. The constructed portfolio beats the market for the considered holding period by a significant margin.

Keywords: Optimum Portfolio, Genetic Algorithm, Portfolio Construction, MATLAB

Algorithm for construction of Optimum portfolio of stocks using Genetic Algorithm

1. Introduction:

The process of genetic evolution which has been verified by the laws of nature since the beginning of earth is proven to be the most intricate and beautiful optimization technique. The key elements of Genetic Algorithm are:

- i) Creation of Chromosomes
- ii) Initiating Parent Species
- iii) Creation of Initial Population
- iv) Reproduction Pool on the basis of Fitness Selection
- v) Genetic Operators: Crossover
- vi) Mutation Operation

This study applies the very concepts of above genetic evolution in constructing an optimum portfolio of stocks selected from a large pool of stocks listed in the single market index. We have put in our utmost efforts to contribute significant analytical conclusion to the application of genetic algorithm on the issue of optimization of weights of stocks selected in the portfolio. The algorithm for portfolio construction involves two stages - selection of stocks by using a priority index function and optimization of the weights of the selected stocks. The process initially selects stocks for the portfolio on the basis of fundamentals of the company using a priority index function and then it optimizes the weights of the selected stocks by a genetic approach, where the selected stocks were allowed to genetically evolve towards the fittest population taking into account both risk and return of stocks. A systematic and computational mode of Darwinian evolution has been applied in this research paper. The optimization of stocks will be adjusted as per the stocks undergo a genetic evolution through the process of reproduction and mating using crossover and mutation.

1.1 Genetic Algorithm

The entire process of optimization by Genetic Algorithm is explained in the following steps:

- i) The process was initiated by starting a random solution called 'Population' which consists of chromosomes.
- ii) Each chromosome represents a solution of the problem with genes representing the weight given to each selected stock.
- iii) The genes of the chromosome are string (not necessarily binary) of symbols, in this case it is representing the randomly assigned weights of selected stocks.

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- iv) These chromosomes are then made to evolve genetically through iterations giving rise to newer populations.
- v) We compare the fitness values of evolving chromosomes with the existing chromosomes, after each iteration. The objective function used to calculate fitness is described in the following section.
- vi) In this study we evaluate the objective function over a large population size such that the objective function attains the maximum possible value.
- vii) After all the iterations are over, we calculate the fitness value of each of the chromosomes and select the chromosome which attains the maximum fitness value , in accordance with Darwin's (1859)theory of survival of the fittest.
- viii) This selected chromosome shall represent the optimization of the weights of the stocks giving us genetically suitable return and risk.
- ix) This genetic procedure is designed to maximize the fitness value under the constraint that the sum of the weights is 1 and that all the weights are less than 1 and greater than zero.

The following diagram explains the above Genetic Evolution Mechanism:

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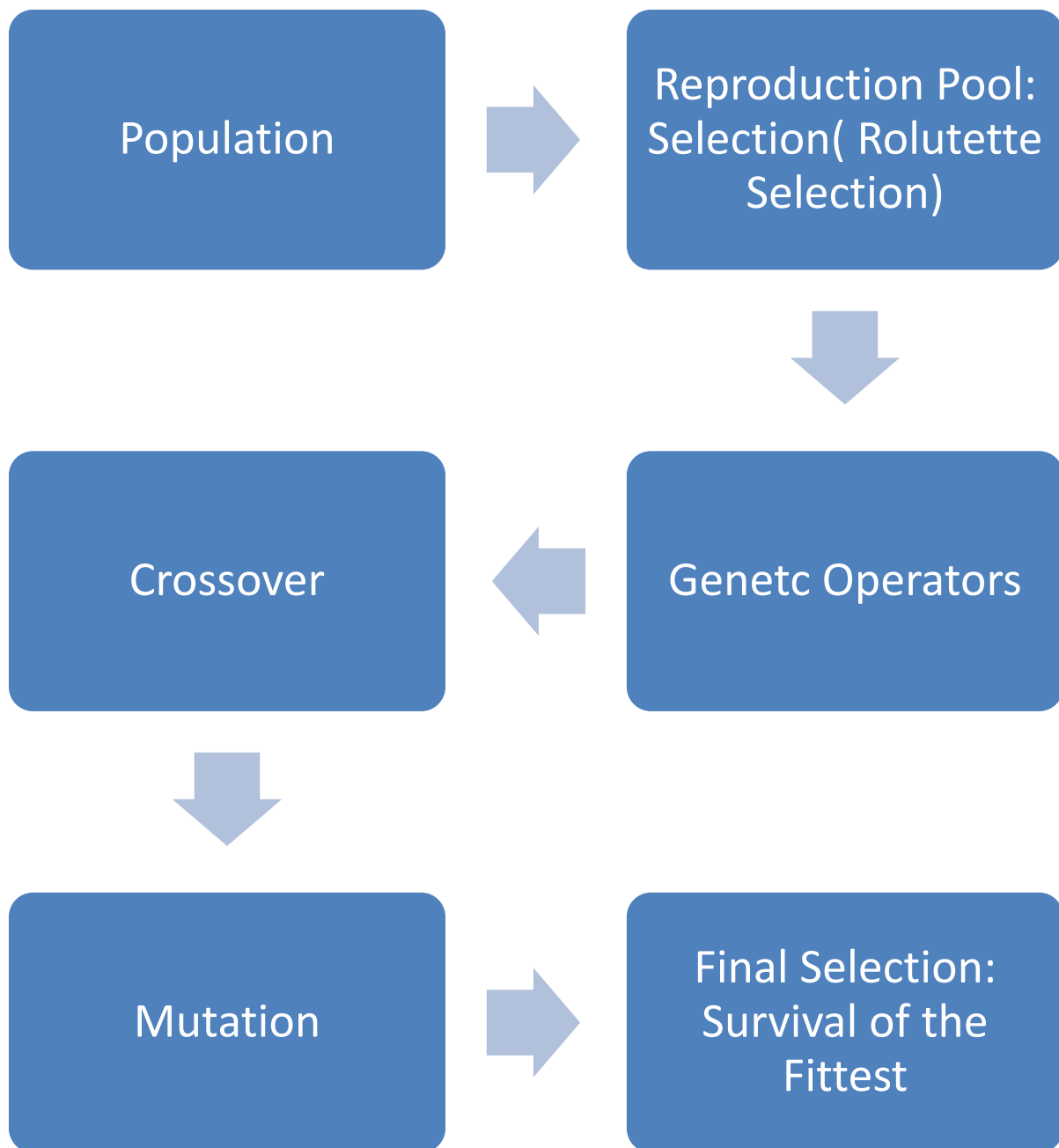


Figure 1: Genetic Evolution Mechanism

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The following various functions are used in the algorithm:

i) Objective Function

Maximize Risk and simultaneously Minimize Risk

$$\text{Portfolio Return} = \sum w_i r_i$$

$$\text{Portfolio Variance } (\sigma_p^2) = \sum w_i^2 \sigma_i^2 + \sum \sum 2 * w_i * w_j * \text{cov}(I, j)$$

$$\text{Such that } \sum w_i = 1$$

Where

w_i = weight of i th stock

R_i = average return of i th stock

$$f(\text{chromosome}) = \text{Portfolio Return} / \text{Portfolio Standard Deviation}$$

Where:

w_i = weight of i th stock

w_j = weight of j th stock

$f(\text{chromosome})$ = fitness value of the chromosome

$\text{Cov}(I, j)$ = Covariance of Returns between the i th and j th stocks

ii) Evaluation Function

This function shall be used to create the reproduction pool from the parent population. The evaluation function is similar to the probability which decides the fitness of the species. It is the ratio of the fitness value of the i th chromosome to the sum of fitness values of all the chromosomes.

$$f(\text{evaluation } i) = f(\text{chromosome } i) / \sum f(\text{chromosome } i)$$

1.2 Population Initiation

- i) A population shall consist of chromosomes which shall be made up of genes. A gene is representative of values which shall return weights of the selected stocks.
- ii) The gene is constructed by random allocation of values using the normal probability distribution.

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- iii) These obtained random values (V_i) are now used to calculate weights (w_i) of the selected stocks by calculating $V_i/(\sum V_i)$. These weights are the stock wise percentages of the total investment which the portfolio manager shall recommend to the user.
- iv) A parent population of 10 random chromosomes representing 10 random initial solutions to the objective function is created.

1.3 Reproduction Pool: The method of selection

- i) The method shall decide the reproduction pool that is the chromosomes which shall undergo mating. There are several available techniques like: Tournament Selection, Roulette Wheel Selection, Stochastic Based Selection and Reward Based Selection. In this paper we have used the Roulette Wheel Selection (also known as the Fitness Proportionate Selection) in order to select the species on the basis of their fitness values mainly.

Roulette Selection:

- ii) Each chromosome is evaluated on the basis of the fitness function (as mentioned above).
- iii) A random number (r_i) is generated from normal distribution and is compared with the chromosome's cumulative probability. The chromosome i having cumulative probability such that $p_{(i-1)} < r_i < p_i$ is selected into the reproduction pool.
- iv) The reproduction pool shall consist of species which have been selected by the Roulette Selection Method defined above.

1.4 Genetic Mating

Crossover:

Crossover can be performed by a variety of methods:

- a) Single Point Crossover
- b) Two Point Crossover
- c) Uniform Crossover
- d) Heuristic Crossover

In our research we have used Arithmetic Crossover due to the proven accuracy of the same.

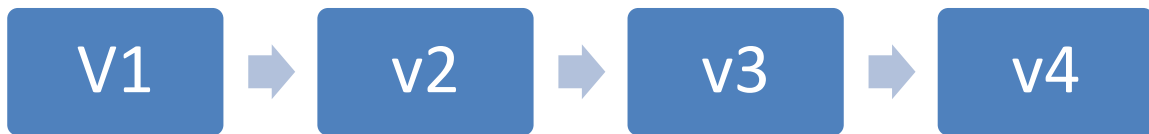
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$$\text{Offspring 1} = a \cdot \text{CH1} + (1-a) \cdot \text{CH2}$$

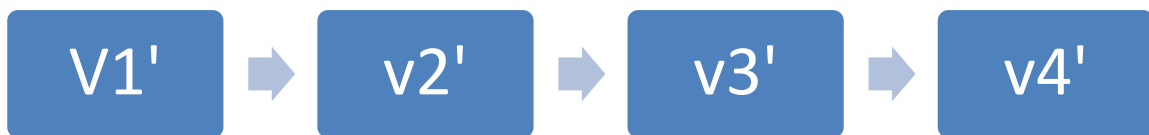
$$\text{Offspring 2} = (1-a) \cdot \text{CH1} + a \cdot \text{CH2}$$

Where a = Any random number belonging to $(0,1)$

Parent Chromosome 1 (CH1):

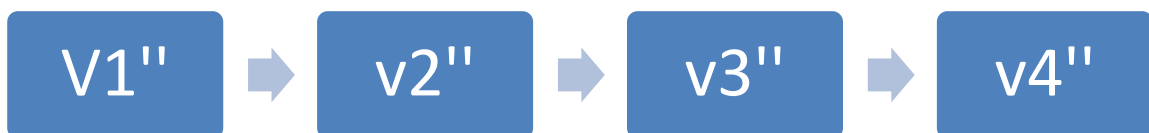


Parent Chromosome 2 (CH2):



Chromosome 1 X Chromosome 2

Offspring 1



Offspring 2

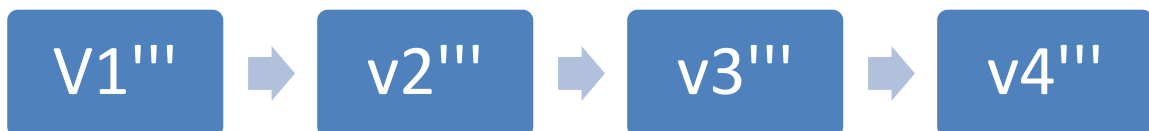


Figure 2: Arithmetic Crossover

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The new values(v'' and v''') can be calculated by the formula given above, where a is a random number between 0 and 1. The Figure above represents arithmetic point crossover and describes how two parent chromosomes mate by crossover to give the two off springs. Each of these values shall now give different weights to the stocks 1,2,3..n, Hence allowing genetic evolution by crossover.

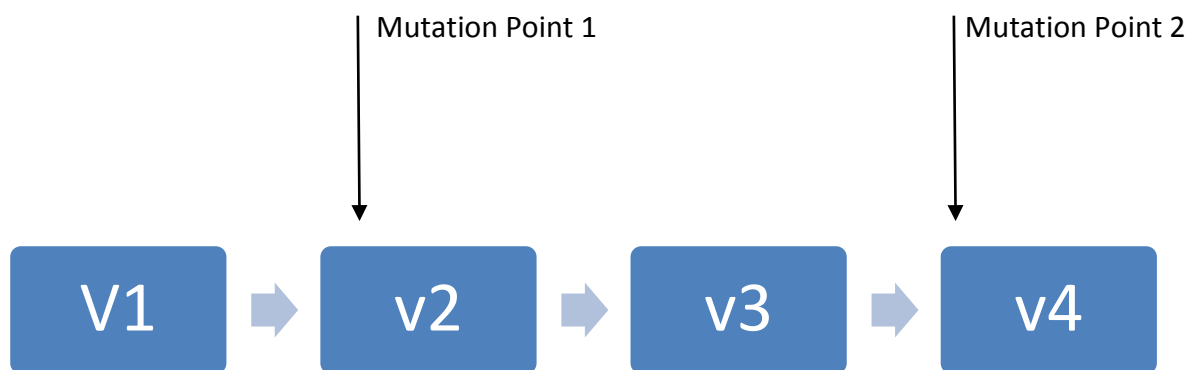
Where:

$$V'' = a*V + (1-a)*v'$$

$$V''' = (1-a)*v + a*v'$$

Mutation:

Parent Chromosome



Offspring

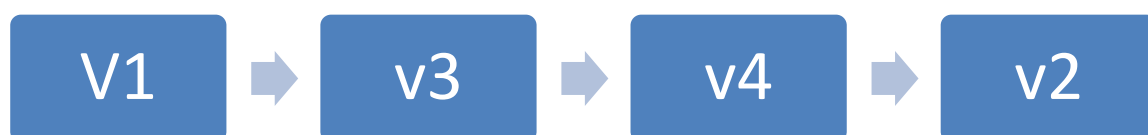


Figure3 : Mutation

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The gene at the mutation point 1 shall move to the mutation point 2 while the others shall shift a position each towards the initial position of mutation point 1.

1.5 Decision of Crossover versus Mutation

This is a question which has seldom been addressed when dealing with a large data set, the problem of how to understand which species shall undergo cross-over and which shall decide to mutate.

We considered the following points before deciding on the approach to tackle this issue:

- i) The solution depends on the nature of problem to a large extent. The solution should be oriented in such a way that the genesis contributes to evolution of the species selecting the fittest species which is decided on the basis of fitness function.
- ii) Crossover is a primarily explorative procedure, it accommodates the features of both the parents and creates a chromosome somewhere in between the parents.
- iii) Mutation is exploitative; it only creates a slight diversion with the parent and hence alters the feature locally.
- iv) Only crossover can combine information from two parents where as mutation shall introduce new information into the offspring. Hence we need a lucky mutation for a perfect genesis.
- v) The paper has found on the basis of probabilistic approach that a mutation probability of $P_m=0.4$ and a crossover probability of $P_c=0.6$ shall yield the optimum solution.
- vi) In case of identical chromosomes being selected in the reproduction pool, the stocks shall mutate in order to obtain a genetic advantage for the next population.

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2. Literature Review

Markovitz (1952), the father of Modern Portfolio theory, has established the role of combining different assets to minimize risk of the portfolio constructed via his publication. We have incorporated his conclusions in our research by keeping diversity a factor while constructing the portfolio. We achieved this by dividing the initial pool of stocks into sectors allowing the representation of each sector in the portfolio thus reducing the risk.

Melanie M. (1998) and M.Gen and R. Cheng (1997) explains the process of genetic algorithm in detail. They have presented various mathematical models for applying evolutionary genetic technique.

Lin and Gen (2007) stresses on the importance of taking risk as well as return into consideration while portfolio selection. The paper suggested that the multi stage genetic algorithm can be used for the portfolio optimization. Pereira (2000) has suggested that Genetic Algorithms are a valid approach to optimization problems in finance. Yang (2006) advocates that Genetic Algorithms can be used to improve the efficiency of the portfolio. Bakhtyar et al (2012) discussed that the choice of crossover and mutation influence the Genetic Algorithm performance. Seflane and Benbouziane(2012) shows that arithmetic crossover is genetically better than single point and two point crossover using an example of five stocks, while considering both return and risk in the objective function.

Sinha and Goyal (2012) develops an algorithm for the portfolio construction from a large pool of stocks listed in a single market index SP CNX 500 using MATLAB code.

Motivation

From review of the earlier research done in this area, it is concluded that the portfolio optimization was not done on an exhaustive genetic scale. An application of a detailed genetic evolutionary algorithm, for constructing an optimum portfolio of stocks selected from a large pool of stocks, is demonstrated in this study. A Priority Index Function is designed to select the stocks on the basis of company fundamentals, and then Genetic Algorithm is used to optimize the weights of the selected stocks. We also test the effectiveness of the Genetic Algorithm for portfolio construction. Steps have been taken to address the practical issues faced during the detailed implementation of genetic algorithm. An effective MATLAB code has been developed for this purpose.

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3. Portfolio Construction: Algorithm

The MATLAB code for the entire process of portfolio construction, as described below, is given in the appendix of this paper.

3.1 Stock Selection Procedure:

1. Diversification of Portfolio

The most vital logic behind our portfolio selection is the inclusion of Diversity in our portfolio. We have achieved this by dividing the pool of stocks into sectors namely IT, Telecom, Automobile et al. Now the stocks will be evaluated on the parameters mentioned below using the criteria of maximum and minimum of that sector. This implies that stocks will get selected if their performance score is high in that sector, and thus only best performers from each sector would comprise of our portfolio resulting in a diversified portfolio.

2. Parameters for Selection

The fundamental factors are those which shall allow the user to construct a portfolio based on business performance principle rather than focusing merely on market sentiments. The following factors are essential for determining the stocks in the final portfolio:

I. Price/Earnings (P/E)

The P/E ratio is an important parameter for understanding the earnings per money invested. A P/E ratio of x shall imply an investment of x units of money for unity profit. Generally, an investor shall prefer to choose stocks which have a lower P/E ratio.

II. Earnings/Share (EPS ratio)

Earnings/Share is defined as the portion of company's profit allotted to each outstanding share of common stock. From an investor's perspective, a higher EPS is desirable.

III. Wealth Creation

Wealth creation is defined as the difference between return on invested capital and the weighted cost of average capital. The investor shall choose stocks of those companies having a higher Wealth creation for investing into stocks.

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IV. Undervaluation

Undervaluation is defined as the situation when the stocks of the company are priced such that the market price is lower than the fair price. An investor shall always look to 'pick up' undervalued stocks. This is measured by the Market Capitalization to Revenue ratio. If the value of this ratio is less than 1, the stock is considered to be undervalued.

V. (Price is to Earning) / Growth (PEG ratio)

We have used this parameter to represent a significant comparison between companies having different Price/Earnings Ratios and Growth Percentages. An investor shall always want to invest into businesses having lower PEG values.

The stocks are selected on the basis of their performance in all these parameters on the basis of historical data. The priority function is such designed that stocks having highest score shall be selected into the pool of stocks comprising the portfolio.

Calculation of Priority Index

The Priority Index Function uses the factors mentioned above as the benchmark, and the stocks are ranked on the basis of their cumulative scores as generated by the Priority Index Function. Equities are ranked and allotted a score as per linear interpolation which can be expressed as follows:

$$S_{ij}=100(X_{ij}-Max)/ (Min-Max)$$

Where:

S_{ij}=Score of *i*th stock on *j*th parameter
X_{ij}=Functional Value of *i*th stock on *j*th function
Min=Minimum Value of *i*th stock on *j*th function
Max=Maximum value of *i*th stock on *j*th function

The formula for values whose minimum is desired is :

$$S_{ij}=100(X_{ij}-Min)/ (Max-Min)$$

The Priority Index (PI)_{*i*} is calculated by the summation of all scores each stock has earned on all the parameters.

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$$PI_i = \sum S_{ij}$$

The stock selection is based on the Priority Index (PI_i) and the algorithm shall create the portfolio from only those stocks which have a Priority Index greater than 3.8 on a 5 point scale.

3.2 Portfolio Optimization

- i) The stocks selected on the basis of Priority Index are added to the portfolio and their weights are operated genetically for optimization. The selected stocks comprise of the genes of a chromosome.
- ii) Each chromosome has genes represented by stocks which are further allotted random values.

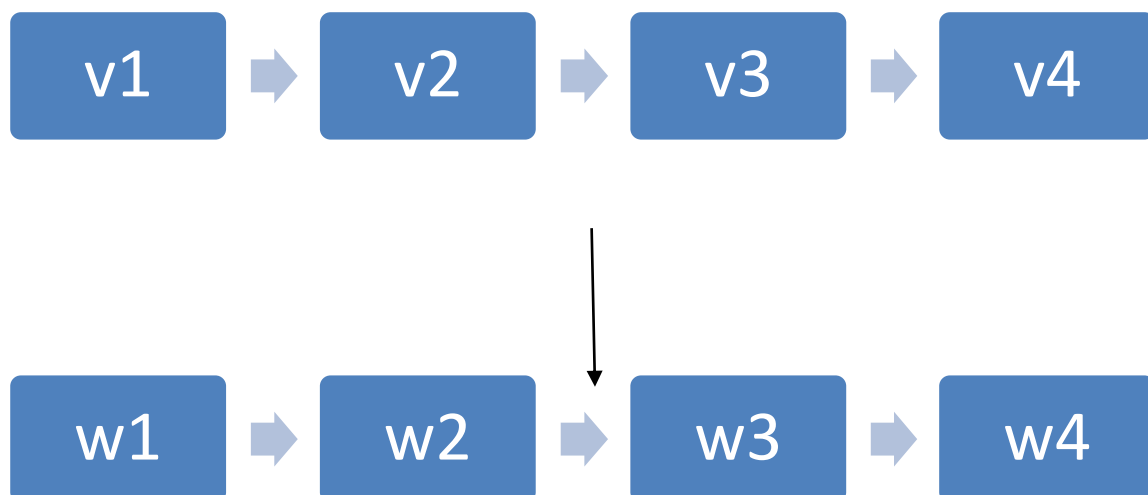


Figure 4: A Chromosome with gene representation

These weights are calculated by the formula $(v_i / \sum v_i)$ which represent the proportion in which the stock i is to be invested in.

- iii) An initial generation of ten such chromosomes is created by the random function.
- iv) The next step is the creation of the reproduction pool which can be understood by the explanation given in the earlier section. A tabular representation is shown below.

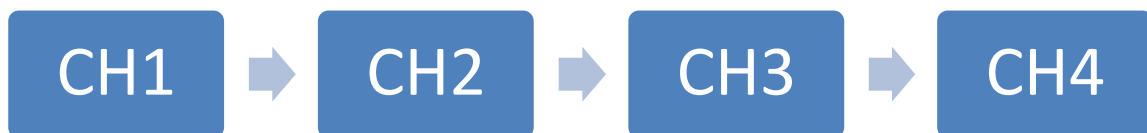
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Chromosome(CH)	Fitness Value	Evaluation Function	Cumulative Probability	Random # generated	Chromosome (CH) selected	# of copies of CH received
CH1	1.79	0.22	0.22	0.36	CH2	1
CH2	1.95	0.24	0.46	0.16	CH1	1
CH3	2.10	0.26	0.72	0.57	CH3	2
Ch4	1.87	0.28	1.00	0.63	CH3	0

Table 1: Generation of Reproduction Pool

The reproduction pool shall now consist of the chromosomes: CH1, CH2, CH3, and CH3

Generation Pool:



Reproduction Pool

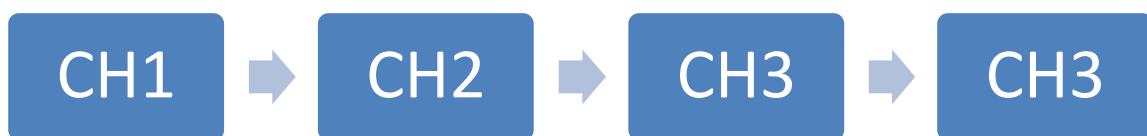


Figure 5: Example to show reproduction pool

- v) After the selection of the reproduction pool, the chromosomes are operated upon by the crossover and mutation operators. This gives rise to the second generation such that each generation has the same number of chromosomes which have been modified genetically.
- vi) For example, after running 100 generations of ten chromosomes each, we get a population size of 1000 chromosomes.

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- vii) The chromosome with the best fitness value is selected and the composition shall be the solution to our portfolio optimization problem.

4. Applied Example of Genetic Algorithm on SP 500 Index: US

Input:

The input shall consist of the stocks from SP 500 Index: US comprising of the daily closing prices, EPS Ratios, PEG Values, Weighted Average Cost of Capital, Market Capitalization/Revenue and Return on Invested Capital in the US Market from the duration December 2011 to December 2012. The input was taken from Bloomberg. The data should be divided sector wise as shown in the column 'Sectors' of the Table 2.

A sample input data file obtained from BLOOMBERG is shown as below.

Stocks	Sectors	EPS	PEG	Return on Invested Capital	WACC EQUITY	P/E	Weighted Average Cost of Capital	Market Cap/Revenue
AAPL US Equity	7	59.144402	0.496343	39.412102	10.481	11.617	28.931102	3.09577
ABC US Equity	5	9.8039	1.148947	14.0395	7.181	15.077	6.8585	0.127797
ABT US Equity	5	1.6779	1.282004	15.3051	6.527	12.986	8.7781	2.608781
ACE US Equity	4	-48.852501	1.104022	4.1135	8.1	9.7919	-3.9865	1.526147
ACN US Equity	7	12.4646	1.278039	81.438202	11.475	16.825	69.963202	1.544408
ADBE US Equity	7	0.5988	2.625089	10.9911	9.841	22.908	1.1501	4.193209
ADM US Equity	2	-41.955799	1.148061	5.2502	7.451	13.599	-2.2008	0.203544
ADP US Equity	7	12.2047	2.02624	3.9673	9.774	20.764	-5.8067	2.561976
ADSK US Equity	7	34.4086	1.039087	17.4709	15.482	30.833	1.9889	3.434506
AEE US Equity	10	270.689697	-3.152977	4.6439	4.752	12.69	-0.1081	1.076148
AEP US Equity	10	28.4585	3.125354	4.8766	4.404	14.311	0.4726	1.400313
AET US Equity	5	25.4118	0.85566	15.5156	8.629	9.0155	6.8866	0.438916
AFL US Equity	4	-15.8	0.674268	13.4208	11.48	8.1184	1.9408	0.995544
AIZ US Equity	4	124.206299	0.643849	3.9461	7.219	5.1462	-3.2729	0.321111
AKAM US Equity	7	12.3711	1.609521	9.2424	13.254	38.194	-4.0116	5.517065
ALL US Equity	4	-12.2093	1.209456	-3.3191	7.688	7.6998	-11.0071	0.5858
ALTR US Equity	7	-6.2745	2.170973	23.070601	12.859	19.223	10.211601	6.111062
ALXN US Equity	5	76.146797	1.135773	21.779301	8.924	79.466	12.855301	17.470169
AMAT US Equity	7	-93.835602	2.343621	3.6268	11.345	15.187	-7.7182	1.559059
AMGN US Equity	5	-15.5602	1.448069	10.3065	5.959	14.802	4.3475	3.931002
AMZN US Equity	1	-46.124001	4.313066	6.3278	12.003	469.12	-5.6752	1.964408
AN US Equity	1	30.6667	0.755868	6.5819	5.894	16.184	0.6879	0.314484
ANF US Equity	1	-14.6199	0.827461	6.6496	13.378	20.561	-6.7284	0.840215
AON US Equity	4	16.4	1.61923	9.5595	7.702	17.691	1.8575	1.57138

Table 2: Input Data File for MATLAB code as obtained from BLOOMBERG

And the data set shall continue in the same order for all the input stocks (SP500 INDEX: US in our study).

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A second input file is a column of the daily market returns for the same duration for the SP 500 Index: US. It is taken as the return of the market portfolio for the calculation of raw Beta of each stock of the index. Adjusted Beta is calculated by using the following relation:

$$\text{Adjusted Beta} = 0.67 * \text{Raw Beta} + 0.33 * 1$$

Output:

The portfolio constructed for a threshold of 3.8 on a 5 point scale is shown in the Table 3 below along with the performance analytics. This threshold value has given us a portfolio of 25 stocks selected by the Priority Index Function and Optimized by Genetic Algorithm. The lower the threshold, the higher will be the number of stocks in the portfolio and vice versa.

Equity	%Annual Return
'ADBE US Equity'	6.435194
'AKAM US Equity'	3.14256
'AMZN US Equity'	2.841326
'AVB US Equity'	1.456976
'BEAM US Equity'	11.7248
'BXP US Equity'	1.237292
'CCL US Equity'	4.894944
'CTL US Equity'	4.456662
'EFX US Equity'	3.865197
'EXPE US Equity'	5.850266
'GAS US Equity'	5.276695
'HCN US Equity'	3.299015
'JNPR US Equity'	3.874859
'KIM US Equity'	3.078482
'KO US Equity'	2.502491
'LRCX US Equity'	2.624299
'NBL US Equity'	0.398242
'NTRS US Equity'	2.002546
'PKI US Equity'	8.936224
'PNR US Equity'	4.292124
'PXD US Equity'	4.428024
'RHT US Equity'	2.723802
'RRC US Equity'	2.623591
'SRCL US Equity'	5.10981
'URBN US Equity'	2.924574

Table 3: Output File showing Weights of Selected Stocks in the Optimum Portfolio

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5. Analysis of Portfolio:

We shall now analyze the constructed portfolio on the basis of the following parameters:

i) **Average Annual return**

The average annual return is calculated from the average daily return as per the following formula:

$$AAR = (1 + ADR)^{252} - 1$$

This shall give us the annual average return for the portfolio allowing the investor to make a decision.

The AAR of the portfolio is 26.01% on the basis of the historical data (Dec' 11 to Dec' 12).

ii) **Beta of Portfolio (β_p)**

The Beta of Portfolio shall be an indicator of the correlated volatility of the asset in relation with the volatility of the index on which the stocks are benchmarked.

The Beta of Portfolio is 0.87 which is less than beta of the market portfolio.

iii) **Treynor's Ratio**

The return which would be earned in excess as compared to a risk free environment is referred to as the Treynor's Ratio. This shall be analyzed for the above portfolio.

$$\text{Treynor's Ratio} = \text{Excess Return} / \text{Beta}$$

The Treynor's Ratio is 0.2976.

iv) **Jenesson's Alpha**

Jenesson's Alpha is calculated by the following formula:

$$\alpha = (R_p - R_f) - \beta_p (R_m - R_f)$$

It is used as an indicator to determine the unusual return on the portfolio as compared to the return on the market indicating the effectiveness of the selection and optimization of the portfolio algorithm.

Jenesson's Alpha of the portfolio constructed is 14.56% which indicates that the portfolio constructed by the algorithm is very effective.

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Performance Parameters:

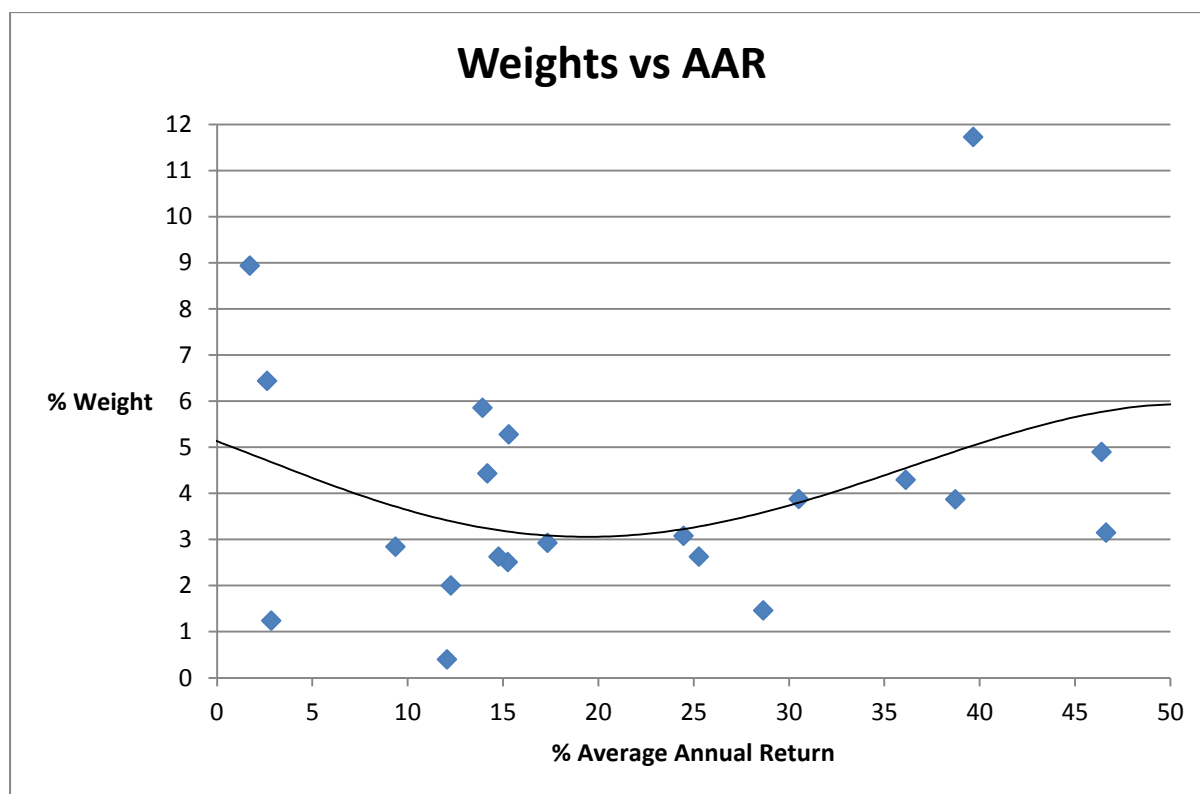
Priority Index Threshold	3.5/5
No of stocks selected	25
Annual Return of Portfolio	26.01%
Beta of Portfolio	0.87
Treyno's Ratio	0.2976
Jensen's Alpha	14.56%

Table 5: Performance Parameters of the Optimum Portfolio constructed for the Period (Dec'11 –Dec'12)

Risk Free Return in the US Market is taken as the yield on a US Treasury Bill for 1 year=0.13%

Graphical Analysis:

Graph 1. The relation between stock weights assigned and %AAR

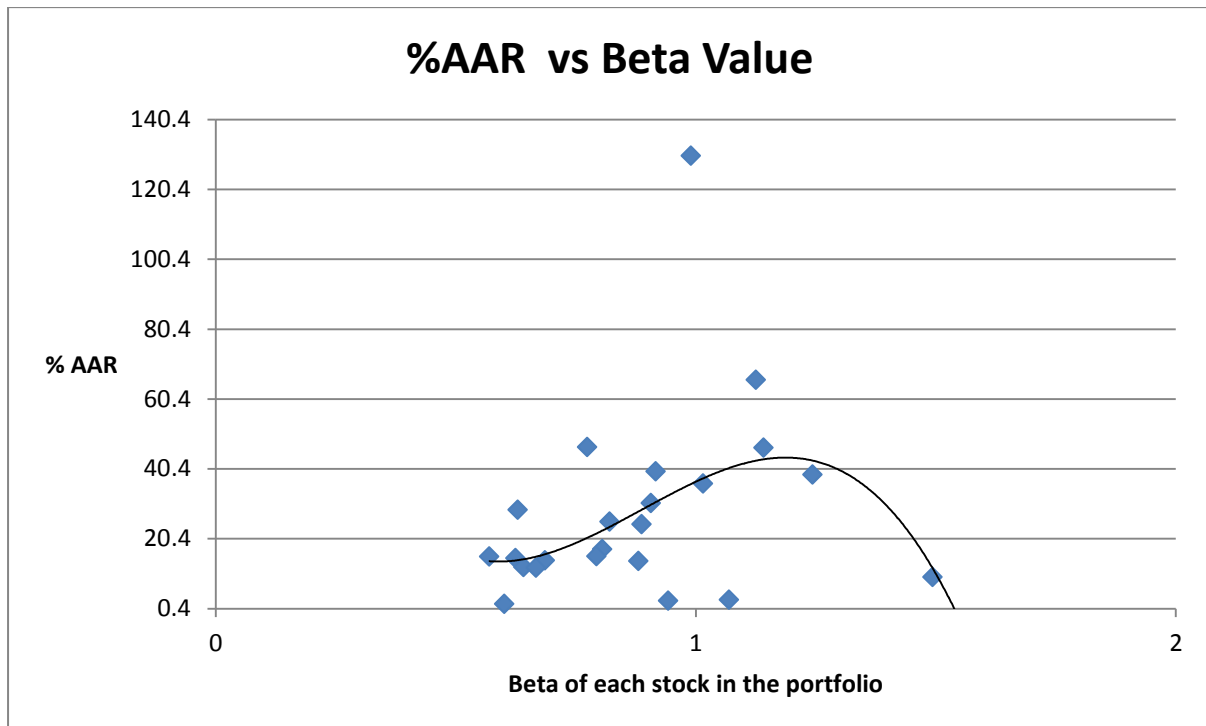


The stock weights assigned show an interesting trend, the Genetic Algorithm allotted weights which were initially decreasing with %AAR and then began increasing after a certain amount. This technique gives insight to the method of optimization of Genetic Algorithm.

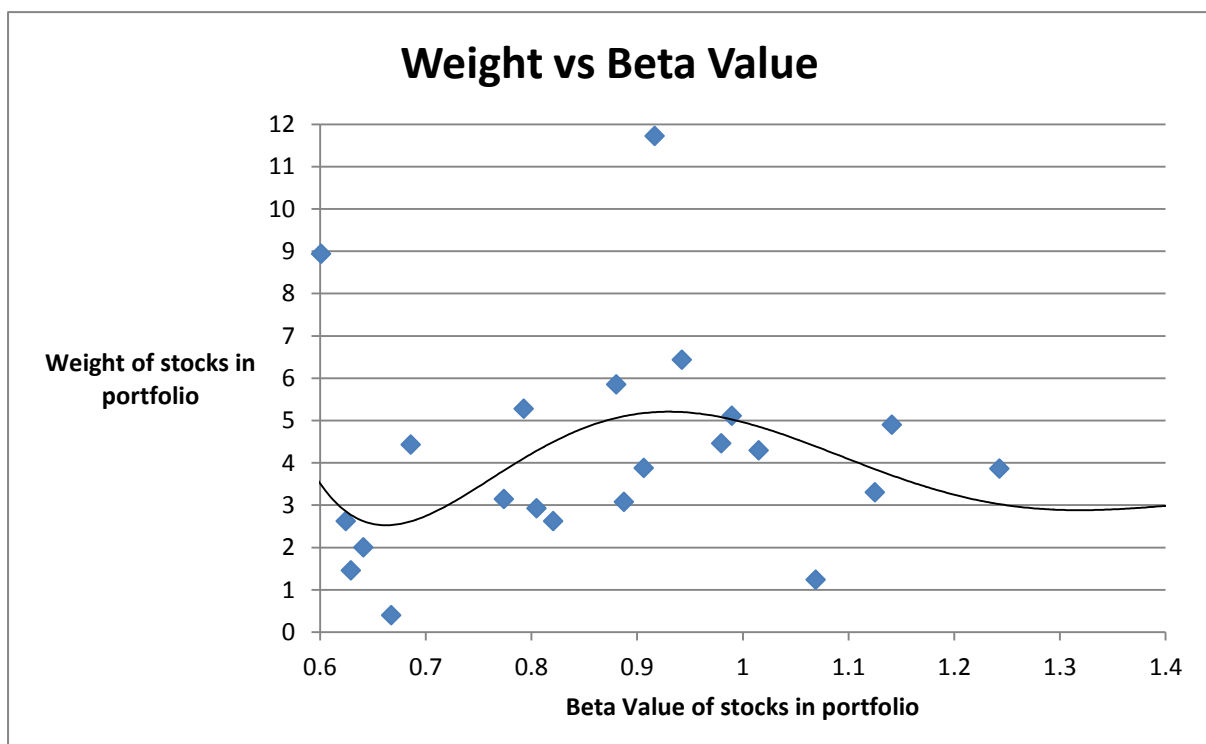
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Graph 2: Variation of %AAR with Beta Value

The Annual return shows an increasing trend with Beta initially and then begins to decrease. The stocks have been chosen at such a manner that we get values with higher AAR and lower beta values.



Graph 3: Weights versus Beta Value



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The weights increase with Beta Value for lower Beta Values but then begin to decrease with higher Beta Values. This shows the optimization technique of the Genetic Algorithm.

We have analyzed the constructed portfolio on the basis of live data also. Please note that the portfolio was constructed using historical data (December 2011- December 2012). We analyze the constructed portfolio by using the futuristic data (January '13 – June'13). The analysis shows that the portfolio has performed remarkably with an % AAR as high as 15.98% which was calculated from the six month yield rate of 7.69%. During this period market shows a %AAR if 10.14%, hence the portfolio constructed above using Genetic Algorithm beats the market.

Name of Equity	%Return as per 6 month holding period
ADBE UW Equity	20.91333188
AKAM UW Equity	4.008895325
AMZN UW Equity	10.57193525
AVB UN Equity	-0.501705071
BEAM UN Equity	3.306263904
BXP UN Equity	-0.321246069
CCL UN Equity	-6.744575008
CTL UN Equity	-9.636814951
EFX UN Equity	8.887625119
EXPE UW Equity	-2.115684313
GAS UN Equity	7.230467441
HCN UN Equity	9.365386173
JNPR UN Equity	-1.830422237
KIM UN Equity	10.92169129
KO UN Equity	10.64765461
LRCX UW Equity	22.72229139
NBL UN Equity	18.0266024
NTRS UW Equity	15.43065818
PKI UN Equity	2.393997109
PNR UN Equity	17.37519993
PXD UN Equity	35.80015595
RHT UN Equity	-9.705376896
RRC UN Equity	23.06221326
SRCL UW Equity	18.3978598
URBN UW Equity	2.185068441

Table 6: The performance of portfolio on 6 month holding period

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6. Conclusion

The study has shown the wide implications of the above two stage process used in portfolio construction:

- i) Selection of stocks on the basis of business fundamentals rather than market sentiments is an important lesson for acquiring long term benefits at lower risks. Our portfolio is based on creating a genetically suitable return and risk at the same time. We have successfully reduced the risk by a selection procedure based upon company fundamentals. Thus the portfolio is said to be optimum giving considerable return by taking a calculated risk.
- ii) The priority index allows users to select the threshold above which the stocks shall enter the portfolio. Higher the threshold, fewer the stocks and vice versa. This indicates that as the threshold goes up, the decision making becomes stern and we chose stocks which are stronger in business fundamentals. This might compromise on the return but shall give us long term stability.
- iii) The research has aptly demonstrated the application of genetic algorithm for optimization of portfolio. This is a novel example of application of Genetic Algorithms in the field of Portfolio optimization in finance.
- iv) The portfolio for a threshold of 3.8 on a five point scale gives an annual average return of 15.98% with respect to a risk free return of 0.13%.
- v) The analysis of the portfolio shows an %AAR of 26.01% from the historical data (December' 11 to December' 12). Now when tested for a six month holding period (Jan' 13- June' 13), the portfolio performed reasonably well giving an %AAR of 15.98% whereas the market return for the same period was 10.14%. Thus the above two stage technique involving genetic algorithm is highly objective, useful and effective for portfolio construction and optimization.

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Appendix

The MATLAB code for the procedure adopted in the optimum portfolio construction is given below:

```
%DEFINITION AND NAMES OF VARIABLES USED

%ndata= numerical data related to stock (EPS, PE, PEG, Wealth Creation,
MC/RE)
%text= name of all stocks
%value= daily adjusted closing price values of all the stocks
%ntms= no of chromosomes to be generated
%l= number of stocks
%m= number of days of which data is available
%value1= daily return of market index(in percentage)
%rtrn= daily return of stocks
%marketrtrn= daily return on market
%k= array used for priority function (i.e. to sort and select stocks)
%z= array used for counting stocks in each sector
%stk= sorted out stocks by priority function (portfolio)
%stkn= number of stocks selected by priority function (portfolio)
%stkrn= return of each stock selected by priority function
%rbs= average daily return of stocks
%rb1= average annual return of stocks
%rb= arithmetic mean of return
%cv= covariance of return of selected stock
%cvr= covariance of return and market return of each stock
%va= variance of market return
%fitness= fitness value of each chromosome
%ro= return of each stock in portfolio
%rtr1= total return of portfolio
%ann= annual return of the portfolio
%betaf= beta of individual stocks
%bta= beta of the portfolio
%rpool= reproduction pool containing chromosome details
(weight,value,random value, selected chromosome for crossover or mutation)
%chr= random value for stocks in a chromosome
%chrw= weight of the stocks in a chromosome
%rc= counter used for number of chromosomes
%treynors= treynors ratio of portfolio
%alpha= alpha of the portfolio
%annmtrn= annual market return
%prfn= threshold value of the Priority Function

%Main Program which is needed to run [Save as: Main.m]

clc;
close all;
clear all;
prfn=3.8;
ntms=[500 , 1000 , 1500 , 2000 ,2500];
[la,lb]=size(ntms);
for ntms=1:lb
    [ndata, text, alldata] = xlsread('Stock_details.xlsx');
    [l,n]=size(ndata);

    %data=transpose(ndata);
```


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```
v=n;
n=n+1;

%adding row number
for i=1:l
    ndata(i,n)=i;
end

%count of all stocks in each sector
for i=1:10
    x=1;
    for j=1:l
        if(ndata(j,1)==i)
            z(i)=x;
            x=x+1;
        end
    end
end

%PE
k=sortrows(ndata,6);
for y=1:10
    j(y)=1;
end
n=n+1;
[k,j,z]= sortp(k,l,z,j,n);

%PEG
k=sortrows(k,3);
for y=1:10
    j(y)=1;
end
n=n+1;
[k,j,z]= sortp(k,l,z,j,n);

%EPS
k=sortrows(k,2);
for y=1:10
    j(y)=z(y);
end
n=n+1;
[k,j,z]= sortm(k,l,z,j,n);

%Wealth Creation
k=sortrows(k,7);
for y=1:10
    j(y)=z(y);
end
n=n+1;
[k,j,z]= sortm(k,l,z,j,n);

%MC/RE
k=sortrows(k,8);
for y=1:10
    j(y)=1;
end
n=n+1;
[k,j,z]= sortp(k,l,z,j,n);
```

Algorithm for construction of Optimum portfolio of stocks using Genetic Algorithm

```
n=n+1;
for i=1:l
    k(i,n)=0;
    for j=v+2:n-1
        k(i,n)=k(i,n)+k(i,j);
    end
end
k=sortrows(k,n);

f=0;
for i=1:l
    if k(i,n)>=prfn;
        f=f+1;
        for j=1:n
            stk(f,j)=k(i,j);
        end
    end
end

k=sortrows(k,1);
stk=sortrows(stk,9);
[stkn, stkr]=size(stk);

[value,name] = xlsread('Stock_return.xlsx');
[nn,m]=size(value);
for j=1:nn
    rtrn(j,1)=j;
end
for i=1:m
    rtrn(:,i+1)=value(:,i)/100;
    rtrn1(:,i)=rtrn(:,i+1);
end

[value1] = xlsread('Market_return.xlsx');
value1=transpose(value1);
for i=1:(m)
    marketrtrn(i)=value1(i)/100;
end

%no of stock screened
y=1;
for i=1:stkn
    for j=1:nn
        if stk(i,9)==rtrn(j,1);
            txt(y)=text(j);
            for x=1:m+1
                stkr(y,x)=rtrn(j,x);
                if x>1;
                    stkrn(y,x-1)=rtrn(j,x);
                end
            end
            y=y+1;
        end
    end
end
end
```

Algorithm for construction of Optimum portfolio of stocks using Genetic Algorithm

```
for i=1:stkn
    stddv(i)=std(stkrn(i,:));
    rbs(i)=1;
    for j=1:m
        rbs(i)=rbs(i)*(1+stkrn(i,j));
    end
    rb1(i)=(rbs(i).^(1/252))-1;
    rb(i)=mean(stkrn(i,:));
end

for i=1:stkn
    for j=1:stkn
        cv(i,j)=0;
        for d=1:252
            temp=(stkrn(i,d)-rb(i))*(stkrn(j,d)-rb(j));
            cv(i,j)=cv(i,j)+temp;
        end
        cv(i,j)=cv(i,j)/252;
    end
end

rbm=mean(marketrtrn);
va=var(marketrtrn);
for i=1:stkn
    cvr(i)=0;
    for d=1:252
        tmp4=(stkrn(i,d)-rb(i)-0.0013)*(marketrtrn(d)-rbm-0.0013);
        cvr(i)=cvr(i)+tmp4;
    end
    cvr(i)=cvr(i)/252;
    bt1(i)=0.67*(cvr(i)/va)+0.33;
end

rc=1;
[rpool,chrw,chrw,fitness,rc]=rpoola(stkn,rb1,cv,rc,stkrn);
for i=1:ntms(ntimes)
    [rpool,chrw,chrw,fitness,rc]
=rpoolb(rpool,chrw,chrw,fitness,stkrn,rb1,cv,rc,stkrn);
end

[mxxx,ind]=max(fitness);
[mx2,ind1]=max(mxxx);
index=((ind(ind1)-1)*10)+ind1;
rtr1(ntimes)=0;
if index >10
    ind3=index-mod(index,10);
    rc=ind3/10;
    ind4=mod(index,10);
    if ind4==0
        ind4=10;
    end
else
    rc=1;
    ind4=index;
end

plt(ntimes,:)=chrw(rc,ind4,:);
```

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```
bta(ntimes)=0;
for i=1:stkn
    ro(ntimes,i)=rb1(i)*chrw(rc,ind4,i);
    rtr1(ntimes)=rtr1(ntimes)+rb1(i)*chrw(rc,ind4,i);
    betaf(ntimes,i)=bt1(i)*chrw(rc,ind4,i);
    bta(ntimes)=bta(ntimes)+betaf(ntimes,i);
end
rtr1(ntimes);
bta(ntimes);
ann(ntimes)=(1+rtr1(ntimes)).^252-1;
treynors(ntimes)=(ann(ntimes))/bta(ntimes);
annmtrn=(1+rbm)^252-1;
alpha(ntimes)=(ann(ntimes)-0.0013)-bta(ntimes)*(annmtrn-0.0013);
end

[v1,ind]=max(ann);
xlswrite('output.xlsx',txt(1:stkn),'A1:A10000');
xlswrite('output.xlsx',bt1(1:stkn),'B1:B10000');
xlswrite('output.xlsx',plt(ind,:),'C1:C10000');
xlswrite('output.xlsx',rb1(1:stkn),'D1:D10000');
xlswrite('output.xlsx',alpha(1:lb),'E1:E10000');
xlswrite('output.xlsx',annmtrn,'F1');
xlswrite('output.xlsx',ann(ind),'G1');

%sortm function to allot priority grades in ascending order as per the
higher value [Save as: sortm.m]
function [k,j,z]= sortm(k,l,z,j,n)
for i=1:l
    if k(i,1)==1 ;
        k(i,n)=j(1)/z(1);
        j(1)=j(1)-1;

    elseif k(i,1)==2 ;
        k(i,n)=j(2)/z(2);
        j(2)=j(2)-1;

    elseif k(i,1)==3 ;
        k(i,n)=j(3)/z(3);
        j(3)=j(3)-1;

    elseif k(i,1)==4 ;
        k(i,n)=j(4)/z(4);
        j(4)=j(4)-1;

    elseif k(i,1)==5 ;
        k(i,n)=j(5)/z(5);
        j(5)=j(5)-1;

    elseif k(i,1)==6 ;
        k(i,n)=j(6)/z(6);
        j(6)=j(6)-1;

    elseif k(i,1)==7 ;
        k(i,n)=j(7)/z(7);
        j(7)=j(7)-1;
```

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```
elseif k(i,1)==8 ;
    k(i,n)=j(8)/z(8);
    j(8)=j(8)-1;

elseif k(i,1)==9 ;
    k(i,n)=j(9)/z(9);
    j(9)=j(9)-1;

else k(i,1)=10 ;
    k(i,n)=j(10)/z(10);
    j(10)=j(10)-1;
end
end

%sortp function to allot priority grades in descending order as per the
higher value [Save as: sortp.m]
function [k,j,z]= sortp(k,l,z,j,n)
for i=1:l
    if k(i,1)==1 ;
        k(i,n)=j(1)/z(1);
        j(1)=j(1)+1;

    elseif k(i,1)==2 ;
        k(i,n)=j(2)/z(2);
        j(2)=j(2)+1;

    elseif k(i,1)==3 ;
        k(i,n)=j(3)/z(3);
        j(3)=j(3)+1;

    elseif k(i,1)==4 ;
        k(i,n)=j(4)/z(4);
        j(4)=j(4)+1;

    elseif k(i,1)==5 ;
        k(i,n)=j(5)/z(5);
        j(5)=j(5)+1;

    elseif k(i,1)==6 ;
        k(i,n)=j(6)/z(6);
        j(6)=j(6)+1;

    elseif k(i,1)==7 ;
        k(i,n)=j(7)/z(7);
        j(7)=j(7)+1;

    elseif k(i,1)==8 ;
        k(i,n)=j(8)/z(8);
        j(8)=j(8)+1;

    elseif k(i,1)==9 ;
        k(i,n)=j(9)/z(9);
        j(9)=j(9)+1;
```

Algorithm for construction of Optimum portfolio of stocks using Genetic Algorithm

```
else k(i,1)=10 ;
      k(i,n)=j(10)/z(10);
      j(10)=j(10)+1;
end
end

%rpoola function to generate the first reproduction pool [Save as:
rpoola.m]

function[rpool,chrw,chrw,fitness,rc] =rpoola(stkn,rbl,cv,rc,stkkn)

for i=1:10
    ss=0;
    for j=1:stkkn
        chrv(rc,i,j)=randn();
        if chrv(rc,i,j)<0;
            chrv(rc,i,j)=chrw(rc,i,j)*(-1);
        end
        ss=ss+chrw(rc,i,j);
    end

    for j=1:stkkn
        chrw(rc,i,j)=(chrw(rc,i,j)/ss);
    end
end

for y=1:10
    sumchr(y)=0;
    newv(y)=0;
    for i=1:stkkn
        val=var(stkkn(i));
        for j=1:stkkn
            sumchr(y)=((sumchr(y)+2*cv(i,j)*chrw(rc,y,i)*chrw(rc,y,j)));
            newv(y)=newv(y)+rbl(j)*chrw(rc,y,j);
        end
        sumchr(y)=sumchr(y)+chrw(rc,y,i).^2*val;
    end
    sumchr(y)=sqrt(sumchr(y));
    fitness(rc,y)=newv(y)/sumchr(y);
end

k=y;
sum1=0;
for i=1:10
    sum1=sum1+fitness(rc,i);
end
sfit(rc)=sum1;

pl=1;
rrp=0;
for i=1:10
    rpool(rc,i,pl)=i;
    rpool(rc,i,pl+1)=fitness(rc,i);
end
```

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```
    rpool(rc,i,pl+2)=rpool(rc,i,pl+1)/sfit(rc);
    rrp=rrp+rpool(rc,i,pl+2);
    rpool(rc,i,pl+3)=rrp;
    rpool(rc,i,pl+4)=rand();
end

for i=1:10
    j=1;
    mx=1;
    while(rpool(rc,j,pl+3)<rpool(rc,i,pl+4))
        mx=j;
        j=j+1;
    end
    rpool(rc,i,pl+5)=mx;
end

for i=1:10
    countrp=histc(rpool(rc,:,6),i);
    rpool(rc,i,pl+6)=countrp;
    if countrp>0
        rpool(rc,i,pl+7)=rpool(rc,i,pl+1);
    else
        rpool(rc,i,pl+7)=0;
    end
end
k=0;

[chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc,yyy] =
crossb(chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc);
if yyy>0
[chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc,yyy] =
crossb(chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc);
end
if yyy>0
[chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc,yyy] =
crossb(chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc);
end

for i=1:10
    while rpool(rc,i,8)>0
        [chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc] =
mutate(chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc);
    end
end
rc=rc+1;

%crossb function for the crossover between two chromosomes [Save as:
crossb.m]
function [chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc,yyy] =
crossb(chrsv,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc)

largest = rpool(rc,1,8);
x=1;
secondLargest = 0;
y=0;
for j=2:10
    number = rpool(rc,j,8);
```

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```
if number > largest
    secondLargest = largest;
    largest = number;
    y=x;
    x=j;
else
    if number > secondLargest
        secondLargest = number;
        y=j;
    end
end
end
YYY=Y;
if y>0
    rpool(rc,x,7)=rpool(rc,x,7)-1;
    rpool(rc,y,7)=rpool(rc,y,7)-1;
    for i=1:10
        if rpool(rc,i,7)<1
            rpool(rc,i,pl+7)=0;
        end
    end
end

i=k+1;
aa=rand();
for j=1:stkn
    chrv(rc+1,i,j)=(1-aa)*chrw(rc,x,j)+(aa*chrw(rc,y,j));
    chrv(rc+1,i+1,j)=(aa)*chrw(rc,x,j)+((1-aa)*chrw(rc,y,j));
end

for i=k+1:k+2
    ss=0;
    for j=1:stkn
        ss=ss+chrw(rc+1,i,j);
    end

    for j=1:stkn
        chrw(rc+1,i,j)=(chrw(rc+1,i,j)/ss);
    end
end

for y=k+1:k+2
    sumchr(y)=0;
    newv(y)=0;
    for i=1:stkn
        for j=1:stkn
            sumchr(y)=(sumchr(y)+cv(i,j)*chrw(rc+1,y,i)*chrw(rc+1,y,j));
            newv(y)=newv(y)+rb1(j)*chrw(rc+1,y,j);
        end
    end
    sumchr(y)=sqrt(sumchr(y));
    fitness(rc+1,y)=newv(y)/sumchr(y);
end
k=k+2;
end
```


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```
%mutate function for the mutation of chromosome [Save as: mutate.m]
function [chrV,fitness,k,stkN,chrW,sumchr,newV,rbl,cv,rpool,pl,rc] =
mutate(chrV,fitness,k,stkN,chrW,sumchr,newV,rbl,cv,rpool,pl,rc)

xx=0;
i=1;
while i<=10 && xx==0
    if rpool(rc,i,7)>0
        rpool(rc,i,7)=rpool(rc,i,7)-1;
        if rpool(rc,i,7)<1
            rpool(rc,i,pl+7)=0;
        end
        xx=i;
    end
    i=i+1;
end

i=k+1;
for j=1:stkN
    chrV(rc+1,i,j)=chrV(rc,xx,j);
end

j1=(17/27)*stkN;
j2=1;
while j2<j1
    j2=j2+1;
end

j3=(14/27)*stkN;
j4=1;
while j4<j3
    j4=j4+1;
end

temp=chrV(2,i,j2+1);
for j=j4:j2
    temp1(j)=chrV(rc+1,i,j);
end
for j=j4+1:j2+1
    chrV(rc+1,i,j)=temp1(j-1);
end
chrV(rc+1,i,13)=temp;

ss=0;
for j=1:stkN
    ss=ss+chrV(rc+1,i,j);
end

for j=1:stkN
    chrW(rc+1,i,j)=(chrV(rc+1,i,j)/ss);
end

y=k+1;
sumchr(y)=0;
newV(y)=0;
for i=1:stkN
    for j=1:stkN
        sumchr(y)=(sumchr(y)+cv(i,j)*chrW(rc+1,y,i)*chrW(rc+1,y,j));
    end
end
```

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```
        newv(y)=newv(y)+rb1(j)*chrw(rc+1,y,j);
    end
end
sumchr(y)=sqrt(sumchr(y));
fitness(rc+1,y)=newv(y)/sumchr(y);
k=k+1;

%rpoolb function to generate the remaining number of reproduction pools
[Save as: rpoolb.m]

function[rpool,chrv,chrw,fitness,rc]
=rpoolb(rpool,chrv,chrw,fitness,stk,rb1,cv,rc,stk)

for i=1:10
    ss=0;
    for j=1:stk
        chrv(rc,i,j)=randn();
        if chrv(rc,i,j)<0;
            chrv(rc,i,j)=chrw(rc,i,j)*(-1);
        end
        ss=ss+chrv(rc,i,j);
    end

    for j=1:stk
        chrw(rc,i,j)=(chrv(rc,i,j)/ss);
    end
end

for y=1:10
    sumchr(y)=0;
    newv(y)=0;
    for i=1:stk
        val=var(stk(i));
        for j=1:stk
            sumchr(y)=((sumchr(y)+2*cv(i,j)*chrw(rc,y,i)*chrw(rc,y,j)));
            newv(y)=newv(y)+rb1(j)*chrw(rc,y,j);
        end
        sumchr(y)=sumchr(y)+chrw(rc,y,i).^2*val;
    end
    sumchr(y)=sqrt(sumchr(y));
    fitness(rc,y)=newv(y)/sumchr(y);
end

k=y;
sum1=0;
for i=1:10
    sum1=sum1+fitness(rc,i);
end
sfit(rc)=sum1;
pl=1;
```

Algorithm for construction of Optimum portfolio of stocks using Genetic Algorithm

```
rrp=0;
for i=1:10
    rpool(rc,i,pl)=i;
    rpool(rc,i,pl+1)=fitness(rc,i);
    rpool(rc,i,pl+2)=rpool(rc,i,pl+1)/sfit(rc);
    rrp=rrp+rpool(rc,i,pl+2);
    rpool(rc,i,pl+3)=rrp;
    rpool(rc,i,pl+4)=rand();
end

for i=1:10
    j=1;
    mx=1;
    while(rpool(rc,j,pl+3)<rpool(rc,i,pl+4))
        mx=j;
        j=j+1;
    end
    rpool(rc,i,pl+5)=mx;
end

for i=1:10
    countrp=histc(rpool(rc,:,6),i);
    rpool(rc,i,pl+6)=countrp;
    if countrp>0
        rpool(rc,i,pl+7)=rpool(rc,i,pl+1);
    else
        rpool(rc,i,pl+7)=0;
    end
end
k=0;

[chr,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc,yyy] =
crossb(chr,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc);
if yyy>0
[chr,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc,yyy] =
crossb(chr,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc);
end
if yyy>0
[chr,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc,yyy] =
crossb(chr,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc);
end

for i=1:10
    while rpool(rc,i,8)>0
        [chr,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc] =
mutate(chr,fitness,k,stk,chrw,sumchr,newv,rbl,cv,rpool,pl,rc);
    end
end
rc=rc+1;
```