Tax evasion, social norms and economic growth

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Abstract

This paper proposes a theoretical model to account for the most relevant micro- and macroeconomic empirical facts in the tax evasion literature. To do so, we integrate tax morale into a dynamic overlapping generations model of capital income tax evasion. Tax morale is modeled as a social norm for tax compliance. It is shown that accounting for such nonpecuniary costs of evasion may not only explain (i) why some taxpayers never evade even if the gamble is profitable, and (ii) how a higher tax rate can increase evasion, but also that (iii) the share of evaded taxes over GDP decreases with the stage of economic development and (iv) that tax morale is positively correlated with the level of GDP per capita as suggested by recent empirical evidence. Finally, a higher tax rate increases aggregate evasion as well as the number of evaders in the economy when taxpayers decisions are interdependent.

Keywords tax evasion, social norms, overlapping generations, economic growth

JEL-Classification H26, Z13, D91

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1 Introduction

Tax evasion is one of the main problems faced by fiscal authorities. For example, Slemrod (2007) estimates that the U.S. income tax gap in 2001 amounts to a total of $345 billion—more than 15% of the estimated actual (paid plus unpaid) tax liability. However, tax evasion is not a particular phenomenon in developed countries. As estimated by Cobham (2005), for instance, the overall level of tax revenue lost due to tax evasion in developing countries is equal to $285 billion per year. Thus, explaining the patterns of tax evasion and identifying tools to reduce it is an important concern in all economies.

The theoretical analysis of tax evasion starts with the seminal papers by Allingham and Sandmo (1972) and Yitzhaki (1974) which model tax evasion as a static portfolio selection issue. Subsequent empirical and experimental findings, however, have revealed important inconsistencies between theory and evidence. Specifically, the literature has identified four main puzzles: first, the finding that tax evasion increases with the tax rate (Clotfelter, 1983, Poterba, 1987, Joulfian and Rider, 1996) while theory predicts the opposite. Second, the finding of low levels of tax evasion in many countries compared to the high level predicted by theory given the low levels of deterrence (see Alm (1999) and Torgler (2002) for a review). Third, the finding that some taxpayers never evade, even if evasion is the profitable option (Baldry, 1986, Alm, 1999). Fourth, the finding that the level of tax evasion and taxpayers’ attitudes towards evasion are related to the behavior of other taxpayers (Gaechter, 2006). This paper sets up a dynamic model of tax evasion in order to reconcile theory with empirical evidence.

While many studies have extended the basic portfolio selection model to explain these puzzles in a similar static framework, only a few recent papers analyze tax evasion in a dynamic context (Lin and Yang, 2001, Chen, 2003, Dzhumashev and Gahramanov, 2011, Levaggi and Menoncin, 2012). However, as the main focus of these papers is on how tax evasion affects the relationship between income taxation and economic growth in the long-run,

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1 See Fuest and Riedel (2009) for a survey of tax evasion in developing countries.
2 Alm (1999), Andreoni et al. (1998), Slemrod and Yitzhaki (2002) and Slemrod (2007) are recent surveys reviewing the main literature on the nature and determinants of tax evasion.
3 See Alm et al. (1992) and Frey and Feld (2002) among others.
4 Most of the papers has focused on explaining the positive relationship between tax rate and evasion. Among these papers, Cowell and Gordon (1988) poses a framework where taxpayers consider the provision of public goods; Landskroner et al. (1990) introduces the possibility of investing in financial assets; Lee (2001) considers the possibility of self-insurance against possible penalties and Bayer (2006) endogenizes the probability of audit.
none of these papers studies the effects of tax evasion behavior in the short-
run and in particular throughout the transition towards the steady state. To
close this gap in the literature is the aim of the present paper.

We set up a general equilibrium model of capital income tax evasion.
More precisely, we integrate the Allingham-Sandmo framework into a dy-
namic two period overlapping-generations setting with production.\footnote{See e.g. Boadway and Keen (1998) for a related approach. They study the role of capital income tax evasion in alleviating welfare losses due to time inconsistent taxation within an open economy model.} Our model departs from a simple version with amoral agents and subsequently,
in order to address the inconsistencies between theory and evidence, consid-
ers a more sophisticated version with tax morale and heterogeneous agents.
Tax morale can be defined as an internalized social norm for tax compliance
which expands the cost incurred by evaders to include not only the fines
payable upon detection, but also certain non-pecuniary considerations (for
an overview, see Torgler (2002) and Dell’Anno (2009)). As in Yitzhaki (1974),
taxpayers are audited with a positive probability and, if caught, have to
pay a penalty on the amount of evaded taxes.\footnote{For reasons of simplicity we assume that tax revenue is wasted. Alternatively, this can be interpreted as financing a public consumption good which increases individual utility.} Aggregate savings of utility
maximizing agents determine the dynamics of the economy. Given a neo-
classical technology, per capita capital increases throughout the transition
towards the steady state, which in turn decreases the rate of return and
therefore the incentives to evade taxes. In such a framework, the amount of
undeclared taxes may increase both in the transition and in the steady state
when the tax rate rises and individuals care about morality. More specifi-
cally, an increase in the tax rate generates two competing effects: a negative
income effect that disincentives to evade and a net increase in the benefit
of being dishonest which encourages taxpayers to evade. Thus, for a reason-
able set of parameters the second effect prevails. Moreover, it is shown that
increases in the strength of the norm to honestly pay what is owed as well
as in the audit probability produce low levels of tax evasion in the long-run
and throughout the transition towards the new steady state.

The main contribution of this paper is to present a simple dynamic model
of tax evasion which accounts simultaneously for well known micro empiri-
cal findings as well as for the latest macro-dynamic observations. Specifi-
cally, our model allows us to derive several new results in the literature on
tax evasion which are consistent with existing empirical evidence.

First, our model predicts that the share of tax evasion is declining dur-
ing the transition towards the steady state level. In this respect, Crane and Nourzad (1986) who study the evolution of aggregate tax evasion in
the United States over the period 1947-81, show that despite tax evasion increases in absolute terms, it has fallen in relative terms when income has risen. This result is further supported by evidence in Schneider et al. (2011). According to their findings, the relative size of the shadow economy\(^7\) for 162 countries over the period 1999 to 2007 has decreased whereas the unweighted average of GDP per capita for the same set of countries and over the same time horizon has increased.\(^8\)

Second, neoclassical growth theory which describes the dynamics of the economies in the model, predicts that countries with low levels of per capita GDP (per capita capital) display high levels of tax evasion. By contrast, high-income countries (high levels of per capita capital) show low levels of tax evasion, for the same size of tax rates and similar technologies and preferences. Gordon and Li (2005), for example, document sharp differences in the ability to generate tax revenue among developed and developing countries: though statutory tax rates are fairly similar across countries, effective tax rates differ widely given the lower fraction of GDP collected by these taxes among poorer countries. For instance, the maximum personal income tax rate in developed countries is on average 1.23 times higher than in developed countries, whereas income tax revenue over GDP is 2.47 times larger in developed countries. A similar pattern is also demonstrated by Easterly and Rebelo (1993a) and Easterly and Rebelo (1993b). According to their findings, income tax evasion is an important phenomenon, in particular for developing countries\(^9\). Consistent with these observations, our model predicts that the size of tax evasion decreases insofar countries accumulate capital and reach higher levels of income.

Finally, we find a positive correlation between per capita GDP and tax morale. More precisely, our model predicts that countries with high levels of tax evasion can be considered to be an integral part of the shadow economy. Though it is difficult to have reliable information about the exact size of tax evasion, since it is an illegal activity and individuals have strong incentives to conceal their cheating, and though the shadow economy is clearly not synonymous with tax evasion, many researchers (Schneider (2005) and Alm and Embaye (2011) among others) frequently use shadow economy estimates as an indicator for the size of tax evasion. See Alm (2012) for a detailed discussion about the measuring of tax evasion.

\(^7\)Tax evasion can be considered to be an integral part of the shadow economy. Though it is difficult to have reliable information about the exact size of tax evasion, since it is an illegal activity and individuals have strong incentives to conceal their cheating, and though the shadow economy is clearly not synonymous with tax evasion, many researchers (Schneider (2005) and Alm and Embaye (2011) among others) frequently use shadow economy estimates as an indicator for the size of tax evasion. See Alm (2012) for a detailed discussion about the measuring of tax evasion.

\(^8\)Specifically, this unweighted average of GDP per capita rose from 5200 US$ to 8400 US$ over the whole period, while the unweighted average size of the shadow economies of all of these 162 countries decreased from 34.0% of official GDP in 1999 to 31.2% of official GDP in 2007. Data on GDP per capita are taken from the World Bank, see http://data.worldbank.org/.

\(^9\)Also, the cross sectional findings about the relative size of the informal economy by Friedman et al. (2000) indicate that informality is (on average) a more severe problem in countries with low GDP per capita, especially in Latin American countries.
per capita income display low levels of evasion and a low share of evaders in the economy. A low share of evaders, in turn, implies larger moral costs of evading since the majority of population pays what they owe and this is perceived as the right behavior. Thus, the more other taxpayers are perceived to be honest, the more willing individuals are to pay their own taxes and reduce evasion. In this respect, Weck (1983) and Torgler (2003) document the existence of a positive relationship between tax evasion and tax morale for a wide sample of countries. Their findings support the hypothesis that the behavior of a taxpayer is influenced strongly by his perception of the behavior of other taxpayers. Moreover, Frey and Torgler (2007) and Torgler and Schneider (2007) find that countries which display higher rates of tax evasion are characterized by low quality institutions or weak direct democratic rights, and Acemoglu et al. (2005) and Bethencourt (2013) among others, show that these countries are typically developing countries with low levels of per capita income. Thus, the empirical facts support the finding that high income countries exhibit high quality institutions, high levels of tax morale and so, low levels of evasion.

Our work relates to the literature analyzing the effects of morality, customs and stigma on tax evasion behavior, see Gordon (1989), Myles and Naylor (1996), Kim (2003) and Traxler (2010). While these papers demonstrate how such non-pecuniary considerations may account for some of the tax evasion puzzles within a static framework, our contribution relative to these studies lies in modeling the dynamics of per capita capital and linking the size of tax evasion to the state of economic development.

Our work also relates to papers studying dynamic models of tax evasion, see e.g. Lin and Yang (2001), Chen (2003), Dzhumashev and Gahramanov (2011) and Levaggi and Menoncin (2012). Relative to these papers, however, we do not focus on the tax evasion-growth nexus, but rather on the dynamics of tax evasion throughout the transition towards the steady state. This in turn allows us to document not only cross country variations in levels of tax evasion but also to account for the development of these levels over time.

The remainder is organized as follows. Section 2 describes the basic model without morality. Section 3 extends the basic framework to account for nonpecuniary costs of evasion and Section 4 concludes. Some proofs and technical considerations are included in the appendix.

10 These results are further supported by evidence in Friedman et al. (2000) suggesting that weak economic institutions imply a large unofficial economy.

11 Note that we refer to these models as static insofar as they do not allow for income dynamics. However, as in the present paper, the share of evaders may well change over time.
2 The Basic Model

The basic framework is a two period overlapping-generations model in the tradition of Diamond (1965). The size of each generation is assumed to be constant and normalized to one. Non-altruistic individuals are endowed with one unit of labor time when young, and are retired during old age. Markets are competitive.

The government collects a proportional tax on capital income which may, however, be evaded by individuals. The reason we focus on capital income tax evasion is twofold. First, the probability of detection is much lower than for other income sources and therefore the opportunities for hiding true income from the tax collector are substantially higher than for example in the case of labor (Poterba, 1987, Sandmo, 2012). Second, capital income tax evasion is indeed a serious problem in many countries (see e.g., Slemrod (2007) for the US). Also, as pointed out by Sandmo (2012), ‘in the theoretical literature, the evasion of taxes on labor income has received considerably more attention than the evasion of taxes on capital. It is not obvious why this should be so; as already noted, it is difficult to argue that capital income evasion is of less empirical importance.’

2.1 Firms

On the production side of the model, perfect competition between a large number of identical firms is assumed. A representative firm in period $t$ produces a homogenous output good according to a Cobb–Douglas production function with capital $K_t$ and homogeneous labour $L_t$ as inputs:

$$ Y_t = AK_t^\alpha L_t^{1-\alpha}, \quad (1) $$

where $1 > \alpha > 0$ is the share parameter of capital.

Each firm maximizes profits under perfect competition, implying that, in equilibrium, production factors are paid their marginal products:

$$ \omega_t = (1 - \alpha)AK_t^\alpha L_t^{-\alpha} = (1 - \alpha)Ak_t^\alpha \quad (2) $$

and

$$ r_t = \alpha AK_t^{a-1}L_t^{1-a} = \alpha Ak_t^{a-1} \quad (3) $$

where $k_t = K_t/L_t$ is the capital intensity.

2.2 Consumers

Each generation consists of agents whose life has two periods of equal lengths: the young adult age during which each agent inelastically supplies one unit
of labor time to work and raises one offspring, and the old age spent in re-
tirement. Since each young adult produces one offspring, the population
remains constant in every generation and is normalized to one. When adult,
working individuals receive the wage \( w_t \). Income is spent on consumption \( c_t \)
and savings \( s_t \):

\[
w_t = c_t + s_t.
\]  

(4)

When old, each individual consumes the return to his savings and may evade
a fraction \( e_{t+1} \in [0,1] \) of this return. The declared income gets taxed with a
proportional income tax at rate \( \tau \). With a fixed probability \( p \) the evasion gets
detected. In this case, the tax evader has to pay the full taxes and a penalty
\( \gamma \) which is proportional to the taxes evaded Yitzhaki (1974). With probability
\( 1 - p \) the evasion remains undetected and the evader only pays taxes on the
declared income. The corresponding levels of second period consumption for
state \( u \) - escaping undetected - or state \( d \) - getting detected - are given by

\[
d^u_{t+1} = R^u_{t+1} s_t
\]  

(5)

\[
d^d_{t+1} = R^d_{t+1} s_t
\]  

(6)

where \( R^u_{t+1} = 1 - \delta + r_{t+1}(1 - \tau + \tau e_{t+1}) \), \( R^d_{t+1} = 1 - \delta + r_{t+1}(1 - \tau - \gamma \tau e_{t+1}) \) and \( \delta \in [0,1] \) denotes the depreciation rate of capital. The life-cycle utility function
of an individual born in \( t \) is

\[
E[U(c_t, d^u_{t+1}, d^d_{t+1})] = u(c_t) + (1 - p)\beta u(d^u_{t+1}) + p\beta u(d^d_{t+1})
\]  

(7)

where \( \beta > 0 \) is a discount factor. For reasons of tractability we will assume
that the expected utility representation \( u \) is logarithmic, i.e., \( u(x) = \ln(x) \).
Each individual maximizes the utility (7), subject to the constraints (4), (5)
and (6), by choosing \( c_t, s_t, e_{t+1}, d^u_{t+1} \) and \( d^d_{t+1} \). With logarithmic preferences,

\[
s_t = \frac{\beta}{1 + \beta} w_t.
\]  

(8)

As a result, the decision on \( e_{t+1} \) does not depend on \( s_t \). The first and second
order conditions with respect to the choice of \( e_{t+1} \) are then given by

\[
E[U(e_{t+1})]' = \beta r_{t+1} \left[ (1-p)u'(R^u_{t+1}) - \gamma p u'(R^d_{t+1}) \right] = 0 \tag{9}
\]

\[
E[U(e_{t+1})]'' = \beta (r_{t+1})^2 \left[ (1-p)u''(R^u_{t+1}) + p \gamma^2 u''(R^d_{t+1}) \right] < 0 \tag{10}
\]

Equation (9) characterizes \( e^*_{t+1} \), the optimal fraction of income concealed. We
will assume in the following that such an interior solution \( e^*_{t+1} \in [0,1] \) always
exists.

7
The fraction of evasion decreases as tax enforcement becomes stricter, i.e. \( \frac{de^*_{t+1}}{dp} < 0 \) and \( \frac{d}{d\gamma} < 0 \). This can be seen by implicitly differentiating the first order condition (9). Furthermore, a marginal increase in the tax rate reduces the optimal share of evasion:

\[
\frac{de^*_{t+1}}{d\tau} = \frac{(1 - p)\tau \beta r^2_{t+1} u'(R^u_{t+1})[\rho(R^u_{t+1})(1 - e_{t+1}) - \rho(R^d_{t+1})(1 + \gamma e_{t+1})]}{-E[U(e_{t+1})]''} < 0
\]

where \( \rho(x) = -u''(x)/u'(x) \) is the Arrow-Pratt measure of absolute risk aversion, which satisfies \( \rho'(x) \leq 0 \) and thereby \( \rho(R^d_{t+1}) \geq \rho(R^u_{t+1}) \) for non-increasing absolute risk aversion.\(^{12}\) Intuitively, a higher tax rate reduces taxpayers’ income and makes them less willing to take risks. There is no substitution effect as the penalty is assumed to be levied on the share of income evaded, implying that marginal gains and marginal costs from evasion exactly offset each other. In addition to the standard model which considers evasion in levels, however, the above result predicts a decreasing share of evasion instead of a decreasing level.

Similarly, for \( \rho(R^d_{t+1}) \geq \rho(R^u_{t+1}) \), an increase in the interest rate (capital income) lowers the optimal share of evasion:

\[
\frac{de^*_{t+1}}{dr_{t+1}} = \frac{(1 - p)\tau \beta r_{t+1} u'(R^u_{t+1})[\rho(R^d_{t+1})(1 - \tau - \gamma e_{t+1}) - \rho(R^u_{t+1})(1 + \tau e_{t+1})]}{-E[U(e_{t+1})]''} \leq 0
\]

Thus, our model predicts that the percentage of evasion over total income decreases. The intuition is similar to the one coming from an increase in the tax rate. First, there is no substitution effect as the penalty is assumed to be levied on the share of income evaded, implying that marginal gains and marginal costs from evasion exactly offset each other. Second, a higher interest rate increases taxpayers’ income and makes them more willing to evade income.\(^{13}\) However, given that concealed income has an income elasticity of demand less than one, the percentage of evaded taxes decreases.

### 2.3 Dynamics and Steady State

We are now able to define the intertemporal equilibrium of the economy. Given a fiscal policy (parameters \( \tau, p \) and \( \gamma \)) and an initial value of the capital stock \( k_0 = K_0/N_{-1} = s_{-1} \), a perfect-foresight intertemporal equilibrium is

\(^{12}\)Note that this assumption is always satisfied for our logarithmic preference representation.

\(^{13}\)It is straightforward to show that the amount of concealed income, \( e_{t+1}r_{t+1} \), increases in \( r_{t+1} \).
characterized by a sequence of quantities and prices:

\[ \{c_t, d_t, k_t, s_t, e_t; w_t, r_t\}_{t \geq 0}. \]

Individuals maximize utility, firms maximize profits, factor markets are competitive, and all markets clear. The market-clearing conditions for the labour and capital markets are

\[
\begin{align*}
L_t &= N_t, \\
K_t &= N_{t-1}s_{t-1}.
\end{align*}
\]

The dynamics of the basic model are characterized by the following first order difference equation (using (2), (3) and (13)):

\[
k_{t+1} = \frac{\beta}{1 + \beta} (1 - \alpha) A k_t^a
\]

which monotonically converges towards a unique steady state \( k^* \). Clearly, the tax rate does not affect the dynamics so that the share of evasion increases throughout the transition towards the steady state, i.e. \( de_{t+1}/dk_t > 0 \).

Therefore, the basic model turns out to be inconsistent with a wide range of empirical findings: First, it predicts a decrease of the share of evasion as a response to a tax increase whereas empirical studies point to an increase of evasion at the individual level (see e.g. Clotfelter (1983) and Joulfaian and Rider (1996)) as well as at the aggregate level (Poterba, 1987). Second, in this model agents will always evade taxes as long as this is the profitable option while empirical evidence shows that there are individuals that never evade (see for example, Baldry (1986) and Alm (1999)). Third, recent empirical literature shows that taxpayers’ attitudes towards evasion are related to the behavior of other taxpayers in the society (see e.g. Gaechter (2006)). Still, in the basic framework taxpayers behavior is absolutely independent of others. Moreover, cross sectional data and longitudinal data suggest that the share of evasion over GDP decreases with the stage of economic development (see for instance, Gordon and Li (2005), Easterly and Rebelo (1993a) and Easterly and Rebelo (1993b) for the cross-sectional case and, Crane and Nourzad (1986) and Schneider et al. (2011) for the longitudinal one). By contrast, the results of the basic model imply an increase of the share of evasion along the transitional path of an economy.

In order to reconcile theory with empirical evidence, the next section introduces moral concerns into the basic model.
3 Morality

In this section we introduce morality and reputation concerns along the lines of Gordon (1989). Accordingly, tax morale is modeled as an internalized social norm for tax compliance. The strength of this norm is assumed to be endogenous and depends on the number of individuals in society adhering to it (Akerlof, 1980, Lindbeck et al., 1999). Hence, if tax evasion becomes more common, the social norm is less powerful as it becomes easier for taxpayers to justify their wrongdoing to themselves, the more other people violate the societies’ code of conduct. Preferences therefore do not only depend on consumption levels but also on the ‘moral costs’ of tax evasion. Consequently, the life-cycle utility function of an individual $i$ born in period $t$ is

$$U_i(c_t,d^u_t,d^d_{t+1}, e_{t+1}) = E[U(c_t,d^u_t,d^d_{t+1})] - e_{t+1}(\theta_i + \mu(1-n_t))$$

(16)

where the expression $(\theta_i + \mu(1-n_t))e_{t+1}$ captures the moral costs of tax evasion. These costs are linearly increasing in the individual degree of norm internalization $\theta_i \geq 0$, which has distribution function $F(\theta_i)$ and support $[0, \bar{\theta}]$.

Furthermore, moral costs depend on individually fixed (marginal) reputation costs $\mu > 0$ and on the share of evaders in society $n_t$. Individuals maximize (16) subject to (8), (5) and (6) taking prices and the number of evaders $n_t$ as given. The first-order condition for an interior solution is

$$E[U(.)]' = \beta \tau r_{t+1} (1-p)u'(R^u_{t+1}) - \gamma pu'(R^d_{t+1}) = \theta_i + \mu(1-n_t)$$

(17)

while the second order condition is the same as (10). Norm guided taxpayers will choose a share of evasion such that the marginal expected utility $E[U(.)]'$ equals $\theta_i + \mu(1-n_t)$, the marginal moral costs from concealing income. An interior solution requires the evasion gamble to be better than fair$^{14}$, i.e.

$$z(r_{t+1}) = E[U(0)]' = \frac{(1-p(1+\gamma))\beta \tau r_{t+1}}{1-\delta + (1-\tau)r_{t+1}} > 0$$

(18)

>From equations (17) and (10) it follows that taxpayers with $\theta_i + \mu(1-n_t) > z(r_{t+1})$ do not conceal any income. This implies the threshold

$$\hat{\theta}(n_t,r_{t+1}) = z(r_{t+1}) - \mu(1-n_t)$$

(19)

which allows us to characterize the optimal individual evasion behavior $e^{*,i}_{t+1}$ for a given level of $n_t$ and $r_{t+1}$:

$$e^{*,i}_{t+1} = \begin{cases} 0 & \text{for } \theta_i \geq \hat{\theta}(n_t,r_{t+1}) \\ e^{i,*}_{t+1} & \text{for } \theta_i < \hat{\theta}(n_t,r_{t+1}) \end{cases}$$

(20)

---

$^{14}$This requires $1-p(1+\gamma) > 0$ or equivalently $\gamma < (1-p)/p$. The opposite case, in which $1-p(1+\gamma) < 0$ is negative, is of little interest, since tax evasion would never take place.
Those individuals with $\theta_i < \hat{\theta}(n_t, r_{t+1})$ will choose an intermediate level of evasion, $e_{t+1}^{i*} \in [0, e_{t+1}^*]$, whereas those with $\theta_i \geq \hat{\theta}(n_t, r_{t+1})$ do not evade as compliance to the norm is the best policy.

Similar to the basic model without morality, evasion decreases when $p$ or $\gamma$ increase for those individuals with $\theta_i < \hat{\theta}$. Moreover, $\hat{\theta}$ falls in both cases, so that the number of individuals choosing $e_{t+1}^* = 0$ increases. Hence, aggregate evasion must fall. The effects of a change in $\tau$ and $r_{t+1}$ are described by the following proposition:

**Proposition 1** Suppose that $\gamma < 2(1 - p)/(1 + 2p)$. Then, there exists some $\tilde{\theta}(n_t, r_{t+1}) < \hat{\theta}(n_t, r_{t+1})$ such that $\partial e_{t+1}^{i*}/\partial \tau \leq 0$, $\partial e_{t+1}^{i*}/\partial r_{t+1} \leq 0$ if $\theta_i < \tilde{\theta}(n_t, r_{t+1})$ and $\partial e_{t+1}^{i*}/\partial \tau \geq 0$, $\partial e_{t+1}^{i*}/\partial r_{t+1} > 0$ if $\theta_i > \tilde{\theta}(n_t, r_{t+1})$ for all $k_t$ and $n_t$.

**Proof:** See Appendix.

The effect of a change in the tax rate on tax evasion has been demonstrated before by Gordon (1989) and Traxler (2010) in a static framework. It is shown here how such a result carries over to a dynamic framework and that it holds along the complete transitional path of the economy. The basic intuition is the following: a higher tax rate increases the marginal benefits, as well as the marginal costs (associated with higher expected fines and with morality concerns). In the model of the previous section (without morality) we show that this marginal gains and marginal costs from evasion exactly offset each other, implying no substitution effect and only a negative income effect which encourages taxpayers to take less risks and so, to reduce tax evasion. However, in this version of the model, given that moral costs of evasion are assumed to depend on the share of income concealed rather than on taxes evaded, costs are not affected by a tax change. As a result, marginal benefits from concealing exceed marginal expected costs implying a substitution effect that provides an incentive to increase evasion. We prove that for those with $\theta_i < \hat{\theta}$, the negative income effect dominates and so, tax evasion reduces as taxes rises. Whereas for those with $\theta_i > \hat{\theta}$ the substitution effect prevails and, tax evasion increases.

An increase in the interest rate produces similar effects to those of the tax rate. It increases the marginal benefits as well as the marginal costs of evasion. However, as moral costs of evasion are assumed to depend on the share of income concealed, the increase in marginal benefits from concealing is above the increase in marginal expected costs, producing a substitution effect which incentives to increase tax evasion. Moreover, similar to the model without morality, there is a positive income effect which increases the total amount of evaded taxes. However, given that tax evasion has an income elasticity of demand less than one, the percentage of evaded taxed decreases.
Thus, the resulting effect depends on a positive income effect as in the basic model and a substitution effect working into the opposite direction. It will be negative for those individuals with $\theta_i < \tilde{\theta}$ and positive otherwise.

As to the behavior of those with $\theta_i > \tilde{\theta}$, differentiation of (19) yields

$$\frac{\partial \hat{\theta}(n_t, r_{t+1})}{\partial \tau} > 0 \quad \text{and} \quad \frac{\partial \hat{\theta}(n_t, r_{t+1})}{\partial r_{t+1}} > 0.$$  (21)

Therefore, the tendency for the share of aggregate evasion to increase is reinforced by the emergence of new evaders (in particular a least honest subset of the initial non-evaders). It is only large evaders who exhibit the standard portfolio response and reduce their holdings of the risky asset. Their more honest peers evade more, the higher tax rate making evasion less of an amoral gamble.

### 3.1 Dynamics and Steady State

The definition of an intertemporal equilibrium is analogous to the basic model without morality. Nevertheless, apart from a fiscal policy (parameters $\tau$, $p$ and $\gamma$) and an initial value of the capital stock $k_0 > 0$, an additional initial value of the share of evaders in society is required, $n_0 \geq 0$. Thus, a perfect-foresight intertemporal equilibrium is characterized by a sequence of quantities and prices:

$$\{c_t, d_t, k_t, s_t, e_t, n_t; w_t, r_t\}_{t \geq 0}.$$  

such that individuals maximize utility, firms maximize profits, factor markets are competitive, and all markets clear.

The capital stock in period $t + 1$ results from individuals’ savings in the preceding period, i.e. $k_{t+1} = s_t$ which implies (using (8)):

$$k_{t+1} = \frac{\beta}{1 + \beta}(1 - \alpha)A k_t^\alpha$$  (22)

The equilibrium share of evaders evolves according to the following dynamic equation:

$$n_{t+1} = F\left(\hat{\theta}(n_t, r_{t+1})\right)$$  (23)

Clearly, the share of evaders in period $t + 1$ is a positive function of the tax rate and the share of evaders in the preceding period whereas it is decreasing in the interest rate. Just note that $F' > 0$ and recall equations (19) and (21).
The aggregate (average) share of evasion in period $t + 1$ is given by

$$
\bar{e}_{t+1} = \int_0^\partial e_{t+1}^i f(\theta_i) d\theta_i = \int_0^\theta(n_t, r_{t+1}) e_{t+1}^i f(\theta_i) d\theta_i.
$$

(24)

The derivatives of $\bar{e}_{t+1}$ with respect to $k_t$ and $\tau$ can be written as follows:\footnote{For the derivation of these formulae, see Leibniz's rule for differentiation of parametric integrals.}

$$
\frac{\partial \bar{e}_{t+1}}{\partial k_t} = \left[ \int_0^\theta(n_t, r_{t+1}) \frac{\partial e_{t+1}^i}{\partial r_{t+1}} f(\theta_i) d\theta_i + e_{t+1}^i \right]_{\theta_i = \hat{\theta}(n_t, r_{t+1})} \frac{\partial \hat{\theta}(n_t, r_{t+1})}{\partial k_t}
$$

(25)

and

$$
\frac{\partial \bar{e}_{t+1}}{\partial \tau} = \left[ \int_0^\theta(n_t, r_{t+1}) \frac{\partial e_{t+1}^i}{\partial \tau} f(\theta_i) d\theta_i + e_{t+1}^i \right]_{\theta_i = \hat{\theta}(n_t, r_{t+1})} \frac{\partial \hat{\theta}(n_t, r_{t+1})}{\partial \tau}
$$

(26)

The second summand in both equations describes the change of aggregate evasion due to the emergence of new evaders in the society. Since $\partial \hat{\theta}/\partial \tau > 0$ and $\partial \hat{\theta}/\partial r_{t+1} > 0$, an increase of the share of evaders should increase the share of tax evasion. However, at the margin, this effect is equal to zero and the second summand vanishes as $e_{t+1}^i$ evaluated at $\theta_i = \hat{\theta}$ is equal to zero according to its definition. The first summand describes the response of existing evaders and can be decomposed into a negative effect for those individuals with $\theta_i < \hat{\theta}(n_t, r_{t+1})$ and a positive effect for those with $\theta_i > \hat{\theta}(n_t, r_{t+1})$, as has been demonstrated in proposition 1. The overall effect thus critically depends on the distribution function $F(\theta_i)$.

As has been demonstrated in Gordon (1989), a sufficient condition for an interior steady state $n_*, \alpha \in (0, 1)$ is given by

$$\max\left\{ \hat{\theta}, \mu \right\} > \hat{\theta}(k_*, 1) > \min\left\{ \hat{\theta}, \mu \right\}$$

(27)

where $k_* = (\beta/(1 + \beta)(1 - \alpha)A)^{1/(1 - \alpha)}$ is the steady state solution of equation (22). Moreover, in the appendix it is shown that a sufficient condition for a stable steady state $(n_*, k_*)$ of the dynamic system defined by equations (22) and (23) is

$$
\frac{1}{F'(\hat{\theta}(n_*, r_*))} > \beta \alpha(1 - \alpha) A k_*^\alpha \left( \frac{\mu}{1 + \beta} + \frac{\tau(1 - \delta)(1 - p(1 + \gamma))}{k_*(1 - \delta + (1 - \tau)r_*)^2} \right)
$$

(28)

The existence of multiple steady states clearly depends on the functional form of $F$.\footnote{See Kim (2003) or Traxler (2010) for an analysis of multiple steady states.} For an uniform distribution, however, there is a unique steady
state \((n, k)\) and the stability condition boils down to assuming that \(\bar{\theta}\) is sufficiently large. As a consequence, in this case it is also possible to explicitly determine the signs of equations (25) and (26). More precisely, we get

\[
\text{sign} \left( \frac{\partial \bar{e}_{t+1}}{\partial k_t} + 1 \right) = - \text{sign} \left( \frac{\partial \bar{e}_{t+1}}{\partial \tau} \right) = \mu(1 - n_t)(\gamma - 1)R_{t+1} - \beta \tau \gamma r_{t+1} + \bar{m}
\]

where \(R_{t+1} = 1 - \delta + (1 - \tau)r_{t+1}\) and

\[
\bar{m} = \sqrt{\beta \tau \gamma r_{t+1} - \mu(1 - n_t)(\gamma + 1)R_{t+1}^2 + 4p \beta \tau \gamma r_{t+1} \mu(1 - n_t)(\gamma + 1)R_{t+1}}.
\]

Straight forward calculations show that

\[
- \text{sign} \left( \frac{\partial \bar{e}_{t+1}}{\partial k_t} \right) = \text{sign} \left( \frac{\partial \bar{e}_{t+1}}{\partial \tau} \right) > 0 \Leftrightarrow \hat{\theta}(n_t, r_{t+1}) > 0.
\]

These findings are summarized in the following proposition:

**Proposition 2** Assume that \(F\) is uniformly distributed with support \([0, \bar{\theta}]\). Then, a higher tax rate increases the aggregate share of evasion, i.e. \(\partial \bar{e}_{t+1}/\partial \tau > 0\) whereas it is decreasing with the level of per capita income, i.e. \(\partial \bar{e}_{t+1}/\partial k_t < 0\).

**Proof:** See Appendix.

The above results are consistent with the empirical findings of Crane and Nourzad (1986) who document the evolution of aggregate evasion in the US over the period 1947-81. Furthermore, interpreting each point along the dynamic path of the economy as a specific set of countries, our model accounts for the observation that tax evasion is a more severe problem in developing countries as compared to developed ones (see e.g. Gordon and Li (2005)). In particular, our simple model predicts that the share of evasion decreases as economies converge towards the (unique) steady state. Moreover, as a consequence, the average moral cost of tax evasion decreases with per capita income, which is consistent with empirical results (see Torgler and Schneider (2007) and Bethencourt (2013)). Finally, the extended version of the model allows to account for existent puzzles in the literature of tax evasion which the basic model without morality could not explain. In fact, predictions of the model are consistent with the following empirical findings: a positive relationship between tax evasion and tax rates at the micro and macro level; the fact that there exists taxpayers that never evade as long as this is the profitable option and the observation that the higher is the share of evaders
in the society, the lower is the morale cost of evading and so, the laxer are taxpayers’ attitudes towards evasion.

Summarizing, the model is able to account simultaneously for well known micro empirical findings as well as for the latest macro-dynamic observations which have been recently documented in the empirical literature on tax evasion.

3.2 A Numerical Example

In order to illustrate how the aggregate share of evasion and the share of evaders in the economy react to an increase in the tax rate and how these variables evolve along the transition towards the steady state, we perform a simple numerical simulation exercise. More specifically, we use the following parameter configuration: $\alpha = 0.3$, a standard value in the literature, $p = 0.05$, $\gamma = 2$ which implies $1 - p(1 + \gamma) = 0.85$ and therefore corresponds to the average value implied by the fiscal systems of most countries (see Kim (2003)), $A = 8$ and $\delta = 0.9$ as in Rivas (2003). Finally, we set $\beta = 0.7$, $\mu = 0.1$ and $\bar{\theta} = 1$. Table 1 summarizes the share of evaders and the share of evaded income in steady state for varying levels of $\tau$. Clearly, an increase in the capital income tax rate by five percentage points raises the share of evaders in society as well as the share of taxes evaded. For example, increasing $\tau$ from 0.3 to 0.35 raises the share of evaders by five percentage points from 13.8% to 19.9% while the aggregate share of evaded income increases by 2.2 percentage points from 7.7% to 9.9%.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_*$</td>
<td>0.0364</td>
<td>0.0842</td>
<td>0.1380</td>
<td>0.1990</td>
<td>0.2688</td>
</tr>
<tr>
<td>$\bar{\epsilon}_*$</td>
<td>0.0170</td>
<td>0.0497</td>
<td>0.0776</td>
<td>0.0994</td>
<td>0.1165</td>
</tr>
</tbody>
</table>

Table 1: Predicted steady state shares of evaded income and of evaders in society for varying levels of the capital income tax.

Figure 2 presents the dynamics of $\bar{\epsilon}_{t+1}$ and $n_{t+1}$ for alternative levels

\footnote{It is worth noting that a calibration exercise for specific countries is beyond the scope of the present paper, whose aim is mainly concerned with the theoretical analysis of the dynamical features of an OLG economy with capital income tax evasion. In this respect, this model may be considered a useful starting point where more realistic components can be added for calibrations analysis and policy exercises in both developing and developed countries.}
Figure 1: Dynamics of the share of evaded income (left) and the share of evaders in society (right) for different levels of $\tau$ and initial values $k_0 = 1$ and $n_0 = 0.3$. Stars indicate steady state levels.

of $\tau$. More precisely, $\tau = 0.2$ corresponds to the lowest transitional paths whereas the highest ones correspond to $\tau = 0.4$. Both the share of evaded income as well as the share of evaders in society monotonically decrease as capital accumulates in line with our theoretical predictions.

Finally, Figure 3 illustrates the evolution of average marginal costs of evasion per percentage points of evaded income, i.e. $\int_0^\theta (\theta_i + \mu(1-n_i))f(\theta_i)d\theta_i$. These costs increase throughout the transition towards the steady state as the share of evaders decreases. Furthermore, the level of these costs increases with the tax rate. As a result, tax morale increases with the stage of economic development when economies accumulate capital.

Figure 2: Average morale costs per percentage points of evaded income for different levels of $\tau$ and initial values $k_0 = 1$ and $n_0 = 0.3$. Stars indicate steady state levels.

More specifically, $\tau$ is increased in steps of 5% percentage points from $\tau = 0.2$ to $\tau = 0.4$. 

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4 Conclusions

This paper integrates non-pecuniary costs of evasion into a dynamic overlapping generations model of capital income tax evasion to explain the empirical observation that evasion is a more severe phenomenon among developing countries as compared to developed countries. It is shown that morale concerns may not only explain why some taxpayers never evade even if the gamble is better than fair, and how a higher tax rate can increase evasion (as has been demonstrated before in static models of tax evasion) but also that the share of evaded taxes over GDP may decrease with the stage of economic development as per capita income increases. By contrast, tax morale increases with per capita income as the number of evaders in society declines. While the overall effect on aggregate evasion in general critically depends on the relative share of two subgroups, namely those individuals who display conventional comparative static behavior and their more honest peers who care about non-pecuniary costs, results for a uniform distribution suggest that the overall effect is indeed negative. Moreover, an increase in the tax rate increases aggregate evasion as well as the number of evaders in the economy when taxpayers decisions are interdependent whereas the number of evaders declines when per capita income increases as a lower interest rate reduces the size of the gamble.

Our findings complement the existing literature on evasion by demonstrating how the size of tax evasion evolves along the transitional path of an economy, whereas previous studies either consider a static environment without production (see e.g. Gordon (1989) or Kim (2003)) or focus on the relationship between tax evasion and economic growth on a balanced growth path (see e.g. Dzhumashev and Gahramanov (2011) or Levaggi and Menoncin (2012)). Furthermore, the present paper documents a positive relationship between per capita income and tax morale consistent with recent empirical evidence (see Torgler and Schneider (2007) and Bethencourt (2013)).

Throughout this paper we have assumed that tax revenue is wasted (or equivalently spent on some public consumption good). Consequently, there is no feedback effect from the aggregate level of evasion on capital accumulation through the government’s budget constraint. The simplest though not convincing way, however, to incorporate such a feedback effect would be to assume that revenue is redistributed lump sum to the young households. In this case, an increase in the share of evaders in the economy exerts a direct negative effect on per capita income as individuals save less due to a smaller
We have also abstracted from wage income taxation. The reason is that individuals savings and evasion decisions became dependent as the amount of taxes evaded in the first period of life affects the potential to save for old age consumption. Numerical simulation results at the individual level for such a case, however, suggests that the main result of proposition 1 carries over to a more elaborated model. We leave a more thorough analysis of these feedback effects for future research.

Acknowledgements

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Appendix

Proof of proposition 1:

For reasons of notational simplicity, set $\tilde{\theta}_i = \theta_i + \mu(1 - n_t)$. First, consider the effect of a tax increase. Implicitly differentiating equation (17) with respect to $\tau$ gives

$$\frac{d\theta_i^{t+1}}{d\tau} = \frac{1}{-E[U(e_i^{t+1})]^{\mu}}((1 - p)\tau r_{t+1}u'(R_{t+1}^u)r_{t+1}[\rho(R_{t+1}^u)(1 - e_i^{t+1})]$$

$$- \rho(R_{t+1}^d)(1 + \gamma e_i^{t+1}]) + \frac{\tilde{\theta}_i}{\beta} \rho(R_{t+1}^d)(1 + \gamma e_i^{t+1})r_{t+1})$$

(33)

Setting the nominator equal to zero and solving for $\theta_i$ yields

$$\tilde{\theta} = \beta \frac{(1 - p)\tau (r_{t+1})^2 u'(\tilde{d}u)[\rho(\tilde{d}u)(1 - \tau + \tau e_i^{t+1})] - \rho(\tilde{d}^d)(1 - \tau - \tau \gamma e_i^{t+1})}{1 - r_{t+1}\rho(\tilde{d}^d)(1 - \tau - \tau \gamma e_i^{t+1})} - \mu(1 - n_t)$$

(34)

Alternatively, one may also consider financing productive public spending which however dramatically increases the model's complexity and precludes analytical solutions.
Hence, we have

\[
\frac{dE[U(e^{i,*}_{t+1})]}{d\tau} \begin{cases} 
> 0 & \text{if } \theta_i > \tilde{\theta} \\
\leq 0 & \text{if } \theta_i \leq \tilde{\theta} \end{cases} \Leftrightarrow \frac{dE[e^{i,*}_{t+1}]}{d\tau} \begin{cases} 
> 0 & \text{if } \theta_i > \tilde{\theta} \\
\leq 0 & \text{if } \theta_i \leq \tilde{\theta} \end{cases} \tag{35}
\]

Similarly, the effect of an increase in the interest rate can be derived as follows:

\[
d e^{i,*}_{t+1} = \frac{1}{-E[U(e^{i,*}_{t+1})]} \left[-(1-p)\tau r_{t+1}u'(R^u_{t+1})\rho(R^u_{t+1})(1-\tau e^{i,*}_{t+1}) - \rho(R^d_{t+1})(1-\tau \gamma e^{i,*}_{t+1}) + \frac{\tilde{\theta}_i}{\beta r_{t+1}} - \tilde{\theta}_i \rho(R^d_{t+1})(1-\tau \gamma e^{i,*}_{t+1}) \right] \tag{36}
\]

Solving the above equation for \(\theta_i\) yields the same threshold \(\tilde{\theta}\) as in equation \((34)\) so that

\[
\frac{dE[U(e^{i,*}_{t+1})]}{dr_{t+1}} \begin{cases} 
> 0 & \text{if } \theta_i > \tilde{\theta} \\
\leq 0 & \text{if } \theta_i \leq \tilde{\theta} \end{cases} \Leftrightarrow \frac{dE[e^{i,*}_{t+1}]}{dr_{t+1}} \begin{cases} 
> 0 & \text{if } \theta_i > \tilde{\theta} \\
\leq 0 & \text{if } \theta_i \leq \tilde{\theta} \end{cases} \tag{37}
\]

We now have to prove that there exists non-emtpiness, this is, that there exists a set of individuals such that, \(\theta_i \in \left[\tilde{\theta}, \hat{\theta}\right]\) and thus, the above results are supported. Given that the framework we use to model the evasion decision is non static and the definitions of \(\tilde{\theta}, \hat{\theta}\) are sensitive to the state variables of the economy, it is needed to prove that \(\tilde{\theta} < \hat{\theta}\) in each period \(t\). The strategy we follow is to prove that \(\tilde{\theta} < \hat{\theta}\) \(\forall k_t\), or alternatively \(\forall r_{t+1}\). First, for \(r_{t+1} \to \infty\) \((k_t \to 0)\) one might get a sufficient condition for \(\tilde{\theta} < \hat{\theta}\) as follows: Note that \(\partial \hat{\theta}/\partial e_{t+1} > 0\). Therefore, set \(e_{t+1} = 1\) and compare the resulting expressions

\[
\hat{\theta} = \frac{\beta \tau (1 - p(1 + \gamma))}{1 - \tau} - \mu(1 - n_t) >\beta(1 - p)\tau^2(1 + \gamma) - \mu(1 - n_t) = \tilde{\theta}(e_{t+1} = 1)
\]

\[
\Leftrightarrow \frac{\beta \tau (1 - p(1 + \gamma))}{1 - \tau} > \beta(1 - p)\tau^2(1 + \gamma)
\]

The inequality is true if \(\gamma < \frac{3(1-p)}{3p+1}\) holds. So if \(k_t\) and \(\gamma\) are sufficiently small we have \(\tilde{\theta} < \hat{\theta}\).

Second, from equations \((19)\) and \((34)\) we get

\[
\frac{\partial \hat{\theta}}{\partial r_{t+1}} = \frac{\beta \tau (1 - p(1 + \gamma)) \left[1 - \delta \right]}{\left[1 - \delta + (1 - \tau) r_{t+1} \right]^2} > 0
\]
\[
\frac{\partial \tilde{\theta}}{\partial r_{t+1}} = \frac{\beta (1-p) t^2 (1+\gamma) e_{t+1} 2 (1-\delta) r_{t+1}}{(1-\delta + (1-\tau + \tau e_{t+1}) r_{t+1})^3} > 0
\]

and
\[
\frac{\partial^2 \tilde{\theta}}{\partial r_{t+1}^2} = \frac{-2 \beta \tau (1-p)(1+\gamma) [1-\delta] (1-\tau)}{[1-\delta + (1-\tau) r_{t+1}]^3} < 0
\]

\[
\frac{\partial^2 \tilde{\theta}}{\partial r_{t+1}^2} = \frac{\beta (1-p) t^2 (1+\gamma) e_{t+1} 2 (1-\delta) [1-\delta - 2(1-\tau + \tau e_{t+1}) r_{t+1}]}{(1-\delta + (1-\tau + \tau e_{t+1}) r_{t+1})^4}
\]

with
\[
\frac{\partial^2 \tilde{\theta}}{\partial r_{t+1}^2} = \begin{cases} 
> 0 & \text{if } r_{t+1} < \varphi(e_{t+1}) \\
= 0 & \text{if } r_{t+1} = \varphi(e_{t+1}) \\
< 0 & \text{if } r_{t+1} > \varphi(e_{t+1})
\end{cases}
\]

where \(\varphi(e_{t+1}) = \frac{1-\delta}{2(1-\tau + \tau e_{t+1})} > 0 \forall e_{t+1}
\]

Notice that \(\tilde{\theta} < \hat{\theta} \forall r_{t+1}\) if the following expression is satisfied
\[
\frac{\partial \tilde{\theta}}{\partial r_{t+1}} (r_{t+1} = \varphi(e_{t+1})) < \frac{\partial \hat{\theta}}{\partial r_{t+1}} (r_{t+1} = \varphi(e_{t+1}))
\]

this is,
\[
\frac{\beta (1-p) t^2 (1+\gamma) e_{t+1} 2 (1-\delta) r_{t+1}}{\left(\frac{3}{2}\right)^3 (1-\delta)^3} < \frac{\beta \tau (1-p)(1+\gamma) [1-\delta]}{[1-\delta + (1-\tau) r_{t+1}]^2}
\]

\[
\Leftrightarrow \frac{\tau e_{t+1}}{(1-\tau + \tau e_{t+1})} < \frac{(1-p)(1+\gamma)}{(1+\gamma) - p(1+\gamma)} \left(\frac{3}{2}\right) (1-\delta) \frac{3}{2}
\]

(38)

Given that the left side of the equation (38) is increasing in both \(\tau\) and \(e_{t+1}\), if we guarantee that equation (38) holds for \(\tau = e_{t+1} = 1\), then it is also true \(\forall \tau, e_{t+1}\). So, evaluating equation (38) at \(\tau = e_{t+1} = 1\), we obtain

\[
\frac{(1-p)(1+\gamma)}{(1+\gamma) - p(1+\gamma)} > \left(\frac{2}{3}\right)^3
\]

which is equivalent to \(\gamma < \frac{2(1-p)}{(1+2p)}\). The proof of proposition 1 follows by noting that \(\frac{3(1-p)}{(1+3p)} > \frac{2(1-p)}{(1+2p)}\).

**Proof of equation (28):**
A steady state \((k_*, n_*)\) of the dynamic system

\[
\begin{align*}
  k_{t+1} &= \Phi(k_t) \\
  n_{t+1} &= \Psi(n_t, k_{t+1})
\end{align*}
\] (39)

is locally stable if the following conditions are met (see de la Croix and Michel (2002))

\[
|1 + D| > |T| \quad \text{and} \quad |D| < 1
\] (41)

where \(T = \Psi_{k_t}(n_*, k_*) = F'(n_*, k_*) \frac{\partial \Phi}{\partial k_t} \frac{\partial r_{t+1}}{\partial k_t}\) is the trace of the Jacobian matrix \(G\) derived from a first order Taylor expansion of the dynamic system around a steady state, i.e.

\[
G = \begin{pmatrix}
  0 & \Phi_{k_t}(k_*) \\
  \Psi_{n_t}(n_*, k_*) & \Psi_{k_t}(n_*, k_*)
\end{pmatrix}
\] (42)

and \(D = -\alpha \frac{\beta}{1+\beta} (1-\alpha) A k_{t-1}^{\alpha-1} F'(n_*, k_*) \mu\). The condition \(|D| < 1\) is equivalent to

\[
\frac{1}{F'(n_*, k_*)} > \frac{\beta}{1+\beta} (1-\alpha) A k_{t-1}^{\alpha-1} \mu.
\] (43)

Similarly, the condition \(|1 + D| > |T|\) is equivalent to (note that \(1 + D > 0\) since \(|D| < 1\))

\[
\frac{1}{F'(n_*, k_*)} > \beta (1-\alpha) A k_{t-1}^{\alpha-1} \left( \frac{\mu}{1+\beta} + \frac{\tau(1-\delta)(1-p(1+\gamma))}{k_*(1-\delta + (1-\tau)r_*)^2} \right).
\] (44)

This proves equation (28).

**Proof of proposition 2:**

In order to prove proposition 2 we need to derive the derivatives of \(\bar{e}_{t+1}\) with respect to \(r_{t+1}\) and \(\tau\). The derivation of these expressions relies on the explicit solution of equation (17). More precisely, solving for \(\bar{e}_{t+1}^{i,*}\) gives\(^{20}\)

\[
\bar{e}_{t+1}^{i,*} = \frac{\beta r_{t+1} \gamma \tau - R_{t+1}(\gamma - 1)(\theta_i + \mu(1-n_t)) - \bar{m}}{2 r_{t+1} \gamma \tau (\theta_i + \mu(1-n_t))}
\] (45)

where \(R_{t+1} = 1 - \delta + (1-\tau)r_{t+1}\) and

\[
\bar{m} = \sqrt{\left( \beta \gamma r_{t+1} - (\theta_i + \mu(1-n_t))(\gamma + 1)R_{t+1} \right)^2 + 4 p \beta \gamma r_{t+1} (\theta_i + \mu(1-n_t))(\gamma + 1)R_{t+1}}.
\] (46)

\(^{20}\)Note that there are two solutions. However, one of them can be excluded from the analysis due to economic reasoning as such a solution is positive for all values of \(\theta_i\).
Derivation of the above expression gives:

\[
\frac{\partial e_{i+1}^*}{\partial r_{t+1}} = \frac{(1 - \delta)(\gamma - 1)\tilde{m} + (\gamma + 1)(R_{t+1}(\gamma + 1)(\theta_i + \mu(1 - n_t)) - \beta r_{t+1}\gamma\tau(1 - 2p)))}{2r_{t+1}^2\gamma\tau\tilde{m}}
\]

and

\[
\frac{\partial e_{i+1}^*}{\partial \tau} = \frac{(1 - \delta + r_{t+1})(\gamma - 1)\tilde{m} - (\gamma + 1)(R_{t+1}(\gamma + 1)(\theta_i + \mu(1 - n_t)) + \beta r_{t+1}\gamma\tau(1 - 2p)))}{2r_{t+1}\gamma^2\tau^2\tilde{m}}
\]

Integrating these expressions over the relevant range ([0, \hat{\theta}]) and assuming an uniform distribution yields:

\[
\frac{\partial \tilde{e}_{t+1}}{\partial r_{t+1}} = \frac{(1 - \delta)[\beta r_{t+1}\gamma\tau - R_{t+1}(\gamma - 1)\mu(1 - n_t) - \tilde{m}]}{2R_{t+1}r_{t+1}^2\gamma\tau}
\]

and

\[
\frac{\partial \tilde{e}_{t+1}}{\partial \tau} = \frac{(1 - \delta + r_{t+1})(\beta r_{t+1}\gamma\tau - R_{t+1}(\gamma - 1)\mu(1 - n_t) - \tilde{m})}{2R_{t+1}r_{t+1}\gamma^2\tau^2}
\]

The sign of these derivatives is determined by the sign of the expression in curly brackets. The proof of proposition 2 follow immediately by noting that \(dr_{t+1}/dk_t < 0\) and by recalling equation (32).
References


