Regional income convergence in India: A Bayesian Spatial Durbin Model approach

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The discussion on regional disparity is essential for addressing politically sensitive policy issues in any federal polity. The research outcome of regional disparity analysis is, however, often ambiguous and is not robust to choice of strategies, namely β and σ convergence analysis. The regression based theoretically appealing β convergence approach have not given adequate attention to spatial effects. Spatial interactions would make the outcomes of this approach less reliable. This study, on reviewing various growth models found that Spatial Durbin Model of Fingleton and Lopez-Bazo(2006) is theoretically useful and empirically appropriate in β convergence analysis. This study estimated parameters of Bayesian Spatial Durbin Model using statewise real per capita GSDP data computed from Central Statistical Organisation (CSO) during the period 1980 – 2010. The study concludes that the later reform period has witnessed beta convergence due to feedback effect. The inclusion of spatial effects, the study contends, helped to explain the contemporary debate in β convergence analysis in India.

Keywords: Convergence, Regional, Spatial Durbin Model and Bayesian econometrics.

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1 Introduction

Increasing economic disparities among both people and regions are always an issue of grave concern. Reducing regional economic disparity and ensuring balanced development is crucial in maintaining political stability of countries with federal polity. The findings of studies concerning regional disparities are thus essential in the promotion of balanced regional development. A study of this kind assumes special significance for India as the sustainability of growth momentum of one of the fast growing economies of the world relies on the political stability of Indian federal polity. The regional convergence analysis adopt two approaches namely β & σ convergence approach. Of the two approaches, β convergence approach (Barro and Sala-i-Martin 1996) is widely preferred for its roots in neoclassical analysis. However, the traditional regression models of β convergence strategy have not acknowledged the spatial effects namely spatial dependence and spatial heterogeneity. This second section of this paper reviews the β convergence approach in regional convergence analysis. The inclusion of spatial effects in this approach, the empirical issues related to that approach, and the interpretation of the model is discussed in this section. This study proposes a framework of β convergence approach that incorporates the concerns of spatial effects and accounts for the presence of spatial dependence and spatial heterogeneity. The section three explains the data source and methodology used in the study i.e Spatial Durbin Model, a variant of spatial autoregression model using state-wise per capita income data during 1980-2010 is estimated and the results of the same is presented. Section four discusses the obtained result and section five concludes.

2 β convergence approach

Conventionally the regional convergence is assessed in σ-convergence and β-convergence framework (Barro and Sala-i-Martin 1995; and Sala-i-Martin 1996). When the dispersion of real per capita income across regions falls over time, then σ-convergence is said to exist. When the correlation between growth in income over time and its initial level is negative, then β-convergence is presumed. Among the two, regression based β convergence approach is widely used compared to dispersion measure based σ convergence approach because of its proximity to neo classical theoretical analysis. In a typical β convergence approach, a neo classical growth equation, on cross sectional data is used. The regression model used in this approach may be given as

$$\left(\frac{y_{it} - y_{i0}}{y_{i0}}\right) = \alpha + \beta\, y_{i0} + u_i$$

where, y_{it} – is the income of i^th^ state at time 't'; y_{i0} – is the income of i^th^ state at the initial year, and so, $[(y_{it} - y_{io})/y_{io}]$ is the growth of i^th^ state at time 't'.

In this approach, the coefficient of y_{i0}, β is assessed for its statistical significance and for its sign to infer about convergence. When the estimate for β is negative and statistically significant, a convergence is confirmed in this approach. In other words, the lower initial income region has a higher growth rate as compared to regions with a higher initial income. The statistical insignificance or the positive co-efficient and its significance would suggest rejection of β convergence.

Another variant growth regression involves logarithm differences and more explanatory variables in addition to the principal variable, initial income (Barro R. and Sala-i-Martin Xavier 1992). The presence of β convergence in this framework is taken as the incomes of all regions converge to each of its steady state (conditional β-convergence).

$$\frac{1}{T}\ \ln\left[\frac{y_{iT}}{y_{i0}}\right] = \alpha + \beta \ln\,[y_{i0}] + x'_i\gamma + \varepsilon_i,$$

where i (i=1, …, n), 0, and T are the indices that denote region, initial period, and final period respectively; y denotes the income; $T^{-1} \times \ln(y_{iT} / y_{i0})$ is the growth rate; x_i is a vector of m structural/ control variables of the region ; ε_i's are i.i.d. errors; α and β are the scalar parameters, and γ is a parameter vector.

2.1 β convergence and spatial effects

The growth is determined by large number of observable and unobservable factors and so parsimonious models are likely to result in specification error. The spatial lag term is likely to imbibe information of those variables. Therefore, the importance of inclusion of spatial effect viz., spatial dependence and spatial heterogeity within the growth equation framework was stressed in convergence analysis (Seya et al., 2012). However, it was pointed out that the spatial dependence issue was handled in an adhoc manner such traditional general econometric analysis (Fingleton and Lopez-Bazo, 2006). A systematic effort was made to include the spatial dependence using economic spillover models (Egger and Pfaffermayr, 2006). It was suggested that various spatial autoregression models(SAR) offer sufficient scope for the inclusion of spatial dependence or spatial spillover effects into growth equation models.

Different spatial auto regression models (SAR) were considered in the literature. The difference was essentially charecterised by the inclusion of spatial lag terms for the different explanatory variables components in the growth regression namely, initial income variable, structural variable and control variables(Lopez-Bazo et al., 2004; Ertur and Koch, 2007; Basile, 2008). Kakamu(2009) has favoured the inclusion of spatial lag for dependend and for all the explanatory variable to address the issue of spatial dependence. This type of models in literature is called Spatial Durbin Models (SDM).

The growth equation model in SDM framework is likely to be afflicted with heteroscedastic error as the growth determinants of spatial units would vary and would be difficult to specify (Seya et al., 2012). In turn, the estimates in the presence of such hertoscedasticity would be inefficient. This is serious in β convergence testing as statistical significance of β is prime concern in deciding on the issue of convergence. Further, the inclusion of spatial lag variables in SDM would tend to increase the risk of multicollinearity problem in the growth regression (Kakamu, 2009).

Different approaches to address those issues of estimation in this framework were considered. One strategy suggested to address the concerns in the estimation was panel data approach (Lopez-Rodrigues, 2008; Parent and LeSage 2010). But this approach suffers from data availability as preparing a data set of explained and explanatory variables for all the years was not always possible. The second approach to address the issue of spatial heterogeneity in the spatial Durbin framework was using Maximum Likelihood Estimators (MLE) but was found to suffer from loss of degrees of freedom (Seya et al., 2012)[1]. The third approach that uses Bayesian Econometrics was found to provide strategy to address the issue of spatial dependence, spatial heteroscedasticity and loss of degrees of freedom at once (Geweke 1993). This strategy is also found to provide robust estimates in the presence of multicollinearity. For this study, the third approach observed to be appealing. The details of the methodology used in this study are discussed below:

2.2 Bayesian approach to estimation of SDM

[1] The MLE would require to estimate as many error variance as the number of spatial units included in the study, apart from the regular parameters in the SDM model for the efficient statistical inference. This may cause severe loss of degrees of freedom in the model.

The Bayesian approach to estimate Spatial Durbin Model was described by Seya et al.(2012). The SDM model is defined as,

$$Y^* = \rho WY^* + \alpha\iota + \beta Y_o + \theta WY_o + X\gamma + WX\xi + \varepsilon,$$

where Y^* is an $n\times1$vector whose elements are given by $T^{-1}\times\ln(y_{i,T}/y_{i,0})$; ι is an $n\times1$ vector with all elements equal to 1; Y_0 is an $n\times1$ vector whose elements are given by $\ln(y_{i0})$; WY^* is an n x 1 vector whose elements are spatial lag for Y^*; X is an $n\times m$ structural and control variables matrix; WX is an n x m matrix whose elements are the spatial lags of structural and control variables; ε is an $n\times1$ vector of i.i.d. errors; W is a row- standardized spatial weight matrix of order n; ρ is the spatial dependence parameter[2]; β and θ are scalar parameters; γ and ξ are $m\times1$parameter vectors.

In sum, in this framework the issue of spatial dependence is accounted by the spatial lag terms of explained and explanatory variables and the issue of spatial heterogeneity is addressed through employing the Bayesian estimates (LeSage, 1997; Pace and Barry, 1998).

The Bayesian estimation approach would require specification of three components namely, the prior distribution, likelihood function and the posterior distribution. The prior distributions are used to express the prior beliefs of the researcher on the parameters in terms of a probability distribution. Each of the parameters in the model needs to be assigned with a prior. The priors are of two types namely non informative / diffuse / ignorant priors and informative priors. The information about each of the parameters may be defined in terms of appropriate prior distributions, viz., normal, inverse gamma and chi-square distributions.

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[2] If the estimate for ρ is positive (negative) and statistically significant, positive (negative) spatial autocorrelation is implied.

The joint probability density function of error terms in the growth equation characterizes the likelihood function. The product of likelihood of each sample point would give likelihood of sample. The likelihood function would be a function of regression co-efficients, error variance and the spatial autocorrelation measure. Hence, the log likelihood function of the sample could be written as,

$L(\beta,\sigma,\rho,Y^*,X) = (2\pi)^{-n/2}\sigma^{-n}|I_n-\rho W|\exp\left\{-\frac{1}{2\sigma^2}(\varepsilon'\varepsilon)\right\}$ where $\varepsilon = [I_n-\rho W](Y^*-X\beta)$

The posterior distributions summarize information about different parameters of the model are drawn from the posterior distributions. The estimation and statistical inference in the Bayesian tradition the posterior distributions are derived by multiplying the likelihood function with the prior distribution function. The conditional posterior distribution of each parameter is derived using either Gibbs Sampling Algorithm or Metropolis-Hastings Algorithm.

2.3 Deriving posterior density for the coefficients of growth equation model in SDM

The Spatial Durbin Model could be rewritten as

$[I_n-\rho W]\ Y^* = Z\varphi + \varepsilon$, where, $Z=[\ \iota\ Y_o\ WY_o\ X\ WX]$ and $\varphi = [\alpha\ \beta\ \theta\ \gamma\ \xi]'$

To derive full prior distribution of this model, all the parameters of the model need to be specified. The parameters of interest in this model consists of regression co-efficient (φ), spatial dependence parameter (ρ), error variance (σ_ε^2) and relative variance co-variance of stochastic error term(V) i.e π (φ, ρ, σ_ε^2, V).

If the prior distributions are assumed to be independent, the joint prior distribution of the parameters used in the model may be given as

$\pi(\varphi, \rho, \sigma_\varepsilon^2, V) = \pi(\varphi).\pi(\rho).\pi(\sigma_\varepsilon^2).\pi(V)$

The priors for the above parameters and justifications for the same is given in Seya et al(2012). The following are the priors for the parameters:

(i). $\rho \sim$ unif(-1,+1) , a uniform prior

(ii). $\varphi \sim$ a diffuse prior

(iii). $\sigma_\varepsilon^2 \sim$ a standard diffuse prior

(iv). $v_i^{-1} \mid q \sim$ iid $\chi^2(q)$, v_i is the i^{th} element in the diagonal of V, the relative variance covariance matrix.

(v). $q \sim \Gamma(a_q\ ,\ b_q)$, a Gamma prior , the parameter q charecterises the distribution of v_i.

Joint posterior distribution function of the parameters may be got from the product of the respective prior and likelihood functions. Full conditional prior for various parameters in the model may be derived as given below:

(a). The full conditional prior for φ

$\pi(\varphi \mid \rho, \sigma_\varepsilon^2, V, q) \propto N(r, S)$, Normal distribution

where $r = [\sigma_\varepsilon^{-2} Z'V^{-1}\tilde{Y}]$; $S = \left[\sigma_\varepsilon^{-2} Z'V^{-1}Z\right]^{-1}$; and $\tilde{Y} = (I_n - \rho W)Y$

(b). The full conditional prior for σ_ε^2

$\pi(\sigma_\varepsilon^2 \mid \varphi, \rho, V, q) \propto IG\left[\frac{n}{2}, \frac{e'V^{-1}e}{2}\right]$, Inverse Gamma distribution

(c). The full conditional posterior for v_i in V

$\pi\left[\frac{-\sigma_\varepsilon^2 e_i^2}{v_i} \mid \varphi, \rho, \sigma_\varepsilon^2, v_{-i}, q\right] \propto$ iid $\chi^2(q+1)$ Chi square distribution; e_i – is the i^{th} element of e & v_{-i} denotes the vector of all diagonal elements except v_i.

(d). The full conditional posterior for ρ.

$$\pi(\rho \mid \varphi, \sigma_\varepsilon^2, V, q) \propto |I - \rho W| \exp\left\{-\frac{1}{2\sigma^2}\left(e'V^{-1}e\right)\right\}$$ is a kernel of distribution.

(e). The log of the full condition posterior distribution for q

$$\pi(q \mid \rho, \varphi, \sigma_\varepsilon^2, V) = \text{constant} + \frac{nq}{2}\ln\left(\frac{q}{2}\right) - n\ln\Gamma\left(\frac{q}{2}\right) - Kq - (a_q - 1)\ln(q)\ ;\ K = \frac{1}{2}\sum_{i=1}^{n}\left\{\ln(v_i) + \frac{1}{v_i}\right\} + b_a$$

The samples for the distributions [a]-[c] are generated with Gibbs Sampler, and the distributions [d]-[e] are generated with Metropolis – Hastings Algorithm (M-H Algorithm). These samples were used for the further analysis.

2.4 Interpreting the Spatial Durbin Model:

The traditional β convergence approach draws its inference solely from the coefficient of initial income variable (Y_0), β. For spatial Durbin model this interpretation is not valid (LeSage and Fischer 2008; Fischer 2010). In this model there would be two effects; one described by Y_0 and the other described by WY_0, as Y^* is affected directly by any change in Y_0 and is also affected by the feedback effect through $Y_{(j,0)}$. Thus, the impact of the initial value varies with location and the neighborhoods described by W. The former effect is the direct effect while the later is the indirect effect. They may be measured using the following:

$M_{direct}=n^{-1}tr(S(W))$, $S(W) = [1-\rho W]^{-1}[\beta I - \theta W]$; $M_{total}=n^{-1}\iota' S(W)\iota$; $M_{indirect}=M_{total}-M_{direct}$, where, $S(W)=(I-\rho W)-1(\beta I+\theta W)$.

3 Methodology and data source

This study analysed the regional disparity among 17 major states viz., Andhra Pradesh, Assam, Bihar, Goa, Gujarat, Haryana, Himachal Pradesh, Karnataka, Kerala, Madhya Pradesh, Maharashtra, Orissa, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh and West Bengal, using the state-wise data on Gross State Domestic Product (GSDP) at 2004-05 constant prices, obtained from the Ministry of Statistics and Programme Implementation. The percapita income was calculated using the projected state-wise population data from the report of the Registrar General of Census, Government of India. The spatial weight matrix was computed based on row standardized binary contiguity matrix.

The Bayesian approach to Spatial Durbin Model was estimated with the help of Spatial Econometric Toolbox for MATLAB developed by LeSage and Pace (2009) for the three periods viz., [a] 1980 – 1991 (pre reform period); [b] 1991 – 2000 (early reform period); and [c] 2000 – 2010 (later reform period).

This study used proportion of agriculture in per capita GSDP, proportion of industry in per capita GSDP and tertiary to industrial sector outputs ratio as structural variables, apart from the usual growth equation variables. The marginal likelihood was computed using method developed by Gelfand and Dey (1994).

4 Results and Discussion:

The results are given in the table 1 for all the 3 periods and the t value of the same is given. The statistic values suggest that the samples were successfully converged to the posterior distribution.

Table 1: Results of parameter estimation of Spatial Durbin Model for various time periods

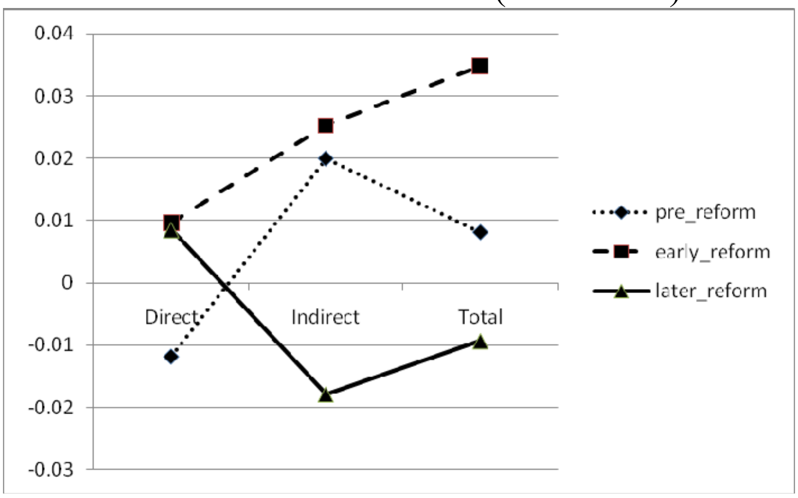
	Pre Reform period	Early Reform period	Later Reform period
Initial Income	-0.0123 (6.2709)	0.0121 (9.1792)	0.0078 (9.7654)
Proportion of Agriculture	-0.0514 (2.1626)	-0.0909 (13.3185)	-0.0908 (6.8347)
Proportion of Industry	0.0016 (0.0319)	-0.0135 (1.0391)	-0.0319 (1.2785)
Tertiary – Industry Ratio	-0.0054 (1.5013)	0.0048 (3.4152)	-0.0026 (1.0669)
Spatially lagged initial income	0.0196 (4.7171)	0.0336 (7.0966)	-0.0184 (7.4311)
Spatially lagged proportion of agriculture	0.035 (0.9213)	-0.0969 (3.2758)	-0.1727 (8.4939)
Spatially lagged proportion of industry	-0.1741 (1.6124)	-0.1239 (2.3915)	-0.3973 (7.0927)
Spatially lagged tertiary industry ratio	-0.0287 (3.6079)	0.0066 (1.3946)	-0.0402 (7.3162)
Constant	0.0636 (0.6921)	-0.3184 (4.6158)	0.4222 (6.3473)
Spatial Dependence measure - Rho (ρ)	0.1030 (14.0658)	-0.3143 (36.4325)	-0.1462 (16.6499)
Error variance (σ_ε^2)	0.0027 (3.0117)	0.0012 (1.8254)	0.0015 (2.9036)
R^2	0.7894	0.4583	0.4279

Source: Author's calculation based on the methodology.
Note: The t – values of the respective coefficients are given in the parenthesis.

The estimation of ρ was positive for the first period. This could mean that the neighboring regions have evolved similarly especially over this period. The estimate for the initial income was negative only for the pre reform period. But for the other two periods the coefficients were positive and significant. However, the β convergence hypothesis should not be tested with these estimates. For all the periods the coefficient of agricultural proportion was negative and significant. In the first period coefficient of industrial proportion and of the tertiary-industry ratio the same was positive and negative respectively but not significant. In the early reform period, the coefficient of industrial proportion was found to be negative and insignificant. The tertiary – industry ratio was significantly positive. In the later reform period, the coefficient of proportion of agriculture was found to be negative and statistically significant but for the other two variables, it was not statistically significant.

As mentioned in the methodology, in the spatial Durbin model, β convergence hypothesis cannot be tested using the values of β in the growth regression. Therefore, the direct, indirect and total effects were derived from results of the analysis.

Figure 1 Decomposition of the overall effect of Y_0 on Y^* into direct and indirect effects (1980–2010)



Source: Authors calculation based on methodology.

Notes: Pre reform period represent the years 1980 to 1991
Early reform period represent the years 1991to 2000
Later reform period represent the years 2000 to 2010

The figure suggests that in the pre reform period direct effect was negative but the indirect effect was found to positive and the overall effect was positive. In the early reform period all the effects (direct / indirect / total) were positive. In the later reform period, though the direct effect was found to be positive, the indirect and over all effect was found to negative and hence a confirmation of beta convergence. In the pre reform and early reform periods the total effect suggesting the negation of beta convergence. The convergence studies of regional income conclude convergence in pre reform and non convergence in post reform periods. The direct effect of the growth equation is observed and interpreted in traditional β convergence studies. However, due to the feedback/ indirect effect, the total effect suggests that the later reform period alone witnessed convergence though the pre reform period witnessed non convergence. The negative indirect effect suggests the non existence of spillover effects. Thus, the result of this paper is able to explain irreconcilable outcomes found in the debate around β convergence analysis.

5 Conclusions

The study reviewed various growth models and contends that Spatial Durbin Model of Fingleton and Lopez-Bazo(2006) was empirically suitable. In this framework the regional income disparity using real percapita GSDP data in India during the pre early and later reform periods is analysed. The study estimated parameters of Bayesian Spatial Durbin Model for the three periods viz., pre reform (1980-1991), early reform (1991-2000) and later reform (2000-2010) periods. The convergence hypothesis is tested in the light of LeSage and Fischer (2008) formulation. The results suggest that the β convergence does not hold from the pre-reform and early reform periods. But the later reform period indicate regional convergence. The later reform period witnessed beta convergence due to feedback effect. The contemporary debate was only involving direct effect, and overlooked the indirect and total effects. The inclusion of spatial effects in β convergence analysis helped to address the econometric issues such as violation of sphericity assumption and to resolve the raging debate in β convergence analysis.

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