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Abstract

This paper analyses the implications of classical liberal and libertarian approaches for distributive justice in the context of social welfare orderings. An axiom capturing a liberal non-interfering view of society, named the Weak Harm Principle, is studied, whose roots can be traced back to John Stuart Mill's essay *On Liberty*. It is shown that liberal views of individual autonomy and freedom can provide consistent foundations for social welfare judgements, in both the finite and the infinite context. In particular, a liberal non-interfering approach can help to adjudicate some fundamental distributive issues relative to intergenerational justice. However, a surprisingly strong and general relation is established between liberal views of individual autonomy and non-interference, and egalitarian principles in the Rawlsian tradition.

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1 Introduction

What are the implications of classical liberal and libertarian approaches for distributive justice? Can liberal views of individual autonomy and freedom provide consistent foundations for social welfare judgements? In particular, can a liberal non-interfering approach help to adjudicate some fundamental distributive issues relative to intergenerational justice? What is the relation between classical liberal political philosophy and the egalitarian tradition stemming from John Rawls's seminal book *A Theory of Justice* ([49])?

This paper addresses these questions, and in so doing it contributes to three different strands of the literature.

In some recent contributions, Mariotti and Veneziani ([47], [43]) have explored a new notion of respect for individual autonomy in social judgements, suited for Social Welfare Orderings (henceforth, SWOs), whose philosophical roots can be traced back to John Stuart Mill's essay *On Liberty*. The *Principle of Non-Interference* embodies the idea that "an individual has the right to prevent society from acting against him in all circumstances of change in his welfare, *provided* that the welfare of no other individual is affected" ([47], p.1).

Formally, the Principle Non-Interference (or Non-Interference, in short) can be illustrated as follows: in a society with two individuals, consider two allocations $u = (u_1, u_2)$ and $v = (v_1, v_2)$, describing the welfare levels of the two agents in two alternative scenarios. Suppose that, *for whatever reason*, u is strictly socially preferred to v. Suppose then that agent 1 either suffers a welfare loss, or enjoys a welfare increase in both allocations, while agent 2's welfare is unchanged, giving rise to two new allocations $u' = (u_1 + \varepsilon_u, u_2)$ and $v' = (v_1 + \varepsilon_v, v_2)$, with $\varepsilon_u \varepsilon_v > 0$. Non-Interference says that, if agent 1 strictly prefers u' to v', then society should *not reverse* the strict preference between u and v to a strict preference for v' over u'. An agent "can veto society from a strict preference switch after a positive or negative change that affects only [her] and nobody else" ([47], p.2).

The veto power accorded to individuals is weak because a switch to indifference is admitted, *and* because Non-Interference is silent in a number of welfare configurations (e.g., if agent 1's welfare changes in opposite directions, $\varepsilon_u \varepsilon_v \ngtr 0$, or if she does not strictly prefer u' to v'). There are numerous non-dictatorial, and even anonymous SWOs that satisfy Non-Interference. Yet, surprisingly, Mariotti and Veneziani ([47]) prove that, in societies with a finite number of agents, dictatorial SWOs are the *only* ones compatible with Non-Interference among those satisfying Weak Pareto.[1] Lombardi and Veneziani ([42]) and Alcantud ([2]) have extended this result to societies with a countably infinite number of agents.

This impossibility proves the limitations of liberal approaches to Paretian social judge-

[1] The Anonymity and Weak Pareto axioms are formally defined in section 2 below.

ments: there cannot be any 'protected sphere' for individuals even if nobody else is affected. As Mariotti and Veneziani ([47], p.2) put it, "Of the appeals of the individuals to be left alone because 'nobody but me has been affected', at least some will necessarily have to be overruled." The first contribution of this paper to the literature on liberal approaches is to analyse a specific, ethically relevant weakening of Non-Interference and provide a series of positive results, both in the finite and in the infinite context.

To be precise, we limit the bite of Non-Interference by giving individuals a veto power only in situations in which they suffer a *decrease* in welfare. Arguably, this captures the most intuitive aspect of a liberal ethics of non-interference, as it protects individuals in situations where they suffer a damage, while nobody else is affected: a switch in society's strict preferences against an individual after she has incurred a welfare loss would represent a double punishment for her.

Formally, in the two-agent example above, we restrict Non-Interference to hold in situations where $\varepsilon_u < 0, \varepsilon_v < 0$. We call this axiom the *Weak Harm Principle* - for it represents a strict weakening of the *Harm Principle* first introduced by Mariotti and Veneziani ([44]) - and show that a limited liberal ethics of non-interference can lead to consistent social judgements.[2]

The implications of liberal principles of non-interference (in conjunction with standard axioms in social choice), however, turn out to be fairly surprising. For there exists a strong formal and conceptual relation between liberal views, as incorporated in the Weak Harm Principle, and egalitarian social welfare relations (henceforth, SWRs). The analysis of this relation is the second main contribution of the paper.

Formally, we provide a number of fresh characterisations of widely used Rawlsian SWRs. Standard characterisations of the difference principle, or of its lexicographic extension, are based either on informational invariance and separability properties (see, e.g., d'Aspremont [21]; d'Aspremont and Gevers [22]) or on axioms with a marked egalitarian content such as the classic *Hammond Equity* axiom (Hammond [31], [32]).[3]

We prove that both the Rawlsian difference principle and its lexicographic extension can be characterised based on the Weak Harm Principle, together with standard efficiency, fairness and - where appropriate - continuity properties. The adoption of SWRs with a strong egalitarian bias can thus be justified based on a liberal principle of non-interference which is logically distinct from informational invariance and separability axioms, has no egalitarian

[2] Mariotti and Veneziani ([45]) analyse different restrictions of Non-Interference and characterise Nash-type orderings. For a related analysis of utilitarianism, see Mariotti and Veneziani ([46]).

[3] See also Tungodden ([59], [60]) and Bosmans and Ooghe ([15]). Similar axioms are used also in the infinite context; see, e.g., Lauwers ([37]), Asheim and Tungodden ([5]), Asheim et al. ([8]), Bossert et al. ([16]), Alcantud ([1]), Asheim and Zuber ([6]).

content and indeed has a marked individualistic flavour (in the sense of Hammond [33]).

This surprising relation between liberal approaches and egalitarian SWRs has been originally established by Mariotti and Veneziani ([44]), who have characterised the leximin SWO in finite societies based on the Harm Principle. We extend and generalise their insight in various directions.

First of all, as noted above, we focus on a strict weakening of the Harm Principle. This is important both formally and conceptually. Formally, it has been argued that the characterisation in Mariotti and Veneziani ([44]) is less surprising than it seems, because under Anonymity the Harm Principle implies Hammond Equity (see Alcantud [2], Proposition 4). This conclusion does not hold with the Weak Harm Principle: even under Anonymity, the Weak Harm Principle and Hammond Equity are logically independent and the original insight of Mariotti and Veneziani ([44]) is therefore strengthened. Conceptually, by ruling out *only* a strict preference switch in social judgements, the Weak Harm Principle captures liberal and libertarian views more clearly than the Harm Principle, for it emphasises the negative prescription at the core of Mill's analysis of non-interference and assigns a significantly weaker veto power to individuals.

Further, based on the Weak Harm Principle, we also provide new characterisations of Rawls's difference principle. Compared to the leximin, the *maximin* SWR may be deemed undesirable because it defines rather large indifference classes. Yet, in a number of settings, its relatively simpler structure is a significant advantage, which allows one to capture the core egalitarian intuitions in a technically parsimonious way. Moreover, unlike the leximin, the maximin satisfies continuity and therefore egalitarian judgements based on the difference principle are more robust to small measurement mistakes, e.g. in empirical analysis. This probably explains the wide use of the maximin in modern theories of equality of opportunity (Roemer [50], [51]; Gotoh and Yoshihara [30]), in experimental approaches to distributive justice (Konow [36]; Bolton and Ockenfels [14]), in the analysis of the ethics of exhaustible resources and global warming (Solow [58]; Cairns and Long [18]; Roemer [53]; Llavador et al. [39]), and in the context of intergenerational justice (Silvestre [57]; Llavador et al. [38]).[4] In the analysis of intergenerational justice and environmental economics, the maximin principle is often taken to embody the very notion of *sustainability* (Llavador et al. [40]).

Indeed, and this is the third main contribution of the paper, we analyse liberal and libertarian approaches to intergenerational justice. On the one hand, the intergenerational context provides a natural framework for the application of liberal principles of non-interference.

[4] Maximin preferences are prominent also outside of normative economics - for example, in decision theory and experimental economics. See, inter alia, the classic papers by Maskin ([48]); Barberà and Jackson ([11]); Gilboa and Schmeidler ([29]); and, more recently, de Castro et al. ([23]); Sarin and Vahid ([55]).

For there certainly are many economic decisions whose effects do not extend over time and leave the welfare of other generations unchanged. Moreover, liberal principles of non-interference seem to capture some widespread ethical intuitions in intergenerational justice (Wolf [62]). In the seminal Brundtland report, for example, sustainable development is defined precisely as "development that meets the needs of the present without compromising the ability of future generations to meet their needs" (Brundtland [17], p.43).

On the other hand, the application of liberal principles to intergenerational justice raises complex theoretical and technical issues. Lombardi and Veneziani ([42]) and Alcantud ([2]) have shown that there exists no fair and Paretian SWR that satisfies a fully non-interfering view in societies with a countably infinite number of agents. More generally, the analysis of distributive justice among an infinite number of generations is problematic for *all* of the main approaches, and impossibility results often emerge (Lauwers [37]; Basu and Mitra [12]; Fleurbaey and Michel [26]; Zame [63]; Hara et al. [34]; Crespo et al. [20]). Several recent contributions have provided characterisation results for SWRs by dropping either completeness (Basu and Mitra [13]; Asheim and Tungodden [5]; Bossert et al. [16]; Asheim et al. [8]) or transitivity (Sakai [54]).[5] But the definition of suitable anonymous and Paretian SWRs is still an open question in the infinite context (for a thorough discussion, see Asheim [3]).

Our main contribution to this literature is a novel analysis of liberal egalitarianism in economies with a countably infinite number of agents.

To be specific, we provide a new characterisation of one of the main extensions of the leximin SWR in infinitely-lived societies, namely the leximin overtaking proposed by Asheim and Tungodden ([5]). As in the finite-horizon case, we show that the Weak Harm Principle can be used to provide a simple and intuitive characterisation, without appealing to any informational invariance or separability property, or to axioms with an egalitarian content. Indeed, although we focus on a specific extension of the leximin that is prominent in the literature on evaluating infinite utility streams, our arguments can be modified to obtain new characterisations for *all* of the main approaches.

We also extend the analysis of Rawls's difference principle to the intergenerational context. As already noted, if the leximin is adopted, social judgements are sensitive to tiny changes in welfare profiles and measurement errors. In the intergenerational context, an additional issue concerns the significant incompleteness of leximin SWRs which may hamper social evaluation in a number of ethically relevant scenarios (see the discussion in Asheim et al. [7]). Therefore we provide a novel characterisation of the maximin *ordering* (more

[5] Asheim and Zuber ([6]) have recently proposed a complete and transitive extension of the leximin SWR which overcomes the impossibility by requiring only sensitivity to the interests of generations whose consumption has finite rank.

precisely, the *infimum rule*, Lauwers [37]) in societies with a countably infinite number of agents: based on the Weak Harm Principle, we identify a complete egalitarian criterion that allows for robust social evaluation of intergenerational distributive conflicts.

Our result differs from other characterisations in the literature in two key respects. Conceptually, the characterisation is again obtained by focusing on standard efficiency, fairness, and continuity properties together with a liberal principle of non-interference: neither egalitarian axioms, nor informational invariance or separability properties are necessary. Formally, unlike in Lauwers' ([37]) seminal paper, the proof of the characterisation result in the infinite context echoes very closely the proof in finite societies: perhaps surprisingly, both the axiomatic framework and the method of proof - and thus the underlying ethical intuitions - are essentially invariant.

In the light of our results, we can provide some tentative answers to the questions posed in the opening paragraph. Liberal and libertarian approaches emphasising individual autonomy and freedom are logically consistent and provide useful guidance in social judgements (including in the analysis of intergenerational justice), provided the notion of non-interference is suitably restricted. Perhaps counterintuitively, however, a liberal non-interfering approach emphasising individual protection in circumstances of welfare losses leads straight to welfare egalitarianism. Based on the Weak Harm Principle, it is possible to provide a unified axiomatic framework to analyse a set of SWRs originating from Rawls's difference principle in a welfaristic framework. Thus, our analysis sheds new light on the normative foundations of standard egalitarian principles and provides a rigorous justification for the label 'liberal egalitarianism' usually associated with Rawls's approach.

The rest of the paper is structured as follows. Section 2 lays out the basic framework. Section 3 introduces our main liberal axiom and characterises the leximin SWO in economies with a finite number of agents. Section 4 analyses the implications of liberal views for robust (continuous) SWOs and derives a characterisation of the difference principle. Sections 5 and 6 extend the analysis to the intergenerational context. Section 7 concludes.

2 The framework

Let $X \equiv [0,1]^{\mathbb{N}}$ be the set of countably infinite utility streams, where $\mathbb{N}$ is the set of natural numbers. An element of X is ${}_1u = (u_1, u_2, ...)$ and u_t is the welfare level of agent t, or - in the intergenerational context - of a representative member of generation $t \in \mathbb{N}$. For $T \in \mathbb{N}$, ${}_1u_T = (u_1, ..., u_T)$ denotes the T-head of ${}_1u$ and ${}_{T+1}u = (u_{T+1}, u_{T+2}, ...)$ denotes its T-tail, so that ${}_1u = \left({}_1u_T,\, {}_{T+1}u\right)$. For $x \in [0,1]$, ${}_{con}x = (x, x, x, ...)$ denotes the stream of constant level of well-being equal to x.[6]

A permutation π is a bijective mapping of $\mathbb{N}$ onto itself. A permutation π of $\mathbb{N}$ is finite if there is $T \in \mathbb{N}$ such that $\pi(t) = t$, $\forall t > T$, and Π is the set of all finite permutations of $\mathbb{N}$. For any ${}_1u \in X$ and any permutation π, let $\pi\left({}_1u\right) = \left(u_{\pi(t)}\right)_{t\in\mathbb{N}}$ be a permutation of ${}_1u$. For any $T \in \mathbb{N}$ and ${}_1u \in X$, ${}_1\bar{u}_T$ is a permutation of ${}_1u_T$ such that the components are ranked in ascending order.

Let $\succcurlyeq$ be a (binary) relation over X. For any ${}_1u$, ${}_1v \in X$, ${}_1u \succcurlyeq {}_1v$ stands for $({}_1u,\ {}_1v) \in \succcurlyeq$ and ${}_1u \not\succcurlyeq {}_1v$ for $({}_1u,\ {}_1v) \notin \succcurlyeq$; $\succcurlyeq$ stands for "at least as good as". The asymmetric factor $\succ$ of $\succcurlyeq$ is defined by ${}_1u \succ {}_1v$ if and only if ${}_1u \succcurlyeq {}_1v$ and ${}_1v \not\succcurlyeq {}_1u$, and the symmetric part $\sim$ of $\succcurlyeq$ is defined by ${}_1u \sim {}_1v$ if and only if ${}_1u \succcurlyeq {}_1v$ and ${}_1v \succcurlyeq {}_1u$. They stand, respectively, for "strictly better than" and "indifferent to". A relation $\succcurlyeq$ on X is said to be: *reflexive* if, for any ${}_1u \in X$, ${}_1u \succcurlyeq {}_1u$; and *transitive* if, for any ${}_1u$, ${}_1v$, ${}_1w \in X$, ${}_1u \succcurlyeq {}_1v \succcurlyeq {}_1w$ implies ${}_1u \succcurlyeq {}_1w$. $\succcurlyeq$ is a quasi-ordering if it is reflexive and transitive. Let $\succcurlyeq$ and $\succcurlyeq'$ be relations on X, we say that $\succcurlyeq'$ is an extension of $\succcurlyeq$ if $\succcurlyeq \subseteq \succcurlyeq'$ and $\succ \subseteq \succ'$.

In this paper, we study some desirable properties of quasi-orderings, which incorporate notions of efficiency, fairness and liberal views of non-interference. In this section, we present some basic axioms that are used in the rest of the paper.

A property of SWRs that is a priori desirable is that they be able to rank all possible alternatives. Formally:

COMPLETENESS, **C**: $\forall {}_1u, {}_1v \in X : {}_1u \neq {}_1v \Rightarrow {}_1u \succcurlyeq {}_1v$ or ${}_1v \succcurlyeq {}_1u$.

$\succcurlyeq$ is an ordering if it is a complete quasi-ordering.

The standard way of capturing efficiency properties is by means of the Pareto axioms.[7]

STRONG PARETO, **SP**: $\forall {}_1u, {}_1v \in X : {}_1u > {}_1v \Rightarrow {}_1u \succ {}_1v$.

WEAK PARETO, **WP**: $\forall {}_1u, {}_1v \in X, \forall \epsilon > 0 : {}_1u \geq {}_1v + {}_{con}\epsilon \Rightarrow {}_1u \succ {}_1v$.

A basic requirement of fairness is embodied in the following axiom, which states that social judgements ought to be neutral with respect to agents' identities.[8]

[6] The focus on the space of bounded vectors is standard in the literature (Lauwers [37]; Basu and Mitra [12], [13]; Zame [63]; Hara et al. [34]; Asheim [3]; Asheim and Banerjee [4]). It is worth noting in passing that, from a theoretical viewpoint, the T-dimensional unit box can be interpreted as the set of all conceivable distributions of *opportunities*, where the latter are conceived of as chances in life, or *probabilities* of success as in Mariotti and Veneziani ([45], [46]).

[7] The notation for vector inequalities is as follows: for any ${}_1u$, ${}_1v \in X$, let ${}_1u \geq {}_1v$ if and only if $u_t \geq v_t$, $\forall t \in \mathbb{N}$; ${}_1u > {}_1v$ if and only if ${}_1u \geq {}_1v$ and ${}_1u \neq {}_1v$; and ${}_1u \gg {}_1v$ if and only if $u_t > v_t$, $\forall t \in \mathbb{N}$.

[8] Observe that the axiom focuses only on finite permutations. For this reason, it is often referred to as *Weak* or *Finite* Anonymity in order to distinguish it from *Strong* Anonymity, which also allows for infinite

Anonymity, **A**: $\forall {}_1u \in X$, $\forall \pi \in \Pi$, $\pi({}_1u) \sim {}_1u$.

Finally, in the analysis of intergenerational justice, we follow the literature and consider two mainly technical requirements to deal with infinite-dimensional vectors (see, e.g., Asheim and Tungodden [5]; Basu and Mitra [13]; Asheim [3]; Asheim and Banerjee [4]).

Preference Continuity, **PC**: $\forall {}_1u, {}_1v \in X : \exists \tilde{T} \geq 1$ such that $({}_1u_T, {}_{T+1}v) \succcurlyeq {}_1v$ $\forall T \geq \tilde{T} \Rightarrow {}_1u \succcurlyeq {}_1v$.

Weak Preference Continuity, **WPC**: $\forall {}_1u, {}_1v \in X : \exists \tilde{T} \geq 1$ such that $({}_1u_T, {}_{T+1}v) \succ {}_1v$ $\forall T \geq \tilde{T} \Rightarrow {}_1u \succ {}_1v$.

These axioms establish "a link to the standard finite setting of distributive justice, by transforming the comparison of any two infinite utility paths to an infinite number of comparisons of utility paths each containing a finite number of generations" (Asheim and Tungodden [5]; p.223).

If there are only a finite set $\{1, ..., T\} = N \subset \mathbb{N}$ of agents, or generations, X_T is the set of utility streams of X truncated at $T = |N|$, where $|N|$ is the cardinality of N. In order to simplify the notation, in economies with a finite number of agents the symbol u is used instead of ${}_1u_T$. With obvious adaptations, the notation and the axioms spelled out above (except for Preference Continuity and Weak Preference Continuity) are carried over utility streams in X_T. In particular, observe that Weak Pareto and Anonymity are logically equivalent to the standard weak Pareto and anonymity axioms in finite economies.

3 The Weak Harm Principle

We study the implications of liberal views of non-interference in fair and Paretian social welfare judgements. In this section, we define and discuss the main liberal principle and then present a novel characterisation of the leximin ordering.

The key features of liberal views in social choice are captured by the Weak Harm Principle, according to which agents have a right to prevent society from punishing them in all situations in which they suffer a welfare loss, *provided no other agent is affected.* Formally:

Weak Harm Principle, **WHP**: $\forall u, v, u', v' \in X_T : u \succ v$ and u', v' are such that

$$\begin{aligned} u'_i &< u_i, v'_i < v_i, \exists i \in N, \text{ and} \\ u'_j &= u_j \,, v'_j = v_j \,, \forall j \neq i, \end{aligned}$$

permutations. Because this distinction is not relevant for our analysis, we have opted for the simpler name for the sake of notational parsimony.

implies $v' \not\succ u'$ whenever $u'_i > v'_i$.

The Weak Harm Principle captures a liberal view of non-interference whenever individual choices have no effect on others. The decrease in agent i's welfare may be due to negligence or bad luck, but in any case the principle states that society should not strictly prefer v' over u': having already suffered a welfare loss in both allocations, an adverse switch in society's strict preferences against agent i would represent an unjustified punishment for her.

The Weak Harm Principle assigns a veto power to individuals in situations in which they suffer a harm and no other agent is affected. This veto power is weak in that it only applies to certain welfare configurations (individual preferences after the welfare loss must coincide with society's initial preferences) and, crucially, the individual cannot force society's preferences to coincide with her own.

The Weak Harm Principle is weaker than the Principle of Non-Interference formulated by Mariotti and Veneziani ([47]) since it only focuses on welfare losses incurred by agents. It also represents a strict weakening of the Harm Principle proposed by Mariotti and Veneziani ([44]) because, unlike the latter, it does not require that society's preferences over u' and v' be identical with agent i's, but only that society should *not reverse* the strict preference between u and v to a strict preference for v' over u' (possibly except when i prefers otherwise). This weakening is important for both conceptual and formal reasons.

Conceptually, the Weak Harm Principle aims to capture - in a welfaristic framework - a *negative freedom* that is central in classical liberal and libertarian approaches, namely, freedom from interference from society, when no other individual is affected. The name of the axiom itself is meant to echo John Stuart Mill's famous formulation in his essay *On Liberty* (see Mariotti and Veneziani [43]). In this sense, by only requiring that agent i should not be punished in the SWR by changing social preferences against her, the liberal content of the axiom is much clearer and the Weak Harm Principle strongly emphasises the negative prescription of Mill's principle.

Formally, our weakening of the Harm Principle has relevant implications. Mariotti and Veneziani ([44]; Theorem 1, p.126) prove that, jointly with Strong Pareto, Anonymity, and Completeness, the Harm Principle characterises the leximin SWO, according to which that society is best which lexicographically maximises the welfare of its worst-off members.

The *leximin ordering* $\succsim^{LM} = \succ^{LM} \cup \sim^{LM}$ on X_T is defined as follows. For all $u, v \in X_T$:

$$u \succ^{LM} v \Leftrightarrow \bar{u}_1 > \bar{v}_1 \text{ or } [\exists i \in N \backslash \{1\} : \bar{u}_j = \bar{v}_j \ (\forall j \in N : j < i) \text{ and } \bar{u}_i > \bar{v}_i];$$
$$u \sim^{LM} v \Leftrightarrow \bar{u}_i = \bar{v}_i, \forall i \in N.$$

The leximin SWO is usually considered to have a strong egalitarian bias, and so a characterisation based on a liberal principle with no explicit egalitarian content is surprising. To clarify this point, note that the classic characterisation by Hammond ([31]) states that a SWR is the leximin ordering if and only if it satisfies Strong Pareto, Anonymity, Completeness, and the following axiom.

HAMMOND EQUITY, **HE**: $\forall u, v \in X_T : u_i < v_i < v_j < u_j \; \exists i, j \in N, \; u_k = v_k \; \forall k \in N \backslash \{i, j\} \Rightarrow v \succcurlyeq u$.

Unlike the Harm Principle, Hammond Equity expresses a clear concern for equality, for it asserts that among two welfare allocations which are not Pareto-ranked and differ only in two components, society should prefer the more egalitarian one.

Although Hammond Equity and the Harm Principle are conceptually distinct and logically independent, it may be argued that the characterisation of the leximin SWO in Mariotti and Veneziani ([44]) is formally unsurprising, because under Anonymity and Completeness, the Harm Principle implies Hammond Equity but the converse is not true (see Alcantud [2], Proposition 4).[9] This objection does not hold if one considers the Weak Harm Principle. To see this, consider the following example.

Example 1 *(*Sufficientarianism*) Suppose that welfare units can be normalised so that a welfare level equal to* $1/2$ *represents a decent living standard. Then one can define a* SWR $\succcurlyeq^s$ *on* X_T *according to which that society is best in which the highest number of people reach a decent living standard. Formally,* $\forall u \in X_T$ *let* $P(u) = \{i \in N : u_i \geq 1/2\}$ *and let* $|P(u)|$ *denote the cardinality of* $P(u)$*. Then* $\forall u, v \in X_T$:

$$u \succcurlyeq^s v \Leftrightarrow |P(u)| \geq |P(v)| \, .$$

It is immediate to see that $\succcurlyeq^s$ *on* X_T *is an ordering and it satisfies Anonymity and the Weak Harm Principle, but violates both Hammond Equity and the Harm Principle.*[10]

Observe that the absence of any conceptual and formal relations between the Weak Harm Principle and Hammond Equity, even under Anonymity, established in Example 1 is not a mere technical artefact. The Suppes-Sen grading principle, for instance, satisfies Anonymity and the Weak Harm Principle and violates Hammond Equity, but one may object that this is due to its incompleteness. In contrast, the SWR in Example 1 is complete and it embodies a prominent approach to distributive justice in political philosophy and social choice (see, for example, Frankfurt [28] and Roemer [52]). Thus, even under Anonymity and Completeness,

[9] The argument is originally due to François Maniquet in unpublished correspondence.

[10] Consider, for example, two welfare profiles $u, v \in X_T$ such that $u = (1, 0, 1, 1, 1, ..., 1)$ and $v = (\frac{1}{3}, \frac{1}{4}, 1, 1, 1, ..., 1)$. By definition $u \succ^s v$, which violates Hammond Equity.

liberal principles of non-interference incorporate substantially different normative intuitions than standard equity axioms. Example 1 also highlights the theoretical relevance of our weakening of the Harm Principle, for the Weak Harm Principle is consistent with a wider class of SWOs, including some - such as the sufficientarian - which embody some widely shared views on distributive justice.

Given this, it is perhaps surprising that the characterisation result provided in Mariotti and Veneziani ([44]) can be strengthened.[11]

Proposition 2 *: A* SWR $\succcurlyeq$ *on* X_T *is the leximin ordering if and only if it satisfies* ***A**, **SP**, **C** and **WHP**.*

In the light of our discussion of the Weak Harm Principle and Example 1, it is worth stressing some key theoretical implications of Proposition 2. First, it is possible to eschew impossibility results by weakening the Principle of Non-Interference proposed by Mariotti and Veneziani ([43]) while capturing some core liberal intuitions. For by Proposition 2 there exist anonymous and strongly Paretian SWOs consistent with liberal non-interfering views, as expressed in the Weak Harm Principle.

Second, by Proposition 2 Hammond Equity and the Weak Harm Principle are equivalent in the presence of Anonymity, Completeness, and Strong Pareto, even though they are logically independent. However, it can be proved that if $N = \{1, 2\}$, then under Strong Pareto and Completeness, Hammond Equity implies the Weak Harm Principle, but the converse is never true (see Mariotti and Veneziani [43]). Together with Example 1, this implies that Proposition 2 is far from trivial. For even under Completeness and either Anonymity or Strong Pareto, the Weak Harm Principle is not stronger than Hammond Equity, and it is actually strictly weaker, at least in some cases.

Third, Proposition 2 puts the normative foundations of leximin under a rather different light. For, unlike in standard results, the egalitarian SWO is characterised without appealing to any axioms with a clear egalitarian content.[12] Actually, Strong Pareto, Completeness, and the Weak Harm Principle are compatible with some of the least egalitarian SWOs, namely the lexicographic dictatorships, which proves that the Weak Harm Principle imposes no significant egalitarian restriction. As a result, Proposition 2 highlights the normative strength of Anonymity in determining the egalitarian outcome, an important insight which is not obvious in standard characterisations based on Hammond Equity.

[11] The properties in Proposition 2 are clearly independent. The proof of Proposition 2 is a generalisation of the proof of Theorem 1 in Mariotti and Veneziani ([44]) and is available from the authors upon request (see the Addendum).

[12] Nor to any invariance or separability axioms.

The next sections extend this intuition significantly and show that the counterintuitive egalitarian implications of liberal non-interfering principles are quite general and robust.

4 Liberal egalitarianism reconsidered

One common objection to the leximin SWO is its sensitivity to small changes in welfare profiles, and so to measurement errors and small variations in policies. Albeit possibly secondary in theoretical analyses, these issues are relevant in empirical applications and policy debates. As Chichilnisky ([19], p.346) aptly noted, "Continuity is a natural assumption that is made throughout the body of economic theory, and it is certainly desirable as it permits approximation of social preferences on the basis of a sample of individual preferences, and makes mistakes in identifying preferences less crucial. These are relevant considerations in a world of imperfect information." In this section, we study the implications of liberal non-interfering approaches for social evaluations that are robust to small changes in welfare profiles.

A standard way of capturing this property is by an interprofile condition requiring the SWO to vary continuously with changes in utility streams.

CONTINUITY, **CON**: $\forall u \in X_T$, the sets $\{v \in X_T | v \succcurlyeq u\}$ and $\{v \in X_T | u \succcurlyeq v\}$ are closed.

By Proposition 2, if Continuity is imposed in addition to the Weak Harm Principle, Completeness, Strong Pareto and Anonymity an impossibility result immediately obtains. Therefore we weaken our efficiency requirement to focus on Weak Pareto. Strikingly, the combination of the five axioms characterises Rawls's difference principle.

The *maximin ordering* $\succcurlyeq^M$ on X_T is defined as follows: $\forall u, v \in X_T$,

$$u \succcurlyeq^M v \Leftrightarrow \bar{u}_1 \geq \bar{v}_1.$$

Theorem 3 states that the standard requirements of fairness, efficiency, completeness, and continuity, together with our liberal axiom characterise the maximin SWO.[13]

Theorem 3 *: A* SWR $\succcurlyeq$ *on* X_T *is the maximin ordering if and only if it satisfies* ***A**, **WP**, **C**, **CON** and **WHP**.*

Proof. ($\Rightarrow$) Let $\succcurlyeq$ on X_T be the maximin ordering, i.e., $\succcurlyeq = \succcurlyeq^M$. It can be easily verified that $\succcurlyeq^M$ on X_T satisfies **A**, **WP**, **C**, **CON**, and **WHP**.

[13] The properties in Theorem 3 are clearly independent.

($\Leftarrow$) Let $\succcurlyeq$ on X_T be a SWR satisfying **A**, **WP**, **C**, **CON** and **WHP**. We show that $\succcurlyeq$ is the maximin SWO. We prove that, $\forall u,\, v \in X_T$,

$$u \succ^M v \Leftrightarrow u \succ v \tag{1}$$

and

$$u \sim^M v \Leftrightarrow u \sim v. \tag{2}$$

Note that as $\succcurlyeq$ on X_T satisfies **A**, in what follows we can focus either on u and v, or on the ranked vectors $\bar{u}$ and $\bar{v}$, without loss of generality.
First, we show that the implication ($\Rightarrow$) of (1) is satisfied. Take any $u, v \in X_T$. Suppose that $u \succ^M v \Leftrightarrow \bar{u}_1 > \bar{v}_1$. We proceed by contradiction, first proving that $v \succ u$ is impossible and then ruling out $v \sim u$.

Suppose that $v \succ u$, or equivalently, $\bar{v} \succ \bar{u}$. As **WP** holds, $\bar{v}_j \geq \bar{u}_j$ for some $j \in N$, otherwise a contradiction immediately obtains. We proceed according to the following steps.

Step 1. Let

$$k = \inf \left\{ l \in N | \bar{v}_l \geq \bar{u}_l \right\}.$$

By **A**, let $v_i = \bar{v}_k$ and let $u_i = \bar{u}_1$. Then, consider two real numbers d_1, $d_2 > 0$, and two vectors u^*, v' - together with the corresponding ranked vectors $\bar{u}^*$, $\bar{v}'$ - formed from $\bar{u}$, $\bar{v}$ as follows: $\bar{u}_1$ is lowered to $\bar{u}_1 - d_1 > \bar{v}_1$; $\bar{v}_k$ is lowered to $\bar{u}_k > \bar{v}_k - d_2 > \bar{u}_1 - d_1$; and all other entries of $\bar{u}$ and $\bar{v}$ are unchanged. By construction u^*, $v' \in X_T$ and $\bar{u}^*_j > \bar{v}'_j$ for all $j \leq k$, whereas by **WHP**, **C**, and **A**, we have $\bar{v}' \succcurlyeq \bar{u}^*$.

Step 2. Let

$$0 < \epsilon < \inf\{\bar{u}^*_j - \bar{v}'_j | j \leq k\}$$

and define $\bar{u}' = \bar{u}^* - {}_{con}\epsilon$. By construction, $\bar{u}' \in X_T$ and $\bar{u}^* \gg \bar{u}'$. **WP** implies $\bar{u}^* \succ \bar{u}'$. As $\bar{v}' \succcurlyeq \bar{u}^*$, by *step* 1, the transitivity of $\succcurlyeq$ implies $\bar{v}' \succ \bar{u}'$.

If $\bar{u}'_j > \bar{v}'_j$ for all $j \in N$, **WP** implies $\bar{u}' \succ \bar{v}'$, a contradiction. Otherwise, let $\bar{v}'_l \geq \bar{u}'_l$ for some $l > k$. Then, let

$$k' = \inf \left\{ l \in N | \bar{v}'_l \geq \bar{u}'_l \right\}.$$

The above steps 1-2 can be applied to $\bar{u}'$, $\bar{v}'$ to derive vectors $\bar{u}''$, $\bar{v}'' \in X_T$ such that $\bar{u}''_j > \bar{v}''_j$ for all $j \leq k'$, whereas $\bar{v}'' \succ \bar{u}''$. By **WP**, a contradiction is obtained whenever $\bar{u}''_j > \bar{v}''_j$ for all $j \in N$. Otherwise, let $\bar{v}''_l \geq \bar{u}''_l$ for some $l > k'$. And so on. After a finite number s of iterations, two vectors $\bar{u}^s$, $\bar{v}^s \in X_T$ can be derived such that $\bar{v}^s \succ \bar{u}^s$, by steps 1-2, but $\bar{u}^s \succ \bar{v}^s$, by **WP**, a contradiction.

Therefore, by **C**, it must be $\bar{u} \succcurlyeq \bar{v}$ whenever $\bar{u} \succ^M \bar{v}$. We have to rule out the possibility that $\bar{u} \sim \bar{v}$. We proceed by contradiction. Suppose that $\bar{u} \sim \bar{v}$. Since $\bar{v}_1 < \bar{u}_1$, there exists $\epsilon > 0$ such that $\bar{u}^\epsilon = \bar{u} - {}_{con}\epsilon$, $\bar{u}^\epsilon \in X_T$, and $\bar{v}_1 < \bar{u}^\epsilon_1$ so that $\bar{u}^\epsilon \succ^M \bar{v}$. However, by **WP** and transitivity of $\succcurlyeq$ it follows that $\bar{v} \succ \bar{u}^\epsilon$. Apply the above reasoning to $\bar{v}$ and $\bar{u}^\epsilon$ to obtain the desired contradiction.

Now, we show that the implication ($\Rightarrow$) of (2) is met as well. Suppose $\bar{u}_1 = \bar{v}_1$. If $\bar{u}_1 = 1$, the result follows by reflexivity. Hence suppose $\bar{u}_1 < 1$. Let $\mathbb{T}(u)=\{t \in N : u_t = \bar{u}_1\}$ and let u^K be such that $u^K_t = u_t$, all $t \notin \mathbb{T}(u)$, and $u^K_t = u_t + K^{-1}$, all $t \in \mathbb{T}(u)$, where K is any natural number such that $u_t + K^{-1} < 1$, all $t \in \mathbb{T}(u)$. By construction, $u^k \in X_T$ and $\bar{u}^k_1 > \bar{v}_1$ for all $k \geq K$. Since $\lim_{k\to\infty} u^k = u$ and $u^k \in \{x \in X_T | x \succcurlyeq v\}$ for all $k \geq K$, **CON** implies $u \succcurlyeq v$. A symmetric argument proves that $v \succcurlyeq u$, and so $u \sim v$. ■

Theorem 3 has two main implications in the context of our analysis. First, if the Principle of Non-Interference proposed by Mariotti and Veneziani ([47]) is replaced by the Weak Harm Principle, then there exist anonymous and (weakly) Paretian liberal SWOs that are also continuous. This is particularly interesting given that the consistency between Weak Pareto, continuity properties, and liberal principles in the spirit of Sen's celebrated *Minimal Liberalism* axiom has been recently called into question by Kaplow and Shavell ([35]).

Second, Theorem 3 provides a novel characterisation of the difference principle that generalises the key insight of section 3. Standard characterisations focus either on informational invariance and separability properties (d'Aspremont and Gevers [22]; Segal and Sobel [56]), or on axioms incorporating a clear inequality aversion such as Hammond Equity (Bosmans and Ooghe [15]) or the Pigou-Dalton principle (Fleurbaey and Tungodden [27]). Theorem 3 characterises an egalitarian SWO by using an axiom - the Weak Harm Principle - that, unlike informational invariance properties has a clear ethical foundation, but it has no egalitarian content as it only incorporates a liberal, non-interfering view of society.

5 A liberal principle of intergenerational justice

In the previous sections, we have studied the implications of liberal principles of non-interference in societies with a finite number of agents and have shown that consistent fair and Paretian liberal social judgements are possible. We now extend our analysis to societies with an infinite number of agents. A liberal non-interfering approach seems particularly appropriate in the analysis of intergenerational distributive issues: although the welfare of a generation is often affected by decisions taken by their predecessors, there certainly are many economic decisions whose effects do not extend over time and leave the welfare of other generations unchanged. In this section (and the next), we explore the implications of fair and Paretian liberal approaches to intergenerational justice.

The extension of the main liberal principle to the analysis of intergenerational justice is rather straightforward and needs no further comment, except possibly noting that in this context, the Weak Harm Principle is weakened to hold only for pairs of welfare allocations whose tails can be Pareto-ranked.

Weak Harm Principle*, **WHP***: $\forall {}_1u,\ {}_1v,\ {}_1u',\ {}_1v' \in X : {}_1u \succ {}_1v$ and $\exists T \geq 1, \exists \epsilon \geq 0$ such that ${}_1v \equiv ({}_1v_T, ({}_{T+1}u + {}_{con}\epsilon))$, and ${}_1u',\ {}_1v'$ are such that

$$\begin{aligned} u_i' &< u_i,\ v_i' < v_i,\ \exists i \leq T, \text{ and} \\ u_j' &= u_j\ ,\ v_j' = v_j\ , \forall j \neq i, \end{aligned}$$

implies ${}_1v' \not\succ {}_1u'$ whenever $u_i' > v_i'$.

As already noted, economies with an infinite number of agents raise several formal and conceptual issues, and different definitions of the main criteria (including utilitarianism, egalitarianism, the Nash ordering, and so on) can be provided in order to compare (countably) infinite utility streams. Here, we derive a novel characterisation of one of the main approaches in the literature, namely the leximin overtaking recently formalised by Asheim and Tungodden ([5]), in the tradition of Atsumi ([10]) and von Weizsäcker ([61]). Yet, as argued at the end of the section, our key results are robust and the Weak Harm Principle can be used to provide normative foundations to *all* of the main extensions of the leximin SWR. Perhaps surprisingly, liberal views of non-interference *in general* lead to egalitarian SWRs even in the intergenerational context.

The leximin overtaking criterion is defined as follows.

Definition 1. (Asheim and Tungodden [5]; Definition 2, p.224) For all ${}_1u,\ {}_1v \in X$,
(i) ${}_1u \sim^{LM^*} {}_1v \Leftrightarrow \exists \tilde{T} \geq 1$ such that ${}_1\bar{u}_T = {}_1\bar{v}_T\ \forall T \geq \tilde{T}$, and
(ii) ${}_1u \succ^{LM^*} {}_1v \Leftrightarrow \exists \tilde{T} \geq 1$ such that $\forall T \geq \tilde{T}, \exists t \in \{1, ..., T\}$: $\bar{u}_s = \bar{v}_s\ \forall 1 \leq s < t$ and $\bar{u}_t > \bar{v}_t$.

In order to characterise the leximin overtaking, we need to weaken completeness and require that the SWR be (at least) able to compare profiles with the same tail.

Minimal Completeness, **MC**: $\forall {}_1u, {}_1v \in X,\ \exists T \geq 1\ \left({}_1u_T,\ {}_{T+1}v\right) \neq {}_1v\ \Rightarrow ({}_1u_T,\ {}_{T+1}v) \succcurlyeq {}_1v$ or ${}_1v \succcurlyeq ({}_1u_T,\ {}_{T+1}v)$.

Theorem 4 proves that Anonymity, Strong Pareto, the Weak Harm Principle*, Minimal Completeness and Weak Preference Continuity characterise the leximin overtaking.[14]

[14] The properties in Theorem 4 are independent (see the Addendum).

Theorem 4 *: $\succcurlyeq$ is an extension of $\succcurlyeq^{LM^*}$ if and only if $\succcurlyeq$ satisfies* ***A****,* ***SP****,* ***MC****,* ***WHP***** and* ***WPC****.*

Proof ($\Rightarrow$) Let $\succcurlyeq^{LM^*} \subseteq \succcurlyeq$. It is easy to see that $\succcurlyeq$ meets **A** and **SP**. By observing that $\succcurlyeq^{LM^*}$ is complete for comparisons between utility streams with the same tail it is also easy to see that $\succcurlyeq$ satisfies **MC** and **WPC**.

We show that $\succcurlyeq$ meets **WHP***. Take any ${}_1u$, ${}_1v$, ${}_1u'$, ${}_1v' \in X$ such that ${}_1u \succ {}_1v$, and $\exists T \geq 1, \exists \epsilon \geq 0$ such that ${}_1v \equiv ({}_1v_T, ({}_{T+1}u + {}_{con}\epsilon))$, and ${}_1u'$, ${}_1v'$ are such that $\exists i \leq T$, $u'_i < u_i$, $v'_i < v_i$, and $u'_j = u_j$, $v'_j = v_j \; \forall j \neq i$. We show that ${}_1u' \succ {}_1v'$ whenever $u'_i > v'_i$.

Because $\succcurlyeq^{LM^*}$ is complete for comparisons between utility streams whose tails differ by a nonnegative constant, ${}_1u \succ^{LM^*} {}_1v$. Then take any $T' \geq \tilde{T}$ that corresponds to part (ii) of Definition 1. Theorem 1 in Mariotti and Veneziani ([44]; 126) implies that there exists $t^* \leq t \leq T'$ such that $\bar{u}'_s = \bar{v}'_s \; \forall 1 \leq s < t^*$ and $\bar{v}'_{t^*} < \bar{u}'_{t^*}$. Since the choice of T' corresponding to part (ii) of Definition 1 was arbitrary, it follows that ${}_1u' \succ {}_1v'$.

($\Leftarrow$) Suppose that $\succcurlyeq$ satisfies **A**, **SP**, **MC**, **WHP*** and **WPC**. We show that $\sim^{LM^*} \subseteq \sim$ and $\succ^{LM^*} \subseteq \succ$. Take any ${}_1u$, ${}_1v \in X$.

Since $\sim^{LM^*} \subseteq \sim$ follows from Asheim and Tungodden ([5]), we only show that $\succ^{LM^*} \subseteq \succ$.

Suppose ${}_1u \succ^{LM^*} {}_1v$. Take any $T \geq \tilde{T}$ that corresponds to part (ii) of Definition 1 and consider ${}_1w \equiv ({}_1u_T, {}_{T+1}v) \in X$. Note that ${}_1w \succ^{LM^*} {}_1v$. We show that ${}_1w \succ {}_1v$. By **A** and transitivity, we can consider ${}_1\bar{w} \equiv ({}_1\bar{u}_T, {}_{T+1}v)$ and ${}_1\bar{v} \equiv ({}_1\bar{v}_T, {}_{T+1}v)$. By **MC**, suppose that ${}_1\bar{v} \succcurlyeq {}_1\bar{w}$. We distinguish two cases.

Case 1. ${}_1\bar{v} \succ {}_1\bar{w}$

As **SP** holds it must be the case that $\bar{v}_l > \bar{w}_l$ for some $l > t$. Let

$$k = \inf\{t < l \leq T | \bar{v}_l > \bar{w}_l\}.$$

By **A**, let $v_i = \bar{v}_k$ and let $w_i = \bar{w}_{k-g}$, for some $1 \leq g < k$, where $\bar{w}_{k-g} > \bar{v}_{k-g}$. Then, let two real numbers d_1, $d_2 > 0$, and consider vectors ${}_1w'$, ${}_1v'$ formed from ${}_1\bar{w}$, ${}_1\bar{v}$ as follows: $\bar{w}_{k-g}$ is lowered to $\bar{w}_{k-g} - d_1$ such that $\bar{w}_{k-g} - d_1 > \bar{v}_{k-g}$; $\bar{v}_k$ is lowered to $\bar{v}_k - d_2$ such that $\bar{w}_k > \bar{v}_k - d_2 > \bar{w}_{k-g} - d_1$; and all other entries of ${}_1\bar{w}$ and ${}_1\bar{v}$ are unchanged. By **A**, consider ${}_1\bar{w}' = ({}_1\bar{w}'_T, {}_{T+1}v)$ and ${}_1\bar{v}' = ({}_1\bar{v}'_T, {}_{T+1}v)$. By construction ${}_1\bar{w}', {}_1\bar{v}' \in X$ and $\bar{w}'_j \geq \bar{v}'_j$ for all $j \leq k$, with $\bar{w}'_{k-g} > \bar{v}'_{k-g}$, whereas **WHP***, combined with **MC** and **A**, implies ${}_1\bar{v}' \succcurlyeq {}_1\bar{w}'$. Furthermore, by **SP**, it is possible to choose d_1, $d_2 > 0$, such that ${}_1\bar{v}' \succ {}_1\bar{w}'$, without loss of generality. Consider two cases:

a) Suppose that $\bar{v}_k > \bar{w}_k$, but $\bar{w}_l \geq \bar{v}_l$ for all $l > k$. It follows that ${}_1\bar{w}' > {}_1\bar{v}'$, and so **SP** implies that ${}_1\bar{w}' \succ {}_1\bar{v}'$, a contradiction.

b) Suppose that $\bar{v}_l > \bar{w}_l$ for some $l > k$. Note that by construction $\bar{v}'_l = \bar{v}_l$ and $\bar{w}'_l = \bar{w}_l$ for all $l > k$. Then, let

$$k' = \inf\{k < l \leq T | \bar{v}'_l > \bar{w}'_l\}.$$

The above argument can be applied to ${}_1\bar{w}'$, ${}_1\bar{v}'$ to derive vectors ${}_1\bar{w}''$, ${}_1\bar{v}'' \in X$ such that $\bar{w}''_j \geq \bar{v}''_j$ for all $j \leq k'$, whereas **WHP***, combined with **MC**, **A**, and **SP**, implies ${}_1\bar{v}'' \succ {}_1\bar{w}''$. And so on. After a finite number of iterations s, two vectors ${}_1\bar{w}^s$, ${}_1\bar{v}^s \in X$ can be derived such that, by **WHP***, combined with **MC**, **A**, and **SP**, we have that ${}_1\bar{v}^s \succ {}_1\bar{w}^s$, but **SP** implies ${}_1\bar{w}^s \succ {}_1\bar{v}^s$, yielding a contradiction.

Case 2. ${}_1\bar{v} \sim {}_1\bar{w}$

Since, by our supposition, $\bar{v}_t < \bar{u}_t \equiv \bar{w}_t$, there exists $\epsilon > 0$ such that $\bar{v}_t < \bar{w}_t - \epsilon < \bar{w}_t$. Let ${}_1\bar{w}^\epsilon \in X$ be a vector such that $\bar{w}^\epsilon_t = \bar{w}_t - \epsilon$ and $\bar{w}^\epsilon_j = \bar{w}_j$ for all $j \neq t$. It follows that ${}_1\bar{w}^\epsilon \succ^{LM^*} {}_1\bar{v}$ but ${}_1\bar{v} \succ {}_1\bar{w}^\epsilon$ by **SP** and the transitivity of $\succcurlyeq$. Hence, the argument of *Case* 1 above can be applied to ${}_1\bar{v}$ and ${}_1\bar{w}^\epsilon$, yielding the desired contradiction.

It follows from **MC** that ${}_1\bar{w} \succ {}_1\bar{v}$. Then **A**, combined with the transitivity of $\succcurlyeq$, implies that $({}_1u_T, {}_{T+1}v) \succ {}_1v$. Since $T \geq \tilde{T}$ is arbitrary, **WPC** implies ${}_1u \succ {}_1v$, as desired. ■

Theorem 4 shows that, if the Principle of Non-Interference analysed by Lombardi and Veneziani ([42]) and Alcantud ([2]) in the intergenerational context is suitably restricted to hold only for welfare losses, then possibility results for liberal, fair and Paretian social judgements do emerge. Indeed, Theorem 4 provides a novel characterisation of one of the main extensions of the leximin to economies with an infinite number of agents, based on the Weak Harm Principle*, thus confirming the striking link between a liberal and libertarian concern for individual autonomy, and egalitarian criteria.

These conclusions are robust and can be extended to alternative definitions of the leximin.[15] For example, if Weak Preference Continuity is replaced with a stronger continuity requirement, a stronger version of the leximin overtaking (the *S-Leximin*, see Asheim and Tungodden, [5]; Definition 1, p.224) can easily be derived. Perhaps more interestingly, Bossert et al. ([16]) have dropped continuity properties and have characterised a larger class of extensions of the leximin criterion satisfying Strong Pareto, Anonymity, and an infinite version of Hammond Equity.[16] Lombardi and Veneziani ([41]) have shown that it is possible

[15] It is worth noting in passing that Theorem 4 can be further strengthened by requiring **WHP*** to hold only for vectors with the same tail, namely $\epsilon = 0$.

[16] Formally, the relationship between the characterisation of the leximin by Bossert et al. ([16]) and that by Asheim and Tungodden ([5]) is analogous to the relationship between the characterisation of the utilitarian SWR by Basu and Mitra ([13]) and the characterisations of the more restrictive utilitarian SWR induced by the overtaking criterion (see the discussion in Bossert et al. [16]; p.580).

to provide a characterisation of the leximin relation defined by Bossert et al. ([16]) based on Strong Pareto, Anonymity, and the Weak Harm Principle. Further, the Weak Harm Principle can be used - instead of various versions of the Hammond equity axiom - to characterise the leximin SWR proposed by Sakai ([54]), which drops transitivity but retains completeness; and the *time-invariant leximin overtaking* proposed by Asheim et al. ([7]).[17]

In summary, in the intergenerational context too, liberalism implies equality.[18]

6 The intergenerational difference principle

In section 4, we argued that a potential shortcoming of the leximin criterion is its sensitivity to infinitesimal changes in welfare profiles and explored the implications of liberal principles together with a continuity requirement that incorporates a concern for robustness in social judgements. In the context of intergenerational distributive justice, a further problem of the various extensions of the leximin criterion is their incompleteness, which makes them unable to produce social judgements in a large class of pairwise comparisons of welfare profiles.

In this section, we complete our analysis of liberal principles of non-interference by analysing the implications of the Weak Harm Principle* for intergenerational justice when social welfare criteria are required to be continuous *and* to be able to adjudicate *all* distributive conflicts. This is by no means a trivial question, for it is well known that continuity is a problematic requirement for SWOs in economies with an infinite number of agents and impossibility results often emerge.[19]

The main axioms incorporating completeness, fairness, efficiency, and liberal non-interference are the same as in previous sections. Further, we follow the standard practice in the literature (see, e.g., Lauwers [37]) and define continuity based on the sup metric.

Sup Continuity, $\mathbf{CON}_{d_\infty}$: $\forall {}_1u \in X$: there is a sequence of vectors $\left\{{}_1v^k\right\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} {}_1v^k = {}_1v \in X$ with respect to the sup metric d_∞, and ${}_1v^k \succcurlyeq {}_1u$ (*resp.*, ${}_1u \succcurlyeq {}_1v^k$) $\forall k \in \mathbb{N} \Rightarrow {}_1u \not\succ {}_1v$ (*resp.*, ${}_1v \not\succ {}_1u$).

Observe that in general $\mathbf{CON}_{d_\infty}$ is weaker than the standard continuity axiom but it is equivalent to the latter if the SWR is complete as in Theorem 5 below.[20]

[17] As compared to the standard overtaking criterion, the time invariant version does not rely on a natural ordering of generations. Thus, it is possible to drop Weak Preference Continuity and replace it with a similar consistency axiom that does not entail a preference for earlier generations.

[18] The proofs of the above claims are available from the authors upon request.

[19] See the classic paper by Diamond ([24]). For more recent contributions see Hara et al. ([34]) and the literature cited therein.

[20] It is also weaker than the *Continuity* axiom recently proposed by Asheim et al. ([9], p.271), although the two properties are equivalent for complete SWRs.

Our next result extends the key insights on liberal egalitarianism to the intergenerational context. Formally, the *maximin* SWO $\succcurlyeq^{M^*}$ on X can be defined as follows:

$$\forall\ {}_1u,\ {}_1v \in X:\ {}_1u \succcurlyeq^{M^*}\ {}_1v\ \Leftrightarrow \inf_{t\in\mathbb{N}} u_t \geq \inf_{t\in\mathbb{N}} v_t.$$

Theorem 5 proves that Anonymity, Weak Pareto, Completeness, Sup Continuity, Weak Harm Principle, and Preference Continuity characterise $\succcurlyeq^{M^*}$ on X.[21]

Theorem 5 *A* SWR $\succcurlyeq$ *on* X *is the maximin* SWO *if and only if it satisfies* ***A***, ***WP***, ***C***, ***CON***$_{d_\infty}$, ***WHP****** *and* ***PC***.

Proof. ($\Rightarrow$) Let $\succcurlyeq$ on X be the maximin SWO, i.e., $\succcurlyeq = \succcurlyeq^{M^*}$. It can be easily verified that $\succcurlyeq^{M^*}$ on X satisfies **A**, **WP**, **C**, **CON**$_{d_\infty}$, **WHP*** and **PC**.
($\Leftarrow$) Let $\succcurlyeq$ on X be a SWR satisfying **A**, **WP**, **C**, **CON**$_{d_\infty}$, **WHP*** and **PC**. We show that $\succcurlyeq$ is the maximin SWO. To this end, it suffices to show that $\forall {}_1u,\ {}_1v \in X$,

$$\inf_{t\in\mathbb{N}} u_t > \inf_{t\in\mathbb{N}} v_t \Rightarrow\ {}_1u \succ\ {}_1v \tag{3}$$

and

$$\inf_{t\in\mathbb{N}} u_t = \inf_{t\in\mathbb{N}} v_t \Rightarrow\ {}_1u \sim\ {}_1v. \tag{4}$$

Consider (3). Take any ${}_1u,\ {}_1v \in X$ such that $\inf_{t\in\mathbb{N}} u_t > \inf_{t\in\mathbb{N}} v_t$. In order to prove that ${}_1u \succ {}_1v$, we first demonstrate that ${}_{con}\hat{x} \succcurlyeq {}_1v$ holds, where

$$\hat{x} = \frac{\inf_{t\in\mathbb{N}} u_t + \inf_{t\in\mathbb{N}} v_t}{2}.$$

To this end, we distinguish two cases.

Case 1. $\sup_{t\in\mathbb{N}} v_t < 1$.

As a first step, we shall prove that

$$\exists T \geq 1, \forall t \geq T : ({}_1\hat{x}_t,\ {}_{t+1}v +\ {}_{con}\epsilon) \succcurlyeq\ {}_1v,\ \forall \epsilon > 0 : ({}_1\hat{x}_t,\ {}_{t+1}v +\ {}_{con}\epsilon) \in X. \tag{5}$$

We proceed by contradiction. Assume that (5) fails. Since $\succcurlyeq$ satisfies **C**, it follows that for any $T \geq 1$ there exist $t \geq T$ and $\epsilon > 0$ such that $({}_1\hat{x}_t,\ {}_{t+1}v +\ {}_{con}\epsilon) \in X$, and ${}_1v \succ ({}_1\hat{x}_t,\ {}_{t+1}v +\ {}_{con}\epsilon)$. Since $\hat{x} > \inf_{t\in\mathbb{N}} v_t$, it follows that there exists $T^* \geq 1$ such that $\hat{x} > v_{T^*} \geq \inf\{v_1, ..., v_{T^*}\}$. By the contradicting hypothesis, and since $\succcurlyeq$ satisfies **C**, there exist $t^* \geq T^*$

[21] The properties in Theorem 5 are independent (see the Addendum). It is worth noting in passing that the characterisation of the maximin SWO can also be obtained without the full force of completeness, by adopting an axiom similar to **MC** above. We thank Geir Asheim for this suggestion.

and $\epsilon > 0$ such that $({}_1\hat{x}_{t^*},\ {}_{t^*+1}v +\ {}_{con}\epsilon) \in X$ and ${}_1v \succ ({}_1\hat{x}_{t^*},\ {}_{t^*+1}v +\ {}_{con}\epsilon)$. For the sake of notational simplicity, let $({}_1\hat{x}_{t^*},\ {}_{t^*+1}v +\ {}_{con}\epsilon) \equiv {}_1x$. Observe that $\hat{x} > \inf\{v_1, ..., v_{T^*}\} \geq \inf\{v_1, ..., v_{t^*}\}$.

Let ${}_1\bar{v} \equiv ({}_1\bar{v}_{t^*}, {}_{t^*+1}v)$. By **A** and transitivity, ${}_1\bar{v} \succ {}_1x$. Suppose that ${}_1x_{t^*} \gg {}_1\bar{v}_{t^*}$. Then, there exists $0 < a < \inf\left\{\inf\{x_t - \bar{v}_t | t \leq t^*\}, \frac{\epsilon}{2}\right\}$ such that $x_t \geq \bar{v}_t + a$ for all $t \in \mathbb{N}$. But then **WP** implies ${}_1x \succ {}_1\bar{v}$ yielding a contradiction.

Therefore, suppose that for some $1 < t \leq t^*$ we have that $\bar{v}_t \geq x_t = \hat{x}$. We proceed according to the following steps.

Step 1. Let

$$q = \inf\{1 < t \leq t^* | \bar{v}_t \geq x_t = \hat{x}\}.$$

Then, consider two real numbers $d_1,\ d_2 > 0$, and two vectors ${}_1x^1$, ${}_1v'$ - together with the corresponding ranked vectors ${}_1\bar{x}^1 = ({}_1\bar{x}^1_{t^*}, {}_{t^*+1}x)$, ${}_1\bar{v}' = ({}_1\bar{v}'_{t^*}, {}_{t^*+1}v)$ - formed from ${}_1x$, ${}_1\bar{v}$ as follows: x_q is lowered to $x^1_q = x_q - d_1 = \hat{x} - d_1 > \bar{v}_1 = \inf\{v_1, ..., v_{t^*}\}$; $\bar{v}_q$ is lowered to $v'_q = \bar{v}_q - d_2$ where $\hat{x} > \bar{v}_q - d_2 > \hat{x} - d_1$; and all other entries of ${}_1x$ and ${}_1\bar{v}$ are unchanged. By construction, ${}_1x^1$, ${}_1v' \in X$ and $\bar{x}^1_t > \bar{v}'_t$ for all $1 \leq t \leq q$, whereas by **WHP***, **C**, **A**, we have ${}_1\bar{v}' \succcurlyeq {}_1\bar{x}^1$.

Step 2. Let

$$0 < k < \inf\left\{\inf\{\bar{x}^1_t - \bar{v}'_t | t \leq q\}, \inf\{1 - \bar{v}'_t | q < t \leq t^*\}, \frac{\epsilon}{2t^*}\right\} < \epsilon \tag{6}$$

and define ${}_1\bar{v}^1 = {}_1\bar{v}' +\ {}_{con}k$. By construction, ${}_1\bar{v}^1 \in X$ and $\bar{v}^1_t \geq \bar{v}'_t + k$ for all $t \in \mathbb{N}$, and so **WP** implies ${}_1\bar{v}^1 \succ {}_1\bar{v}'$. Since ${}_1\bar{v}' \succcurlyeq {}_1\bar{x}^1$, then transitivity implies that ${}_1\bar{v}^1 \succ {}_1\bar{x}^1$.

Suppose that ${}_1\bar{x}^1_{t^*} \gg {}_1\bar{v}^1_{t^*}$. Then, since $\inf_{t\in\mathbb{N}} \bar{x}^1_t > \inf_{t\in\mathbb{N}} \bar{v}^1_t$ and ${}_{t^*+1}\bar{x}^1 \equiv {}_{t^*+1}v +\ {}_{con}\epsilon \gg {}_{t^*+1}\bar{v}^1 \equiv {}_{t^*+1}v +\ {}_{con}k$, there exists $a \in \left(0, \inf\left\{\inf\{\bar{x}^1_t - \bar{v}'_t | t \leq t^*\}, \frac{k}{2t^*}\right\}\right)$ such that $\bar{x}^1_t \geq \bar{v}^1_t + a$ for all $t \in \mathbb{N}$. **WP** implies ${}_1\bar{x}^1 \succ {}_1\bar{v}^1$ yielding a contradiction. Otherwise, let $\bar{v}^1_t \geq \bar{x}^1_t$ for some t, with $q < t \leq t^*$. Let

$$q' = \inf\left\{q < t \leq t^* |\ \bar{v}^1_t \geq \bar{x}^1_t\right\}.$$

Noting that by (6), $\epsilon - k = \epsilon' > 0$ so that ${}_{t^*+1}\bar{x}^1 -\ {}_{t^*+1}\bar{v}^1 = {}_{con}\epsilon' \gg {}_{con}0$, the above steps 1-2 can be applied to ${}_1\bar{x}^1$, ${}_1\bar{v}^1$ to derive vectors ${}_1\bar{x}^2$, ${}_1\bar{v}^2 \in X$ such that $\bar{x}^2_t > \bar{v}^2_t$ for all $1 \leq t \leq q'$, whereas ${}_1\bar{v}^2 \succ {}_1\bar{x}^2$. By **WP**, a contradiction can be obtained whenever ${}_1\bar{x}^2_{t^*} \gg {}_1\bar{v}^2_{t^*}$. Otherwise, let $\bar{x}^2_t \leq \bar{v}^2_t$ for some $q' < t \leq t^*$. And so on. After a finite number $s \leq t^*$ of iterations, two vectors ${}_1\bar{x}^s, {}_1\bar{v}^s \in X$ can be derived such that ${}_1\bar{v}^s \succ {}_1\bar{x}^s$, by steps 1-2, but ${}_1\bar{x}^s_{t^*} \gg {}_1\bar{v}^s_{t^*}$, and so ${}_1\bar{x}^s \succ {}_1\bar{v}^s$ can be obtained by applying **WP**, a contradiction. This completes the proof of (5).

Next, we prove that ${}_{con}\hat{x} \succcurlyeq {}_1v$ holds. To this end, define $H \in \mathbb{N}$ such that ${}_1v + {}_{con}h^{-1} \in X$ for all $h \in \mathbb{N}$, $h \geq H$: the existence of H is guaranteed by the assumption $\sup_{t\in\mathbb{N}} v_t < 1$. Because (5) holds, it follows that there exists $T \geq 1$ such that $({}_1\hat{x}_t,\ {}_{t+1}v + \ {}_{con}h^{-1}) \in X$ and $({}_1\hat{x}_t,\ {}_{t+1}v + \ {}_{con}h^{-1}) \succcurlyeq {}_1v$ for all $t \geq T$ and all $h \geq H$. Fix any $t \geq T$. Then, since $\lim_{h\to\infty}$ $({}_1\hat{x}_t,\ {}_{t+1}v + \ {}_{con}h^{-1}) = ({}_1\hat{x}_t,\ {}_{t+1}v) \in X$ and $({}_1\hat{x}_t,\ {}_{t+1}v + \ {}_{con}h^{-1}) \succcurlyeq {}_1v$ for any $h \geq H$, $\mathbf{CON}_{d_\infty}$ and **C** imply that $({}_1\hat{x}_t,\ {}_{t+1}v) \succcurlyeq {}_1v$. Because $t \geq T$ is arbitrary, it follows that $({}_1\hat{x}_t,\ {}_{t+1}v) \succcurlyeq {}_1v$ for all $t \geq T$, and so **PC** implies that ${}_{con}\hat{x} \succcurlyeq {}_1v$, as sought.

Case 2. $\sup_{t\in\mathbb{N}} v_t = 1$.

As $\inf_{t\in\mathbb{N}} u_t > \inf_{t\in\mathbb{N}} v_t$, choose $K \in \mathbb{N}$ large enough such that the set $\mathbb{T}(K)$ defined below is non-empty:

$$\mathbb{T}(K) \equiv \left\{ t \in \mathbb{N} | 1 - \frac{1}{K} < v_t \leq 1, v_{t'} < v_t - \frac{1}{K} \text{ for some } t' \in \mathbb{N} \right\}.$$

Consider ${}_1v^K$ formed from ${}_1v$ as follows: $v_t^K = v_t - \frac{1}{K}$, for all $t \in \mathbb{T}(K)$, and $v_t^K = v_t$ for all $t \notin \mathbb{T}(K)$. By construction, ${}_1v^K \in X$, $\sup_t v_t^K \leq 1 - \frac{1}{K}$ and $\inf_t u_t > \inf_t v_t^K = \inf_t v_t$. By (5), **C** and $\mathbf{CON}_{d_\infty}$, it follows that for some $T \geq 1$, $({}_1\hat{x}_t, {}_{t+1}v^K) \succcurlyeq {}_1v^K$ for all $t \geq T$. Since the above arguments hold for any $k \geq K$, then $({}_1\hat{x}_t, {}_{t+1}v^k) \succcurlyeq {}_1v^k$ for all $t \geq T$ and all $k \geq K$. Further, $\lim_{k\to\infty} \left({}_1v^k\right) = {}_1v$ and $\lim_{k\to\infty}({}_1\hat{x}_t, {}_{t+1}v^k) = ({}_1\hat{x}_t, {}_{t+1}v)$, and so **C** and $\mathbf{CON}_{d_\infty}$ imply that $({}_1\hat{x}_t, {}_{t+1}v) \succcurlyeq {}_1v$ for all $t \geq T$. The desired result then follows from **PC** as in *Case 1*.

We have established that ${}_{con}\hat{x} \succcurlyeq {}_1v$. In order to complete the proof of (3), we note that by construction, ${}_1u \gg {}_{con}\hat{x}$ and $\inf_{t\in\mathbb{N}} u_t > \hat{x}$, and so **WP** implies that ${}_1u \succ {}_{con}\hat{x}$. By transitivity we conclude that ${}_1u \succ {}_1v$, as sought.

Next, we show that (4) holds as well. Suppose that $\inf_{t\in\mathbb{N}} u_t = \inf_{t\in\mathbb{N}} v_t$. If $\inf_{t\in\mathbb{N}} u_t = 1$, then the result follows by reflexivity. Hence suppose $\inf_{t\in\mathbb{N}} u_t < 1$. Choose $\delta > 0$ small enough such that the set $\mathbb{T}({}_1u; \delta)$ defined below is non-empty:

$$\mathbb{T}({}_1u; \delta) \equiv \{t' \in \mathbb{N} | 1 > \inf_t u_t + \delta > u_{t'} \geq \inf_t u_t\}.$$

Fix $\epsilon > 0$ such that $\delta \geq \epsilon$, and consider ${}_1u^\epsilon$ formed from ${}_1u$ as follows: $u_t^\epsilon = u_t + \epsilon$, all $t \in \mathbb{T}({}_1u; \delta)$, and $u_t' = u_t$, all $t \notin \mathbb{T}({}_1u; \delta)$. By construction, ${}_1u^\epsilon \in X$ and $\inf_t u_t^\epsilon > \inf_t v_t$, and so ${}_1u^\epsilon \succ {}_1v$ by (3). Since it holds for any $\epsilon > 0$ such that $\delta \geq \epsilon$ and since $\lim_{\epsilon\to 0}$ ${}_1u^\epsilon = {}_1u$, **C** and $\mathbf{CON}_{d_\infty}$ imply ${}_1u \succcurlyeq {}_1v$. A similar argument proves ${}_1v \succcurlyeq {}_1u$, and thus we obtain ${}_1u \sim {}_1v$. ■

Theorem 5 establishes an interesting possibility result for liberal approaches in economies with an infinite number of agents. For it proves that there exist fair, Paretian and continuous social welfare orderings that respect a liberal principle of non-interference. Indeed, the maximin SWO satisfies even the stronger version of the Weak Harm Principle (analogous to that presented in section 3) extended to hold for *any* countably infinite streams.

Further, Theorem 5 provides a novel, and interesting characterisation of the maximin SWO in the intergenerational context. Lauwers ([37]) characterises the maximin SWO in the infinite context by focusing on Weak Pareto, Anonymity,[22] Continuity, Repetition Approximation and either a strong version of Hammond Equity,[23] or Ordinal Level Comparability. Theorem 5 provides a completely different foundation to the maximin SWO, because the Weak Harm Principle* is logically and theoretically distinct both from axioms with an egalitarian content, such as Hammond Equity, and from informational invariance conditions.

Theorem 5 thus confirms the main intuitions concerning the relation between liberal and egalitarian approaches: the Weak Harm Principle, together with standard fairness, efficiency, and continuity properties leads straight to intergenerational welfare egalitarianism.

7 Conclusions

A number of recent contributions have raised serious doubts on the possibility of a fair and efficient liberal approach to distributive justice that incorporates a fully non-interfering view. This paper has shown that possibility results do emerge, in societies with both a finite and an infinite number of agents, provided the bite of non-interference is limited in an ethically relevant way. Anonymous and Paretian criteria exist which incorporate a notion of protection of individuals (or generations) from unjustified interference, in situations in which they suffer a welfare loss, provided no other agent (or generation) is affected.

A weaker version of a liberal axiom - the Harm Principle - recently proposed by Mariotti and Veneziani ([44]), together with standard properties, allows us to derive a set of new characterisations of the maximin and of its lexicographic refinement, including in the intergenerational context. This is surprising, because the Weak Harm Principle is meant to capture a liberal and libertarian requirement of non-interference and it incorporates no obvious egalitarian content. Thus, our results shed new light on the ethical foundations of the egalitarian approaches stemming from Rawls's difference principle, and provide new meaning to the label of *liberal egalitarianism* usually attached to Rawls's theory.

From the viewpoint of liberal approaches emphasising a notion of individual autonomy, or freedom, however, our results have a rather counterintuitive implication. For they prove that,

[22] Actually, the characterisation by Lauwers ([37]) relies on a *Strong* Anonymity axiom that considers all permutations of the utility vectors.

[23] Formally, for any two bounded infinite vectors ${}_1u, {}_1v$ such that $u_i \geq v_i \geq v_j \geq u_j$ for some $i, j \in \mathbb{N}$ and $u_k = v_k$ $\forall k \in \mathbb{N} \backslash \{i, j\}$, ${}_1v \succcurlyeq {}_1u$ (Lauwers [37], p.46).

in various contexts, liberal non-interfering principles lead straight to welfare egalitarianism.

References

[1] Alcantud, J.C. (2013): The impossibility of social evaluations of infinite streams with strict inequality aversion. *SAET Bulletin*. doi: 10.1007/s40505-013-0005-5.

[2] Alcantud, J.C. (2013): Liberal approaches to ranking infinite utility streams: When can we avoid interference? *Social Choice and Welfare* 41: 381–396.

[3] Asheim, G. B. (2010): Intergenerational equity. *Annual Review of Economics* 2: 197–222.

[4] Asheim, G. B., Banerjee, K. (2010): Fixed-step anonymous overtaking and catching-up. *International Journal of Economic Theory* 6: 149–165.

[5] Asheim, G. B., Tungodden, B. (2004): Resolving distributional conflicts between generations. *Economic Theory* 24: 221–230.

[6] Asheim, G. B., Zuber, S. (2013): A complete and strongly anonymous leximin relation on infinite streams. *Social Choice and Welfare.* Forthcoming. doi: 10.1007/s00355-012-0705-z.

[7] Asheim, G. B., d'Aspremont, C., Banerjee, K. (2010): Generalized time-invariant overtaking. *Journal of Mathematical Economics* 46: 519–533.

[8] Asheim, G.B., Mitra, T., Tungodden, B. (2007): A new equity condition for infinite utility streams and the possibility of being Paretian. In: Roemer, J.E., Suzumura, K. (eds.) *Intergenerational Equity and Sustainability*, pp. 55–68, MacMillan, London.

[9] Asheim, G. B., Mitra, T., Tungodden, B. (2012): Sustainable recursive social welfare functions. *Economic Theory* 49: 267–292.

[10] Atsumi, H. (1965): Neoclassical growth and the efficient program of capital accumulation. *Review of Economic Studies* 32: 127–136.

[11] Barberà, S., Jackson, M.O. (1988): Maximin, leximin, and the protective criterion: characterizations and comparisons. *Journal of Economic Theory* 46: 34–44.

[12] Basu, K., Mitra, T. (2003): Aggregating infinite utility streams with intergenerational equity: the impossibility of being Paretian. *Econometrica* 71: 1557–1563.

[13] Basu, K., Mitra, T. (2007): Utilitarianism for infinite utility streams: a new welfare criterion and its axiomatic characterization. *Journal of Economic Theory* 133: 350–373.

[14] Bolton, G.E., Ockenfels, A. (2006): Inequality Aversion, Efficiency, and Maximin Preferences in Simple Distribution Experiments: Comment. *American Economic Review* 96: 1906–1911.

[15] Bosmans, K., Ooghe, E. (2006): A Characterization of maximin. Mimeo, Katholieke Universiteit Leuven.

[16] Bossert, W., Sprumont, Y., Suzumura, K. (2007): Ordering infinite utility streams. *Journal of Economic Theory* 135: 179–189.

[17] Brundtland, G.H. (ed.) (1987): *Our Common Future*, Oxford University Press, Oxford.

[18] Cairns, R., Long. N.V. (2006): Maximin: A direct approach to sustainability. *Environment and Development Economics* 11: 275–300.

[19] Chichilnisky, G. (1982): Social Aggregation Rules and Continuity. *Quarterly Journal of Economics* 97: 337–352.

[20] Crespo, J.A., Nuñez, C., Rincón–Zapatero, J.P. (2009): On the impossibility of representing infinite utility streams. *Economic Theory* 40: 47–56.

[21] d'Aspremont, C. (1985): Axioms for social welfare orderings. In: Hurwicz, L., Schmeidler, D., Sonnenschein, H. (eds.) *Social Goals and Social Organization: Essays in Memory of Elisha Pazner*, pp.19–76, Cambridge University Press, Cambridge.

[22] d'Aspremont, C., Gevers, L. (2002): Social Welfare Functionals and Interpersonal Comparability. In: Arrow, K.J., Sen, A.K., Suzumura, K. (eds.) *Handbook of Social Choice and Welfare vol. 1*, pp.459–541, North Holland, Amsterdam.

[23] de Castro, L.I., Pesce, M., Yannelis, N.C. (2011): Core and equilibria under ambiguity. *Economic Theory* 48: 519–548.

[24] Diamond, P.A. (1965): The evaluation of infinite utility streams. *Econometrica* 33: 170–177.

[25] Engelmann, D., Strobel, M. (2004): Inequality Aversion, Efficiency, and Maximin Preferences in Simple Distribution Experiments. *American Economic Review* 94: 857–69.

[26] Fleurbaey, M., Michel, P. (2003): Intertemporal equity and the extension of the Ramsey Criterion. *Journal of Mathematical Economics* 39: 777–802.

[27] Fleurbaey, M., Tungodden, B. (2010): The tyranny of non-aggregation versus the tyranny of aggregation in social choices: a real dilemma. *Economic Theory* 44: 399–414.

[28] Frankfurt, H. (1987): Equality as a Moral Ideal. *Ethics* 98: 21–43.

[29] Gilboa, I., Schmeidler, D. (1989): Maximin expected utility with non-unique prior. *Journal of Mathematical Economics* 18: 141–153.

[30] Gotoh, R., Yoshihara, N. (2003): A Class of Fair Distribution Rules à la Rawls and Sen. *Economic Theory* 22: 63–88.

[31] Hammond, P.J. (1976): Equity, Arrow's conditions and Rawls' difference principle. *Econometrica* 44: 793–804.

[32] Hammond, P.J. (1979): Equity in two–person situations: Some consequences. *Econometrica* 47: 1127–1135.

[33] Hammond, P.J. (1996): Consequentialist decision theory and utilitarian ethics. In: Farina, F., Hahn, F., Vannucci, S. (eds.) *Ethics, Rationality, and Economic Behaviour*, pp.92–118, Clarendon Press, Oxford.

[34] Hara, C., Shinotsuka, T., Suzumura, K., Xu, Y. (2008): Continuity and egalitarianism in the evaluation of infinite utility streams. *Social Choice and Welfare*, 31: 179–191.

[35] Kaplow, L., Shavell, S. (2001): Any Non-welfarist Method of Policy Assessment Violates the Pareto Principle. *Journal of Political Economy* 109: 281–286.

[36] Konow, K. (2002): Which is the fairest one of all? A positive analysis of justice theories. *Journal of Economic Literature* 41: 1188–1239.

[37] Lauwers, L. (1997): Rawlsian equity and generalised utilitarianism with an infinite population. *Economic Theory* 9: 143–150.

[38] Llavador, H., Roemer, J.E., Silvestre, J. (2010): Intergenerational justice when future worlds are uncertain. *Journal of Mathematical Economics* 46: 728–761.

[39] Llavador, H., Roemer, J.E., Silvestre, J. (2011): A Dynamic Analysis of Human Welfare in a Warming Planet. *Journal of Public Economics*, 95: 1607–1620.

[40] Llavador, H., Roemer, J.E., Silvestre, J. (2013): *Sustainability for a warming planet*, ms. in preparation.

[41] Lombardi, M., Veneziani, R. (2009): Liberal Egalitarianism and the Harm Principle. WP 649, Queen Mary University of London.

[42] Lombardi, M., Veneziani R. (2012): Treading a fine line: characterizations and impossibilities for liberal principles in infinitely-lived societies. *The B.E. Journal of Theoretical Economics*, 9. doi: 10.1515/1935-1704.1899.

[43] Mariotti, M., Veneziani, R. (2009): The paradoxes of the liberal ethics of non-interference. WP 653, Queen Mary University of London.

[44] Mariotti, M., Veneziani, R. (2009): Non-interference implies equality. *Social Choice and Welfare* 32: 123–128.

[45] Mariotti, M., Veneziani, R. (2012): Opportunities as chances: maximising the probability that everybody succeeds, DP 9, University of Massachusetts, Amherst.

[46] Mariotti, M., Veneziani, R. (2012): Allocating chances of success in finite and infinite societies: The Utilitarian criterion. *Journal of Mathematical Economics* 48: 226–236.

[47] Mariotti, M., Veneziani, R. (2013): On the impossibility of complete non-interference in Paretian social judgements. *Journal of Economic Theory*, forthcoming.

[48] Maskin, E. (1979): Decision-making under ignorance with implications for social choice. *Theory and Decision* 11: 319–337.

[49] Rawls, J. (1971): *A Theory of Justice*, Harvard University Press, Cambridge.

[50] Roemer, J.E. (1998): *Equality of opportunity*, Harvard University Press, Cambridge.

[51] Roemer, J.E. (2002): Equality of opportunity: A progress report. *Social Choice and Welfare* 19: 455–471.

[52] Roemer, J.E. (2004): Eclectic Distributional Ethics. *Politics, Philosophy and Economics* 3: 267–281.

[53] Roemer, J.E. (2011): The ethics of distribution in a warming planet. *Environmental and resource economics* 48: 363–390.

[54] Sakai, T. (2010): Intergenerational equity and an explicit construction of welfare criteria. *Social Choice and Welfare* 35: 393–414.

[55] Sarin, R., Vahid, F. (2001): Predicting how people play games: a simple dynamic model of choice. *Games and Economic Behavior* 34: 104–122.

[56] Segal, U., Sobel, J. (2002): Min, Max, and Sum. *Journal of Economic Theory* 106: 126–150.

[57] Silvestre, J. (2002): Progress and conservation under Rawls's maximin principle. *Social Choice and Welfare* 19: 1–27.

[58] Solow, R. M. (1974): Intergenerational equity and exhaustible resources. *Review of Economic Studies* 41: 29–45.

[59] Tungodden, B. (1999): Hammond equity: A generalization. Discussion paper 20/99, Norwegian School of Economics and Business Administration.

[60] Tungodden, B. (2000): Egalitarianism: Is leximin the only option? *Economics and Philosophy* 16: 229–245.

[61] von Weizsäcker, C.C. (1965): Existence of optimal program of accumulation for an infinite time horizon. *Review of Economic Studies* 32: 85–104.

[62] Wolf, C. (1995): Contemporary Property Rights, Lockean Provisos, and the Interests of Future Generations. *Ethics* 105: 791–818.

[63] Zame, W.R. (2007): Can intergenerational equity be operationalized? *Theoretical Economics* 2: 187–202.

Addendum

Proof of Proposition 2

Proof of Proposition 2

($\Rightarrow$) Let $\succcurlyeq$ on X_T be the leximin ordering, i.e., $\succcurlyeq = \succcurlyeq^{LM}$. It is clear that leximin ordering satisfies **C**, **SP** and **A**. Moreover, since **WHP** is weaker than **HP**, the proof that $\succcurlyeq^{LM}$ on X_T meets **WHP** follows from the proof of necessity of **HP** provided by Mariotti and Veneziani (2009, Theorem 1, p.126).

($\Leftarrow$) Let $\succcurlyeq$ on X_T be a SWO satisfying **SP**, **A**, **C**, and **WHP**. We show that $\succcurlyeq$ on X_T is the leximin SWO. Thus, we should prove that, $\forall u,\, v \in X_T$,

$$u \sim^{LM} v \Leftrightarrow u \sim v \tag{7}$$

and

$$u \succ^{LM} v \Leftrightarrow u \succ v \tag{8}$$

First, we prove the implication ($\Rightarrow$) of (7). If $u \sim^{LM} v$, then $\bar{u} = \bar{v}$, and so $u \sim v$, by **A**. Next, we prove the implication ($\Rightarrow$) of (8). Suppose that $u \succ^{LM} v$, and so, by definition $\bar{u}_1 > \bar{v}_1$ or $\exists t \in \{2, ..., T\}$ such that $\bar{u}_s = \bar{v}_s\ \forall 1 \leq s < t$ and $\bar{u}_t > \bar{v}_t$. Suppose, by contradiction, that $v \succ u$. Note that since $\succcurlyeq$ satisfies **A**, in what follows we can focus, without loss of generality, either on u and v, or on the ranked vectors $\bar{u}$ and $\bar{v}$. Therefore, suppose $\bar{v} \succ \bar{u}$. As **SP** holds it must be the case that $\bar{v}_l > \bar{u}_l$ for some $l > t$. Let

$$k = \min\{t < l \leq T | \bar{v}_l > \bar{u}_l\}.$$

By **A**, let $v_i = \bar{v}_k$ and let $u_i = \bar{u}_{k-g}$, for some $1 \leq g < k$, where $\bar{u}_{k-g} > \bar{v}_{k-g}$. Then, let two real numbers $d_1, d_2 > 0$, and consider vectors u', v' and the corresponding ranked vectors $\bar{u}'$, $\bar{v}'$ formed from $\bar{u}$, $\bar{v}$ as follows: first, $\bar{u}_{k-g}$ is lowered to $\bar{u}_{k-g} - d_1$ such that $\bar{u}_{k-g} - d_1 > \bar{v}_{k-g}$; next, $\bar{v}_k$ is lowered to $\bar{v}_k - d_2$ such that $\bar{u}_k > \bar{v}_k - d_2 > \bar{u}_{k-g} - d_1$; finally, all other entries of $\bar{u}$ and $\bar{v}$ are unchanged. By construction u', $v' \in X_T$ and $\bar{u}'_j \geq \bar{v}'_j$ for all $j \leq k$, with $\bar{u}'_{k-g} > \bar{v}'_{k-g}$, whereas **WHP**, combined with **C**, and **A**, implies $\bar{v}' \succcurlyeq \bar{u}'$. By **SP**, $d_1, d_2 > 0$ can be chosen so that $\bar{v}' \succ \bar{u}'$, without loss of generality. Consider two cases:

a) Suppose that $\bar{v}_k > \bar{u}_k$, but $\bar{u}_l \geq \bar{v}_l$ for all $l > k$. It follows that $\bar{u}' > \bar{v}'$, and so **SP** implies that $\bar{u}' \succ \bar{v}'$, a contradiction.

b) Suppose that $\bar{v}_l > \bar{u}_l$ for some $l > k$. Note that by construction $\bar{v}'_l = \bar{v}_l$ and $\bar{u}'_l = \bar{u}_l$ for all $l > k$. Then, let

$$k' = \min\{k < l \leq T | \bar{v}'_l > \bar{u}'_l\}.$$

The above argument can be applied to $\bar{u}'$, $\bar{v}'$ to derive vectors $\bar{u}''$, $\bar{v}''$ such that $\bar{u}''$, $\bar{v}'' \in X_T$ and $\bar{u}''_j \geq \bar{v}''_j$ for all $j \leq k'$, whereas **WHP**, combined with **A**, **C**, and **SP**, implies $\bar{v}'' \succ \bar{u}''$. And so on. After a finite number of iterations s, two vectors $\bar{u}^s$, $\bar{v}^s \in X_T$ can be derived such that, by **WHP**, combined with **A**, **C**, and **SP**, we have that $\bar{v}^s \succ \bar{u}^s$, but $\bar{u}^s > \bar{v}^s$ so that **SP** implies $\bar{u}^s \succ \bar{v}^s$, yielding a contradiction.

We have proved that if $u \succ^{LM} v$ then $u \succcurlyeq v$. Suppose now, by contradiction, that $v \sim u$, or equivalently $\bar{v} \sim \bar{u}$. Since, by our supposition, $\bar{v}_t < \bar{u}_t$, there exists $\epsilon > 0$ such that $\bar{v}_t < \bar{u}_t - \epsilon < \bar{u}_t$. Let $\bar{u}^\epsilon \in X_T$ be a vector such that $\bar{u}^\epsilon_t = \bar{u}_t - \epsilon$ and $\bar{u}^\epsilon_j = \bar{u}_j$ for all $j \neq t$. It follows that $\bar{u}^\epsilon \succ^{LM} \bar{v}$ but $\bar{v} \succ \bar{u}^\epsilon$ by **SP** and the transitivity of $\succcurlyeq$. Hence, the above argument can be applied to $\bar{v}$ and $\bar{u}^\epsilon$, yielding the desired contradiction. ■

Independence of Axioms

The proofs of the independence of the axioms used to characterise the finite maximin and leximin SWOs are obvious and therefore they are omitted. It is worth noting, however, that some of the examples below can be easily adapted to apply to the finite context.

Independence of axioms used in Theorem 4

In order to complete the proof of Theorem 4, we show that the axioms are tight.

For an example violating *only* **A**, define $\succcurlyeq$ on X as follows: $\forall {}_1u,\, {}_1v \in X$,

$$\begin{aligned} {}_1u &\sim {}_1v \Leftrightarrow {}_1u = {}_1v\,, \\ {}_1u &\succ {}_1v \Leftrightarrow \text{ either } u_1 > v_1, \text{ or } \exists T \in \mathbb{N}\setminus\{1\} : u_t = v_t \ \forall t < T \text{ and } u_T > v_T. \end{aligned}$$

The SWR $\succcurlyeq$ on X is not an extension of the leximin SWR $\succcurlyeq^{LM^*}$. The SWR $\succcurlyeq$ on X satisfies all axioms except **A**.

For an example violating *only* **SP**, define $\succcurlyeq$ on X as follows: $\forall {}_1u,\, {}_1v \in X,\, {}_1u \sim {}_1v$. The SWR $\succcurlyeq$ on X is not an extension of the leximin SWR $\succcurlyeq^{LM^*}$. The SWR $\succcurlyeq$ on X satisfies all axioms except **SP**.

For an example violating *only* **WHP***, define $\succcurlyeq$ on X as follows: $\forall {}_1u, {}_1v \in X$,

$$\begin{aligned} {}_1u &\sim {}_1v \Leftrightarrow \exists \tilde{T} \geq 1 \text{ such that } \forall T \geq \tilde{T} : \ {}_1\bar{u}_T = {}_1\bar{v}_T, \\ {}_1u &\succ {}_1v \Leftrightarrow \exists \tilde{T} \geq 1 \text{ such that } \forall T \geq \tilde{T}\,, \exists t \in \{1, ..., T\} \text{ with } \bar{u}_s = \bar{v}_s \ (\forall t < s \leq T) \text{ and } \bar{u}_t > \bar{v}_t. \end{aligned}$$

The SWR $\succcurlyeq$ on X is not an extension of the leximin SWR $\succcurlyeq^{LM^*}$. The SWR $\succcurlyeq$ on X satisfies all axioms except **WHP***.

For an example violating *only* **MC**, let for any $T \in \mathbb{N}$ and ${}_1u \in X$, $\rho_T({}_1u_T)$ be a permutation of ${}_1u_T$. Then define $\succcurlyeq$ on X as follows: $\forall {}_1u, {}_1v \in X$,

$$\begin{aligned} {}_1u &\sim {}_1v \Leftrightarrow \exists \tilde{T} \geq 1 \text{ such that } \forall T \geq \tilde{T} : \ {}_1u_T = \rho_T({}_1v_T) \text{ for some permutation } \rho_T; \\ {}_1u &\succ {}_1v \Leftrightarrow \exists \tilde{T} \geq 1 \text{ such that } \forall T \geq \tilde{T} : \ {}_1u_T > \rho_T({}_1v_T) \text{ for some permutation } \rho_T. \end{aligned}$$

The SWR $\succcurlyeq$ on X is not an extension of the leximin SWR $\succcurlyeq^{LM^*}$. The SWR $\succcurlyeq$ on X satisfies all axioms except **MC**.

For an example violating *only* **WPC**, let $\succcurlyeq$ on X be the leximin defined in Bossert et al. (2007; p. 586). The SWR $\succcurlyeq$ on X is not an extension of the leximin SWR $\succcurlyeq^{LM^*}$. The SWR $\succcurlyeq$ on X satisfies all axioms except **WPC**. [To see that **WPC** is violated, for all x,$y \in \mathbb{R}$, let ${}_{rep}(x, y) \equiv (x, y, x, y, ...)$ and consider the profiles ${}_1u = \left(\frac{1}{2}, {}_{rep}\left(\frac{1}{4}, \frac{1}{8}\right)\right)$ and ${}_1v = \left(\frac{3}{4}, {}_{rep}\left(0, \frac{3}{20}\right)\right)$. Then, $({}_1u_T, {}_{T+1}v) \succ {}_1v, \forall T \in \mathbb{N}\setminus\{1\}$ but ${}_1u \not\succ {}_1v$.].

Independence of axioms used in Theorem 5

In order to complete the proof of Theorem 5, we show that the axioms are tight.

For an example violating *only* **A**, define $\succsim$ on X as follows: $\forall {}_1u,\ {}_1v \in X$,

$$ {}_1u \succsim {}_1v \Leftrightarrow u_1 \geq v_1. $$

$\succsim$ is a SWO on X and it satisfies all axioms except **A**.

For an example violating *only* **WP**, define $\succsim$ on X as follows: $\forall {}_1u,\ {}_1v \in X$, ${}_1u \sim {}_1v$. $\succsim$ is a SWO on X and it satisfies all axioms except **WP**.

For an example violating *only* **PC**, define $\succsim$ on X as follows: $\forall {}_1u, {}_1v \in X$,

$$ {}_1u \succsim {}_1v \Leftrightarrow \liminf_{t\in\mathbb{N}} u_t \geq \liminf_{t\in\mathbb{N}} v_t. $$

$\succsim$ is a SWO on X and it satisfies all axioms except **PC**. [To see that **PC** is violated, consider the profiles ${}_1u = {}_{con}0$ and ${}_1v = {}_{con}1$. By construction, $({}_1u_T,\ {}_{T+1}v) \sim {}_1v\ \forall T \geq 2$, but ${}_1v \succ {}_1u$.]

Let the following notation hold for the next two examples. Define X^* as follows:

$$ X^* = \{{}_1u \in X |\ \min_{t\in\mathbb{N}} u_t \text{ exists}\}. $$

For all ${}_1u \in X^*$, let $t({}_1u)$ be one of the generations such that $u_{t({}_1u)} = \min_{t\in\mathbb{N}} u_t$.

For an example violating *only* **WHP***, define $\succsim$ on X as follows: $\forall {}_1u,\ {}_1v \in X$,

(i) if ${}_1u,\ {}_1v \in X^*$, then ${}_1u \succsim {}_1v \Leftrightarrow \frac{\min_{t\in\mathbb{N}} u_t + \inf_{t\in\mathbb{N}\setminus\{t({}_1u)\}} u_t}{2} \geq \frac{\min_{t\in\mathbb{N}} v_t + \inf_{t\in\mathbb{N}\setminus\{t({}_1v)\}} v_t}{2}$;

(ii) if ${}_1u \in X^*,\ {}_1v \in X\setminus X^*$, then ${}_1u \succsim {}_1v \Leftrightarrow \frac{\min_{t\in\mathbb{N}} u_t + \inf_{t\in\mathbb{N}\setminus\{t({}_1u)\}} u_t}{2} \geq \inf_{t\in\mathbb{N}} v_t$;

(iii) otherwise, ${}_1u \succsim {}_1v \Leftrightarrow \inf_{t\in\mathbb{N}} u_t \geq \inf_{t\in\mathbb{N}} v_t$.

$\succsim$ is a SWO on X and it satisfies all axioms except **WHP***. [To see that **WHP*** is violated, consider the profiles ${}_1u = (\frac{1}{6},\ {}_{con}1)$, ${}_1v = {}_{con}\frac{1}{2}$, ${}_1u' = (\frac{1}{6}, \frac{1}{2},\ {}_{con}1)$, and ${}_1v' = (\frac{1}{2}, \frac{1}{3},\ {}_{con}\frac{1}{2})$. By the definition of $\succsim$, ${}_1u \succ {}_1v$, but ${}_1v' \succ {}_1u'$, which contradicts **WHP***.]

For an example violating *only* $\mathbf{CON}_{d_\infty}$, define $\succsim$ on X as follows: $\forall {}_1u,\ {}_1v \in X$,

(i) if $\inf_{t\in\mathbb{N}} u_t > \inf_{t\in\mathbb{N}} v_t$, then ${}_1u \succ {}_1v$;

(ii) if ${}_1u,\ {}_1v \in X^*$ and $u_{t({}_1u)} = v_{t({}_1v)}$, then ${}_1u \succsim {}_1v \Leftrightarrow \inf_{t\in\mathbb{N}\setminus\{t({}_1u)\}} u_t \geq \inf_{t\in\mathbb{N}\setminus\{t({}_1v)\}} v_t$;

(iii) if ${}_1u \in X\setminus X^*,\ {}_1v \in X^*$, and $\inf_{t\in\mathbb{N}} u_t = \min_{t\in\mathbb{N}} v_t$, then ${}_1u \succ {}_1v$;

(iv) if ${}_1u,\ {}_1v \in X\setminus X^*$, and $\inf_{t\in\mathbb{N}} u_t = \inf_{t\in\mathbb{N}} v_t$, then ${}_1u \sim {}_1v$.

$\succcurlyeq$ is a SWO on X and it satisfies all axioms except $\mathbf{CON}_{d_\infty}$. [To see that $\mathbf{CON}_{d_\infty}$ is violated, consider the profiles ${}_1u^k = (\frac{1}{k},\ {}_{con}\frac{1}{2})$, $k \in \mathbb{N}$, and ${}_1v = (0,\ {}_{con}1)$. Observe that ${}_1v \in X^*$, ${}_1u^k \in X^*$ $\forall k \in \mathbb{N}$ and $\lim_{k\to\infty} {}_1u^k = (0,\ {}_{con}\frac{1}{2}) \in X^*$. By the definition of $\succcurlyeq$, ${}_1u^k \succcurlyeq {}_1v$ $\forall k \in \mathbb{N}$, but ${}_1v \succ (0,\ {}_{con}\frac{1}{2})$, which contradicts $\mathbf{CON}_{d_\infty}$.]

For an example violating *only* $\mathbf{C}$, define $\succcurlyeq$ on X as follows: $\forall {}_1u, {}_1v \in X$,

$$\begin{aligned} {}_1u &\sim {}_1v \Leftrightarrow {}_1u = \pi\left({}_1v\right) \text{ for some } \pi \in \Pi; \\ {}_1u &\succ {}_1v \Leftrightarrow \exists \epsilon > 0 :\ {}_1u \geq \pi\left({}_1v\right) + {}_{con}\epsilon, \text{ for some } \pi \in \Pi. \end{aligned}$$

$\succcurlyeq$ is a SWR on X and it satisfies all axioms except $\mathbf{C}$.