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# Patents, R&D Subsidies and Endogenous Market Structure in a Schumpeterian Economy

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## Abstract

This study explores the different implications of patent breadth and R&D subsidies on economic growth and endogenous market structure in a Schumpeterian growth model. We find that when the number of firms is fixed in the short run, patent breadth and R&D subsidies serve to increase economic growth as in previous studies. However, when the number of firms adjusts endogenously in the long run, R&D subsidies increase economic growth but decrease the number of firms, whereas patent breadth expands the number of firms but reduces economic growth. Therefore, R&D subsidy is perhaps a more suitable policy instrument than patent breadth for the purpose of stimulating long-run economic growth.

*JEL classification:* O30, O40

*Keywords:* economic growth, endogenous market structure, patents, R&D subsidies

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# 1 Introduction

What are the different implications of patent breadth and R&D subsidies on economic growth and market structure? To explore this question, we consider a second-generation R&D-based growth model, pioneered by Peretto (1998), Young (1998), Howitt (1999) and Segerstrom (2000). To our knowledge, this is the first study that analyzes patent breadth in a second-generation R&D-based growth model. The model features two dimensions of technological progress. In the vertical dimension, firms improve the quality of existing products. In the horizontal dimension, firms invent new products. In this scale-invariant Schumpeterian growth model with endogenous market structure (EMS) measured by the number of firms in equilibrium, we find some interesting differences between patent breadth and R&D subsidies. At the first glance, these two policy instruments should have similar effects on innovation and economic growth. On the one hand, patent breadth improves the incentives for innovation by increasing the private return to R&D investment. On the other hand, R&D subsidies improve the incentives for innovation by reducing the private cost of R&D investment. For example, an interesting study by Li (2001) shows that both of these policy instruments contribute to increasing innovation and economic growth in a quality-ladder growth model that features exogenous market structure. However, in a Schumpeterian growth model with EMS, we find that patent breadth and R&D subsidies have drastically different implications on economic growth and market structure. Specifically, when the number of firms is fixed in the short run, patent breadth and R&D subsidies have positive effects on economic growth as in previous studies. Interestingly, when the number of firms adjusts endogenously in the long run, patent breadth expands the number of firms but decreases economic growth, whereas R&D subsidies increase economic growth but reduce the number of firms.

Intuitively, R&D subsidies decrease the cost of R&D investment and improve the incentives for R&D; therefore, a higher rate of R&D subsidies increases economic growth in the short run and in the long run. As for an increase in patent breadth, it raises the profit margin of monopolistic firms and provides more incentives for R&D in the short run. In the long run, it encourages the entry of new firms, which reduces average firm size measured by the number of workers per firm. Given that firm size determines the incentives for R&D in the second generation model,<sup>1</sup> a larger patent breadth decreases long-run economic growth. These contrasting long-run implications of patent breadth and R&D subsidies suggest that R&D subsidy is perhaps a more suitable policy instrument than patent breadth for the purpose of stimulating long-run economic growth. The negative effect of patent protection on innovation is consistent with case-study evidence in Jaffe and Lerner (2004), Bessen and Meurer (2008) and Boldrin and Levine (2008). As for the positive effect of R&D subsidies on innovation, it is also consistent with evidence; see for example, Hall and Van Reenen (2000) for a survey of empirical studies.

This study relates to the literature on R&D-driven economic growth; see Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) for seminal studies. Subsequent studies in this literature often apply variants of the R&D-based growth model to analyze the effects of policy instruments, such as patent breadth and

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<sup>1</sup>Laincz and Peretto (2006) provide empirical evidence for a positive relationship between average firm size and economic growth. See also Ha and Howitt (2007) and Madsen (2008) for other empirical studies that support the second generation model.

R&D subsidies, on economic growth and innovation; see for example, Segerstrom (2000), Li (2001), Goh and Olivier (2002), Iwaisako and Futagami (2013), Chu (2011) and Chu and Furukawa (2011). However, these studies do not analyze the effects of patent policy on EMS;<sup>2</sup> therefore, the present study contributes to the literature with a novel analysis of patent breadth in a Schumpeterian growth model with EMS. Furthermore, we contrast the different effects of patent breadth and R&D subsidies and find that in a scale-invariant Schumpeterian growth model with EMS, the long-run effects of patent breadth and R&D subsidies are drastically different suggesting the importance of taking into consideration EMS when performing policy analysis in R&D-based growth models. O’Donoghue and Zweimuller (2004), Horii and Iwaisako (2007), Furukawa (2007, 2010), Chu (2009), Chu *et al.* (2012) and Chu and Pan (2013) also find that increasing the strength of other patent policy levers, such as blocking patents and patentability requirement, could have a negative effect on economic growth. The present study differs from these previous studies that mostly focus on the long-run effects of patent policy and contributes to the literature by highlighting the different short-run and long-run implications of patent protection on economic growth.

The rest of this study is organized as follows. Section 2 presents the Schumpeterian growth model with EMS. Section 3 analyzes the effects of patent breadth and R&D subsidies. Section 4 concludes.

## 2 A Schumpeterian growth model with EMS

In summary, the growth-theoretic framework is based on the Schumpeterian model with in-house R&D and EMS in Peretto (2007, 2011). In this model, labor is used as a factor input for the production of final goods. Final goods are either consumed by households or used as a factor input for R&D, entry and the production of intermediate goods. We incorporate patent breadth into the model and analyze its different implications from R&D subsidies on economic growth and market structure.

### 2.1 Household

In the economy, the population size is normalized to unity, and there is a representative household who has the following lifetime utility function:

$$U = \int_0^{\infty} e^{-\rho t} \ln C_t dt, \quad (1)$$

where  $C_t$  denotes consumption of final goods (numeraire) at time  $t$ . The parameter  $\rho > 0$  determines the rate of subjective discounting. The household maximizes (1) subject to the following asset-accumulation equation:

$$\dot{A}_t = r_t A_t + (1 - \tau) w_t L - C_t. \quad (2)$$

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<sup>2</sup>See Peretto (1996, 1999) for seminal studies in R&D-based growth models with EMS and Etro (2012) for an excellent textbook treatment of this topic.

$A_t$  is the real value of assets owned by the household, and  $r_t$  is the real interest rate. The household has a labor endowment of  $L$  units and supplies them inelastically to earn a real wage rate  $w_t$ . The household also pays a wage-income tax  $\tau w_t L$  to the government. From standard dynamic optimization, the familiar Euler equation is

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \quad (3)$$

## 2.2 Final goods

We follow Aghion and Howitt (2005) and Peretto (2007, 2011) to assume that final goods  $Y_t$  are produced by competitive firms using the following production function:

$$Y_t = \int_0^{N_t} X_t^\theta(i) [Z_t^\alpha(i) Z_t^{1-\alpha} l_t(i)]^{1-\theta} di, \quad (4)$$

where  $\{\theta, \alpha\} \in (0, 1)$  and  $X_t(i)$  denotes intermediate goods  $i \in [0, N_t]$ . The productivity of labor  $l_t(i)$  using intermediate good  $X_t(i)$  depends on the quality  $Z_t(i)$  of good  $i$  and also on the average quality  $Z_t \equiv \frac{1}{N_t} \int_0^{N_t} Z_t(i) di$  of all intermediate goods. The degree of technology spillovers is determined by  $1 - \alpha$ . From profit maximization, the conditional demand function for  $l_t(i)$  is

$$l_t(i) = \left( \frac{1 - \theta}{w_t} \right)^{1/\theta} X_t(i) [Z_t^\alpha(i) Z_t^{1-\alpha}]^{(1-\theta)/\theta}, \quad (5)$$

and the conditional demand function for  $X_t(i)$  is

$$X_t(i) = \left( \frac{\theta}{p_t(i)} \right)^{1/(1-\theta)} Z_t^\alpha(i) Z_t^{1-\alpha} l_t(i), \quad (6)$$

where  $p_t(i)$  is the price of  $X_t(i)$  and the price of  $Y_t$  is normalized to unity. Perfect competition implies that final goods producers pay  $\theta Y_t = \int_0^{N_t} p_t(i) X_t(i) di$  to intermediate goods firms and pay  $(1 - \theta) Y_t = \int_0^{N_t} w_t l_t(i) di$  to labor.<sup>3</sup>

## 2.3 Intermediate goods and in-house R&D

Existing intermediate goods firms produce differentiated goods with a technology that requires one unit of final goods to produce one unit of intermediate goods  $X_t(i)$ . Following Peretto (2011), we assume that the firm in industry  $i$  incurs  $\phi Z_t$  units of final goods as a fixed operating cost, where  $Z_t$  is aggregate technology as defined above. This specification implies that managing facilities are more expensive to operate in a technologically more advanced economy. To improve the quality of its products, the firm invests  $R_t(i)$  units of final goods in R&D. The innovation process is

$$\dot{Z}_t(i) = R_t(i). \quad (7)$$

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<sup>3</sup>Free movement of labor across firms implies that wages must be equal across firms.

The value of the monopolistic firm in industry  $i$  is

$$V_t(i) = \int_t^\infty \exp\left(-\int_t^u r_v dv\right) \pi_u(i) du. \quad (8)$$

The profit flow  $\pi_t(i)$  at time  $t$  is

$$\pi_t(i) = [p_t(i) - 1]X_t(i) - \phi Z_t - (1 - s)R_t(i), \quad (9)$$

where the parameter  $s \in (0, 1)$  is the rate of R&D subsidies. The monopolistic firm maximizes (8) subject to (6) and (7). The current-value Hamiltonian for this optimization problem is

$$H_t(i) = \pi_t(i) + \lambda_t(i)\dot{Z}_t(i). \quad (10)$$

We solve this optimization problem in the Appendix and find that the unconstrained profit-maximizing markup ratio is  $1/\theta$ . To analyze the effects of patent breadth, we impose an upper bound  $\mu > 1$  on the markup ratio.<sup>4</sup> Therefore, the equilibrium price becomes

$$p_t(i) = \min\{\mu, 1/\theta\}. \quad (11)$$

For the rest of this study, we assume that  $\mu < 1/\theta$ . In this case, a larger patent breadth  $\mu$  leads to a higher markup, and this implication is consistent with Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price". Finally, the return to in-house R&D is increasing in firm size measured by employment  $l_t(i) = l_t$  for  $i \in [0, N_t]$ .

**Lemma 1** *The return to in-house R&D is given by*

$$r_t = \frac{\alpha}{1-s} \left[ (\mu - 1) \left( \frac{\theta}{\mu} \right)^{1/(1-\theta)} l_t \right]. \quad (12)$$

**Proof.** See the Appendix. ■

## 2.4 Entrants

A firm that is active at time  $t$  must have been born at some earlier date. Following the standard treatment in the literature, we consider a symmetric equilibrium in which  $Z_t(i) = Z_t$  for  $i \in [0, N_t]$ , by assuming that any new entry at time  $t$  has access to the level of aggregate

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<sup>4</sup>Intuitively, the presence of monopolistic profits attracts potential imitators. However, stronger patent protection increases the production cost of imitative products and allows monopolistic firms to charge a higher markup without losing market share to imitators; see also Li (2001), Goh and Olivier (2002), Chu (2011), Chu and Furukawa (2011) and Iwaisako and Futagami (2013) for a similar formulation.

technology  $Z_t$ .<sup>5</sup> A new firm pays a setup cost  $X_t(i)F$ , where  $F > 0$  is a cost parameter, to set up its operation and introduce a new variety of products to the market.<sup>6</sup> We refer to this process as entry. Suppose entry is positive (i.e.,  $\dot{N}_t > 0$ ). Then, the no-arbitrage condition is<sup>7</sup>

$$V_t(i) = X_t(i)F. \quad (13)$$

The familiar Bellman equation implies that the return to entry is

$$r_t = \frac{\pi_t}{V_t} + \frac{\dot{V}_t}{V_t}. \quad (14)$$

## 2.5 Government

The government chooses an exogenous rate of R&D subsidies  $s \in (0, 1)$ . The government collects tax revenue  $T_t$  from the household, and the balanced-budget condition is

$$T_t = G_t + s \int_0^{N_t} R_t(i) di, \quad (15)$$

where  $G_t$  is unproductive government consumption that changes endogenously as in Peretto (2007). The amount of tax revenue is  $T_t = \tau w_t L = \tau(1 - \theta)Y_t$ , where  $\tau \in (0, 1)$  is a stationary tax rate on wage income.

## 2.6 Aggregation

Applying symmetry, we derive the labor market clearing condition as

$$L = N_t l_t. \quad (16)$$

The resource constraint on final goods is

$$Y_t = C_t + N_t(X_t + \phi Z_t + R_t) + \dot{N}_t X_t F. \quad (17)$$

Substituting (6) into (4) and imposing symmetry yield the aggregate production function:<sup>8</sup>

$$Y_t = \left( \frac{\theta}{p_t(i)} \right)^{\theta/(1-\theta)} Z_t N_t l_t = \left( \frac{\theta}{\mu} \right)^{\theta/(1-\theta)} Z_t L, \quad (18)$$

where the second equality uses (16) and markup pricing  $p_t(i) = \mu$ .

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<sup>5</sup>See Peretto (1998, 1999, 2007) for a discussion of the symmetric equilibrium being a reasonable equilibrium concept in this class of models.

<sup>6</sup>The setup cost is proportional to the new firm's initial volume of output. This assumption captures the idea that the setup cost depends on the amount of productive assets required to start production; see Peretto (2007) for a discussion.

<sup>7</sup>We follow the standard approach in this class of models to treat entry and exit symmetrically (i.e., the scrap value of exiting an industry is also  $X_t(i)F$ ); therefore,  $V_t = X_t(i)F$  always holds. If  $V_t > X_t(i)F$  ( $V_t < X_t(i)F$ ), then there would be an infinite number of entries (exits).

<sup>8</sup>We could follow Peretto (2007) to consider an extension that allows for a positive externality effect of  $N_t$  on  $Y_t$  by modifying (4) to  $Y_t = N_t^{\sigma(1-\theta)} \int_0^{N_t} X_t^\theta(i) [Z_t^\alpha(i) Z_t^{1-\alpha} l_t(i)]^{1-\theta} di$ , where  $\sigma \in (0, 1)$ . In this case, (18) becomes  $Y_t = N_t^\sigma (\theta/\mu)^{\theta/(1-\theta)} Z_t L$ . Our main results are robust to this modification although the dynamic analysis becomes much more complicated. Derivations are available upon request.

## 2.7 Decentralized equilibrium

The equilibrium is a time path of allocations  $\{A_t, C_t, Y_t, l_t(i), X_t(i), R_t(i)\}$  and prices  $\{r_t, w_t, p_t(i), V_t(i)\}$ . Also, at each instant of time, the following conditions hold:

- Households maximize utility taking  $\{r_t, w_t\}$  as given;
- Competitive final goods firms maximize profits taking  $\{p_t(i), w_t\}$  as given;
- Incumbents in the intermediate goods sector choose  $\{p_t(i), R_t(i)\}$  to maximize  $\{V_t(i)\}$  taking  $\{r_t\}$  as given;
- Entrants make entry decisions taking  $\{V_t(i)\}$  as given;
- The value of all existing monopolistic firms adds up to the value of household's assets such that  $A_t = N_t V_t$ ;
- The market-clearing condition of labor holds such that  $L = N_t l_t$ ;
- The market-clearing condition of final goods holds such that  $Y_t = C_t + N_t(X_t + \phi Z_t + R_t) + \dot{N}_t X_t F$ .

## 2.8 Dynamics

In this subsection, we analyze the dynamics of the model. In the Appendix, we show that the consumption-output ratio  $C_t/Y_t$  jumps to a unique and stable steady-state value. This equilibrium property simplifies the analysis of transition dynamics.

**Lemma 2** *The consumption-output ratio jumps to a unique and stable steady-state value:*

$$(C/Y)^* = (1 - \tau)(1 - \theta) + \frac{\rho\theta F}{\mu}. \quad (19)$$

**Proof.** See the Appendix. ■

Equations (18) and (19) imply that for any given  $\mu$  and  $\tau$ ,

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} = r_t - \rho, \quad (20)$$

where the last equality uses the Euler equation in (3). Combining (12), (16) and (20), we derive the equilibrium growth rate given by

$$g_t \equiv \frac{\dot{Z}_t}{Z_t} = \max \left\{ \frac{\alpha}{1-s} \left[ (\mu - 1) \left( \frac{\theta}{\mu} \right)^{1/(1-\theta)} \frac{L}{N_t} \right] - \rho, 0 \right\}, \quad (21)$$



which is increasing in firm size  $l_t$  determined by employment per firm  $L/N_t$ .<sup>9</sup> From (21), the growth rate  $g_t$  is strictly positive if and only if

$$N_t < \bar{N} \equiv \frac{\alpha(\mu - 1)(\theta/\mu)^{1/(1-\theta)} L}{(1-s)\rho}.$$

Intuitively, innovation requires each firm's market size to be large enough so that it is profitable for firms to do in-house R&D. A sufficient market size requires the number of firms to be below a critical level  $\bar{N}$ . If  $N_t > \bar{N}$ , then there are too many firms diluting the return to R&D. As a result, firms do not invest in R&D, and the growth rate of vertical innovation is zero. In the Appendix, we provide the derivations of the dynamics of  $N_t$ .

**Lemma 3** *The dynamics of  $N_t$  is determined by a one-dimensional differential equation.*<sup>10</sup>

$$\frac{\dot{N}_t}{N_t} = \begin{cases} \frac{\mu-1}{F} - \left[ \phi + (1-s) \frac{\dot{Z}_t}{Z_t} \right] \frac{N_t/L}{(\theta/\mu)^{1/(1-\theta)} F} - \rho & \text{if } N_t < \bar{N} \\ \frac{\mu-1}{F} - \phi \frac{N_t/L}{(\theta/\mu)^{1/(1-\theta)} F} - \rho & \text{if } N_t > \bar{N} \end{cases}. \quad (22)$$

**Proof.** See the Appendix. ■

The differential equation in (22) shows that given any initial value,  $N_t$  gradually converges to its steady-state value denoted as  $N^*$ .<sup>11</sup> On the transition path, the number of varieties  $N_t$  determines firm size  $L/N_t$  and the equilibrium growth rate  $g_t$  according to (21). When  $N_t$  evolves toward the steady state,  $g_t$  also gradually converges to its steady-state value  $g^*$ . The following proposition derives the steady-state values  $\{N^*, g^*\}$ .

**Proposition 1** *Under the parameter restrictions  $\rho < \min\{\phi/(1-s), (1-\alpha)(\mu-1)/F\}$ ,<sup>12</sup> the economy is stable and has a positive and unique steady-state value of  $N_t$ . The steady-state values  $\{N^*, g^*\}$  are given by*

$$N^*(\mu, s) = \left[ (1-\alpha) \frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\rho F}{\mu^{1/(1-\theta)}} \right] \frac{\theta^{1/(1-\theta)}}{\phi - (1-s)\rho} L > 0, \quad (23)$$

$$g^*(\mu, s) = \frac{\alpha(\mu-1)}{(1-\alpha)(\mu-1) - \rho F} \left( \frac{\phi}{1-s} - \rho \right) - \rho > 0. \quad (24)$$

**Proof.** See the Appendix. ■

<sup>9</sup>Considering data on employment, R&D personnel, and the number of establishments in the US for the period from 1964 to 2001, Laincz and Peretto (2006) provide empirical evidence that is consistent with the theoretical prediction from this class of models that economic growth is increasing in average firm size.

<sup>10</sup>It is useful to note that  $\dot{Z}_t/Z_t$  is a function of  $N_t$  given by (21).

<sup>11</sup>In this model, we have assumed zero population growth, so that  $N_t$  converges to a steady state. If we assume positive population growth, it would be the number of firms per capita that converges to a steady state instead, and our main results would be unchanged.

<sup>12</sup>These parameter restrictions would depend on a larger set of parameters if we parameterize R&D productivity in (7) and the productivity in producing intermediate goods from final goods. For simplicity, we have implicitly normalized these productivity parameters to unity.

### 3 Patent breadth versus R&D subsidies

In this section, we analyze the effects of patent breadth and R&D subsidies. In Section 3.1, we analyze the effects of patent breadth on the number of firms, firm size and economic growth. In Section 3.2, we analyze the effects of R&D subsidies. In Section 3.3, we introduce a subsidy to entry and analyze its effects on the number of firms, firm size and economic growth.

#### 3.1 Effects of patent breadth

In this subsection, we analyze the effects of patent breadth. Equation (21) shows that the initial impact of a larger patent breadth  $\mu$  on the equilibrium growth rate  $g_t$  is positive because  $N_t$  is fixed in the short run. This result captures the standard positive effects of patent breadth on monopolistic profits and innovation as in previous studies, such as Li (2001), Chu (2011), Chu and Furukawa (2011) and Iwaisako and Futagami (2013). However, in the long run, the market structure is endogenous and the number of firms adjusts. In particular, the higher profit margin attracts the entry of firms, which in turn reduces firm size  $L/N_t$  and decreases the incentives for innovation. This negative entry effect dominates the positive profit-margin effect such that the steady-state equilibrium growth rate  $g^*$  becomes lower than the original steady-state level. Therefore, allowing for the endogeneity of market structure, the present study extends previous studies in the literature by demonstrating the contrasting short-run and long-run effects of patent breadth on economic growth. Proposition 2 summarizes the results. Figures 1 and 2 plot the transition paths of  $\{g_t, N_t\}$  when  $\mu$  increases at time  $t$ .

**Proposition 2** *The initial effect of a larger patent breadth on economic growth is positive as a result of increased monopolistic profits. In the long run, higher profit margin attracts the entry of firms and reduces firm size. The smaller firm size decreases incentives for innovation and the steady-state growth rate.*

**Proof.** Equation (21) shows that for a given  $N_t$ ,  $\partial g_t / \partial \mu > 0$ . Equations (23) and (24) show that  $\partial N^* / \partial \mu > 0$  and  $\partial g^* / \partial \mu < 0$ . ■

[Insert Figures 1 and 2 here]

#### 3.2 Effects of R&D subsidies

In this subsection, we analyze the effects of R&D subsidies. Equation (21) shows that the initial impact of a higher rate of R&D subsidies  $s$  on the equilibrium growth rate  $g_t$  is positive given  $N_t$ . On the transition path, the higher rate of R&D subsidies makes in-house

R&D more attractive relative to entry. As a result, resources reallocate from entry to in-house R&D, and the number of firms decreases. The smaller number of firms increases firm size, which further improves the incentives for in-house R&D. This positive firm size effect strengthens the initial positive effect of R&D subsidies such that the steady-state equilibrium growth rate  $g^*$  increases further above the initial level. Therefore, the endogeneity of market structure amplifies the positive effects of R&D subsidies on economic growth. Peretto (1998) and Segerstrom (2000) also analyze the effects of R&D subsidies in a scale-invariant Schumpeterian growth model. Segerstrom (2000) finds that R&D subsidies can have either positive or negative effects on economic growth, and this interesting result is driven by the tradeoff between quality improvement and variety expansion on economic growth. In contrast, economic growth is solely based on quality improvement in the present study and in Peretto (1998), who also finds a positive effect of R&D subsidies on economic growth; see Peretto and Connolly (2007) who show that quality improvement is the only plausible engine of economic growth in the long run. Proposition 3 summarizes the results. Figures 3 and 4 plot the transition paths of  $\{g_t, N_t\}$  when  $s$  increases at time  $t$ .

**Proposition 3** *The initial effect of a higher rate of R&D subsidies on economic growth is positive. In the long run, firms exit the market, and firm size increases. The larger firm size further strengthens the incentives for innovation and increases the steady-state growth rate.*

**Proof.** Equation (21) shows that for a given  $N_t$ ,  $\partial g_t / \partial s > 0$ . Equations (23) and (24) show that  $\partial N^* / \partial s < 0$  and  $\partial g^* / \partial s > 0$ . ■

[Insert Figures 3 and 4 here]

### 3.3 Extension: Effects of entry subsidies

In this subsection, we extend the baseline model by allowing for a subsidy to entry denoted by  $e \in (0, 1)$ . In this case, the entry condition in (13) becomes

$$V_t(i) = (1 - e)X_t(i)F. \quad (25)$$

Furthermore, the government's balanced-budget condition is modified to

$$T_t = G_t + s \int_0^{N_t} R_t(i)di + e\dot{N}_t X_t F. \quad (26)$$

The rest of the model is the same as before. Following the same procedures as before,<sup>13</sup> we derive the same equilibrium growth rate in (21) and the steady-state equilibrium number of varieties given by

$$N^*_+(e) = \left[ (1 - \alpha) \frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{(1 - e)\rho F}{\mu^{1/(1-\theta)}} \right] \frac{\theta^{1/(1-\theta)}}{\phi - (1 - s)\rho} L > 0, \quad (27)$$

<sup>13</sup>The derivations are available upon request.

which is naturally increasing in the entry subsidy rate  $e$ . Given that the equilibrium growth rate is given by (21) as before and does not directly depend on  $e$ , an increase in entry subsidies does not affect economic growth initially. However, given that entry subsidies attract entry and reduce firm size, the equilibrium growth rate gradually decreases during the transition path and converges to a lower steady-state value. If we think of entry as horizontal R&D, then this analysis implies that horizontal R&D subsidies can be harmful to economic growth, and this finding is consistent with Peretto (2007). In other words, in order for R&D subsidies to have a positive effect on economic growth, policymakers need to design a subsidy (or tax-deduction) system that distinguishes between different types of R&D activities.

For the rest of this subsection, we consider symmetric R&D and entry subsidies by setting  $e = s = \bar{s}$ . Given that entry subsidies  $e$  have no effect on the initial growth rate, an increase in  $\bar{s}$  must have the same initial positive effect on the growth rate  $g_t$  as R&D subsidies. As for the long-run effect on the number of firms, (27) becomes

$$N^*(\bar{s}) = \left[ \frac{(1 - \alpha)(\mu - 1) - (1 - \bar{s})\rho F}{\phi - (1 - \bar{s})\rho} \right] \left( \frac{\theta}{\mu} \right)^{1/(1-\theta)} \quad L > 0, \quad (28)$$

which is increasing (decreasing) in  $\bar{s}$  if the following inequality holds:

$$(1 - \alpha)(\mu - 1) < (>) \phi F.$$

If  $N^*$  is decreasing in  $\bar{s}$ , then the long-run effect of  $\bar{s}$  on  $g^*$  must be positive. If  $N^*$  is increasing in  $\bar{s}$ , then a higher rate of subsidies  $\bar{s}$  would have a negative indirect effect on long-run growth through entry partly offsetting the direct positive effect of  $\bar{s}$  on growth. Substituting (28) into (21) yields

$$g = \underbrace{\frac{\alpha(\mu - 1)}{1 - \bar{s}}}_{\text{direct effect of } \bar{s}} \underbrace{\frac{\phi - (1 - \bar{s})\rho}{(1 - \alpha)(\mu - 1) - (1 - \bar{s})\rho F}}_{\text{indirect effect of } \bar{s}} - \rho, \quad (29)$$

which is increasing in  $\bar{s}$  if and only if the following inequality holds:<sup>14</sup>

$$\phi(1 - \alpha)(\mu - 1) - (1 - \bar{s})[2\phi - (1 - \bar{s})\rho]\rho F > 0.$$

Given the parameter restriction  $\phi/(1 - \bar{s}) > \rho$  in Proposition 1, this inequality holds if  $\rho$  is sufficiently small. In other words, the overall long-run growth effect of symmetric R&D and entry subsidies  $\bar{s}$  is generally ambiguous. If the discount rate  $\rho$  is sufficiently small, then an increase in  $\bar{s}$  would have a positive effect on long-run growth.

## 4 Conclusion

In this study, we have analyzed the different implications of two important policy instruments, patent breadth and R&D subsidies, on economic growth and market structure in a

<sup>14</sup>It can be shown that  $(1 - \alpha)(\mu - 1) > \phi F$  is sufficient (but not necessary) for this inequality to hold.

scale-invariant Schumpeterian growth model with EMS. We find that when the number of firms is fixed in the short run, patent breadth and R&D subsidies serve to increase economic growth as in previous studies. However, when the number of firms adjusts endogenously in the long run, these two commonly discussed policy instruments have surprisingly opposing effects on economic growth and market structure. Specifically, patent breadth decreases economic growth but expands the number of firms, whereas R&D subsidies reduce the number of firms but increase economic growth. These contrasting effects of patent breadth and R&D subsidies suggest that R&D subsidy is perhaps a more suitable policy instrument than patent breadth for the purpose of stimulating economic growth. This finding is consistent with evidence from empirical studies and case studies discussed in the introduction. Given our result that the endogeneity of market structure leads to different short-run and long-run effects of patent breadth, it is important for policymakers to take into consideration the different implications of patent policy reform in the short run and in the long run.

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## Appendix A

**Proof of Lemma 1.** Substituting (6), (9) and the constraint  $p_t(i) \leq \mu$  into (10) yields

$$H_t(i) = [p_t(i) - 1] \left( \frac{\theta}{p_t(i)} \right)^{1/(1-\theta)} Z_t^\alpha(i) Z_t^{1-\alpha} l_t(i) - \phi Z_t - (1-s)R_t(i) + \lambda_t(j)R_t(i) + \eta_t(j)[p_t(i) - \mu], \quad (\text{A1})$$

where  $\eta_t(j)$  is the multiplier on  $p_t(i) \leq \mu$  and  $\eta_t(j) = 0$  if  $p_t(i) < \mu$ . The first-order conditions include

$$\frac{\partial H_t(i)}{\partial p_t(i)} = 0 \Leftrightarrow p_t(i) = \min \{ \mu, 1/\theta \}, \quad (\text{A2})$$

$$\frac{\partial H_t(i)}{\partial R_t(i)} = 0 \Leftrightarrow \lambda_t(i) = 1 - s, \quad (\text{A3})$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha [p_t(i) - 1] \left( \frac{\theta}{p_t(i)} \right)^{1/(1-\theta)} Z_t^{\alpha-1}(i) Z_t^{1-\alpha} l_t(i) = r_t \lambda_t(i) - \dot{\lambda}_t(i). \quad (\text{A4})$$

Substituting (A3) and the constrained monopolistic price  $p_t(i) = \mu < 1/\theta$  from (A2) into (A4) yields

$$r_t = \frac{\alpha}{1-s} \left[ (\mu - 1) \left( \frac{\theta}{\mu} \right)^{1/(1-\theta)} l_t \right], \quad (\text{A5})$$

where we have also applied the symmetry condition  $Z_t(j) = Z_t$ . ■

**Proof of Lemma 2.** Substituting  $V_t = X_t F$  from (13) into  $A_t = N_t V_t$  yields

$$A_t = N_t X_t F = \frac{p_t N_t X_t}{p_t} F = \frac{\theta Y_t}{\mu} F, \quad (\text{A6})$$

where the last equality uses  $p_t = \mu$  and  $p_t X_t N_t = \theta Y_t$ . Using (A6) and (2), we obtain

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} = r_t + \mu \frac{(1-\tau)w_t L - C_t}{\theta Y_t F}. \quad (\text{A7})$$

Substituting the Euler equation and  $w_t L = (1-\theta)Y_t$  into (A7) yields

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} = \mu \frac{C_t/Y_t}{\theta F} - \left[ \mu \frac{(1-\tau)(1-\theta)}{\theta F} + \rho \right]. \quad (\text{A8})$$

Therefore, the dynamics of  $C_t/Y_t$  is characterized by saddle-point stability such that  $C_t/Y_t$  must jump to its steady-state value in (19). ■

**Proof of Lemma 3.** Substituting (9), (13) and  $p_t(i) = \mu$  into (14) yields

$$r_t = \frac{\mu - 1}{F} - \frac{\phi Z_t + (1-s)R_t}{X_t F} + \frac{\dot{X}_t}{X_t}, \quad (\text{A9})$$



where we have applied  $\dot{V}_t/V_t = \dot{X}_t/X_t$ . Substituting (16) and  $p_t(i) = \mu$  into (6) yields

$$X_t = \frac{Z_t}{N_t} \left( \frac{\theta}{\mu} \right)^{1/(1-\theta)} L, \quad (\text{A10})$$

where we have applied  $Z_t(i) = Z_t$ . Substituting (7) and (A10) into (A9) yields

$$r_t = \frac{\mu - 1}{F} - \left[ \phi + (1 - s) \frac{\dot{Z}_t}{Z_t} \right] \frac{N_t/L}{(\theta/\mu)^{1/(1-\theta)} F} + \frac{\dot{Z}_t}{Z_t} - \frac{\dot{N}_t}{N_t}, \quad (\text{A11})$$

where we have used  $\dot{X}_t/X_t = \dot{Z}_t/Z_t - \dot{N}_t/N_t$ . Substituting (20) into (A11) yields the dynamics of  $N_t$  given by

$$\frac{\dot{N}_t}{N_t} = \frac{\mu - 1}{F} - \left[ \phi + (1 - s) \frac{\dot{Z}_t}{Z_t} \right] \frac{N_t/L}{(\theta/\mu)^{1/(1-\theta)} F} - \rho. \quad (\text{A12})$$

Equation (A12) describes the dynamics of  $N_t$  when  $N_t < \bar{N}$ . When  $N_t > \bar{N}$ ,  $\dot{Z}_t/Z_t = 0$  as shown in (21). ■

**Proof of Proposition 1.** This proof proceeds as follows. First, we prove that under  $\rho < \min \{ \phi/(1-s), (1-\alpha)(\mu-1)/F \}$ , there exists a stable, unique and positive steady-state value of  $N_t$ . Substituting (21) into the first equation of (22) yields

$$\frac{\dot{N}_t}{N_t} = \frac{(1-s)\rho - \phi}{(\theta/\mu)^{1/(1-\theta)} F} \frac{N_t}{L} + \frac{(1-\alpha)(\mu-1)}{F} - \rho. \quad (\text{A13})$$

Because  $N_t$  is a state variable, the dynamics of  $N_t$  is stable if and only if  $(1-s)\rho < \phi$ . Solving  $\dot{N}_t = 0$ , we obtain the steady-state value of  $N_t$  in an economy with positive in-house R&D given by

$$N^* = \left[ \frac{(1-\alpha)(\mu-1)}{F} - \rho \right] \frac{(\theta/\mu)^{1/(1-\theta)} F}{\phi - (1-s)\rho} L. \quad (\text{A14})$$

Given  $(1-s)\rho < \phi$ , (A14) shows that  $N^* > 0$  if and only if  $\rho < (1-\alpha)(\mu-1)/F$ . Combining this inequality with  $(1-s)\rho < \phi$ , we have

$$\rho < \min \left\{ \frac{\phi}{1-s}, \frac{(1-\alpha)(\mu-1)}{F} \right\}.$$

Finally, substituting (A14) into (21) yields

$$g_t = \frac{\alpha(\mu-1)}{(1-\alpha)(\mu-1) - \rho F} \left( \frac{\phi}{1-s} - \rho \right) - \rho, \quad (\text{A15})$$

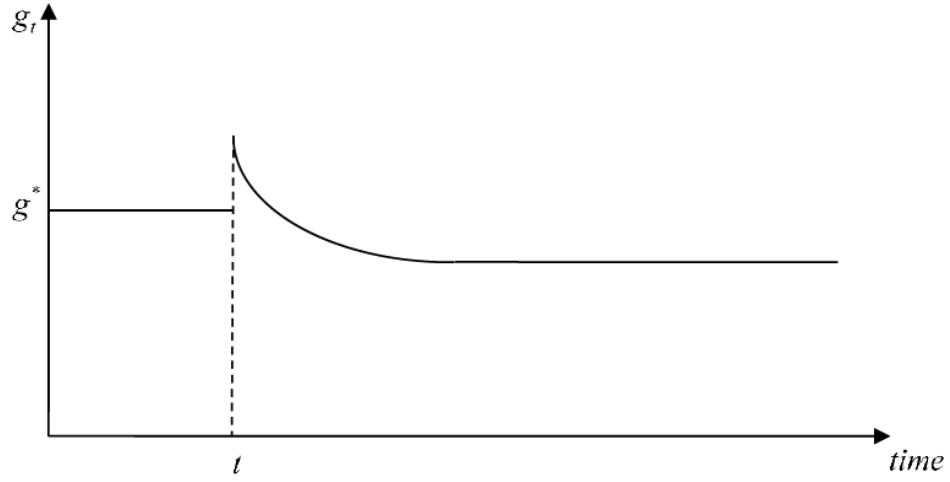
which is positive if and only if the following inequality holds:

$$(1-s)F\rho^2 - (1-s)(\mu-1)\rho + \phi\alpha(\mu-1) > 0,$$

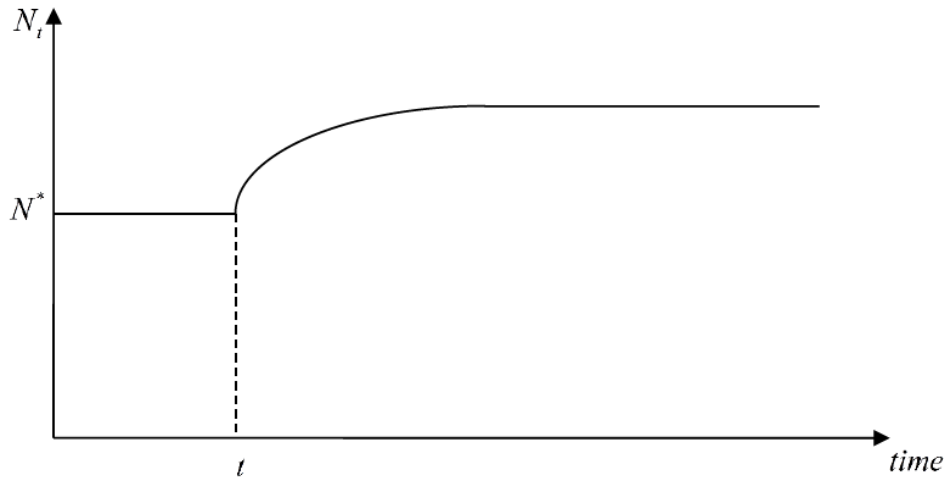
and this inequality holds if  $\rho$  is sufficiently small (or sufficiently large). ■

# Figures

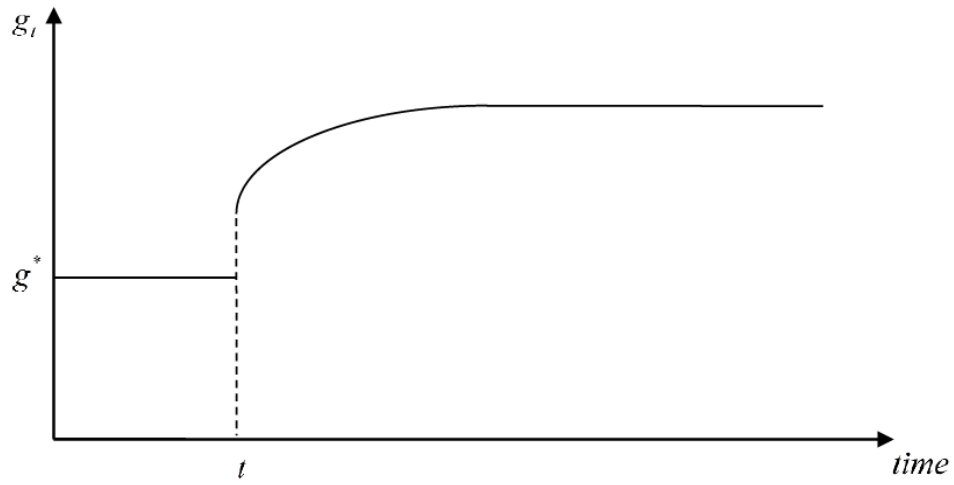
**Figure 1: Transitional effects of patent breadth on economic growth**



**Figure 2: Transitional effects of patent breadth on the number of firms**



**Figure 3: Transitional effects of R&D subsidies on economic growth**



**Figure 4: Transitional effects of R&D subsidies on the number of firms**

