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Abstract

In this paper, we studied the spillover effect among four financial markets from MENA area during a period that was characterized by political instability. The countries chosen are also signatories of an agreement of free trade in order to liberalize the movement flowing of their capitals. As the linear correlation is unable to capture nonlinear relation between variables, it also suffers from many shortcomings. Reason why, we used copula functions to understand better the dependence structure between markets and to be able to detect spillover effect in that period. The results show that Egyptian Exchange and Casablanca Stock Exchange are highly correlated. We observed the same thing between Amman Stock Exchange and Egyptian Exchange. It seems that Egyptian market transmitted its volatility to the Moroccan and Jordanian markets.

Keywords: Spillover effect, Copulas, Contagion, Interdependence
JEL Classification Codes: F3, D53, G15, N25, N27

1. Introduction

Due to the development of financial globalization, financial markets are becoming more and more connected and interdependent. This may explain the dynamic of these markets and the turbulence of its fluctuations. Moreover, when the number of actors is very important in markets it become very hard to understand the complexity of fluctuations since actors behave heterogeneously and nonlinearly. In addition, markets seem to be unpredictable in the way that any crisis can cause a big collapse in the financial system. For this reason, many authors have been interested to study interdependence and contagion among markets especially in time of crisis.

King & Wadhwani (King & Wadhwani, 1990) note that all stock markets fell together despite widely differing economic circumstances. They consider that any mistake in one market may be transmitted to others. The empirical evidence suggests that an increase in volatility leads in turn to an increase in the size of the contagion effects. The rise in the correlation between markets just after the crash is evidence of this (King & Wadhwani, 1990). (Pindyck & Rotemberg, 1993) test whether comovements of individual stock prices can be justified by economic fundamentals. The test of the present value model of security valuation with the constraint that changes in discount rates depend only on changes in macroeconomic variables. They rejected the hypothesis that the stocks of companies whose earnings are uncorrelated should move together only in response to changes in current or expected future macroeconomic conditions (Pindyck & Rotemberg, 1993). (Karolyi & Stulz, 1996) investigate daily return comovements between Japanese and U.S. stocks. They demonstrate that there is a nonlinear relation between covariances and large shocks. The joint dynamics of US and Japanese stock returns are also affected in that large shocks have spillover effects on covariances. (Karolyi & Stulz, 1996) show that neither macroeconomic announcements nor interest rate shocks significantly affect comovements between US and Japan share returns. Controlling for industry effects also has little or no impact on the magnitude of stock return comovements. In contrast, stock return comovements exhibit day-of-the-week effects, and more importantly, using a variety of methods, they show that comovements are high when contemporaneous absolute returns of national market indices are high. The evidence of authors shows that correlations and covariances are high when markets move a lot. The analysis of authors also suggests that covariances change over time and can be forecasted various instrumental variables (Karolyi & Stulz, 1996). Forbes & Rigobon (2002) focused on the study of interdependence and contagion. They define this later as a significant increase in cross-market linkages after a shock to one country (or group of countries). If the comovement does not increase significantly, then any continued high level of market correlation suggests strong linkages between the two economies that exist in all states of the world. (Forbes & Rigobon, 2002) use the term interdependence to refer to this situation.

After adjusting for hesteroskedasticity and according to the contagion test as computed by (Forbes & Rigobon, 2002), there is only evidence of contagion from the Hong Kong crash to one other country in the sample (versus 15 cases of contagion when tests are based on the conditional correlations). The authors also analyze the contagion during the 1994 Mexican Peso Crisis. They found that conditional correlation coefficients show many patterns similar to the East Asian case. Once again, the adjustment of correlation coefficients has a significant impact on estimated correlations and the resulting tests for contagion. When tests for contagion are performed on these unconditional correlations, there is not one case which the correlation coefficient increases significantly during the turmoil period. In other words, according to this testing methodology, there is no longer evidence of a significant change in the magnitude of the propagation mechanism from Mexico to any other country in the sample. When analyzing contagion during the 1987 US Stock Market Crash, authors show that most patterns are similar to those found after the 1997 East Asian crisis and the 1994 Mexican devaluation (Forbes & Rigobon, 2002). (Bradley & Taqqu, 2004) present an alternative definition of contagion between financial markets, which is based on a measure of local correlation. (Bradley & Taqqu, 2004) found evidence of contagion from the US equity markets to equity markets of several developed countries. they also find evidence of flight to quality from the US equity market to the US government bond market (Bradley & Taqqu, 2004). Another study determined whether China's financial market becomes more related with other world markets using time-varying copula models (Wang, Chen, & Huang, 2011). They assume that this high dependence can be attributed to the close geographical proximities and trading relationships between China and the Pacific nations. Furthermore, they advance that the greater dynamic dependences during bear markets imply that opportunities for portfolio diversification are reduced at such times (Wang, Chen, & Huang, 2011). By exploring studies in this area, we note that authors used many methods to measure links among markets as cross-market correlation coefficient, ARCH and GARCH models, cointegration techniques, switching regimes, or copula functions.

In this work, we use copula functions to measure the links among four Arabs markets during the Arab spring. The countries chosen are signatory of the Agadir agreement concerning the establishment of a free trade area comprising Arab Mediterranean countries. We chose 200 daily logarithmic returns before December, 31st, 2012. That will give us an idea about the interdependence of these markets and the importance of contagion effect among them in context of political instability. Thus, the paper is organized as follows, in section 2 we present some dependence measures and copula functions, in section 3, we expose the main families of copulas. Then, we present the results about volatility spillover in MENA area in section 4 and finally we conclude.

2. Copulas and Dependence Measures

There are many measures to use for computing association between variables, the famous one is the linear correlation coefficient ρ it capture only linear dependence. So, we don't have an idea about the dependence structure. The use of copulas allows to have this information and to explain better correlations based on functions rather than a simple coefficient.

In finance, Pearson correlation is very used, it is defined as,

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var\left(X\right).Var(Y)}} \tag{1}$$

This measure is characterized by its tractability. However, it had many of theoretical shortcomings especially in finance settings (Chollete, de la Peña, & Lu, 2011).

As (Embrechts, McNeil, & Straumann, Correlation And Dependence In Risk Management: Properties And Pitfalls, 2002) point out, there are some shortcomings of linear correlation:

- The variances of X nd Y ust be finite or the linear correlation is not defined. This is not ideal for a dependence measure and causes problems when we work with heavy-tailed distributions.
- Independence of two random variables implies they are uncorrelated (linear correlation equal to zero) but zero correlation does not in general imply independence.
- Linear correlation has the serious deficiency that it is not invariant under non-linear strictly increasing transformations $T : R \rightarrow R$ For two real-valued random variables we have in general

$$\rho\left(T\left(X\right),T\left(Y\right)\right) \neq \rho\left(X,Y\right)..$$

rst, a major shortcoming is that correlation is not invariant to monotonic transformations. Thus, the correlation of two return series may differ from the correlation of the squared returns or log returns. Second, there is mixed evidence of infinite variance in financial data. If either X r Y as infinite variance, the estimated correlation may give little information on dependence, since it will be undefined or close to zero. A third drawback concerns estimation bias: by definition the conditional correlation is biased and spuriously increases during volatile periods. Fourth, correlation is a symmetric measure and therefore may overlook important asymmetric dependence. It does not distinguish, for example, between dependence during up and down markets. Finally, correlation is a linear measure of dependence, and may not capture important nonlinearities (Chollete, de la Peña, & Lu, 2011).

Spearman's ρ_s s the rank correlation, in the sense of correlation of the integral transforms, of X nd Y Cherubini, Luciano, & Vecchiato, 2004).

Since $\frac{1}{2}$ nd $\frac{1}{12}$ re the mean and variance of standard uniforms, it follows that

$$\rho_s = \frac{Cov\left(F_1\left(X\right),F_2\left(Y\right)\right)}{\sqrt{Var\left(F_1\left(X\right)\right).Var(F_2\left(Y\right))}} \tag{2}$$

This is more robust than the traditional correlation. The rank correlation is especially useful when analyzing data with a number of extreme observations, since it is independent of the levels of the variables, and therefore robust to outliers. (Chollete, de la Peña, & Lu, 2011)

Another nonlinear correlation measure is one termed downside risk, $d(u)$ This function measures the conditional probability of an extreme event beyond some threshold u For simplicity, normalize variables to the unit interval $[0,1]$ Hence

$$d\left(u\right) \equiv \Pr(F_X\left(X\right) \leq u \mid F_Y\left(Y\right) \leq u) \tag{3}$$

A final nonlinear correlation measure is left tail dependence, $\lambda(u)$ which is the limit of downside risk as losses become extreme,

$$\lambda\left(u\right) = \lim_{u \downarrow 0} \Pr(F_X\left(X\right) \leq u \mid F_Y\left(Y\right) \leq u) \tag{4}$$

Tail dependence is important because it measures the asymptotic likelihood that two variables go down or up at the same time (Chollete, de la Peña, & Lu, 2011).

To overcome the shortcomings of traditional measures of dependence, one should look for measures that could capture nonlinearity in the series and that assess risk better. For this reason, we chose functions of copula. These latter have many good properties that are adapted to the dynamic of markets. The essence of the copula approach is that a joint distribution of random variables can be expressed as a function of the marginal distributions (Clemen & Reilly, 1999).

Copulas has been used in many fields like actuarial science, finance, biomedical studies to model correlated event times and competing risks and engineering (multivariate process and hydrological modeling).

Next, we present the main characteristics of copulas. We start with the mathematical definition of a copula.

Definition: For every $d \geq 2$ a d-dimensional copula (shortly, d-copula) is a d-variate d.f. on I^d hose univariate marginals are uniformly distributed on $\mathbb{I}$ Thus, each d-copula may be associated with a r.v.

Sklar's Theorem (Sklar A. , 1959) (Jaworski, Duante, Härdle, & Rychlik, 2010)

Sklar's theorem is the building block of the theory of copulas; without it, the concept of copula would be one in a rich set of joint distribution functions.

Theorem 1. Let F be a d-dimensional d.f. with univariate margins $F_1, F_2, \ldots, F_d$ Let A_j enote the range of F_j $A_j := F_j\left(\overline{\mathbb{R}}\right)\left(j = 1, 2, \ldots, d\right)$. hen there exists a copula C uch that for all

$$\left(x_1, x_2, \ldots, x_d\right) \in \overline{\mathbb{R}}^d, \tag{5}$$

Such a C s uniquely determined on $A_1 \times A_2 \times \ldots \times A_d$ nd, hence, it is unique when $F_1, F_2, \ldots, F_d$ re all continuous.

Sklar's theorem has been announced in (Sklar, 1959), however its first proof for the bivariate case appeared in (Schweizer & Sklar, 1974).

Theorem 1 also admits the following converse implication, usually very important when one wants to construct statistical models by considering, separately, the univariate behaviour of the components of a random vector and their dependence properties as captured by some copula.

Theorem 2. If $F_1, F_2, \ldots, F_d$ re univariate d.f.'s, and if C s any d-copula, then the function $F : \overline{\mathbb{R}}^d \to \mathbb{I}$ defined by (1) is a d-dimensional distribution function with margins $F_1, F_2, \ldots, F_d$ By summarizing, from any d-variate d.f. F ne can derive a copula C ia (5). Specifically, when F_i s continuous for every $i \in \left\{1, 2, \ldots, d\right\}$ C an be obtained by means of the formula

$$C\left(u_1, u_2, \ldots, u_d\right) = F\left(F_1^{-1}\left(u_1\right), F_2^{-1}\left(u_2\right), \ldots, F_d^{-1}\left(u_d\right)\right), \ \forall\left(u_1, u_2, \ldots, u_d\right) \in (0,1)^d \tag{6}$$

Where F_i^{-1} enoted the pseudo-inverse of F_i iven by $F_i^{-1}(s) = \inf\{t|F_i(t) \geq s\}$ Thus, copulas are essentially a way for transforming the r.v. $(X_1, X_2, \ldots, X_d)$ nto another r.v. $(U_1, U_2, \ldots, U_d) = (F_1(X_1), F_2(X_2), \ldots, F_d(X_d))$ aving the margins uniform on $\mathbb{I}$ nd preserving the dependence among the components. (Jaworski, Duante, Härdle, & Rychlik, 2010)

The copula C ompletely specifies the distribution F Sklar A. , 1996) in as much as

$$\forall (x_1, x_2, \ldots, x_d) \in \overline{\mathbb{R}}^d \; F(x_1, x_2, \ldots, x_d) = C(F_1(X_1), F_2(X_2), \ldots, F_d(X_d)). \quad (7)$$

From Eqs. (6) and (7), we observe that the copula represents the multivariate dependence structure which links univariate uniform marginals. When F s not continuous, there still exists the copula representation of F but it is not unique anymore (Schweizer & Sklar, Probabilistic metric spaces, 1983). The copula of any F aptures and summarizes different types of dependence between variables even when they had been rescaled by strictly monotone transformations. This invariance property guarantees, for example, that financial returns and their logarithms have the same copula (de Melo Mendes & de Souza, 2004).

The function C s called a *copula*. Sklar's Theorem is completely general: Any joint distribution can be written in copula form.

Given that F_i nd C re differentiable, the joint density $f(x_1, x_2, \ldots, x_d)$ an be written as

$$f(x_1, x_2, \ldots, x_d) = f_1(x_1) \times f_2(x_2) \times \ldots \times f_d(x_d)\, c(F_1(x_1), F_2(x_2), \ldots, F_d(x_d)) \quad (8)$$

Where $f_i(x_i)$ s the density corresponding to $F_i(x_i)$ and $c = \partial^n C / (\partial F_1, \ldots, \partial F_n)$ s called the copula density. This is another essential result, which states that, under appropriate conditions, the joint density can be written as a product of the marginal densities and the copula density. For example, if the X_i are independent, then $c = 1$ nd $f(x_1, x_2, \ldots, x_d) = f_1(x_1) \times f_2(x_2) \times \ldots \times f_d(x_d)$ the familiar formula for n ndependent random variables. From the representation in (4) it is clear that the copula density c ncodes information about the dependence among the X_i. For this reason c is sometimes called a dependence function (Clemen & Reilly, 1999).

On the other hand, any copula can be combined with different univariate d.f.'s in order to obtain a d ariate d.f. by using (1). In particular, copulas can serve for modelling situations where a different distribution is needed for each marginal, providing a valid alternative to several classical multivariate d.f.'s such Gaussian, Pareto, Gamma, etc.

This fact represents one of the main advantages of the copula's idea, as underlined by Mikosch:

There is no simple alternative to the Gaussian distribution in the non-Gaussian world. In particular, one needs multivariate models for portfolios with different marginal distributions (including different tail behavior) and a dependence structure which is determined not only by covariances. Many of the well known multivariate distributions are not flexible enough to allow for different tail behavior in different components. Therefore copulas seem to be the right tools in order to overcome the mentioned difficulties: they generate all multivariate distributions with flexible marginals.

3. Families of Copulas

As underlined, copulas play an important role in the construction of multivariate d.f.'s and, as a consequence, having at one's disposal a variety of copulas can be very useful for building stochastic models having different properties that are sometimes indispensable in practice (e.g., heavy tails, asymmetries, etc.). Therefore, several investigations have been carried out concerning the construction of different families of copulas and their properties. Here, we focus on the families that seem to be more popular in the literature (Jaworski, Duante, Härdle, & Rychlik, 2010).

Before presenting these examples, we would like briefly to discuss the use of these families for copula-fitting procedures. For several researchers and practitioners it seems enough to try to fit any stochastic model with the most convenient family and check that the fitting procedure is not so "bad". However, it should be stressed that any fitting procedure may be misleading if one were to describe any situation with families of copulas satisfying some unnecessary assumptions (e.g., exchangeability, light tails, etc.). According to (Jaworski, Duante, Härdle, & Rychlik, 2010), before applying any statistical tool, one should not forget to analyse the main characteristics of the model under consideration. At the same time, one should have clearly in mind the final output of the investigation. Usually, in fact, fitting a copula (or a joint d.f.) to some data is just a tool for deriving some quantities of interest for the problem at hand (e.g., VaR of a portfolio, return period of an extreme event, etc.). For such problems, a dramatic underestimation of the risk can be obtained when one tries to fit with copulas that do not exhibit any peculiar behavior in the tails. This was exactly one of the main criticisms to the use of Li's model for credit risk using Gaussian copulas (Jaworski, Duante, Härdle, & Rychlik, 2010).

The general properties that a "good" family of multivariate copulas $\{C_\theta\}$ where θs a parameter belonging to a (usually, compact) subset $\Theta \subseteq \mathbb{R}^p$ $\left(p \geq 1\right)$should have for being considered "interesting" in statistical applications (Jaworski, Duante, Härdle, & Rychlik, 2010).

3.1. Elliptical Copulas

A random vector $\boldsymbol{X} = \left(X_1, X_2, \ldots, X_d\right)$s said to have an elliptical distribution with mean vector $\mu \in \mathbb{R}^d$ covariance matrix $\Sigma = (\sigma_{ij})$nd generator $g:[0,+\infty[\rightarrow[0,+\infty[$ and one writes $\boldsymbol{X} \sim \mathcal{E}(\mu, \Sigma, g)$ if it can be expressed in the form

$$\boldsymbol{X} = \mu + R\boldsymbol{A}\boldsymbol{U} \tag{9}$$

Where $AA^T = \Sigma$s the Cholesky decomposition of Σ, Us a d-dimensional random vector uniformly distributed on the sphere $\mathbb{S}^{d-1} = \{u \in \mathbb{R}^d : u_1^2 + \ldots + u_d^2 = 1\}$ and Rs a positive random variable independent of U with density given, for every $r > 0$ by

$$f_g\left(r\right) = \frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{\mathrm{d}}{2}\right)} r^{d-1} g\left(r^2\right). \tag{10}$$

The density function (if it exists) of an elliptical distribution is given, for every $x \in \mathbb{R}^d$ y

$$h_g\left(x\right) = \left|\Sigma\right|^{-\frac{1}{2}} g\left(\left(x-\mu\right)^T \Sigma^{-1}\left(x-\mu\right)\right). \tag{11}$$

For instance, when $g\left(t\right) = 2\pi^{-\frac{d}{2}} \exp(-\frac{t}{2})$ then X as a multivariate Gaussian distribution. Similarly, $g\left(t\right) = c(1+\frac{t}{\nu})^{-(d+\nu)/2}$ for a suitable constant c generates the multivariate t-Student distribution with ν egrees of freedom.

One of the characteristics of an elliptical distribution is that the scaled components $\frac{X_1}{\sqrt{\sigma_{11}}}, \ldots, \frac{X_d}{\sqrt{\sigma_{dd}}}$ re identically distributed according to a d.f. F_g This fact represents a limitation to the use of such distributions for modeling stochastic systems when the components are not similar. In order to avoid this, it is useful to calculate the copula of a multivariate elliptical distributions and use it, together with some univariate marginal d.f.'s, for obtaining more flexible models. These distributions, constructed by means of Theorem 2, are usually called meta-elliptical distribution. For these reasons, we give the following

Definition: Let X be an elliptical random vector, $X \sim \mathcal{E}_d(\mu, \Sigma, g)$ Suppose that, for every $i \in \{1,2,\ldots,d\}, \left(\frac{X_i}{\sqrt{\sigma_{ii}}}\right) \sim F_g$ We call elliptical copula the distribution function of the random vector

$$\left(F_g\left(\frac{X_1}{\sqrt{\sigma_{11}}}\right), F_g\left(\frac{X_2}{\sqrt{\sigma_{22}}}\right), \ldots, F_g\left(\frac{X_d}{\sqrt{\sigma_{dd}}}\right)\right). \quad (12)$$

An **elliptical copula** is typically not available in closed form.

3.1.1. Bivariate Gaussian Copula

The bivariate Gaussian copula is given by

$$C_{Ga}^{\rho}(u_1,u_2) = \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{\left(\frac{s^2-2\rho st+s^2}{2(1-\rho^2)}\right)} dsdt, \quad (13)$$

Where $\rho \in [-1,1]$ and Φ^{-1} enotes the inverse of the univariate Gaussian distribution.

This is the copula pertaining to a bivariate normal distribution with standard normal margins and Pearson correlation coefficient $\rho(\Phi_2^{\rho})$ From Eq. $F(x_1,x_2) = \Phi_2^{\rho}(x_1,x_2) = C_{Ga}^{\rho}(\Phi(u_1),\Phi(u_2))$ The Pearson correlation coefficient $\widetilde{\rho}$ f the copula C_{Ga}^{ρ} s slightly smaller than the ρ f the normal bivariate distribution :

$$\rho(\Phi(X_1),\Phi(X_2)) = \frac{6}{\pi}\arcsin\left(\frac{(\rho(X_1,X_2))}{2}\right)$$ (Joag-dev, Perlman, & Pitt, 1983)

Bivariate t-Student Copula

$$C_t^{\rho,\nu}(u_1,u_2) = \int_{-\infty}^{t_\nu^{-1}(u_1)} \int_{-\infty}^{t_\nu^{-1}(u_2)} \frac{(2\pi)^{-1}}{\sqrt{1-\rho^2}} e^{\left(1-\frac{s^2-2\rho st+t^2}{\nu(1-\rho^2)}\right)^{\frac{-(\nu+2)}{2}}} dsdt, \quad (14)$$

Where $t_\nu^{-1}(.)$ epresents the quantile function of a (univariate) standard t-student distribution with ν egrees of freedom (t_ν) This copula depends only on θ nd ν and the value for ν s common for marginal and joint distributions. This is the copula pertaining to a bivariate t-student distribution $(t_2^{\rho,\nu})$ ith ν egrees of freedom and correlation coefficient ρ Like the normal case, the Pearson correlation coefficient $\widetilde{\rho}$ f the copula is slightly smaller than the ρ f the $t_2^{\rho,\nu}$ **Archimedean Copulas**

Here we present the basic properties and examples of the Archimedean class of copulas. Basically, (Jaworski, Duante, Härdle, & Rychlik, 2010) follow the approach in (McNeil & Neslehova, 2009) (see also (MvNeil & Neslehova, 2010)). In (Genest & MacKay, 1986), authors give properties of Archimedean copulas. They show how these copulas can be used to illustrate the existence of distributions with singular components and to give a geometric interpretation to Kendall's tau.

Most thinking on bivariate distributions is centered about the normal case, which has been studied intensively since the times of Bravais and Karl Pearson (Gumbel, 1960).

First, we introduce some notations. We call *Archimedean generator* any decreasing and continuous function $\psi : [0,\infty[\to \mathbb{I}$ hat satisfies the conditions $\psi(0) = 1$, $\lim_{t\to\infty} \psi(t) = 0$ nd which is strictly decreasing on $[0, \inf\{t \mid \psi(t) = 0\}[$ By convention, $\psi(+\infty) = 0$ nd $\psi^{-1}(0) = \inf\{t \geq 0 \mid \psi(t) = 0\}$ where ψ^{-1} enotes the pseudo-inverse of ψ **Definition:** A d imensional copula C is called *Archimedean* if it admits the representation

$$C(\mathbf{u}) = \psi\left(\psi^{-1}(u_1) + \psi^{-1}(u_2) + \ldots + \psi^{-1}(u_d)\right) \quad (15)$$

For all $\mathbf{u} \in \mathbb{I}^d$ nd for some Archimedean generator ψ Functions of type (10) have been widely considered and studied in the literature, especially in connection with the investigations about semi-groups of the unit interval.

Theorem: Let ψ e an Archimedean generator. Let C_ψ e the function given by (10). Then C_ψ s a d-dimensional copula if, and only if, the restriction of ψ o $]0,\infty[$ s d-monotone, i.e. it satisfies:

(a)ψ s differentiable up to the order $d-2$ n $]0,+\infty[$ nd the derivatives satisfy

$$(-1)^k \psi^{(k)}(t) \geq 0 \qquad for\, k \in \{0,1,\ldots,d-2\}.$$

s decreasing and convex in $]0,+\infty[$

3.1.2. Bivariate Gumbel Copula (Gumbel, 1960) (de Melo Mendes & de Souza, 2004)

$$C_{Gu}^{\delta}(u_1,u_2) = e^{\left[-\left\{\tilde{u}_1^{\delta}+\tilde{u}_2^{\delta}\right\}^{\frac{1}{\delta}}\right]} \quad (16)$$

The exponential distribution holds for distances in time, especially between the happenings of rare events (Bortkiewicz, 1913). It can be used as the starting point for the theory of extreme values (Gumbel, 1960).

Where $\tilde{u} = -\log u$ The parameter $\delta \geq 1$xpresses the degree of dependency—when $\delta = 1$ X_1 nd X_2 re independent; when $\delta \to \infty$ the degree of dependency approaches that of perfect dependence. The Gumbel copula is an extreme value copula which captures a different sensation of risk occurring during stress (bear and bull markets) periods: the upper tail dependence. Its value is equal to $2-2^{1/\delta}$ **Bivariate Clayton copula** (Clayton, 1978) (Genest & MacKay, 1986) (Venter, 2001) (Aas, Czado, Frigessi, & Bakken, 2009)

This model have been developed and applied to the problem of demonstrating association in disease incidence in ordered pairs of individuals by Clayton (Clayton, 1978).

The density of the bivariate Clayton copula is given by

$$C(u_1,u_2) = (1+\delta)(u_1.u_2)^{-1-\delta}(u_1^{-\delta}+u_2^{-\delta}-1)^{-\frac{1}{\delta-2}} \quad (17)$$

Where $0 < \delta < \infty$ s a parameter controlling the dependence. Perfect dependence is obtained when $\delta \to \infty$ while $\delta \to 0$ mplies independence.

3.1.3. Bivariate Frank Copula (Frank, 1979)

The Frank copula is a symmetric Archimedean copula given by

$$C(u_1,u_2) = -\frac{1}{\delta}\log\left(1+\frac{\left(e^{-\delta u_1}-1\right)\left(e^{-\delta u_2}-1\right)}{e^{-\delta}-1}\right) \quad (18)$$

And its generator is:

$$\varphi_\delta(t) = -\log\left(\frac{e^{-\delta t}-1}{e^{-\delta}-1}\right).$$

where

$$\delta \in (-\infty,\infty)\setminus\{0\}.$$

4. Latility Spillover of Four Arabs Countries in MENA Area

The MENA area knows many economical, social and political changes since many decades, and the movement of liberalization is newly adopted in the financial markets of this area. So, it is important to show the dynamic of these markets and their dependence between each other. For this reason the choice was done on Egypt, Morocco, Jordan and Tunisia, these four countries are signatory of an agreement[1] aiming to establish a Free Trade Area in accordance with the provisions of the General Agreement on Tariffs and Trade of 1994 (GATT).

Globally, the MENA capital markets are generally perceived as less developed than the Asian or Latin American emerging markets and suffer from a number of institutional underdevelopments (Henry and Springborg, 2004). First, derivatives are not available and foreign access to the market was liberalized only in the last decade. Second, market makers are missing due to the consider-able involvement of governments in economic activities. Third, short selling remains illegal, and information disclosure requirement are lax (Lagoarde-Segot & Lucey, 2008).

Table1: Stock Markets Capitalization (Million $U.S.) of Egypt, Morocco, Jordan and Tunisia

	October 2011	November 2011	December 2011
Egyptian Exchange	55,973	52,458	48,679
Casablanca Stock Exchange	64,868	61,355	60,092
Amman Stock Exchange	27,250	26,368	27,210
Tunis Stock Exchange	10,026	9,772	9,648

Source: Arab Monetary Fund, Arab Capital Markets, 4th Quarter Bulletin 2011

Table 2: Number of Listed Companies in Stock Exchange of Egypt, Morocco, Jordan and Tunisia

	October 2011	November 2011	December 2011
Egyptian Exchange	214	214	214
Casablanca Stock Exchange	75	75	76
Amman Stock Exchange	248	248	247
Tunis Stock Exchange	57	57	57

Source: Arab Monetary Fund, Arab Capital Markets, 4th Quarter Bulletin 2011

Jordan has the highest numbers of listed companies, followed by Egypt, Morocco and Tunisia. But in term of capitalization, the Moroccan market occupies the first place even if the number of listed companies is not important as Jordan and Egypt (see Table 1 and 2).

Table 3: MSCI Equity Market Indices[2] of Egypt, Morocco, Jordan and Tunisia

	2011			
	Q1	Q2	Q3	Q4
Egyptian Exchange	-23.7%	-3.2%	-20.7%	-12.5%
Casablanca Stock Exchange	5.5%	-5.3%	-9.7%	-10%
Amman Stock Exchange	-9.7%	-4.2%	-4.6%	0.5%
Tunis Stock Exchange	-10.5%	1.2%	5.6%	-6.4%

Source: MSCI

During 2011, performance achieved by the four countries was not good. For the Egyptian case, the MSCI index was negative for all the four quarters and more deeply in the 1st and the 3rd quarters. The Moroccan market realized bad performance according to MSCI index except in the first quarter $+5,5\%$(Besides, Jordan market has bad MSCI index, but the last quarter was slightly positive $+0,5\%$()Even if the context in Tunisia seems to be unstable, the MSCI index was positive in the 2nd and the 3rd quarters.

Next, we study bivariate correlation between these four markets using copula functions in period of political instability. This will allow us to have a bird's eye about spillover effect among these markets.

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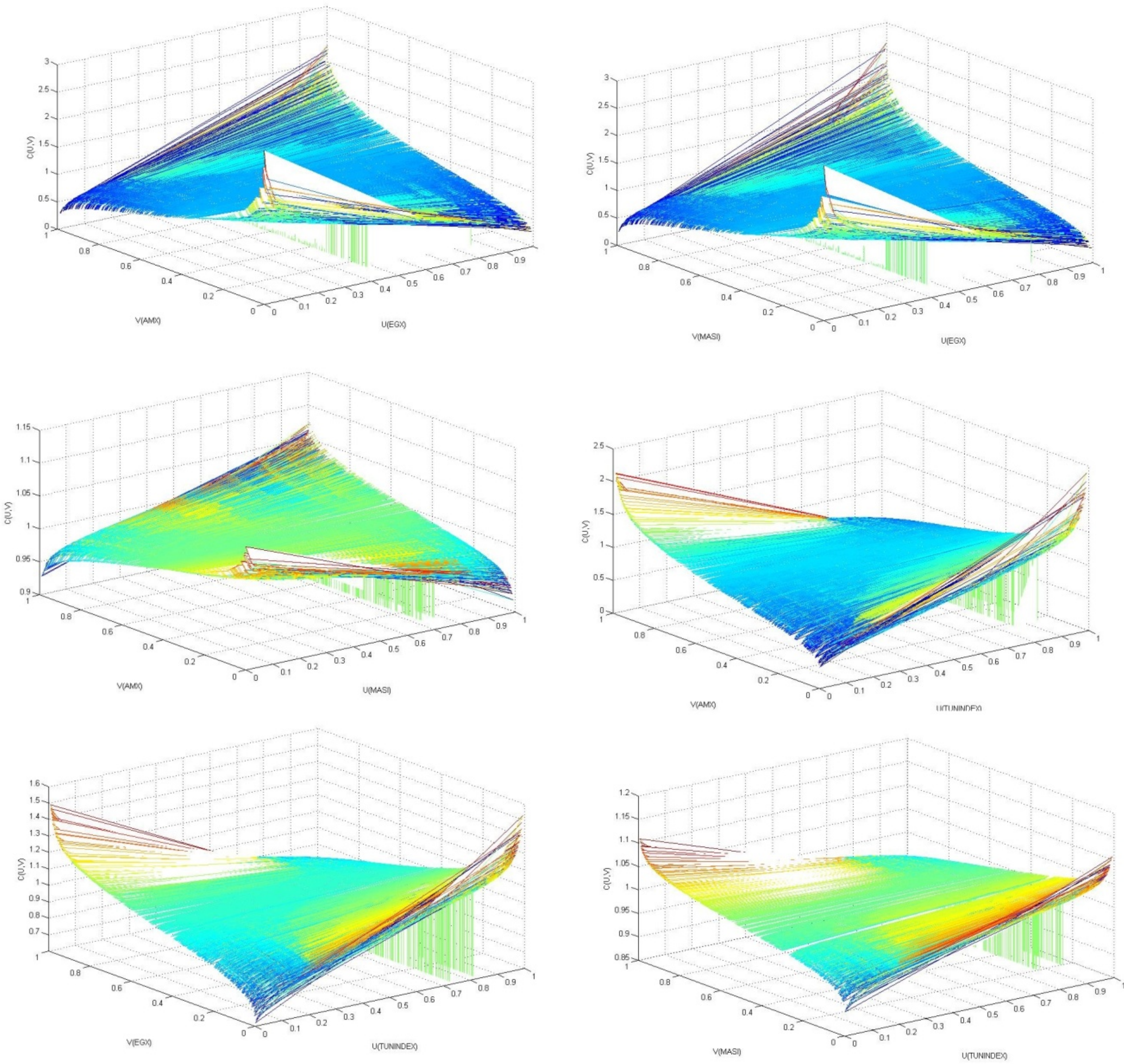
1 The Agadir agreement was signed on 25th February, 2004 and really implemented since March 27th, 2007, after the publishing of customs circulars of the four member countries.

2 The MSCI Emerging Markets Index is a free float-adjusted market capitalization index that is designed to measure equity market performance in the global emerging markets.

Bivariate Gaussian Copulas of MENA Markets

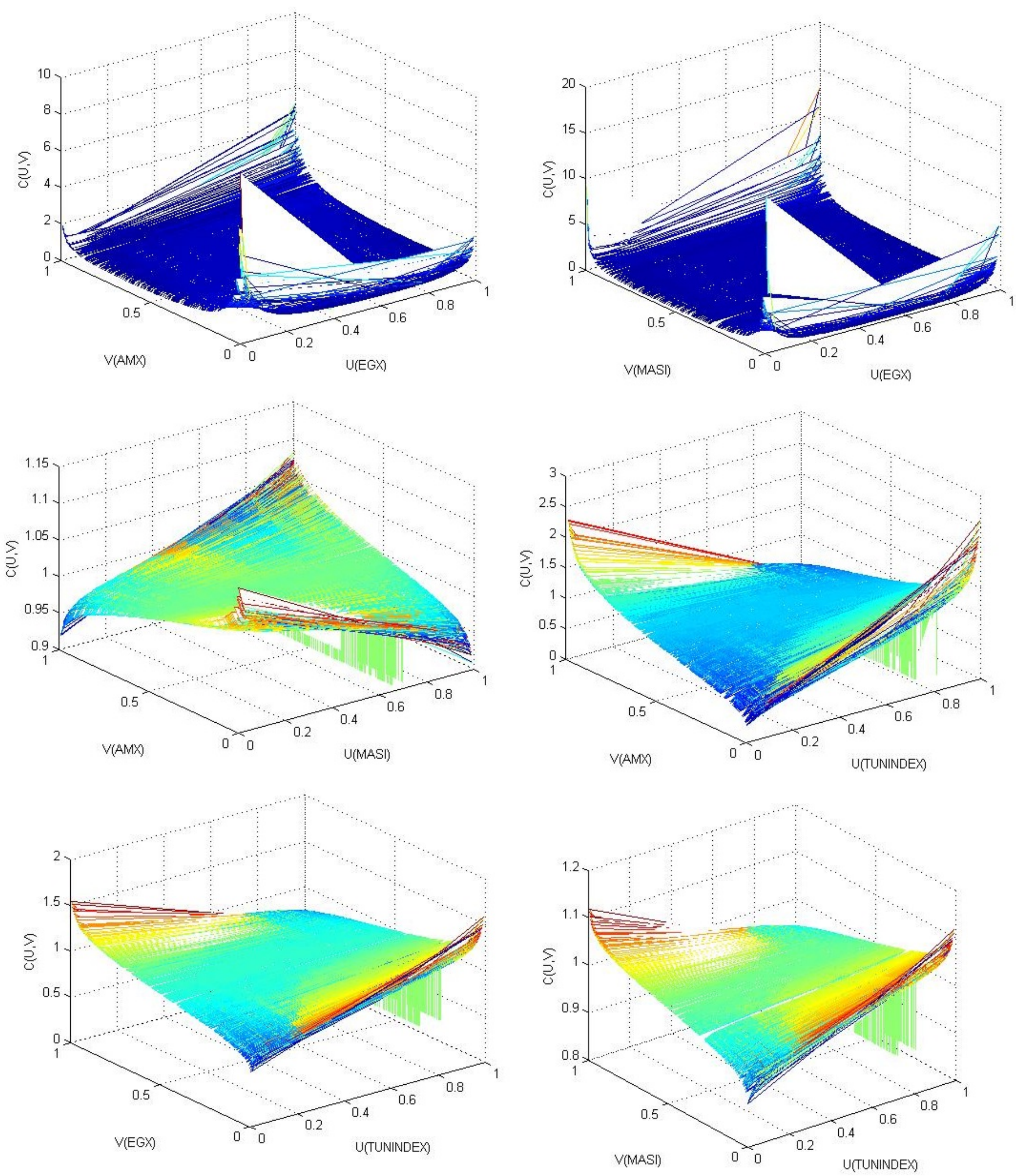
Figure 1: Bivariate Gaussian copula



We observe that there is a significant correlation between Jordanian and Egyptian and between Moroccan and Egyptian markets near the areas of lower and upper tails. There is also the same thing between Moroccan and Jordanian markets but correlation is less significant and the wave is much scatter. However, correlation between Tunisian and Jordanian markets, between Tunisian and Egyptian markets and between Tunisian and Moroccan markets is very small near the areas of lower and upper tails.

Bivariate t-Student Copulas of MENA Markets

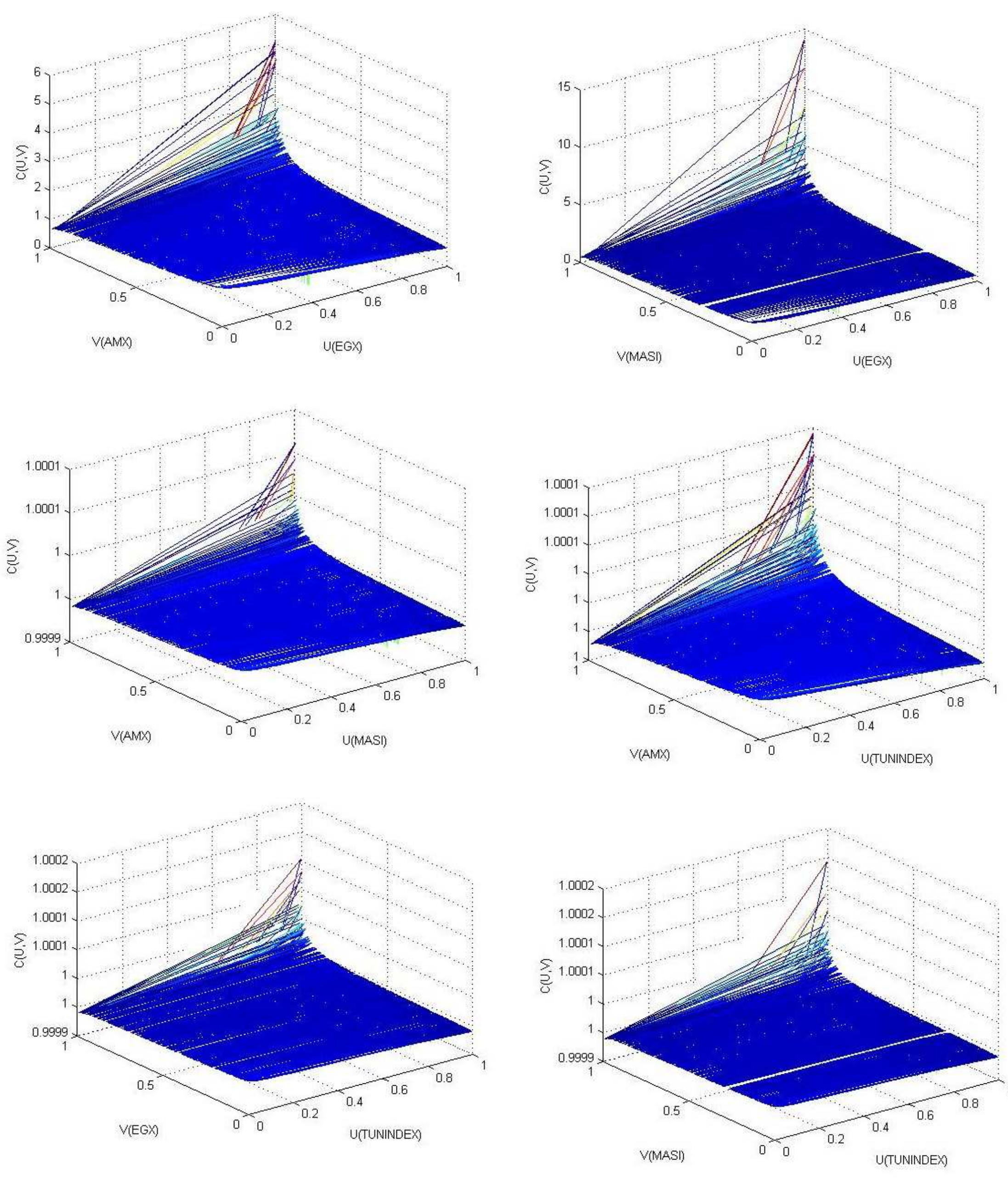
Figure 2: Bivariate t-Student copula



There is a very important correlation between Jordanian and Egyptian and between Moroccan and Egyptian markets near of lower and upper tails, the wave is more sharper than the obtained one by Gaussian copula. Less importantly, there is correlation in lower and upper tails between Moroccan and Jordanian markets. However, correlation between Tunisian and Jordanian markets, between Tunisian and Egyptian markets and between Tunisian and Moroccan markets is close to zero near the areas of lower and upper tails.

Bivariate Gumbel Copulas of MENA Markets

Figure 3: Bivariate Gumbel copula

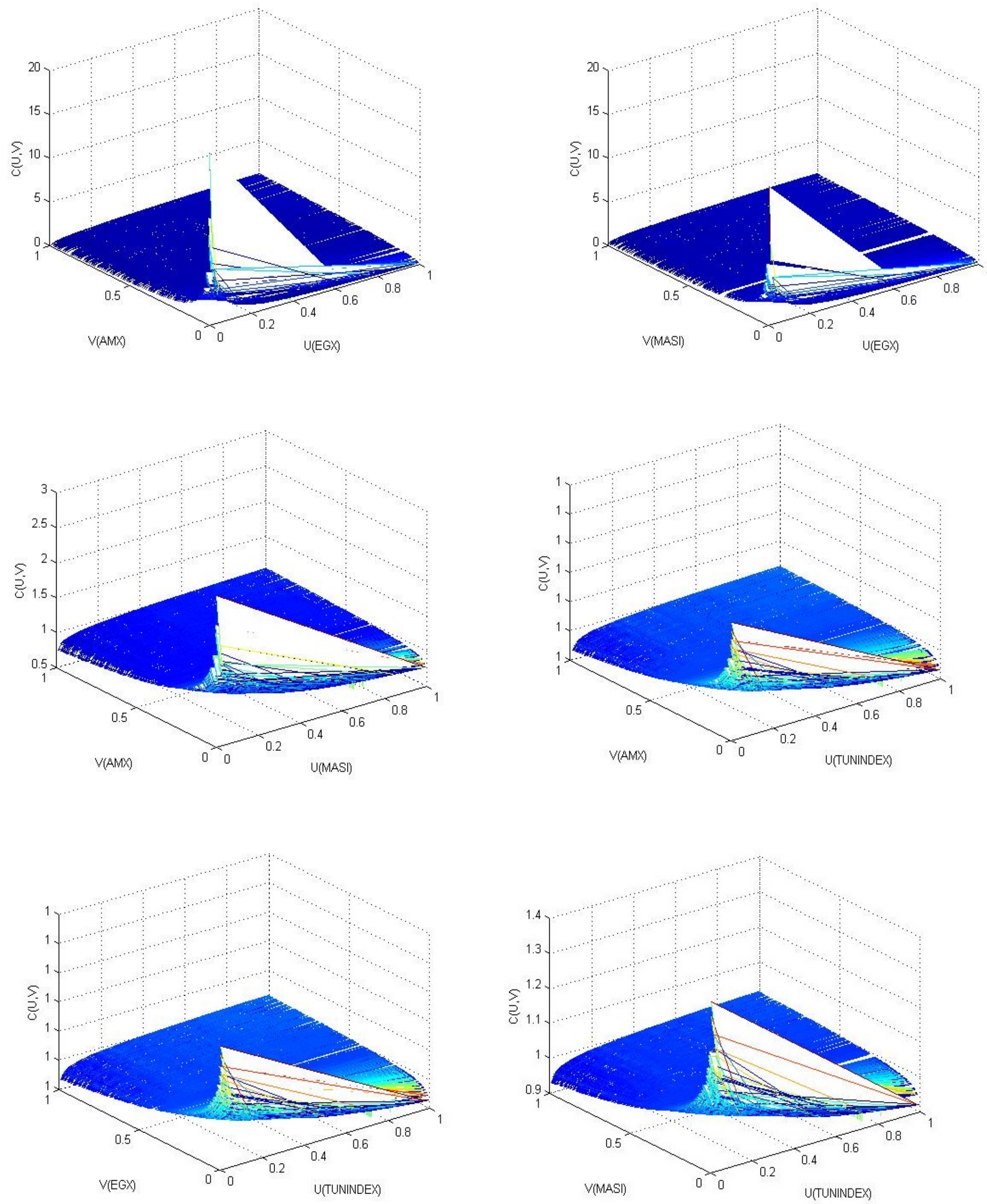


The Gumbel copula is an extreme value copula which captures a different sensation of risk in the upper tail dependence.

The Gumbel copula measures correlation between high positive variations. Once again, there is an important relation between Jordanian and Egyptian and between Moroccan and Egyptian markets in their upper tails. However, correlation between Moroccan and Jordanian markets, between Tunisian and Jordanian markets, between Tunisian and Egyptian markets and between Tunisian and Moroccan markets is very small near the upper tail.

Bivariate Clayton Copulas of MENA Markets

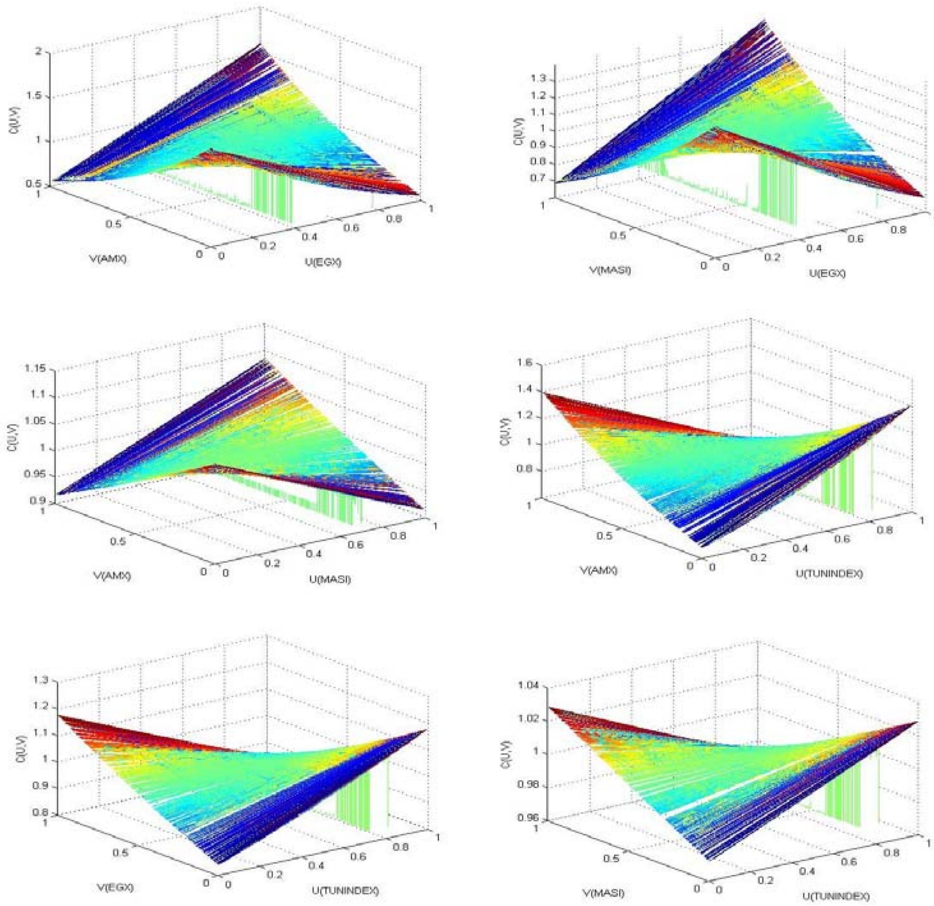
Figure 4: Bivariate Clayton copula



The Clayton copula measures correlation between lower tail dependence. Once again, there is an important relation between Jordanian and Egyptian and between Moroccan and Egyptian markets near the area of lower tail. On the other hand, correlation between Moroccan and Jordanian markets, between Tunisian and Jordanian markets, between Tunisian and Egyptian markets and between Tunisian and Moroccan markets is very small near the lower tail.

Bivariate Frank Copulas of MENA Markets

Figure 5: Bivariate Frank copula



The density function of Frank Copula function has symmetrical distribution. This copula has no tail dependence.

We note that there is a significant correlation between Jordanian and Egyptian and between Moroccan and Egyptian markets near the areas of upper and lower tail. There is also the same thing between Moroccan and Jordanian markets but in less significant way. The correlation between Tunisian and Jordanian markets, between Tunisian and Egyptian markets and between Tunisian and Moroccan markets still very small near the areas of upper and lower tail.

5. Conclusion

As we have seen, the bivariate correlation is important for the case of Egyptian financial market with Moroccan and Jordanian ones. These countries signed an agreement of free trade, so the flowing of capital is liberalized which is assumed to ensure a development of their activities.

The transmission of volatility was obvious between Egyptian Market and Casablanca Stock Exchange, and between Egyptian Exchange and Amman Stock Exchange. It seems that Egypt has exported his volatility to Moroccan and Jordanian Markets. However, Tunisian market has not been affected by this transmission. The small capitalization of Tunis Stock Exchange may its disconnection and its low-importance. Consequently the spillover effect has been insignificant for this market.

The spillover effect would be more important if these markets were developed enough, as having derivatives market or authorizing short selling.

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References

[1] Aas, K., Czado, C., Frigessi, A., & Bakken, H. (2009). Pair-copula constructions of multiple dependence. *Insurance: Mathematics and Economics , 44* (2), 182-198.

[2] Bonfiglioli, A., & Favero, C. A. (2005). Explaining co-movements between stock markets: The case of US and Germany. *Journal of International Money and Finance , 24*, 1299-1316.

[3] Bortkiewicz, L. v. (1913). *Die radioaktive Strahlung als Gegenstand Wahrscheinlichkeits theoretischer Untersuchunge*. Berlin: Springer.

[4] Bradley, B. O., & Taqqu, M. S. (2004). Framework for Analyzing Spatial Contagion between Financial Markets. *Finance Letters , 2* (6), 8-15.

[5] Cherubini, U., Luciano, E., & Vecchiato, W. (2004). *Copula methods in finance* . Chichester: John Wiley & Sons Ltd.

[6] Chollete, L., de la Peña, V., & Lu, C.-C. (2011). International diversification: A copula approach. *Journal of Banking & Finance , 35*, 403-417.

[7] Clayton, D. G. (1978). A Model for Association in Bivariate Life Tables and Its Application in Epidemiological Studies of Familial Tendency in Chronic Disease Incidence. *Biometrika , 65* (1), 141-151.

[8] Clemen, R. T., & Reilly, T. (1999). Correlations and Copulas for Decision and Risk Analysis. *Management Science , 45* (2), 208-224.

[9] de Melo Mendes, B. V., & de Souza, R. M. (2004). Measuring financial risks with copulas. *International Review of Financial Analysis , 13*, 27-45.

[10] Embrechts, P., McNeil, A., & Straumann, D. (2002). Correlation And Dependence In Risk Management: Properties And Pitfalls. Dans M. A. Dempster, *Risk Management: Value at Risk and Beyond* (pp. 176-223). Cambridge: Cambridge University Press.

[11] Embrechts, P., Resnick, S. I., & Samorodnitsky, G. (1999). Extreme value theory as a risk management tool. *North American Actuarial Journal , 3* (2), 30-41.

[12] Forbes, K. J., & Rigobon, R. (2002). No Contagion, Only Interdependence: Measuring Stock Market Comovements. *The Journal of Finance , LVII* (5), 2223-2261.

[13] Forbes, K., & Rigobon, R. (2001). Measuring Contagion: Conceptual and Empirical Issues. Dans S. Claessens, & K. Forbes, *International Financial Contagion* (pp. 43-66). Massachussetts: Springer.

[14] Frank, M. (1979). On the simultaneous associativity of F(x, y) and x+y–F(x, y). *Aequationes Mathematicae , 19* (2-3), 194-226.

[15] Genest, C., & MacKay, J. (1986). The Joy of Copulas: Bivariate Distributions with Uniform Marginals. *The American Statistician , 40* (3), 280-283.

[16] Gumbel, E. J. (1960). Bivariate Exponential Distributions. *Journal of the American Statistical Association , 55* (292), 698-707.

[17] Jaworski, P., Duante, F., Härdle, W., & Rychlik, T. (2010). *Copula Theory and Its Applications.* Heidelberg: Springer-Verlag Berlin Heidelberg.

[18] Joag-dev, K., Perlman, M., & Pitt, L. (1983). Association of normal random variables and Slepian's inequality. *Annals of Probability , 11*, 451-455.

[19] Karolyi, G. A., & Stulz, R. M. (1996). Why Do Markets Move Together? An Investigation of U.S.-Japan Stock Return Comovements. *The Journal of Finance , LI* (3), 951-986.

[20] King, M. A., & Wadhwani, S. (1990). Transmission of Volatility between Stock Markets. *The Review of Financial Studies , 3* (1), 5-33.

[21] Lagoarde-Segot, T., & Lucey, B. M. (2008). Efficiency in emerging markets—Evidence from the MENA region. *International Financial Markets, Institutions and Money , 18* (1), 94–105.

[22] McNeil, A., & Neslehova, J. (2009). Multivariate Archimedean copulas, d-monotone functions and λ1-norm symmetric distributions. *Ann. Stat.*, *37* (5B), 3059-3097.
[23] MvNeil, A., & Neslehova, J. (2010). From Archimedean to Liouville copulas. *Journal of Multivariate Analysis*, *101* (8), 1772-1790.
[24] Pindyck, R. S., & Rotemberg, J. J. (1993). The Comovement of Stock Prices. *The Quarterly Journal of Economics*, *108* (4), 1073-1104.
[25] Schweizer, B., & Sklar, A. (1974). Operations on distribution functions not derivable from operations on random variables. *Studia Math.*, *52*, 43-52.
[26] Schweizer, B., & Sklar, A. (1983). *Probabilistic metric spaces.* New York: North-Holland.
[27] Sklar, A. (1959). Fonctions de répartition à n dimensions et leurs marges. *Publ. Inst. Stat. Univ. Paris*, *8*, 229-231.
[28] Sklar, A. (1996). Random Variables, Distribution Functions, and Copulas: A Personal Look Backward and Forward. *Lecture Notes-Monograph Series*, *28*, 1-14.
[29] Venter, G. G. (2001). Tails of copulas. *Proceedings ASTIN Washington*, (pp. 68-113). Washington.
[30] Wang, K., Chen, Y.-H., & Huang, S.-W. (2011). The dynamic dependence between the Chinese market and other international stock markets: A time-varying copula approach. *International Review of Economics and Finance*, *20*, 654-664.

Appendix

Gumbel n-copula (Cherubini, Luciano, & Vecchiato, 2004)

The generator is given by $\varphi^{-1}\left(t\right)=e^{-t^{1/\delta}}$ it is completely monotonic if $\delta>1$ The Gumbel n-copula is therefore:

$C_{Gu}^{\delta}(\mathbf{u})=e^{\left\{-\left[\sum_{i=1}^{n}(-\log u_i)^{\delta}\right]^{\frac{1}{\delta}}\right\}}$ with $\delta>1$ **Clayton n-copula** (Cherubini, Luciano, & Vecchiato, 2004)

The generator is given by $\varphi^{-1}\left(t\right)=(t+1)^{-\frac{1}{\theta}}$ it is completely monotonic if $\theta>0$ The Clayton n-copula is therefore:

$C_{\theta}^{Cl}\left(\mathbf{u}\right)=\left[\sum_{i=1}^{d}u_i^{-\theta}-n+1\right]^{-\frac{1}{\theta}}$ with $\theta>0$ **Frank copula** (Frank, 1979) (Jaworski, Duante, Härdle, & Rychlik, 2010) (Cherubini, Luciano, & Vecchiato, 2004)

The standard expression for members of this family of d-copulas is

$$C_{\theta}^{Fr}\left(\mathbf{u}\right)=-\frac{1}{\theta}\log(1+\frac{\prod_{i=1}^{d}(e^{-\theta u_i}-1)}{\left(e^{-\theta}-1\right)^{d-1}}$$ with $\theta>0$ hen $d\geq 3$

Where $\theta>0$ The limiting case $\theta=0$ orresponds to $\prod_d$ For the case $d=2$ the parameter θ an be extended also to the case $\theta<0$ Copulas of this type have been introduced by Frank (Frank, 1979) in relation with a problem about associative functions on $\mathbb{I}$ They are absolutely continuous.

The Archimedean generator is given by $\varphi^{-1}\left(t\right)=-\frac{1}{\theta}\log\left(1-\left(1-e^{-\theta}\right)e^{-t}\right).$