Understanding Profit and the Markets: The Canonical Model

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Abstract
Neither Walrasians nor Keynesians have a clear idea of the fundamental economic concepts income and profit, nor of the interdependence of qualitatively different markets. Critique of these approaches is necessary but not overly productive. A real breakthrough requires a new set of premises because no way leads from the accustomed behavioral assumptions to the understanding of how the economy works. More precisely, the hitherto accepted behavioral axioms have to be replaced by structural axioms. Starting from new formal foundations, this paper gives a comprehensive and consistent account of the objective interrelations of the monetary economy’s elementary building blocks.

JEL B59, D00, E00

Keywords new framework of concepts; structure-centric; axiom set; income; profit; full employment; price stability; quantitative adaptation; primary markets; secondary markets; financial markets

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1 Premises and consequences

Walrasians are comparatively stronger on the formal leg, Keynesians on the material leg; inseparable because of the micro-macro yoke they limp along together. There is not much to choose between vacuous logic and platitudinous realism.

Dissatisfaction with the current state calls for improvements. The question is, can the known weak spots of the familiar approaches be removed or are they beyond repair? An unbiased look on the history of economic thought and the actual state of theoretical economics makes the conclusion inescapable that there is no potential left for real improvements. That orthodoxy has run its course and that heterodoxy has no convincing alternative to offer is the premise of this paper.

Because of the logical architecture of theories, to change a theory means to change its premises. With regard to the accustomed approaches the key point therefore is: the formal foundations of theoretical economics must be nonbehavioral and epitomize the interdependence of real and nominal variables that constitutes the monetary economy (for details see 2013b).

Accordingly, the first three structural axioms are introduced in Section 2. In Sections 3 and 4 the properties of the pure consumption economy and the trajectory of the market clearing product price in a random environment are established. The introduction of transaction money in Section 5 completes the sketch of the elementary consumption economy. Lifting the balanced budget condition in Sections 6 and 7 brings monetary profit into existence. This amounts to a terminatory clarification of the hitherto muddled profit theory. In Sections 8 and 9 the relations between profit, distributed profit, retained profit, monetary saving and investment are formally determined. The General Complementary states that retained profit is equal to the difference of investment and saving. The old contention that saving equals investment is thereby refuted. In Section 10 the household and business sector’s stock of money and the quantity of money are directly derived from the axioms. Since each firm is a price setter the price setter’s task is precisely defined in Sections 11 and 12. From the attempt to keep the inventory close to a target stock follow the flexible price adaptations in Section 13. In Sections 14 and 15 we turn to the determination of full employment. The ideal economy is characterized by full employment, quantitative adaption to exogenous demand shocks and perfect price stability in the product market. The wage rate moves with productivity. The quantity mechanism takes precedence over the price mechanism. Changes of the business sector’s inventory necessitates the distinction between monetary and nonmonetary profits in Section 16. Section 17 is devoted to differentiation of the business sector which entails the heterogeneity of firms. In Section 18 the financial sector and the household sector’s portfolio of assets and liabilities is differentiated. In Section 19 the bond market is introduced and in Section 20 the real assets market. Both markets are shown to be entirely different from the product market. The idea of a generic market is refuted. Section 21 concludes.
2 All you need is axioms – but which are the proper ones?

Central to the question of formalisation is the role of the rationality axioms. The internal goal, derived from a particular form of mathematics, of developing a closed, axiomatic, mathematically-expressed theoretical system which yielded equilibrium solutions required reductionist axioms of deterministic individual behaviour. (Dow, 1997, p. 83)

For a proper formalization it is important to appreciate that there exists no such thing as a rationality axiom, only a rationality assumption. The following set of structural axioms implies the denial of the status of an axiom to the assumptions of rationality, equilibrium, and deterministic individual behavior. For methodological reasons, these kinds of assumptions cannot be taken into the premises. A behavioral axiom is a contradiction in terms (for details see 2013b). This is not a minor formal point. Success or failure of a theory depends in the last instance on the axioms.

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. For the remainder of this inquiry the period length is conveniently assumed to be the calendar year. Simplicity demands that we have at first one world economy, one firm, and one product. Quantitative and qualitative differentiation is obviously the next logical step.

Total income of the household sector \( Y \) in period \( t \) is the sum of wage income, i.e. the product of wage rate \( W \) and total working hours \( L \), and distributed profit, i.e. the product of dividend \( D \) and the number of shares \( N \).

\[
Y = WL + DN \quad |t
\]

Output of the business sector \( O \) is the product of productivity \( R \) and working hours.

\[
O = RL \quad |t
\]

The productivity \( R \) depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures \( C \) of the household sector is the product of price \( P \) and quantity bought \( X \).

\[
C = PX \quad |t
\]

\(^{1}\) Taken in isolation, each of the following three equations is not unheard-of in theoretical economics. The 1st axiom compares to Debreu’s private ownership economy (1959, p. 79), see also (Ingrao and Israel, 1990, p. 23), the 2nd is known from textbooks, e.g. (Blanchard, 2000, p. 251, eq. 13.1), and the 3rd is known since Verri (Schumpeter, 1994, p. 307, fn. 13). The economic content of the structural axioms, which are indeed unique as a set, therefore needs no further elaboration. The originality of the structural axiom set consists in the rigorous exclusion of all behavioral or nonempirical concepts.
A set of axioms is, since Nassau William Senior, the formal starting point in theoretical economics (Stigum, 1991, p. 4), (Schumpeter, 1994, p. 575). An unaxiomatized theory belongs without further ado to the realm of political economics. There, ‘nothing is clear and everything is possible’ (Keynes, 1973, p. 292). Under the scientific perspective political economics has always been a nuisance. The distinction between assumptionism and axiomatization is therefore crucial for theory building. There is no way around it:

What particular reality is described by a given theory can be ascertained only from that theory’s axiomatic foundation. (Georgescu-Roegen, 1966, p. 361)

The axioms’ assessment proceeds with the interpretation of the logical implications of the formal world and the comparison with selected data and phenomena of the real world. Axioms should have an intuitive economic interpretation (von Neumann and Morgenstern, 2007, p. 25). Only in pure mathematics any reference to a real domain is superfluous or arbitrary. The economic meaning is rather obvious for the set of structural axioms. What deserves mention is that total income in (1) is the sum of wage income and distributed profit and not of wage income and profit. Profit and distributed profit have to be thoroughly kept apart.

By choosing objective structural relationships as axioms behavioral assumptions are not ruled out. The structural axiom set is open to any behavioral assumption and not restricted to the standard optimization calculus. A theory that deals only with optimizing behavior is evidently not general.

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms (Boylan and O’Gorman, 2007, p. 431). With (4) wage income $Y_W$ and distributed profit income $Y_D$ is defined:

$$Y_W \equiv WL \quad Y_D \equiv DN \quad |t.$$  \(4\)

With (5) the expenditure ratio $\rho_E$, the sales ratio $\rho_X$, the distributed profit ratio $\rho_D$, and the factor cost ratio $\rho_F$ is defined:

$$\rho_E \equiv \frac{C}{Y} \quad \rho_X \equiv \frac{X}{O} \quad \rho_D \equiv \frac{Y_D}{Y_W} \quad \rho_F \equiv \frac{W}{PR} \quad |t.$$  \(5\)

The axioms and definitions are consolidated to one single equation:

$$\rho_F \frac{\rho_E}{\rho_X} (1 + \rho_D) = 1 \quad |t.$$  \(6\)

The period core (6) as absolute formal minimum determines the interdependencies of the measurable key ratios for each period. The core is purely structural, i.e. free
of any behavioral assumptions, unit-free\(^2\) because all real and nominal dimensions cancel out, and contingent. Contingency means that it is open until explicitly stated which of the variables are independent and which is dependent. The form of (6) precludes any notion of causality; the equation states that the interdependence of the key ratios is subject to a conservation law.

The factor cost ratio \(\rho_F\) summarizes the internal conditions of the firm. A value of \(\rho_F < 1\) signifies that the real wage \(\frac{W}{P}\) is lower than the productivity \(R\) or, in other words, that unit wage costs \(\frac{W}{R}\) are lower than the price \(P\) or, in still other words, that the value of output per hour \(PR\) exceeds the value of input \(W\). In this case the profit per unit is positive. Then we have the conditions in the product market. An expenditure ratio \(\rho_E = 1\) indicates that consumption expenditures \(C\) are equal to income \(Y\), in other words, that the household sector’s budget is balanced. A value of \(\rho_X = 1\) of the sales ratio means that the quantities produced \(O\) and sold \(X\) are equal in period \(t\) or, in other words, that the product market is cleared. In the special case \(\rho_E = 1\) and \(\rho_X = 1\) with budget balancing and market clearing the factor cost ratio \(\rho_F\) and with it the profit per unit is determined solely by the distributed profit ratio \(\rho_D\). The period core (6) covers the key ratios about the firm, the market, and the income distribution and determines their interdependencies. The period core represents the pure consumption economy, that is, no investment expenditures, no foreign trade, and no government. It is impossible to go behind the period core: eq. (6) is the shortest possible formal description of the elementary economy.

### 3 Fundamentals

There can be no doubt whatsoever that a problem which has not yet been solved in all its aspects under its simplest conditions will be still more difficult to tackle if other, "more realistic" assumptions are being made. (Morgenstern, 1941, p. 373)

After the implementation of the conditions of market clearing \(\rho_X = 1\), budget balancing \(\rho_E = 1\), and zero distributed profit \(Y_D = 0\) the pure consumption economy looks as shown in Figure 1. It is important to notice that the three conditions are not a constituent part of the axiom set but an – in principle – arbitrary addendum. It is not to be expected that we will find the consumption economy in this very special state. The merit of this state consists in its unsurpassable theoretical simplicity. The complexity of the real thing is reduced to the bare minimum.

The market clearing price and the real wage follow directly from (6) and the three conditions:

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\(^2\) "This procedure is in accordance with the principle of objectivity requiring that the whole theory and its interpretations have to be independent of the choice of the units of measurement. And this requirement is met, if the theory is unit-free, the necessary condition stated in Buckingham’s Π-theorem." (Schmiechen, 2009, p. 176)
Figure 1: The pure consumption economy with product market clearing, budget balancing, and zero distributed profit

\[ \rho_F = 1 \implies P = \frac{W}{R} \quad \text{and} \quad \frac{W}{P} = R \]

if \( \rho_X = 1, \rho_E = 1, \rho_D = 0 \) i.e. \( Y = Y_W \mid t \).

The market clearing price \( P \) is equal to unit wage costs \( \frac{W}{R} \) under the additional conditions of budget balancing and zero distributed profit. Hence profit per unit of output is zero and therefore overall profit is zero. The real wage \( \frac{W}{P} \) is equal to the productivity \( R \).

This configuration is reproducible. The firm sells its period output completely and fully recoups its wage costs. It is worth emphasizing that the market clearing price is unequivocally determined by the three axioms and the three conditions. In order to avoid over-determination it is therefore inadmissible to add independent demand and supply functions. There is simply no formal room left for additional behavioral assumptions or some occult market forces that supposedly equalize price and unit wage costs. The same holds for the real wage which is determined with (7) and not by “demand and supply” in the labor market.

The market clearing price in (7) is independent of employment. Hence, if employment \( L_u \) changes in subsequent periods while wage rate \( W \) and productivity \( R \) remain unaltered then the price \( P \) remains constant. This case is depicted in Figure 2.

It is therefore possible that the economy moves from unemployment \( L_u \) to full employment \( L_f \) without any change of the market clearing price provided wage rate and productivity are fix in the relevant range. If productivity and wage rate change
on the move to full employment this affects the market clearing price according to (7).

Since profit is zero if the price is equal to unit wage costs it is of no consequence for the business sector whether the economy operates at full employment or unemployment. Profit is zero in both cases. Business could therefore be indifferent about various employment levels. A wage rate reduction is no precondition for attaining full employment, it would only lower the market clearing price.

Whatever happens to the wage rate is of no consequence for the real wage which is invariably equal to the productivity. The wage earners, which include all employees of the firm, are well-off if the productivity is high. At the moment, the wage rate is equal for all employees. This is a provisional simplification (the differentiation is carried out in Section 17.3).

Under the condition of increasing returns the move from unemployment to full employment entails an increasing real wage. With a constant productivity in the relevant range the real wage does not change at all. Under this condition the move to full employment is indifferent for the already employed and beneficial for the hitherto unemployed. How the productivity develops in the relevant range is an empirical question. It is in any case inadmissible to take decreasing returns a priori as given. A theory that puts decreasing returns in the premises cannot claim to be general. Apart from this decisive formal point it is worth to recall Adam Smith’s praise of the division of labor and the factual importance of increasing returns for the promotion of the wealth of nations (Alam, 2013).

Under the conditions of product market clearing, budget balancing, and zero distributed profit a move from unemployment to full employment that is perfectly
indifferent for both the already employed wage earners and the business sector presupposes a constant productivity. In this benchmark case wage rate changes in either direction are immaterial. Wage stickiness does not play any role. If the business sector is not indifferent and does not move to a higher employment level unless profit is greater than zero then full employment is unattainable, that is, the business sector prevents a Pareto-optimal employment expansion. The consumption economy is, in principle, reproducible at any employment level; unemployment is always a failure of the business sector.

It should be noted in passing that, with perfect indifference between employment levels, the notion of equilibrium has no meaning. This is why equilibrium cannot be put into the premises in the first place. In other words:

\[ \ldots \text{you shouldn’t find the fixed equilibria first and then see if an economy converges to it; rather, the convergence process will itself constitute the equilibrium, if any exist. (Mirowski, 1989, p. 459)} \]

For methodological reasons alone, all of equilibrium economics has to be rejected as far as it puts equilibrium into the premises. This mistake is known as petitio principii (Mill, 2006, p. 820).

4 The market clearing price in a random environment

The period values of the variables are formally connected by the familiar growth equation, which is added to the structural set as the 4th axiom:

\[ Z_t = Z_{t-1} \left(1 + \ddot{Z}_t\right). \]  

(8)

The path of the representative variable \(Z_t\) is then determined by the initial value \(Z_0\) and the rates of change \(\ddot{Z}_t\) for each period:

\[ Z_t = Z_0 (1 + \ddot{Z}_1)(1 + \ddot{Z}_2) \ldots (1 + \ddot{Z}_t) = Z_0 \prod_{i=1}^{t} (1 + \ddot{Z}_i). \]  

(9)

Equation (9) describes the path of a variable with the rates of change as unknowns. These unknowns are in need of determination and explanation. This has a straightforward methodological consequence:

The simplest hypothesis is that variation is random until the contrary is shown, the onus of the proof resting on the advocate of the more complicated hypothesis \(\ldots\) (Kreuzenkamp and McAleer, 1995, p. 12)
The conditions of market clearing, budget balancing, and zero distributed profit reduce the number of the independent variables to three, i.e. employment, wage rate and productivity. It is assumed now for a start that these variables vary at random and, more specifically, that the variations are symmetrical around zero. This produces a drifting economy that over a longer time span neither grows nor shrinks. The respective probability distributions are given by:

\[
\begin{align*}
Pr \left( \{-1\% \leq \bar{L} \leq 1\%\} \right) \\
Pr \left( \{-3\% \leq \bar{W} \leq 3\%\} \right) \\
Pr \left( \{-3\% \leq \bar{R} \leq 3\%\} \right) \\
\end{align*}
\]

(10)

For the simulation the random variates for each period are taken from the worksheet random number generator and are then appropriately adapted. The assumed probability distributions can at any time be replaced by distributions that have been observed over a reasonable time span. Average wage rate changes, for example, lie normally between zero and five percent. Those empirical distributions bring the simulation closer to reality. There is, though, no need at this early stage to discuss the merits and demerits of different probability distributions (Mirowski, 2004).

With the rates of change of wage rate and productivity drawn from (10) and inserted in (8) the price in each period is then given by (7). Employment plays no role for the price determination. The random shocks to the consumption economy together with the axioms and conditions produce the path of the market clearing price as shown in Figure 3.

Figure 3: Path of the product price under the conditions of market clearing, budget balancing, and zero distributed profit, with symmetric random variations of employment, productivity, and wage rate
Note that output and sales follow identical paths which is to say that the product market is cleared in each period. The same holds for income and consumption expenditures which is to say that the household sector’s budget is balanced.

The drifting economy arrives in the selected simulation after fifty periods at a configuration that closely resembles the initial configuration. This, of course, is due to the assumption of symmetric random distributions (10). It has to be emphasized that the economy is not in any way drawn towards this configuration. If the endpoint after fifty periods resembles the initial configuration this is due to pure chance and not to some occult equilibrating forces. This said, it is clear that an outcome like Figure 3 could be interpreted as a self-equilibrating system – which it is definitively not at the moment. The drifting economy simply returns infinitely often in infinite time to the initial configuration if we chose the appropriate random distributions. It is much like Poincaré’s recurrence theorem which, however, is not directly applicable because it presupposes differential equations.

The simulation provides a benchmark because it displays all desirable properties, above all market clearing and budget balancing in each period. Figure 3, which follows in direct lineage from Figure 2, fully replaces the familiar demand-supply-equilibrium cross which lacks the dimension of time and some other essential features. Note that the simulation yields a definite numerical value for the market clearing price in each period. As an analytical model it shows that continuous market clearing and budget balancing is, in principle, possible in a random environment. However, since the price setter in our single firm cannot know the market clearing price, Figure 3 is not descriptive. What we will see in the real world is that the market is not cleared and the budget is not balanced in any single period. The chief merit of Figure 3 is to provide a better tool than demand-supply-equilibrium for the comprehensive analysis of market interactions.

The path of the market clearing price follows deterministically from random variations of the independent variables and the structure of the pure consumption economy which is ultimately given with the axiom set. The market clearing price is unequivocally determined but not predictable.

5 Transaction money

In order to reduce the monetary phenomena to the essentials it is supposed that all financial transactions between the household and the business sector are carried out by the central bank. Money then takes the form of current deposits and current overdrafts. Initial endowments are set to zero. Then, if the household sector owns current deposits the current overdrafts of the business sector are of equal amount, and vice versa. Money and credit are at first symmetrical. From the central bank’s perspective the quantity of money is then given by the actual stock of current deposits. It is assumed at first that the central bank plays an accommodative role.
and simply supports the autonomous market transactions between the household and the business sector as exemplarily given by the income and expenditure paths in Figure 3. For the time being, money is the dependent variable.

By sequencing the initially given period length of one year into months the idealized transaction pattern that is displayed in Figure 4a results. It is assumed that the monthly income $\frac{Y}{12}$ is paid out at mid-month. In the first half of the month the daily spending of $\frac{Y}{360}$ increases the current overdrafts of the households. At mid-month the households change to the positive side and have current deposits of $\frac{Y}{24}$ at their disposal. This amount reduces continuously towards the end of the month. This pattern is exactly repeated over the rest of the year. At the end of each subperiod, and therefore also at the end of the year, both the stock of money and the quantity of money is zero.

![Figure 4: Household sector’s idealized transaction pattern for different nominal incomes in two periods](image)

In period 2 the wage rate, the dividend, and the price is doubled. Since no cash balances are carried forward from one period to the next, there results no real balance effect provided the doubling takes place exactly at the beginning of period 2.

From the perspective of the central bank it is a matter of indifference whether the household or the business sector owns current deposits. Therefore, the pattern of Figure 4a translates into the average amount of current deposits in Figure 4b. This average stock of transaction money depends on income according to the transaction equation

$$\hat{M}_T \equiv \kappa Y \quad |t$$

which resembles Pigou’s Cambridge equation. For the regular transaction pattern that is here assumed as a idealization the index is $\frac{1}{18}$. Different transaction patterns are characterized by different numerical values of the transaction pattern index. Taking (11) and (5) together one gets the explicit transaction equation for the limiting case of market clearing and budget balancing:
We are now in the position to substantiate the notion of accommodation as a money-growth formula. According to (i) the central bank enables the average stock of transaction money to expand or contract with the development of productivity, employment, and price. In other words, the real average stock of transaction money, which is a statistical artifact and no physical stock, is proportional to output (ii) if the transaction index is given and if the ratios $\rho_X$ and $\rho_E$ are unity. Under these initial conditions money is endogenous (Desai, 1989, p. 150) and neutral (Patinkin, 1989) in the structural axiomatic context.

The average stock of transaction money follows graphically via (12) from Figure 3. The quantity of money is zero in the initial period and at the end of period $t_0$ because consumption expenditures are equal to income in each period; only the average stock of transaction money is $>0$ throughout. It is obvious from (7) that neither the quantity of money nor the average stock of transaction money has any effect on the market clearing price. Money performs at the moment solely the task of a transaction medium. It vanishes from the economy when the job is done and reappears when needed. Money is not conceived in its most primitive form as a given physical stock, neither is it thrown from a helicopter. Money comes into being with the autonomous transactions between the household and the business sector. This is the correct way if money is to be absolutely neutral.

6 The logical emergence of profit

If profits are not derived as herein stated, will not some one undertake to show whence they do come and by what forces they are determined and limited as to amount? (Walker, 1887, p. 288)

The business sector’s monetary profit in period $t$ is defined with (13) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditures $C$ – and costs – here identical with wage income $Y_W$:

$$
\Delta \bar{Q}_m \equiv C - Y_W \mid t.
$$

In explicit form, after the substitution of (3) and (4), this definition is identical with that of the theory of the firm:

$$
\Delta \bar{Q}_m \equiv PX - WL \mid t.
$$
Using the first axiom (1) and the definitions (4) one gets:

\[ \Delta \tilde{Q}_m \equiv C - Y + Y_D \mid t. \]  \hfill (15)

The three definitions are formally equivalent. Profit can be seen from different perspectives. Taken together, the three perspectives make a comprehensive view. If distributed profit \( Y_D \) is set to zero in (15), then profit or loss of the business sector is determined solely by consumption expenditures and wage income. For the business sector as a whole to make a profit consumption expenditures \( C \) have in the simplest case to be greater than wage income \( Y \) as shown in Figure 5.

![Figure 5: The emergence of monetary profit under the condition of market clearing and zero distributed profit](image)

The price \( P \) is determined by the axioms (1) to (3) and the condition of market clearing:

\[ P = \frac{C}{RL} \quad \text{if} \quad \rho_X = 1 \mid t. \]  \hfill (16)

Together with the definition of the expenditure ratio (5) this yields the market clearing price in a more convenient form:

\[ P = \rho_E \frac{W}{R} \quad \text{if} \quad \rho_X = 1, \rho_D = 0 \mid t. \]  \hfill (17)

The market clearing price, which compares to (7), is higher or lower than unit wage costs depending on the expenditure ratio \( \rho_E \). In Figure 5 the profit per unit is positive because \( \rho_E > 1 \); it is graphically given as the difference between the two rays from the origin at \( X = 1 \).
So that profit comes into existence for the first time in the pure consumption economy the household sector must run a deficit at least in one period. As long as the households spend their wage incomes fully the business sector will not make a loss but it will not see any profits either. The existence of profit for the economy as a whole does neither depend on the working hours, the wage rate, nor on productivity. Variations of these variables are compensated for by the market clearing price. Profit is in no way dependent on profit maximization or other forms of wishful thinking.

The household sector’s initial deficit in turn makes the inclusion of the financial sector mandatory. A theory that does not include at least one bank that supports the concomitant credit expansion cannot capture the essential features of the market economy. It is impossible to derive profit in a real model.

Mention should be made that neither Walrasians nor Keynesians nor Marxians nor Institutionalist, not to speak of Austrians or Sraffiaians, ever came to grips with profit (Desai, 2008), (Tómasson and Bezemer, 2010), (Kakarot-Handtke, 2013a). This is of no consequence for political economics where anything goes and substance only hampers a lively discussion. For theoretical economics, though, it has been a real disadvantage to operate over two hundred years without a correct conception of income and profit. As Schumpeter observed in his History of Economic Analysis:

Sometimes the difficulties are not in the things but in our own minds. (1994, p. 560)

7 Beyond the horizon

The determinants of profit look essentially different depending on the perspective. For the individual firm price $P$, quantity $X$, wage rate $W$, and employment $L$ in (14) appear to be all important; under the broader perspective of (15) these variables play no role at all. These phenomenological differences should not cause any irritation because both equations are formally equivalent.

The individual firm is blind to the structural relationships as given by (15). On the firm’s level profit is therefore subjectively interpreted as a reward for innovation or superior management skills or higher efficiency or toughness on wages or for risk taking or capitalizing on market imperfections or as the result of monopolistic practices. These and other factors play a role when it comes to the distribution of profits between firms and these phenomena become visible when similar firms of an industry are compared. Firms do not create profit, they redistribute it. The case is perfectly clear when there is only one firm. It is a matter of indifference whether the firm’s management thinks that it needs profit to cover risks or to finance growth or whether it realizes the profit maximum or not. If the expenditure ratio is unity and distributed profit is zero, profit as defined by (15) will invariably be zero, no matter what the agents want or plan. Hence there is no need to speculate about it.
From the structural axioms and definitions follows in direct lineage:

- The business sector’s revenues can only be greater than costs if, in the simplest of all possible cases \( Y_D = 0 \), consumption expenditures are greater than wage income.

- In order that profit comes into existence for the first time in the pure consumption economy the household sector must run a deficit at least in one period.

- Profit is, in the simplest case, determined by the increase and decrease of household sector’s debt.

- Wage income is the factor remuneration of labor input \( L \). Profit \( \Delta \bar{Q}_m \) is not a factor income. Since capital is nonexistent in the pure consumption economy profit is not functionally attributable to capital.

- Profit has no real counterpart in the form of a piece of the output cake. Profit has a monetary counterpart.

- The existence and magnitude of overall profit does not depend on profit maximizing behavior of the business sector but solely on the expenditure ratio of the household sector.

- The value of output is, in the general case, different from the sum of factor incomes. This is the defining property of the monetary economy.

- Only in the limiting case \( Y_D = 0 \), \( \rho_X = 1 \) and \( \rho_E = 1 \) is the value of output equal to factor income, i.e. \( C = Y_W \). This is the zero profit case which has no practical relevance.

The fundamental error of value theory is to start from the premise that the value of the output of goods and services is always equal to the sum of factor incomes. This error can be traced back to Adam Smith (2008, pp. 50, 155).

Under the condition \( C = Y \), profit \( \Delta \bar{Q}_m \) is according to (15) numerically equal to distributed profit \( Y_D \). The fundamental difference between the two variables does not catch the eye in this limiting case. The equality of profit and distributed profit is an implicit feature of equilibrium models. These have no counterpart in reality. In the real world holds \( C \neq Y \), hence profit and distributed profit are never equal. In addition, profits are never fully distributed. This fact is sufficient to reject Debreu’s model (1959, p. 79) and with it the standard approach in toto.

All models that are based on the definition total income \( \equiv \) wages + profits are fatally flawed because profit and distributed profit is not the same thing. To treat profit as factor income is a category mistake. Models that conceive profit in real terms as some kind of surplus are deficient from the outset.
8 Retained profit, saving, and investment

8.1 The Special Complementarity

Profits can either be distributed or retained. If nothing is distributed, then profit adds entirely to the financial wealth of the firm. Retained profit $\Delta \tilde{Q}_{re}$ is defined for the business sector as a whole as the difference between profit and distributed profit in period $t$:

$$
\Delta \tilde{Q}_{re} \equiv \Delta \tilde{Q}_m - Y_D |_{t} .
$$

(18)

Using (15) it follows:

$$
\Delta \tilde{Q}_{re} \equiv C - Y |_{t} .
$$

(19)

Retained profit $\Delta \tilde{Q}_{re}$ is the residual $C - Y$ as it appears at the firm that represents the business sector. The same residual appears at the central bank as a change of the business sector’s stock of money (see (32)).

Monetary saving is given by (20) as the difference of income and consumption expenditures. This definition is identical with Keynes’s (1973, p. 63), only the notation is different.

$$
\Delta \tilde{S}_m \equiv Y - C |_{t} .
$$

(20)

Monetary saving $\Delta \tilde{S}_m$ is the residual $Y - C$ as it appears at the household sector. The same residual appears at the central bank as a change of the household sector’s stock of money (see (29)).

Saving (20) and retained profit (19) always move in opposite directions, i.e.

$$
\Delta \tilde{Q}_{re} \equiv -\Delta \tilde{S}_m |_{t} .
$$

(21)

Let us call this the Special Complementarity. It says that the complementary notion to saving is not investment but negative retained profit. Positive retained profit is the complementary of dissaving.

Eq. (21) tells us that the plans of households and firms are in the general case not compatible. If, in the pure consumption economy, the households realize their saving plans, firms cannot realize their profit plans. This poses a serious theoretical problem for approaches that define equilibrium in behavioral terms as compatibility of all individual plans. In the structural axiomatic context this self-produced problem does not arise because the notion of equilibrium is not put into the premises and no attempt is made to read the minds of agents.
8.2 The General Complementarity

We proceed now from the pure consumption economy briefly to the investment economy (for more details see 2011f). Based on the differentiated formalism it is assumed that the investment goods industry, which consists of one firm, produces \( O_I = X_I \) units of an investment good, which is bought by the consumption goods industry to be used for the production of consumption goods in future periods. The households buy but the output of the consumption goods industry. From (10) then follows for the monetary profit of the consumption and investment goods industry, respectively:

\[
\Delta \bar{Q}_{mC} \equiv C - Y_{WC} \\
\Delta \bar{Q}_{mI} \equiv I - Y_{WI} 
\]

Total monetary profit, defined as the sum of both industries, is then given by the sum of consumption expenditure and investment expenditure minus wage income which is here expressed, using (1), as the difference of total income minus distributed profit:

\[
\Delta \bar{Q}_m \equiv C + I - (Y - Y_D) \\
\text{with } Y_W \equiv Y_{WC} + Y_{WI} |_{t}. 
\]

From this and the definition of monetary saving (20) follows:

\[
\Delta \bar{Q}_m \equiv I - \Delta \tilde{S}_m + Y_D |_{t}. 
\]

Higher total monetary profits on the one side demand as a corollary, i.e. as a logical implication of the definition itself, higher investment expenditure and distributed profits and lower saving on the other side. By finally applying the definition of retained profit (18) the General Complementarity follows:

\[
\Delta \bar{Q}_{re} \equiv I - \Delta S_m |_{t}. 
\]

This compares to (21). If retained profit \( \Delta \bar{Q}_{re} \) is zero, that is, if profit and distributed profit happen to be equal in (18), then, as a corollary, investment expenditure and household saving in (25) must be equal too. Vice versa, if it happens that household saving is equal to investment expenditure then, as a corollary, profit and distributed profit must be equal too. In reality, though, profit and distributed profit are never equal and correspondingly household saving and business investment are not equal either. The fact that retained profit is different from zero in the real world can be taken as an empirical proof of the logically equivalent inequality of household saving and business investment. In other words, all \( I=S/I=S \)-models are logically deficient and therefore a priori inapplicable in the world we live. IS-LM, for example, is simply an intellectual embarrassment.
9 Profit distribution

We return to the pure consumption economy. If, with distributed profit at first set to zero, consumption expenditures get ahead of wage income, i.e. $\rho E > 1$, the household and business sector’s transaction patterns diverge in period 2 as shown in Figure 6. The household sector’s current overdrafts increase until period end and, as a perfect mirror image, the business sector’s current deposits increase, too. Profit is equal to dissaving.

Figure 6: Dissaving leads in period 2 to an increase of the household sector’s current overdrafts and the business sector’s current deposits; at the beginning of period 3 et seq. profits are fully distributed

It is assumed that the subset of households with an expenditure ratio greater unity consolidates their overdrafts and takes up a loan at the (banking unit of the) central bank exactly at the beginning of period 3. This reduces the overdrafts of the subset to zero. The household sector as a whole switches from short term liabilities, in fact the shortest possible term, to longer term liabilities. With this we have put the initial overdrafts of period 2 out of sight and have brought the household sector back to the baseline.

The business sector posts a profit at the end of period 2 according to (15). It is assumed that this profit is fully distributed at the beginning of period 3. This reduces the business sector’s current deposits to zero and at the same time increases the deposits of a another subset of households by the same amount. It therefore holds in this example that distributed profit in period 3 is exactly equal to profit in period 2:

$$Y_{D3} = \Delta Q_{m2}. \quad (26)$$

This, of course, is the simplest case; the amount of distributed profit is at the discretion of the firm and need not be equal to profit (in the previous or current period).
period). In period 3 the households no longer dissave but spend their distributed profits. Total consumption expenditures are equal to total income, i.e. $\rho_E = 1$, as they were in period 1. From this follows the profit in period 3 as:

$$\Delta \bar{Q}_{m3} \equiv C_3 - Y_3 + Y_{D3} \Rightarrow \Delta \bar{Q}_{m3} = \Delta \bar{Q}_{m2}. \quad (27)$$

Profit in period 3 is exactly equal to the profit of the previous period. From (18) in turn follows that retained profit is zero. This pattern is repeated in period 4 and it is evident that this configuration is reproducible for an indefinite time span, provided that profits are, as a limiting case, fully distributed and fully spent in each successive period. The transaction pattern index $\kappa$ in (12), assumes different numerical values in period 2 and period 3. Subsequently it remains constant. This entails an increase of the average stock of transaction money beginning with period 2.

By applying the 1st axiom and the definitions (5) to eq. (15) one arrives at the general profit equation for the pure consumption economy:

$$\Delta \bar{Q}_m \equiv \left( \rho_E - \frac{1}{1 + \rho_D} \right) Y \mid t. \quad (28)$$

Overall monetary profit is positive if the expenditure ratio $\rho_E$ is > 1 or the distributed profit ratio $\rho_D$ is > 0, or both. In the case of budget balancing $\rho_E = 1$ profit for the business sector as a whole is alone determined by distributed profit. Profit becomes perfectly circular under the condition of full profit distribution and budget balancing. This limiting case, which is implicit in Debreu’s model, is reminiscent of what Keynes called the widow’s cruse.

10 The stock of money

If income is higher than consumption expenditures the household sector’s stock of money increases. The change in period $t$ is defined as:

$$\Delta \bar{M}_H \equiv^m Y - C \equiv^m Y (1 - \rho_E) \mid t. \quad (29)$$

The identity sign’s superscript $m$ indicates that the definition refers to the monetary sphere. There no change of stock if the expenditure ratio is unity.

The stock of money $\bar{M}_H$ at the end of an arbitrary number of periods $\bar{t}$ is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{M}_H \equiv \sum_{t=1}^{\bar{t}} \Delta \bar{M}_H + \bar{M}_{H0}. \quad (30)$$

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The interrelation between the expenditure ratio and the households sector’s stock of money, is then given by:

\[ \bar{M}_{Ht} = \sum_{t=1}^{T} Y_t (1 - \rho_{Et}) \quad \text{if} \quad \bar{M}_{H0} = 0. \]  

(31)

Formally, the expenditure ratio takes the role of the first derivative with \( \rho_E = 1 \leftrightarrow \frac{dy}{dx} = 0. \)

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

\[ \Delta \bar{M}_B \equiv m \cdot C - Y \mid t. \]  

(32)

The business sector’s stock of money at the end of an arbitrary number of periods is accordingly given by:

\[ \bar{M}_{Bt} = \sum_{t=1}^{T} \Delta \bar{M}_B + \bar{M}_{B0}. \]  

(33)

The development of the stock of money follows without further assumptions from the axioms and is determined by variations of the elementary variables \( P, X, W \) and \( L \). While the stock of money can be either positive or negative the quantity of money is always positive and given by:

\[ \bar{M}_t = \left| \sum_{t=1}^{T} \Delta \bar{M}_t \right| \quad \text{if} \quad \bar{M}_0 = 0. \]  

(34)

The quantity of money follows either from (31) or from (33).

From the definitions follows a strict parallelism between the stocks of money and the nominal magnitudes saving/dissaving and negative/positive retained profit:

\[ \Delta \bar{M}_B \equiv -\Delta \bar{M}_H \quad \text{if} \quad \bar{M}_0 = 0. \]  

(35)

Changes of the business sector’s stock of money are complementary to the household sector’s stock and in step with retained profit. Changes of the household sector’s stock of money are in turn complementary to the business sector’s stock and in step with saving/dissaving. Ultimately, these changes sum up to the quantity of money. The development of the household sector’s stock of money is depicted in Figure 8. The business sector’s stock is symmetrical.
11 The price setter’s task in a random environment

In the small and well-arranged world of (6) all variables are either set by the household or the business sector, except the sales ratio $\rho_X$ which resides in the no man’s land between the sectors. Accordingly, we treat it as the dependent variable and have now:

$$\rho_X = \frac{WR}{R} (1 + \rho_D) \frac{\rho_E}{P} \mid t. \tag{36}$$

This equation confronts the business sector with three possible outcomes. The sales ratio is $\rho_X < 1$, that is, the quantity bought by the household sector is less than the quantity produced by the business sector. As a consequence the stock of unsold products increases. Vice versa if $\rho_X > 1$. What we would like to see is, of course, market clearing $\rho_X = 1$ because this is our preconceived idea of how efficient markets operate. From this idea, however, does not follow that the product market has to be cleared in the period under consideration, but, loosely speaking, in the course of time.

The sales ratio $\rho_X$ depends on unit wage costs $\frac{WR}{R}$, on the income distribution $\rho_D$, on effective demand as defined by the expenditure ratio $\rho_E$, and finally on the price $P$. We first focus on the interdependency between demand and price and how they interact in the process of market clearing. By blanking out the rest of the consumption economy the formal representation simplifies to:

$$\rho_X = \frac{\rho_E}{P} \Theta \mid t. \tag{37}$$

with $\Theta = \frac{WR}{R} \left(1 + \frac{DN}{WL}\right)$ constant $\mid t$.

There are two ways to keep the factor $\Theta$ constant. Either each single variable remains fix or the respective proportions of the variables remain unaltered while the variables themselves change. If, for example, the wage rate $W$ moves in tandem with the productivity $R$, unit wage costs remain fix over time. For the beginning each variable of $\Theta$ is kept constant. This implies that labor input is fixed and by consequence the firm’s output too. Real supply stays put.

It is assumed now that the expenditure ratio fluctuates randomly around unity, such that over a longer time span there is no bias or permanent deviation in one direction or the other, i.e.

$$\frac{1}{n} \sum_{t=1}^{n} \rho_{Et} \sim 1. \tag{38}$$

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The households alternately save, i.e. $\rho_E < 1$, and dissave, i.e. $\rho_E > 1$, in an irregular sequence and after $n$ periods it is open whether cumulated consumption expenditures are greater, less or equal to cumulated income, i.e. the symmetric variations of the expenditure ratio do not necessarily lead to cumulated budget balancing, i.e.

$$\sum_{t=1}^{n} C_t \preceq \sum_{t=1}^{n} Y_t. \quad (39)$$

Logic demands that budget balancing must occur at some date before the end of time. At some date in the future, cumulated consumption expenditures must be equal to cumulated income or, in other words, cumulated saving and dissaving must be zero (for details see 2011a).

In order to clear the market in period $t$ the firm must set the price in (37) such that $\rho_X = 1$. Analytically this is no problem. We take market clearing as a condition and get the market clearing price as:

$$P^* = \Theta \rho_E \quad \text{if} \quad \rho_X = 1 \mid t. \quad (40)$$

The market clearing price moves in parallel with the random changes of the expenditure ratio or, in loose terms, with demand. All exogenous demand shocks are absorbed by the price, the rest of the system is not affected. There is no split of an expansive or contractive effect between price and output. The price setting has to be carried out at the beginning of period $t$ because the axioms refer to a period of a suitably defined length. It almost goes without saying that the firm does not know at period beginning what $\rho_E$ is going be. There is no foreknowledge of a random event. Hence $P \neq P^*$ and therefore $\rho_X \neq 1$. The product market is never cleared because it is beyond the price setter’s faculties to divine the market clearing price. What applies to the current period applies a fortiori to future periods. Because the product market is not cleared, the stock of products changes.

### 12 The stock of products

The change of the stock of – durable – products in period $t$ is defined as the excess between output $O$ and the quantity bought $X$ by the households:

$$\Delta \bar{O} \equiv O - X \equiv O(1 - \rho_X) \mid t. \quad (41)$$

The stock at the end of an arbitrary number of periods $\bar{t}$ is given by definition as the numerical integral of all previous stock changes plus the initial endowment:

$$\bar{O}_{\bar{t}} \equiv \sum_{t=1}^{\bar{t}} \Delta \bar{O}_t + \bar{O}_0. \quad (42)$$
The resulting interrelation between the sales ratio and the stock is given by

\[ \bar{O}_t \equiv \sum_{i=1}^{t} O_i (1 - \rho_{Xt}) \quad \text{if} \quad \bar{O}_0 = 0. \tag{43} \]

From this in combination with (37) follows that the stock of products ultimately depends on the development of \( \rho_E \) and \( P \).

Seen from the firm’s perspective, the stock at the end of period \( \bar{t} \) is either too large, too small, or just right. This depends on the firm’s target stock which is denoted by \( \bar{O}^\theta \). The firm’s objective is not to clear the market in the period under consideration, that is, to sell exactly the current output \( O \), but to bring the actual stock as close as possible to the target stock, i.e.

\[ \bar{O}_t - \bar{O}_t^\theta \to 0. \tag{44} \]

Only if the actual stock is exactly equal to the target stock the task in the subsequent periods reduces to market clearing in the narrow sense, i.e. to

\[ O - X = 0. \tag{45} \]

The development of the stock of products depends on \( \rho_X \), and that of the stock of money on \( \rho_E \). Both variables are connected via (37).

### 13 Price flexibility in the evolving product market

The fact that it has not been possible to build a process for the formation of equilibrium prices is disastrous when it is recalled that the fundamental task of theory is precisely to make coordination in the market intelligible. (Benetti and Cartelier, 1997, p. 213)

As the next logical step on our way to full generality we lift the simplifying assumption of a fixed employment and other restrictions, but keep the resulting factor \( \Theta \) unchanged. Eq. (36) turns to:

\[ \rho_X = \frac{W}{R} (1 + \rho_V \rho_N) \frac{\rho_E}{P} \]

with \( \rho_V \equiv \frac{D}{W}, \quad \rho_N \equiv \frac{N}{L} \mid t. \tag{46} \]

The factor \( \Theta \) remains constant if, first, the wage rate \( W \) moves in step with the productivity \( R \). This keeps unit wage costs stable. Second, the dividend \( D \) is
assumed to follow the wage rate \( W \), hence \( \rho_V \) is a constant. Finally, the number of shares \( N \) follows employment \( L \), hence \( \rho_N \) is also a constant. Both conditions together make that the income distribution does not change while the economy either grows or shrinks because \( \rho_D \equiv \rho_V \rho_N \). Distribution is a separate issue that has been dealt with elsewhere (for details see 2012a). While all constituent parts of the factor \( \Theta \) change, they do it in such a way that the factor itself remains constant.

Since the wage rate has, according to the inner logic of the system, been given the task of compensating random productivity variations it cannot be used for the coordination of the labor market. It is assumed here that employment \( L \) follows demand or, more precisely, the expenditure ratio. If \( \rho_E > 1 \) then \( L \) grows with a random rate of change \( L \) and vice versa if demand decreases (cf. (49)). This implies that additional labor is hired at the going wage rate \( W \). Changes of the wage rate depend ultimately on productivity variations and not on the accustomed conception of demand–supply in the labor market.

The price setting rule that is applied for simulation says that the very price is taken as an anchor in period \( t \) that would have sold current output plus the excess inventory in period \( t - 1 \) given the consumption expenditures in period \( t - 1 \). The precisely calculated price is then slightly modified by a symmetric random disturbance to account for all kinds of errors or frictions. Hence this rule of thumb lies somewhere in between a stochastic and a deterministic rule.

\[
P_t = P_{t-1} \left(1 + Pr \{ -x\% \leq \bar{P} \leq x\% \} \right)
\]

with \( P_{t-1} = \frac{C_{t-1}}{(\bar{O}_{t-1} - \bar{O}_{t-1}^p) + \bar{O}_{t-1}} \)

Figure 7 summarizes the resulting development in the product market. While the quantities produced and sold grow over the time span of observation with employment, the inventory keeps close to its target level. The price reacts in each period and straightens out all exogenous random variations. However, the flexible price remains roughly constant over the whole time span. Whether the market is cleared at the end of an arbitrary period \( \bar{t} \) or not can be read off the stock of products. Figure 7 shows the three market dimensions quantity, price, and time. It fully replaces demand-supply-equilibrium as theoretical representation of the evolving product market.

The price stability over the time span of observation is, of course, due to our assumptions. In the general case it follows from (46) under the condition of market clearing:

\[
P = \frac{W}{R} (1 + \rho_V \rho_N) \rho_E \quad \text{if} \quad \rho_X = 1 \quad |t.
\]

If the wage rate rises faster than productivity, if the dividend rises faster than the wage rate, if the number of shares rises faster than employment and if the
expenditure ratio rises above unity, then the market clearing price goes up. Eq. (48) captures the structural axiomatic theory of inflation. Three elements are crucial: unit wage costs, distribution expressed by the distributed profit ratio, and demand expressed by the expenditure ratio. Note that the quantity of money (34) is not among the price determinants of (48). This amounts to a repudiation of the quantity theory.

14 Employment as dependent variable

The period core (6) is neutral with regard to the direction of dependency and the notion of causality has no meaning in the structural axiomatic context. This physical analogy is illegitimate and misleading. Dependency is not a property of the axiom set but an add-on assumption that has to be justified on its own merits.

For analytical purposes we now change the direction of dependency. The crucial alteration in comparison to (36) consists in making employment \( L \) the dependent variable and imposing market clearing. The firm does not react with the price to random changes of the expenditure ratio but with an adaptation of employment. This implies that there is, ideally, no practical hindrance to the flexible adaptation of total working hours. We now have:

\[
L = \frac{DN}{\frac{PR}{PE} - W} \quad \text{if} \quad \rho_X = 1 \quad |t.
\]  

(49)
Under the condition of market clearing, employment in the pure consumption economy is dependent on distributed profit, the expenditure ratio, price, productivity and the wage rate. Rather unsurprisingly, employment moves in step with demand, expressed by the expenditure ratio. If the latter is taken as an indicator of effective demand eq. (49) can be applied as open interface to a Keynesian employment theory (for details see 2012b).

What, indeed, runs against the accustomed belief of market clearing in the labor market is that wage rate and employment also move in step, that is, cutting the wage rate is not conductive to higher employment according to (49), just the contrary. The application of the demand–supply–equilibrium scheme to the labor market is therefore more than ever beside the point because it ignores the interdependence with the product market. The point to take home is: all other variables in (49) fixed, an increase of the wage rate increases overall employment. This is a systemic property that does not depend upon what employers and employees (or economists, for that matter) think about wage rate changes. Eq. (49) is testable in principle.

From (49) follows for the real wage:

\[
\frac{W}{P} = \frac{R}{(1 + \rho_D)\rho_E} \quad \text{if} \quad \rho_X = 1 \mid t. \tag{50}
\]

The real wage is determined by the productivity, the income distribution, and the expenditure ratio. It does not depend on some fictional demand and supply schedules for the labor market.

Employment grows in Figure 7. Whether this leads to full employment depends on the concurrent growth of labor supply.

15 Full employment and price stability

There is little or nothing in existing micro- or macroeconomics texts that is of value for understanding real markets. (McCauley, 2006, p. 16)

Eq. (49) can be rewritten in the general form as:

\[
L = \frac{D}{P} \frac{N}{W} \frac{\rho_X}{\rho_E} \mid t. \tag{51}
\]
Now, let us fix all variables except the expenditure ratio $\rho_E$ and the price $P$. As before, the expenditure ratio varies at random and increases in the period under consideration by $x$ percent. If the price setter happens to increase the price also by $x$ percent the variations cancel out in the denominator and there is no effect on employment. If the price increase is less than $x$ percent employment increases, under the condition that all other variables in (51) stay put. The expansive demand effect is split between the nominal variable price and the real variable employment. It is obvious that any mixture is possible. The real effect of the demand expansion depends on the parallel price change. Ultimately, the price setting in the product market determines, for purely systemic reasons, employment in the labor market. This leads quite naturally to the question of what the full employment price looks like.

All that is formally necessary is to take (51) and to make the price the dependent variable. With full employment labor input $L^\theta$ inserted one gets:

$$P^* = \frac{\rho_E}{\rho_X} \rho_U \left( 1 + \rho_v \frac{N}{L^\theta} \right)$$

with $\rho_v \equiv \frac{D}{W}$, $\rho_U \equiv \frac{W}{R}$ and $\rho_{N}^\theta \equiv \frac{N^\theta}{L^\theta}$ | $t$.

In the full employment consumption economy, the ratio of $\rho_E$ and $\rho_X$ is unity. In other words, the random variations of the expenditure ratio are accompanied by exactly symmetric variations of the sales ratio. If demand goes up, i.e. $\rho_E > 1$, it is satisfied out of the stock of products, i.e. $\rho_X > 1$, the quantity sold is greater than the quantity produced. Vice versa, if the expenditure ratio falls below unity, price, output, and employment remain unchanged. It has to be emphasized that, contrary to the accustomed stories of the price mechanism, the price does not react to demand variations as in Figure 7. There is no price signal of any sort, and no market clearing in any given period.

The purely quantitative adaptation works satisfactorily if the inventory changes cancel out over a reasonable time span. This, of course, can pose a problem to the business sector. Condition (39) guarantees budget balancing but not necessarily within a time span that may appear reasonable for the business sector. This, of course, was the gaping hole in the argument of the ‘classics’ that Keynes exposed with his famous long-run-all-dead quip. From a simultaneous equilibrium model nothing can legitimately be inferred about an unspecified long run. Keynes, no doubt, was right on this crucial point and the ‘classics’ were wrong. Their intellectual heirs still are.

As a limiting case we have market clearing, i.e. $\rho_X = 1$, and budget balancing, i.e. $\rho_E = 1$, instead of $\frac{\rho_E}{\rho_X} = 1$. If budget balancing is assumed for each single period no random variations of the expenditure ratio around unity can occur. For the price $P^*$ this makes no difference.
Unit wage costs $\rho_U$ and the dividend-wage ratio $\rho_V$ are kept constant. This entails that the wage rate moves with productivity and the dividend with the wage rate. Random productivity changes are therefore completely neutralized. They have no effect on the price $P^*$. Remains employment. If we start from unemployment $L < L^\theta$ and increase labor input until full employment is established the price must fall under the condition that the number of shares is fixed. However, we can take the number of shares as a proxy for the firm’s size. Then, it is quite plausible that the number of shares moves in parallel with employment. This in turn implies that the ratio $\rho_N$ is a constant. Hence, as soon as the full employment ratio $\rho_N^\theta$ has been established, labor supply $L^\theta$ may vary for whatever reason, this is of no consequence for the price $P^*$. This quintessential price guarantees full employment in each period and is compatible with budget balancing and product market clearing in the course of time. The price remains unaltered from the first to the last period while the expenditure ratio, the full employment level and the productivity change at random. In the well-behaved consumption economy, ‘taste and technology’ are subject to random shocks, yet the price is an absolute systemic invariant as can be seen in Figure 8.

Figure 8: The well-behaved consumption economy; the arrows indicate (cumulated) budget balancing/market clearing in a regime of continuous full employment and price stability with random variations of labor supply, productivity and the expenditure ratio.

The expenditure ratio hovers randomly around unity. Saving and dissaving alternate unpredictably. Accordingly, the household sector’s stock of money (31) switches in the course of time between deposits and overdrafts. The business sector’s stock of money (33), which is not shown here, is symmetrical. Changes of the quantity of money are adaptive and have no effect on the price. This conforms to the ideal that money should have no influence on the ‘real’ economy. The arrows in Figure 8 indicate when the quantity of money is zero. Problems arise if the central bank,
or the banking industry in the general case, does not accommodate. In principle, this problem should not arise because the business sector’s overdrafts are always ‘covered’ by the stock of products. This, it will be remembered, was a central tenet of the banking school. However, this argument presupposes that inventories liquidate within a short time span at the actual market price. At this point risk comes into the picture because the length to the next market clearing is unknown.

The business sector reacts to all demand variations with the quantity sold and never with the price. The product market is not cleared in any single period, the business sector’s inventory varies as shown in Figure 8. The variations of the stock of products and the stock of money are coupled via (37). In each period, the monetary and the real side are synchronized because of \( \rho_E \rho_X = 1 \).

Budget balancing and market clearing are achieved uno actu in the course of time without any price adaptations. The arrows in Figure 8 indicate the return of the monetary and real stocks to their initial levels. All depends on whether the business sector can cope with the inventory fluctuations that are caused by the random alterations of the households’ saving and dissaving. Problems arise whenever the saving phase lasts too long. Note that the timing of saving and dissaving, which has been taken as random, may as well be due to intertemporal optimization of the households’ consumption pattern. For the business sector it makes no difference at all whether chance rules or the households optimize.

In sum, the well-behaved consumption economy exhibits all desirable properties including an invariant income distribution. It is important to recall that absolute price stability presupposes that the wage rate moves with productivity and the dividend with the wage rate. In the final analysis the salient point is that random budget balancing recurs within ‘reasonable’ intervals. It is not sufficient that the variations of the expenditure ratio are symmetric. If the equilibrists tell us that Figure 8 is what they always talked about and meant with natural (Smith), necessary (Mill), or normal price (Marshall) but could not express in a formally correct way, this would be acceptable. After all, they lacked structural axioms. Figure 8 adumbrates that quantitative adaptation is more in keeping with structural conditions than price adaptation. Product price flexibility is not the alpha and omega of the market system. It is, rather, an indication that it is beyond the price setter’s capabilities to find and implement the quintessential price. Seen from an ideal economy, product price flexibility is an error correction mechanism which, however, does not eliminate error. In the ideal market economy the price mechanism plays no role at all in the product market, only the quantity mechanism.

Eq. (28) tells us that monetary profit, too, varies with the expenditure ratio. The well-behaved consumption economy entails therefore that the firm does not react with price or employment adaptations to changes of monetary profit. These changes are, under certain conditions, exactly compensated by changes of nonmonetary profit.
16 Nonmonetary profit

For the full specification of profit the set of axioms is extended because additional variables have to be introduced. The 5th axiom states that total profit has a monetary and nonmonetary component:

\[ \Delta \bar{Q} = \Delta \bar{Q}_m + \Delta \bar{Q}_n \]  \hspace{1cm} (53)

Nonmonetary profit is defined as the difference between the valued increase of the stock of products in period \( t \) and the increase or decrease of the existing stock’s value due to changes of quantities and valuation prices which is captured by \( G_B \):

\[ \Delta \bar{Q}_n \equiv P(O - X) + G_B | t. \]  \hspace{1cm} (54)

If more goods are produced than sold in period \( t \), i.e. \( O > X \), the stock of products rises according to (41) and accumulates according to (42). It is, of course, possible that more units are sold than produced in a period, i.e. \( O < X \). In this case the products are taken from the inventory. The period changes build up or take down the stock of products that therefore consists of different vintages. Initially, the valuation price of each vintage is \( P \) but it change over time. These changes come as appreciation or depreciation:

\[ G_B \equiv G_B^+ - G_B^- | t. \]  \hspace{1cm} (55)

Changes of the inventory’s value originate from the change of the quantity and the valuation price \( B \) of each hitherto unsold vintage. For the subset of items with a decrease in value taken together the depreciation is given by:

\[ G_B^- \equiv \sum_{h=1}^{t} (B_{ht} \bar{X}_{ht} - B_{ht-1} \bar{X}_{ht-1}) \quad \text{with} \quad B_{ht} < B_{ht-1} | t. \]  \hspace{1cm} (56)

For the subset of items with an increase of value taken together the appreciation is given by:

\[ G_B^+ \equiv \sum_{h=1}^{t} (B_{ht} \bar{X}_{ht} - B_{ht-1} \bar{X}_{ht-1}) \quad \text{with} \quad B_{ht} \geq B_{ht-1} | t. \]  \hspace{1cm} (57)

The valuation price \( B \) is introduced as a new variable with the 5th axiom (53). The firm has some leeway in the valuation of its stock of products. So \( B \) usually differs from the market price \( P \). Whether the firm’s internal valuation prices are realistic or not remains to be seen until the respective vintage is brought to market.
In periods with an increase of the stock of products total profit (53) is higher than
monetary profit and vice versa when the stock decreases. Summed over all periods
nonmonetary profits and losses are zero when the market is momentarily cleared in
some period $t$. In this case the sum of total profits is equal to the sum of monetary
profits. Nonmonetary profits cancel out. By the same token arbitrary valuations
automatically cancel out over time and produce not much more than a time shift of
nonmonetary profits.

Taking (15) and (54) into account the profit axiom (53) in its explicit form finally
reads:

$$\Delta \tilde{Q} = (C - Y + Y_D) + P(O - X) + G_B | t. \quad (58)$$

The equation summarizes the twofold process that generates the business sector’s
valued stock of products and the stock of money until period $t$. This boils down to
the explicit form of the 5th axiom:

$$\Delta \tilde{Q} = PO - Y + Y_D + G_B | t. \quad (59)$$

Total profit is given as the difference of the valued output and total income, plus
distributed profit, plus changes of the value of the stock of products. Value changes
of inventory cancel out over time. If they are zero in a certain period total profit is
given by:

$$\Delta \tilde{Q} = PO - Y_W | t. \quad (60)$$

In the simplest case total profit is the difference between the market value of output
and wage income. Hence, if the random variations of the expenditure ratio affect but
the quantity sold $X$, changes of monetary profit are compensated for by changes of
nonmonetary profit and total profit remains constant according to (60). This implies
the assumption that the firm’s behavior depends on total and not on monetary profit.
This makes no difference if the market is cleared in the period under consideration,
which is a rare event indeed.

17 Differentiation and heterogeneity

The assumption that the business sector consists of one single firm can only be
justified if it is eventually removed and if it can be demonstrated that the results so far
derived still hold as limiting cases of more general relationships. The differentiation
of axioms does not change them, it makes them only a bit more complex.
17.1 Two firms, zero profit

At first, the axioms and definitions have to be differentiated. Period income changes from (1) to:

\[
Y = \frac{W_A}{w} L_A + \frac{W_B}{w} L_B + \frac{D_A N_A + D_B N_B}{g_0} | t. \tag{61}
\]

The total labor input \( L \) is now allocated between two firms:

\[
L = L_A + L_B | t. \tag{62}
\]

It is assumed that the initial full employment labor input \( L_0 \) remains unchanged.

Since distributed profits are set to zero in order to keep things simple for the beginning, and the wage rates of the two firms are assumed to be identical, total income does not change with a reallocation of labor input between firms. And since initial full employment is maintained by assumption only the composition of the business sector’s output changes with a reallocation of labor input.

The partitioning of the consumption expenditures in period \( t \) is given by:

\[
C_A = P_A X_A \quad C_B = P_B X_B. \tag{63}
\]

For the relative prices of two products then follows directly from (63) in combination with the differentiated sales ratio (5):

\[
P_A = \frac{R_B L_B C_A}{R_A L_A C_B} \quad P_B = \frac{R_B L_B C_B}{R_A L_A C_B} \tag{64}
\]

if \( \rho_{XA} = 1, \rho_{XB} = 1 \) \( | t \).

If the markets for both products are cleared the relation of prices is inversely proportional to the relation of productivities and the relation of labor inputs and directly proportional to the relation of consumption expenditures for the two products. A straightforward result materializes if the labor inputs of the two firms stand in the same proportion as the household sector’s expenditures for both products:

\[
\frac{P_A^0}{P_B^0} = \frac{R_B}{R_A} \quad \text{if} \quad \frac{L_A}{L_B} = \frac{\rho_{EA}}{\rho_{EB}} \quad \text{and} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad | t. \tag{65}
\]
If labor input is allocated according to the consumers’ preferences, which are revealed by their consumption expenditures, or what amounts to the same, their expenditure ratios, and markets are cleared then relative prices $\frac{p_A}{p_B}$ are inversely proportional to the productivities in the two lines of production. The productivities are measurable in principle. Relative prices depend in the simplest case only on the objective ratio of productivities. The subjective partitioning of consumption expenditures has no effect on relative prices if it corresponds to the allocation of labor input. We refer to this unique configuration of labor inputs and expenditure ratios as the *competitive structure*.

From (65) in combination with (62) follows under the condition of budget balancing:

$$\frac{L_A}{L_0 - L_A} = \frac{\rho_{EA}}{1 - \rho_{EA}} \Rightarrow L_A = \rho_{EA} L_0$$

(66)

if $\rho_{EA} + \rho_{EB} = 1$ \(\forall t\).

The employment of firm $A$ is determined by that part of total income that the households spend on product $A$. Under the condition of full employment the labor input of firm $B$ is then also known.

Since distributed profit is zero according to (61) and the overall expenditure ratio is unity according to (66), profit for the business sector as a whole is zero according to (15).

For firm $A$ the profit equation (14) reads in the case of market clearing:

$$\Delta \bar{Q}_{mA} \equiv P_A R_A L_A \left(1 - \frac{W_A}{P_A R_A}\right)$$

(67)

if $\rho_{XA} = 1$ \(\forall t\).

Monetary profit of firm $A$ is zero under the condition that the quotient of wage rate, price, and productivity is unity, i.e. $\rho_{FA} = 1$. This holds independently of the level of employment or the size of the firm. From the zero profit condition follows:

$$P_A^\circ = \frac{W_A}{R_A} \mid t.$$  

(68)

The price is, unsurprisingly, equal to unit wage costs. In the same way we get the price $P_B^\circ$. Taken together, the zero profit condition yields for relative prices:

$$\frac{P_A^\circ}{P_B^\circ} = \frac{W_A}{R_A} = \frac{W}{R_A} = \frac{R_B}{R_B} = \frac{R_B}{R_B}$$

(69)

if $\Delta \bar{Q}_{mA} = 0, \Delta \bar{Q}_{mB} = 0, \rho_{EA} + \rho_{EB} = 1, W_A = W_B = W$
In general, it holds for the zero profit case that relative prices are equal to the relation of unit wage costs. In the limiting case of equal wage rates this boils down to the equality of relative prices and inverse productivities. In the latter case we have then both, zero profit according to (69) and a competitive structure according to (65).

In other words, in the competitive structure with employment in each line of production in strict proportion to the expenditure ratios, with zero profits of both firms, and equal wage rates, the relation of prices is unequivocally determined by the inverse productivities, independently of the partitioning of the consumption expenditures. No matter how the household sector distributes total income between the two products, the markets are cleared, wage rates are equal, and profits are zero. The full employment labor input is allocated according to the preferences of the households. In this benchmark consumption economy, relative prices do not depend on tastes, only on technology, i.e. on productivities. The subjective element plays a role in the allocation of labor but is ultimately irrelevant for the exchange value. Subjective value theories remain on the surface.

17.2 Two firms, equal profit ratios

Total income in the general case with two firms follows from (1) as:

\[ Y = W_A L_A + W_B L_B + D_A N_A + D_B N_B \]  \hspace{1cm} (70)

Monetary profit is given by (15). To get rid of all absolute magnitudes, the profit ratio \( \rho_Q \) is defined with (71) and this gives a succinct summary of the structural interrelation of the profit ratio, the expenditure ratio, and the distributed profit ratio for the business sector as a whole:

\[ \rho_Q \equiv \frac{\Delta \tilde{Q}_m}{Y_W} \Rightarrow \rho_Q \equiv \rho_E (1 + \rho_D) - 1 \hspace{1cm} (71) \]

The overall profit ratio is positive if the expenditure ratio \( \rho_E \) is > 1 or the distributed profit ratio \( \rho_D \) is > 0, or both.

For the comparison of firms with different size and different absolute profits the respective profit ratios are required. The profit ratio for the business sector as a whole is adapted for a single firm as follows:

\[ \rho_{QA} \equiv \frac{\Delta \tilde{Q}_{mA}}{W_A L_A} \hspace{1cm} \]  \hspace{1cm} (72)  

We now turn the question around and ask for the implications of equal profit ratios in the general case of different wage rates. Eq. (72) can be rewritten for both firms:
\[
\rho_{QA} \equiv \frac{\rho_{EA}Y}{W_AL_A} - 1
\]

\[
\rho_{QB} \equiv \frac{\rho_{EB}Y}{W_BL_B} - 1
\]

Under the condition of equal profit ratios this yields:

\[
\frac{W_AL_A}{W_BL_B} = \frac{\rho_{EA}}{\rho_{EB}} |t. \quad (74)
\]

Equalization demands that the weighted labor inputs must be in proportion to the expenditure ratios. If wages rates are equal we are back at the initial condition for the competitive structure (65). Eq. (74) is taken as the general condition for the competitive structure. Inserted in (64) the price relation in the competitive structure with equalized profit ratios by consequence is:

\[
\frac{P_A^c}{P_B^c} = \frac{W_A}{R_A} \frac{R_B}{W_B}
\]

if \( \frac{W_AL_A}{W_BL_B} = \frac{\rho_{EA}}{\rho_{EB}} \) and if \( \rho_{XA} = 1, \rho_{XB} = 1 \) |t. \quad (75)

Relative prices are equal to the relation of unit wage costs. This is the general case. If wage rates are equal, then relative prices are equal to the inverse productivities as in (65). In both cases the profit ratios are equal, in the simplest case they are zero.

Different profit ratios in different lines of production do not jeopardize the functioning of the system as a whole but must be taken as empirical normality. Profit ratio equalization is a theoretical requirement. The market economy can exist for an indefinite time without equal profit ratios but not with losses. Positive profit is the sine qua non. If the expenditure ratio \( \rho_E \) is unity and the distributed profit ratio \( \rho_D \) is zero then the profit ratio for the business sector is zero. If profit ratios are not equal in this zero-profit economy the profit of one firm is equal to the loss of the other and this is not a sustainable configuration.

When market clearing, budget balancing, and the equalization of profit ratios is assumed then the only subjectively chosen variable is the expenditure ratio for one product. The rest of the system is in this case determined by objective conditions. This economy deserves the predicate optimal because the partitioning of consumption expenditures can always be interpreted as optimal. In the case of budget balancing total profit is equal to distributed profit.

The optimal competitive structure can obviously be generalized for an arbitrary number of firms and products. In marked contrast to the classical approach the
structural axiomatic approach asserts that a perfect competitive structure with all desirable properties is possible but not that the economy will attain this state sooner or later. This, though, is not a matter of primary concern. With regard to the proper functioning of the market economy the critical condition is that the expenditure ratio has to be greater than unity and/or the distributed profit ratio has to be greater than zero because a zero-profit economy – Walras’s ‘Ni bénéfice ni perte’ – is not reproducible with more than one firm (for details see 2011e, pp. 10-14).

While an economy with equal profit ratios is an ideal but acceptable analytical construct because it could work, it is an unresolved mystery in the history of economic thought how Walras’s zero-profit equilibrium could ever be taken serious for more than one second.

17.3 Wage differentiation within the firm

That the wage rate of the 1st axiom is equal for all employees is, of course, only a preliminary simplification. For one firm with \( n \) employees with individual wage rates and working hours we differentiate as follows:

\[
W_L \equiv W_1L_1 + W_2L_2 + \ldots + W_nL_n \mid t. \tag{76}
\]

The average wage rate is then given by the weighted individual wage rate:

\[
W \equiv \frac{W_1L_1}{L} + \frac{W_2L_2}{L} + \ldots + \frac{W_nL_n}{L} \mid t. \tag{77}
\]

All wage rates, which are sorted in descending order, are then expressed in relation to the lowest wage rate:

\[
W \equiv \left( \frac{W_1L_1}{W_nL} + \frac{W_2L_2}{W_nL} + \ldots + \frac{L_n}{W_nL} \right)W_n \mid t. \tag{78}
\]

The average wage rate is then composed of a dimensionless weighting factor, that takes different values depending on whether the wage structure is more equal or unequal, and the minimum wage rate:

\[
W \equiv (w_1l_1 + w_2l_2 + \ldots + l_n)W_{\min} \mid t. \tag{79}
\]

The average wage rate for each firm can then be expressed as a combination of a wage structure index and the minimum wage rate for the economy as a whole:

\[
W_A \equiv \Omega_{WA}W_{\min} \mid t. \tag{80}
\]
The market clearing price in the simplest case (7) reads now:

\[ P = \frac{\Omega_W}{R} \underbrace{W_{\text{min}}}_{\text{real}} \]

(81)

if \( \rho_X = 1, \rho_E = 1, \rho_D = 0 \mid t. \)

The market clearing price is dependent on a real and a nominal magnitude. The real magnitude in turn depends on the actual wage structure, which is defined by the firm, and the production conditions, which are objectively given in the period under consideration. The nominal magnitude is in principle arbitrary and has to be defined exogenously. It is assumed here that the minimum wage rate is treated as the nominal numéraire and initially fixated by the central bank. As a matter of principle, eq. (81) enables the central bank to directly stabilize the price level without touching the firm’s wage structure or manipulating the rate of interest or the quantity of money, that is, without violating the neutrality of money.

For the average real wage (7) it makes no difference whether the central bank fixates the minimum wage rate at one, ten, or hundred Euro, Dollar or Yen. The minimum real wage rate depends in the simplest case on the actual productivity and the wage structure index:

\[ \frac{W_{\text{min}}}{P} = \frac{R}{\Omega_W} \]

(82)

if \( \rho_X = 1, \rho_E = 1, \rho_D = 0 \mid t. \)

The minimum real wage is, given the production conditions, the higher the more equal the wage rates among employees are. This holds mutatis mutandis in the general case (50):

\[ \frac{W_{\text{min}}}{P} = \frac{R}{\Omega_W (1 + \rho_D) \rho_E} \]

(83)

if \( \rho_X = 1 \mid t. \)

For the real minimum wage it makes no difference how the central bank fixates the nominal minimum wage rate because its determinants are productivity, the wage structure, the distributed profit ratio, and the expenditure ratio. The nominal minimum wage rate, though, is decisive for the development of the market clearing price. Hence, eq. (81) determines the conditions for price stability in the product market with the minimum wage rate as an instrument variable.
18 Financial services, interest, portfolios

Neo-Walrasian analysis cannot accommodate money because it cannot accommodate any kind of endogenously sensible institutional set-up. (Clower and Howitt, 1997, p. 29)

18.1 The price of money transactions

The business sector consists now of a consumption goods producing firm A and the central bank as the second firm B. The central bank stands here for the whole banking industry. To begin with, the central bank handles only the money transactions. Total employment is given by (62). To focus exclusively on relative prices variations of total employment are excluded. Initial full employment labor input remains unchanged.

Total income consists according to (1) of wage income and distributed profit. To simplify the analysis the wage rates for all firms are set equal. Distributed profits are at first zero:

\[
Y = \frac{W_A}{W} L_A + \frac{W_B}{W} L_B + \underbrace{(D_A N_A + D_B N_B)}_{Y_0 = 0} \quad |t. \]  

(84)

The household sector apportions its consumption expenditures between the purchase of consumption goods and the purchase of transaction services. With \(X_B\) the number of transactions per period that are carried out by the central bank on behalf of the households is denoted:

\[
C = P_A X_A + P_B X_B \quad |t. \]  

(85)

Consumption expenditures are equal to total income, i.e. \(\rho_E = 1\). The households neither save nor dissave.

Overall monetary profit (14) is differentiated for the two firms:

\[
\Delta \bar{Q}_{mA} = P_A X_A - W L_A \quad |t. \]  

\[
\Delta \bar{Q}_{mB} = P_B X_B - W L_B \quad |t. \]  

(86)

Under the condition of market clearing, i.e. \(\rho_X = 1\), this can be rewritten as:

\[
\Delta \bar{Q}_{mA} = P_A R_A L_A \left(1 - \frac{W}{P_A R_A}\right) \quad \text{if} \quad \rho_{XA} = 1 \quad |t. \]  

\[
\Delta \bar{Q}_{mB} = P_B R_B L_B \left(1 - \frac{W}{P_B R_B}\right) \quad \text{if} \quad \rho_{XB} = 1 \]  

(87)
Overall profits are zero because of \( \rho_E = 1 \) and \( \rho_D = 0 \). The zero profit condition for a single firm reads \( \frac{W}{\tau_E} = 1 \). Under this conditions follows from (87) that absolute prices are equal to unit wage costs, i.e. \( P_A = \frac{W}{\tau_A} \) respectively \( P_B = \frac{W}{\tau_B} \).

In sum: both markets are cleared, the household sector’s budget is balanced and profits are zero for both the consumption goods producing firm and the transaction unit of the central bank. Money transactions consume resources, the less so the higher the productivity of the transaction unit is. The price the households pay for each money transaction \( P_B \) follows from (87) and the zero profit condition. In the general case, the banking industry makes a profit or a loss on selling financial services taken in the broadest sense. This profit or loss is independent of the rate of interest or of value changes of assets and liabilities.

18.2 Household loans and the interest rate

The inclusion of the banking unit entails that the given resources of the business sector \( L \) have first to be reallocated:

\[
L \equiv L_A + L_B + L_C \quad |t.
\]  

(88)

As a consequence total income is then given by:

\[
Y = \frac{W_A}{W} L_A + \frac{W_B}{W} L_B + \frac{W_C}{W} L_C + \left( \frac{D_A N_A + D_B N_B + D_C N_C}{\gamma_D = 0} \right) \quad |t.
\]  

(89)

The interest payments of the household sector to the banking unit for the processing of one-period loans have to be subsumed under consumption expenditures:

\[
C = P_A X_A + P_B X_B + l_c \bar{\lambda}_C \quad |t.
\]  

(90)

\[
C = C_A + C_B + C_C
\]

The quantity bought from the banking unit \( X_C \) can here be replaced by the total amount of the loans \( \bar{\lambda}_C \) (for a more detailed derivation of the rate of interest from the differentiated axiom set see 2011b, Sec. 7).

The reallocation of labor input is neutral with regard to the price of the consumption good. When labor input \( L_C \) is taken away from firm \( A \) output falls. At the same time consumption expenditures are redirected away from purchases of consumption goods to purchases of the loan services of the banking unit, i.e. \( C_A \) goes down and \( C_C \) goes up. This leaves the price of the consumption good unaffected under the given conditions. The household sector buys less consumption goods and more banking services. According to this demand shift the unaltered total labor input is reallocated.
Profit for each firm is zero in the first approximation, i.e. \( \frac{W}{P_R} = 1 \):

\[
\Delta Q_{mA} = P_A R_A L_A \left( 1 - \frac{W}{P_A R_A} \right) \quad \text{if} \quad \rho_{XA} = 1
\]

\[
\Delta Q_{mB} = P_B R_B L_B \left( 1 - \frac{W}{P_B R_B} \right) \quad \text{if} \quad \rho_{XB} = 1
\]

\[
\Delta Q_{mC} = l_C \tilde{A}_C \left( 1 - \frac{W}{\tilde{A}_C L_C} \right) \quad \text{if} \quad \rho_{XC} = 1
\]

The zero profit conditions and the market clearing condition define the product price, the transaction price and the rate of interest. All are equal to the respective unit wage costs. The rate of interest is in the simplest case given by:

\[
l_C = \frac{W}{\tilde{A}_C} = \frac{W}{R_C^e} = \frac{I_C}{L_C}
\]

The production of loans is not much different from the production of goods and services. The inclusion of the banking unit and the appearance of interest results in a reallocation of demand and resources. The loan interest rate is, at first, alone determined by the production conditions of the banking unit. The same holds for the price of the consumption good \( P_A \) and the price of a monetary transaction \( P_B \). All firms recoup their costs. Interest payments of the households on the loans are equal to wage income in the banking unit. All relative prices are objectively determined by the respective productivities. The case for business loans is analogous (for details see 2011d, Sec. 1).

The relation of the rate of interest to the product price is given by:

\[
\frac{l_C}{P_A} = \frac{R_A}{R_C^e}
\]

What may be called the real rate of interest depends, in the elementary zero profit case, on the production conditions in the consumption goods industry and the banking industry, respectively.

### 18.3 Savers, dissavers, and the average expenditure ratio

Changes of the quantity of money depend on the transactions between the household and the business sector as given by (29) and (32). This, however, is only part of the story. By differentiation of the household sector one gets:
If households $A$ dissave and households $B$ save, such that saving exactly equals dis-saving in period $t$, then the sum of (94) is zero, the overall expenditure ratio is unity, and the business sector is in no way affected. Overall consumption expenditures do not change but the stocks of deposits and overdrafts do and with it the quantity of money. The current deposits grow as shown in the upper half of Figure 9.

In periods 1-10 saving and dissaving within the household sector is perfectly symmetrical. This is an idealized configuration. Actually, such a perfect synchronization is impossible. Either saving exceeds dissaving or vice versa. That is, the overall expenditure ratio is invariably different from unity and this in turn affects the business sector. A general theory must take this into account.

In periods 10-20 the expenditure ratio of households $B$ is lowered. Higher saving accelerates the growth of deposits in the decade under consideration. The dissaving of households $A$ remains unchanged. The respective stocks are calculated analogous to (30). Total consumption expenditures fall. Under the condition of market clearing the price is reduced according to (17) and the business sector, coming from an initial zero profit situation, now makes a loss according to (15). The loss in each period successively increases the business sector’s overdrafts. Both sides of the central bank’s balance sheet are, trivially, equal at any point in time. For every borrower there is a lender, albeit indirectly.

In periods 20-30 the situation is reversed. The expenditure ratio of households $B$ is increased. The household sector as a whole now dissaves and the business sector
makes a profit. These profits successively reduce the business sector’s overdrafts. At the end of period $30$ the overdrafts are zero. Now the business sector switches to the other side. Subsequently, it accumulates current deposits. At the end of period $40$ the quantity of money consists of the deposits of households $B$ and the business sector’s retained profits.

The general formula for the quantity of money becomes a bit more complex. Eq. (34) evolves to:

$$\bar{M}_t \equiv H [\bar{M}_{HA}] \bar{M}_{HA} + H [\bar{M}_{HB}] \bar{M}_{HB} + H [\bar{M}_{B}] \bar{M}_{B}. \quad (95)$$

The quantity of money is the sum of the deposits of households $A$ and $B$ and of the business sector. To exclude overdrafts in any period, the discrete Heaviside function is applied:

$$H [\bar{M}_t] = \begin{cases} 0, & \bar{M}_t < 0 \\ 1, & \bar{M}_t \geq 0 \end{cases}. \quad (96)$$

Note well that from the foregoing analysis follows a relation between the quantity of money and the market clearing price. This relation, though, does not confirm the commonplace quantity theory, but completely replaces it.

From

$$C \equiv C_A + C_B \quad \Rightarrow \quad \rho_E \equiv \rho_{EA} + \rho_{EB} \quad (97)$$

in combination with (17) follows

$$P = (\rho_{EA} + \rho_{EB}) \frac{W}{R} \quad \text{if} \quad \rho_X = 1, \rho_D = 0 \quad |t. \quad (98)$$

The expenditure ratios in turn affect the development of the respective stocks in (95). In the limiting case of $\rho_{EA} + \rho_{EB} = 1$ the quantity of money changes according to (94), yet there is no effect on the price according to (98). In the general case of $\rho_{EA} + \rho_{EB} \neq 1$ a correlation between product price and changes of the quantity of money emerges from the structural axiomatic set. This, of course, does not amount to a corroboration of the quantity theory.

### 18.4 Differentiation of portfolios

In the first step the transactions between the sectors and the apportionment of saving and dissaving within the household sector affects the development of current deposits and overdrafts at the central bank. In the second step additional possibilities to hold money and to incur debt have to be introduced. In the real world the owners
of deposits have a virtually unlimited variety of financial products at their disposal. All of them have to be, in the course of time, formally integrated into the structural axiomatic framework.

For a start we offer the households interest bearing savings account at the central bank as an alternative to simply holding current deposits. Figure 10a gives an example of the households’ dispositions and their effects on the quantity of money.

![Graph](image1)

(a) Changing magnitude and composition of the household sector’s portfolio

(b) Switches from current deposits into savings accounts and back (refers to (a))

**Figure 10:** The household sector restructures over time its portfolio of financial assets and liabilities which is the exact mirror image of the central bank’s balance sheet

To simplify matters, the interactions with the consumption goods producing industry are blanked out. The expenditure ratio is set to unity in each period and this implies that saving and dissaving are equal throughout. In the first ten periods the complementary groups of households simply accumulate deposits and overdrafts.

In period 10 the households with deposits at their disposal put some money in savings accounts at the central bank, which represents at the moment the banking industry. The basic characteristic of these accounts is that the interest rate is fixed for a
predetermined time span. The households give up liquidity and get interest in return. In period $t_{14}$ the households reduce their current deposit to zero. Figure 10b isolates the development of the savings accounts. It is assumed that the central bank offers a rate of interest and that the households decide about how much of their total deposits to invest in savings accounts of different maturity.

The households with overdrafts take up a longer term loan in period $t_{12}$ and then again in period $t_{18}$. At the end of period $t_{20}$ the total portfolio of the household sector consists of longer term loans $b$, overdrafts, savings accounts $a$, and current deposits. The term structure of the central bank’s balance sheet is not congruent, i.e. the longer term loans are not matched by longer term savings accounts.

None of the financial transactions of Figure 10 has any effect on the rest of the economy. The changing quantity of money, in particular, has no effect on the price of the consumption good. The complete decoupling of the financial sphere from the production sphere is due to an expenditure ratio of unity and the neglect of interest payments (for details see 2011c; 2011d).

Figure 10 shows transactions on the financial markets but has no similarity with the demand-supply-equilibrium cross. There is no similarity with the product market of Figure 1 either. It could be said that the market for longer term savings $a$ in Figure 10a is complementary to the money market, and that the market for longer term loans $b$ is complementary to the short term overdrafts market, and finally, that the interdependence of markets is established by the fact that the market for assets always has the same total volume as the market for liabilities. However, the indiscriminate use of the term market tends to evoke the impression that there is such a thing as a generic market and that this market can be represented by demand and supply schedules. This is an optical illusion. There are at least two markets that display an entirely different structure. The product market is, in the strict sense, not comparable to, say, the money market or the bond market to which we turn next. The market system, then, consists of interrelated markets that are qualitatively different and not representable by a one-fits-all demand-supply cross. The economist’s box of tools urgently needs retooling.

19 The bond market

We take Figure 10a again as a starting point and assume now that the central bank sells a bond to the household sector in period $t_{10}$. Basically, bonds, savings accounts or deposits are only different forms of central bank liabilities. The initial stock of total deposits $c$ is owned by three households. For the sake of simplicity it is assumed that the amounts are equal. The initial situation of the households $A, B, C$ is depicted as step $0$ in Figure 11. Each household starts with 150 money units.

With step $1$ household $A$ buys one bond from the central bank and pays 100 units. Analogous to Figure 10 the stock of deposits decreases and the stock of bonds
Wealth creation in the bond market increases by the same amount. Financial wealth, here defined as the sum of money (=

= deposits) and bonds, remains unchanged by this financial transaction.

The terms of the bond are as follows: irredeemable but with with a clause to the effect that repayment at par is possible in the case of a national emergency. This eventuality is regarded by the market participants as an extremely improbable event and simply ignored in the sequel. The coupon is fix until redemption.

With the issuance of the the first bond the bond market comes into existence and gains a life of its own. Buying and selling bonds among households is independent from the central bank. However, the central bank may buy the bond at the current market price whenever it wishes and thereby increase the liquidity of the market.

With step 2 household A sells to B for 110 units. Why B pays 110 percent is left open here. One possible motive is for instance that B expects a fall of the rate of interest and that he can sell the bond at a higher price in the future. Agent A, of course, does not think so. Whatever the subjective motivations of buyer and seller, the bond vanishes from A’s portfolio and the current deposit increases to 160 units. The household sector’s financial wealth increases from 450 to 460 units. There is no change at all of the sum of deposits. The wealth effect increases via nonmonetary saving, analogous to (107), the net worth of the household sector. The quantity of money remains constant and there is no effect on the product market. Household A keeps the extra money in the portfolio and does not spend it on the consumption good.
With step 3, household B sells to C for 120 units. Financial wealth as sum of deposits and bonds increases again. For the further development several roads are open. The simplest case is that C finds a next buyer who pays 120 units. In this case household C has only passed on liquidity for a limited time span. The market price can remain unchanged over a couple of transactions. With these transactions the individual households switch between a lower and higher liquidity. Nothing else changes. The belief in the existence of potential next buyers stabilizes the market price at the level of 120. The household sector has effectively created wealth out of nothing for a certain time span. The effects last as long as there is a potential next buyer at the current or a higher price.

Let us assume, for the sake of argument, that there are more than three households and that the market price continues to increase with every transaction. In this linear expansion the price cannot rise higher than 150. However, if we allow for a circle A could now pay 160 units. The market price does not depend on some intrinsic value, the value is in the head of the potential next buyers and their ability to put (own or other people’s) money on the table. The question of whether the price in the bond market is an indication of a bubble is empty. As long as there are potential next buyers at the actual price, whatever it is, no bubble exists. When the potential next buyers vanish, for whatever reason, the nonexistent bubble bursts.

Note that credit is excluded at the moment. There is no injection of additional money. The total amount of deposits remains constant. The wealth effect depends alone on the subjective valuation of market participants, with the distribution of deposits defining the upper constraint. It is not the sheer number of participants that is decisive for price formation but the more or less equal distribution of the quantity of money. With an unequal distribution the market price can be higher but the number of potential next buyers tends to zero, therefore the price is less stable. If we change the initial distribution of deposits to 100, 110, 240, the upper limit for the bond price changes from 150 to 240 in the simple case of a linear sequence of transactions. If the central bank buys at this price it transforms the increased financial wealth of all participants completely into money. In this case the central bank acts as a money pump.

With step 4, the central bank unexpectedly redeems the bond at the nominal value 100. This reduces household C’s wealth to 130. After this, total financial wealth is again equal to the quantity of money, just as in the initial period. The net result of all bond market transactions taken together is a redistribution of money among the households: A and B are better off and C is worse off. For the household sector and the business sector as a whole this has no further consequences in real terms. In the case of a redemption at par the households ultimately play a zero sum game among themselves. In the meantime all feel wealthier.
The real assets market

To recall, monetary saving is defined as the difference of income and consumption expenditures.

\[ \Delta \bar{S}_m \equiv Y - C | t \]  \hspace{1cm} (99)

In combination with (29) this yields the straightforward relation:

\[ \Delta \bar{S}_m \equiv Y - C \equiv m \Delta \bar{M} \mid t. \]  \hspace{1cm} (100)

Monetary saving and the change of the household sector’s stock of money are two aspects of the same flow residual.

For the determination of the nonmonetary component of saving first real consumption is needed as new variable. With \( U \), that part of the quantity bought denoted by \( X \) that vanishes for good from the household sector’s stock of commodities because it has been used up completely in the current period. Nonmonetary saving is defined as the valued increase of the commodity stock \( X - U \) and the change of valuation of the already existing stock in period \( t \), which is captured by \( \Delta \bar{G}_H \):

\[ \Delta \bar{S}_n \equiv P(X - U) + \Delta \bar{G}_H | t. \]  \hspace{1cm} (101)

If the quantity bought is used up completely in each period, i.e. \( X = U \), the first part of nonmonetary saving is always zero. This is the case when the whole output consists of nondurables. Under this condition there is no addition to the stock of commodities, which is initially zero. That means we have, as limiting case, a pure hand-to-mouth economy with no stocks at all. Real residuals fill this empty economy with an ever increasing stock of durable commodities (cars, notebooks, TV sets, furniture, houses, etc.).

Not before \( X > U \) the household sector’s stock of commodities starts to grow. Then, a new vintage of commodities is added to the stock as long as \( X_t > U_t \). At any point in time the household sector’s stock is therefore composed of \( l \) commodities with a vintage index.

The symbol \( l \) denotes the ‘finite number of distinguishable commodities’ or the ‘universe of discourse’ that ‘must always be explicitly listed at the outset’ (Debreu, 1959, pp. 32, 3). The quantity of the \( h \)th commodity is given by:

\[ \bar{X}_{ht} \equiv \sum_{t=1}^{l} (X_h - U_h - \bar{X}_{hu})_t \]  \hspace{1cm} (102)

The symbol \( h \) defines a subset of \( l \), in this case the commodities that the households possess. The meaning of \( h \) depends here and in the following on the actual context.
The subscript \( u \) denotes that the quantity in question has vanished from the household sector’s stock in period \( t \).

There are items among the household sector’s stock of commodities whose value increases over time, but the greater part decreases in value because of wear and tear. The complete stock of commodities is therefore divided in each period into two mutually exclusive parts. The overall change of value of the existing stock is then given by appreciation \( \Delta \bar{G}_H^+ \) and depreciation \( \Delta \bar{G}_H^- \):

\[
\Delta \bar{G}_H \equiv \Delta \bar{G}_H^+ - \Delta \bar{G}_H^- \mid t.
\]  

(103)

For all items with a loss of value taken together the depreciation is given by:

\[
\Delta \bar{G}_H^- t \equiv \sum_{h=1}^{l} (B_{ht} \bar{X}_{ht} - B_{ht-1} \bar{X}_{ht-1}) \quad \text{with} \quad B_{ht} < B_{ht-1}.
\]  

(104)

Depreciation is the difference of the valued stock of remaining items in the current period and the valued stock of the previous period. The valuation price \( B \) is introduced as a new variable \( \text{uno actu} \) with the 6th axiom:

\[
\Delta \bar{S} = \Delta \bar{S}_m + \Delta \bar{S}_n \mid t.
\]  

(105)

The 6th axiom states that total saving in period \( t \) is the sum of monetary and nonmonetary saving.

The households have some leeway in the valuation of their stock. Whether the valuation prices are ‘realistic’ or not remains to be seen until the respective commodity is offered on the secondary market. Normally, the households do not care much about the valuation of their stock of commodities. They simply keep it, thus indicating that they value each item higher than the price attainable on the secondary market.

As a rough and ready first approximation the valuation price can be calculated from the purchase price, the life expectancy of the item in question, and the time that has elapsed since the purchase. This entails that \( B_t < B_{t-1} \). The minimum price is one cent. If the life expectancy of the item in question is virtually infinite this pragmatic calculation leads to \( B_t = B_{t-1} \). Until some good reasons for a re-evaluation appear over time the valuation price is therefore equal to the price that has been paid on occasion of the purchase out of current production.

Some items of the households’ stock of commodities may increase in value. For these items the appreciation is, as a total, given by:

\[
\Delta \bar{G}_H^+ \equiv \sum_{h=1}^{l} (B_{ht} \bar{X}_{ht} - B_{ht-1} \bar{X}_{ht-1}) \quad \text{with} \quad B_{ht} \geq B_{ht-1}.
\]  

(106)
Appreciation or depreciation of the stock of commodities in each period originates therefore from the largely subjective change of the valuation price $B$ of each hitherto not consumed vintage. Over- or under-valuations automatically cancel out as the end of the life expectancy is approached and the valuation price tends to zero. Subjective valuations therefore produce not much more than self-correcting time shifts of nonmonetary saving. This does not apply to commodities with an infinite life expectancy.

Equation (101) can be rewritten in combination with (103) as:

$$\Delta S_n \equiv PX - PU - \Delta \bar{G}_H + \Delta \bar{G}_H^+ \mid t.$$  (107)

Consumption $K$ is finally defined as the sum of the valued quantity that is used up in the current period and the decrease of the value of the not yet consumed stock of durable commodities. Depreciation gives a rough measure of the services which the durable commodities yield in one period.

$$K \equiv PU + \Delta \bar{G}_H^+ \mid t.$$  (108)

The greater the accumulated stock of durable commodities, the greater $\Delta \bar{G}_H^+$ becomes.

Nonmonetary saving (107), then, is the difference between consumption expenditures and consumption plus the appreciation of the remaining stock of commodities:

$$\Delta S_n \equiv C - K + \Delta \bar{G}_H^+ \mid t.$$  (109)

There can be consumption without consumption expenditures. In this case one has nonmonetary dissaving and the valued stock of commodities decreases.

Consumption expenditures $C$ include all products bought by the household sector, be it nondurables or durables. By consequence, the consumption of the services of durables, which is measured by the depreciation, progressively takes a greater share of consumption $K$ as the economy develops. If consumption and consumption expenditures are equal in period $t$ nonmonetary saving is equal to the appreciation of the existing stock.

The households satisfy their needs and wants in the current period by physical consumption of $U$ units and by usage of the existing stock of commodities. Consumption $K$ embraces both sources of satisfaction and is the formal interface between the axiom set and consumption theory. With $C$ households buy consumption goods for the current period and a stream of future consumption that is subsequently realized. This realization is coarsely expressed by $\Delta \bar{G}_H^+$.

The 6th axiom finally takes the explicit form:

\[ \text{49} \]
\[ \Delta \bar{S} = (Y - C) + (C - K + \Delta \bar{G}_{H}) = Y - K + \Delta \bar{G}_{H} |_{t}. \]  

(110)

Total saving as the sum of monetary and nonmonetary saving is in period \( t \) given as the difference of income \( Y \) and consumption \( K \) plus the appreciation of commodities that the household sector possesses. If there is no appreciation total saving is given by:

\[ \Delta \bar{S} = Y - K |_{t}. \]  

(111)

In the simplest case total saving is the difference between income and consumption.

With the final step the household sector’s net worth \( \bar{S} \) at the end of an arbitrary number of periods is now defined as the numerical integral of the changes of monetary and nonmonetary saving from the first period onwards plus the initial endowment:

\[ \bar{S}_{t} \equiv \sum_{t=1}^{T} \Delta \bar{S}_{t} + \bar{S}_{0}. \]  

(112)

Taking (110) and (30) into account this reads in explicit form:

\[ S \equiv \bar{M}_{H}^{\text{stock of money}} + \sum_{t=1}^{T} \left( C - K + \Delta \bar{G}_{H} \right)_{t} + B_{0}\bar{S}_{0} + \bar{S}_{0} |_{\bar{t}}. \]  

(113)

This equation summarizes the twofold process that generates the household sector’s stock of nonmonetary assets and stock of money until period \( t \). The latter may actually consist of either current deposits or current overdrafts.

The stock of money is set at first to zero, which implies \( \rho_{E} = 1 \) in all periods up to \( t \). This does not exclude that there is a group of households, which has accumulated current deposits, and a complementary group, which has accumulated current overdrafts of exactly the same amount as in Figure 9. In other words, overdrafts are at any moment the zero-sum complement of deposits:

\[ \bar{M}_{H} = 0 \Rightarrow \bar{M}_{d H} - \bar{M}_{o H} = 0 |_{\bar{t}}. \]  

(114)

It is obvious that this strong condition is only needed to keep nominal residuals out of focus for a while.

The 1st axiom contains the number of shares \( \bar{N} \). For completion the value of these shares has finally to be substituted for \( \bar{S}_{0} \) in (113):

50
Whatever exists in the economy for more than one period are, apart from all initial endowments, cumulated real and nominal residuals. Accordingly, the structural framework of the secondary market is given by (115) with the available stock of current deposits and the stock of nonmonetary assets derived in direct lineage from the structural axiom set. The quantitative frame consists of

\[
\tilde{S} = M_{HA}^d - M_{HB}^o + \sum_{h} B_h X_h + B_{N0} N |\bar{f}.
\]  

(115)

and entails the distribution of deposits and commodities among the households. The secondary market concerns only the households; the business sector is not involved. This is the defining characteristic in comparison to the product market. Hence we have all current deposits on the side of potential demand and all nonmonetary assets on the side of potential supply. For the general case the demand side has to be supplemented by free overdraft lines.

It is eq. (116) and not the simple-minded demand-supply-equilibrium schema that is the formally correct description of the secondary market which may in turn consist of many submarkets. The price determination is analogous to the bond market and completely different from the primary market which is depicted in Figures 1 to 3.

Ricardo already recognized that there are two entirely different kinds of markets and gave a description of the secondary market.

There are some commodities, the value of which is determined by their scarcity alone. . . . Some rare statues and pictures, scarce books and coins, wines of a peculiar quality, . . . . Their value. . . varies with the varying wealth and inclinations of those who are desirous to possess them. (Ricardo, 1981, p. 12)

This is a rough verbal translation of (106). Ricardo’s insight that there exists a primary and a secondary market with entirely different properties was completely lost on the marginalists who never realized that demand-supply-equilibrium provides no explanation of price formation at all. The champions of the market economy never had a proper understanding of how qualitatively different interrelated markets work.
21 Conclusion

This paper provides the toolkit for a consistent analysis of the monetary economy. It should have become clear by now to everybody that demand-supply-equilibrium is a vacuous conception. General Equilibrium Theory has led into an impasse. The Keynesian Revolution on the other hand has ultimately not resulted in a full emancipation from the behavioral axioms of received theory. Heterodoxy was so occupied with debunking standard economics that no resources were left for building up a viable alternative. Therefore, an entirely new approach is required.

The chief analytical tool is the set of structural axioms which fully replaces the accustomed behavioral axioms. The achievement of the foregoing analysis is threefold and consists in:

- the terminatory clarification of the foundational concepts income and profit which is the precondition of any substantial economic analysis,
- the replacement of the static demand-supply cross by a three-dimensional product market whose evolution is ultimately determined by structural factors,
- the refutation of the idea of a one-fits-all generic market-format.

The structural axiomatic toolkit enables the comprehensive analysis of the major interrelated economic phenomena that comprise the markets for products and services, the orthogonal labor markets, the financial markets, and the secondary markets. The application of the structural axiom set enables economists to really understand what they have hitherto interpreted perfunctorily. Proper axiomatization is the only way to true understanding.

References


AXEC-Project Website: http://www.axec.org

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