Financial Deepening and Economic Growth

Bhattarai, Keshab

University of Hull Business School

29 July 2013

Online at https://mpra.ub.uni-muenchen.de/48696/
MPRA Paper No. 48696, posted 11 Feb 2019 14:34 UTC
Financial Deepening and Economic Growth

Keshab Bhattarai*
University of Hull Business School

Abstract

A Venn diagram is used to show the efficient allocation of resources in terms of the core of Shapley-Shubik games and general equilibrium models. These concepts are applied to study the role of real finance in growth with and endogenous, cash in advance, money in utility function and applied dynamic general equilibrium model of Germany, France and UK. Computations of the general equilibrium model confirms the over financing hypothesis. The actual financial deepening was 3.5, 2.4 and 5.1 times more than optimal financial deepening for France, Germany and the UK respectively. This explains the widespread impacts of financial crises on growth and employment in these economies that was observed after the 2008 recessions. Shocks in financial deepening ratio cause massive macroeconomic fluctuations. Smooth and sustainable growth of the economy requires adoptions of the separating equilibrium in line of Miller-Stiglitz-Roth mechanisms to avoid the problem of asymmetric information in process of financial intermediation.

JEL Classification: F41, O11, O33, O41
Keywords: financial deepening, growth

*Business School, University of Hull, HU6 7RX, Hull, UK. Phone: 441482463207; Fax: 441482463484; email: K.R.Bhattarai@hull.ac.uk. This paper was presented to the EEFS-Berlin 2013 conference in June 2013.
1 Financial Crises

Economic crises of 2008/09 that originated from the burst of housing market bubble and the credit crises in the US spread around the globe. Recessions hit the US, UK, EU, Japan and many other advanced countries hard and slowed down growth and other economic activities in these countries. Output, employment, investment, capital accumulation, exports, imports shrank causing alarming loss of income, deterioration in living standards of households and loss of business or profit prospects of small, medium and large scale firms. Governments have attempted to stimulate the aggregate demand by expanding the public expenditure and cutting down the taxes despite growing risk of accumulation of public debt. Central banks have reduced the basic interest rate to a record low rate since the beginning of central banking (on January 2009 Federal fund rate has remained close to zero, Bank of England’s basic rate is 0.5 percent since 2009 and ECB now has 0.5 rate1) in order to expand the liquidity is the system. Sources of credit levels of banks have been expanded under the quantitative easing.

The fact that the process of capital accumulation and growth in modern economies is enhanced substantially by the financial markets that channel resources of millions of risk adverse savers to millions of risk neutral borrowers is well recognised. Schumpeter (1911) argued for financial development for economic growth but Robinson (1952) viewed that financial development a by-product of economic growth process. Importance of risk minimisation and efficiency of portfolio allocation was noted by Sproul (1947) Smith (1958) and Chiang (1959). Then Tobin (1969) linked the balancesheet of the financial system to economic growth. Klein (1971) had a theory of Banking firm. The process of financial deepening was discussed by Shaw (1973) applied in the context of developing economies by McKinnon (1973), Fry (1978), Boycko, Shleifer and Vishny (1996), Champ, Smith and Williamson (1996). King and Levine (1993) and Levine (1997) tested these propositions empirically across countries. Hills and Thomas and Dimsdale (2010) and Davies et al. (2010) had put issues of recessions and banking evolution in the context of the UK economy.


1 http://www.ecb.int/suru/intro/html/map.en.html


Classical economists had put capital accumulation at the centre of economic growth (Figure 1). For them higher degree of financial deepening through saving and investment activities promote the level of income and raises the rates of economic growth. No economist can disagree that the economic advancement is impossible without a reasonable degree of financial deepening as measured in the ratios of capital stock to GDP ratio or less precisely by M3/GDP ratios (for France, Germany and UK see Figure 2). This paper aims to analyse these issues using theoretical and applied methods. Section 2 motivates the paper with a short discussion of the underlying concept of an efficient competitive equilibrium mechanism contained in non-blocking core in Shapley-Shubik game and Pareto optimal core in a general equilibrium model that could be applied to analyse efficient allocations both in goods and asset markets. It illustrates the Schumpetarian view that growth of the financial sector is linked to the growth of the rest of the economy over time. Then it investigates possibilities of fluctuations using analytical solutions in models with exogenous and exogenous growth of money in the economy. These are illustrated with standard cash in advance (CIA) and money in utility (MIU) models of the form in Williamson (2008) and Walsh (1998) in sections 3 and 4. Paper proceeds further taking clues from these prototype models to dynamic multisectoral and multi-household general equilibrium models of Germany, France and UK in section 5 to establish the efficient and optimal path of capital output ratios implied by such equilibrium process refining concepts illustrated in (Bhattacharyya 1997 and 2005). The conclusions, references and appendices supporting the study are in the final section.
Table 1: Financial Deepening in Three EU Economies

<table>
<thead>
<tr>
<th></th>
<th>FA</th>
<th>GDP</th>
<th>FDratio</th>
<th></th>
<th>FA</th>
<th>GDP</th>
<th>FDratio</th>
<th></th>
<th>FA</th>
<th>GDP</th>
<th>FDratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>20.52</td>
<td>1.89</td>
<td>10.88</td>
<td>Germany</td>
<td>19.34</td>
<td>2.43</td>
<td>7.96</td>
<td>United Kingdom</td>
<td>21.27</td>
<td>1.41</td>
<td>15.06</td>
</tr>
<tr>
<td>2007</td>
<td>20.52</td>
<td>1.89</td>
<td>10.88</td>
<td>2008</td>
<td>19.44</td>
<td>1.93</td>
<td>10.06</td>
<td>2009</td>
<td>20.39</td>
<td>1.89</td>
<td>10.81</td>
</tr>
<tr>
<td>2008</td>
<td>19.44</td>
<td>1.93</td>
<td>10.06</td>
<td>2009</td>
<td>19.54</td>
<td>2.47</td>
<td>7.90</td>
<td>2010</td>
<td>21.31</td>
<td>1.94</td>
<td>11.00</td>
</tr>
<tr>
<td>2010</td>
<td>21.31</td>
<td>1.94</td>
<td>11.00</td>
<td>2011</td>
<td>21.97</td>
<td>2.00</td>
<td>10.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data Source: OECD (national accounts section). FA = Nonconsolidated Financial Assets and Y = GDP both in Trillions; FDratio = FA/GDP

FA and GDP are in Trillions of National Currencies.

1: Financial Deepening and Economic Growth

2: Financial Deepening Index

Stocks of total financial assets are much larger than stocks of M3 assets. From the OECD data summarised in Table 1, one can observe that the financial deepening ratios (FDratios) are twice as
large in the UK than those in Germany. French FDratios seem to a bit higher than in Germany but much smaller than in the UK. The non-consolidated financial assets include currency and deposits, financial derivatives, securities, shares and equities. Using four indicators of financial development for about 119 countries for 1960 to 1989 King and Levin (1993) had showed empirical support for the Schumpeterian hypothesis that financial development leads to economic growth in contrast to the Robinsonian argument that growth rate of output had little connection to the financial development and the long run growth is a function of real physical capital not the financial leverages or derivatives that promotes the artificial financial deepening. Over-financing however is a phenomenon that has become more serious in the last two decades. Our general equilibrium computations shows that there is little difference between the real financial deepening across these countries - much else is caused by casino capitalism (Sinn 2009, 2010) or asset bubbles and collective illusion (Miller and Stiglitz (2010)). Farmer (2013) shows adverse consequences on growth when asset prices are unbounded above.

Thus the main aim of this paper is to show how economies are vulnerables of good or bad financial sector policies, degree of over-financing and wide ranging inefficiencies, fluctuations in growth of output and other economic activities because of this.

2 Core and Efficient Allocations

Game theory and general equilibrium models analyse optimal choices of consumers and producers faced with resource constraints in which the essential process involves finding the core of bargaining over the gains from the intra and intertemporal trade on goods, services and financial assets. The core in a bargaining game is given by the payoff from a non-blocking coalition. It is a Pareto efficient point. Similarly core of a general equilibrium lies in the contract curve where it is difficult to make one economic agent better off without making another worse off. The core of the coalition in the game and core in a general equilibrium model represent basically the same efficient point and relative prices. The optimal allocation of resources to economic agents possible with given endowments confirm to the first and second theorems of welfare economics. Solutions of both models characterise the optimal allocation of resources after more complex bid and offer interactions among economic agents. Debreu and Scarf (1963) had proven the equivalence of a competitive equilibrium to the core of the game for economies with and without production by contradiction when preferences are non-satiable, strictly convex and continuous. Scarf (1967) theorem states that a balanced n person game has a non-empty core. Financial markets open each time, bid-offer process sets the prices of assets, exchange takes place in the core. This process continues forever. Thus the competitive equilibrium is equivalent to the allocation at the core, “An exchange economy with convex preferences always gives rise to a balanced n person game and such will always have a nonempty core (Scarf (1967)).” This model is best illustrated in terms of a Venn diagram with three players as given in Figure 3 below.

\[
\sum_{i} x_i = \sum_{i} \sum_{S \supset \{i\}} \delta_i x_i^S = \sum_{T=\{S\}} \sum_{i \in S} \delta_S x_i^S = \sum_{T=\{S\}} \sum_{i \in S} \delta_S \sum_{i \in S} \omega_i = \sum_{i} \sum_{T=\{S\}} \delta_S \sum_{i \in S} \omega_i = \sum_{i} \sum_{S \supset \{i\}} \omega_i = n \sum_{i} \sum_{S \supset \{i\}} \omega_i
\]

It is natural that economic agents play a zero sum and non-cooperative game until they realise the benefits of coalition and cooperation (Gale (1986)). When an agreement is made and cooperation is achieved there is a question on whether such coalition is stable or not. There are always incentives
at least for one of the player to cheat others from this cooperative agreement in order to raise its own share of the gain. However, it is unlikely that any player can fool all others at all the times. Others will discover such cheating sooner or later. A coalition of players should fulfil individual rationality, group rationality and coalition rationality. These can be ascertained by the supper-additivity property of coalition where the maximisation of gain requires being a member of the coalition rather than playing alone. Superadditivity condition implies that the value of the game in a coalition is greater than the sum of the value of the game of playing alone by those individual members. In case of three players this means:

\[ v(1 \cup 2 \cup 3) \geq v(1) + v(2) + v(3) \]  

Coalitions (parties) playing together generate more value, \( v(1 \cup 2 \cup 3) \), for each of its member than when they play alone with payoffs \( v(1), v(2), \) and \( v(3) \). Team spirit generates extra benefits. This is a tiny set of core equilibrium as illustrated by the intersection of 1, 2, and 3 in the Venn diagram.

The dynamic economy implied by this model can better be explained using a diagram as in Figure 4 where the E-E is allocations at the core; LL market valuations of lenders; BB the market valuation of borrowers. The gap between LL and BB reflects the subjective difference in the assessment of prospects of financial assets and reason for trades among lenders and borrowers.
Market equilibrium presented above assumes complete information among lenders and borrowers ignoring the asymmetry of information in financial markets, which is the underlying case of deviation of asset accumulation path of borrowers (BB) and lenders (LL) of the equilibrium path (EE) in Figure 4. The main intuitive points are as follows:

1. Assets are results of consumption saving behavior resulting from the intertemporal optimisation of households or firms.
2. There is an equilibrium allocation EE for each time period of the economy that is at the core of the equilibrium.
3. Lenders and borrowers start with different amounts of endowments and bargain continuously in order to gain more from the transaction.
4. Underlying productivity and preferences cause differentiation in valuation by the buyers and sellers in the asset markets. Therefore the valuation can be generalised in n number of cases.
5. Corrective measures are taken by individuals or the policy makers when these valuations significantly deviate away from the underlying equilibrium destabilising the whole financial system.
6. The asset accumulation profile can contain overlapping generations and has infinite life in contrast to individual traders with finite life.
7. There are gains from trading in the financial markets. Whether the lenders or the borrowers get the larger shares of this gain depend on their bargaining power, which changes over time.

Above dynamic economy can be expressed with a simple stochastic technology $Y_t = z_t K_t$ where $z_t \sim N(0, \sigma^2)$, capital accumulation takes the form $I_t = K_{t+1} - (1 - \delta) K_t$, and amount of investment deviates from saving depending on the efficiency of financial markets ($0 < \phi < 1$), $I_t = \phi S_t$ in spirit of Wickens (1995) or Price (1997) or more recently in Levin et al. (2013). Assuming market clearing $Y_t = C_t + S_t$ and a steady economy $K_{t+1} = (1 + g) K_t$ and $\frac{I}{Y} = \frac{\phi S}{Y}$ and the parameters $z, \phi, s$ and $\delta$ in Table 2 determine the growth rate of the economy (Bhattarai, 1997, 2005) as shown in Fig. 5.

$$g = z \frac{I}{Y} - \delta = z\phi s - \delta \tag{3}$$

$$\therefore I_t = \phi S_t = (1 + g) K_t - (1 - \delta) K_t = (g + \delta) K = (g + \delta) \frac{Y}{z}$$

It is important to show that financial and real sectors of the economy are mirror images of each other using an asset accumulation equation as

$$A_t (1 + \hat{r}_t) + W_t - C_t = A_{t+1} \tag{4}$$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\delta$</th>
<th>$\phi$</th>
<th>$y_0$</th>
<th>$z$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIA</td>
<td>0.02</td>
<td>0.95</td>
<td>1</td>
<td>(0.15, 0.05)</td>
<td>0.15</td>
</tr>
</tbody>
</table>
where \( C_t \) is consumption, \( A_t \) financial assets, \( W_t \) endowment, and \( \tilde{r}_{t+1} \) return to asset net of tax and depreciation rate; \( \tilde{r}_t = (1 - \tau_k) (r - \delta) \) with \( r \) real interest rate, \( \delta \) rate of depreciation and \( \tau_k \) capital income tax. When \( \tau_k = 0 \), (4) can be written as

\[
A_t r_t + W_t - C_t - \{ A_{t+1} - (1 - \delta) A_t \} = 0
\]

(5)

Now replacing \( A_t \) by \( K_t \) and using definition of income \( Y_t = A_t r_t + W_t = C_t + I_t \)

\[
Y_t - C_t - (K_{t+1} - (1 - \delta) K_t) = 0; \quad \Longrightarrow Y_t = C_t + I_t
\]

(6)

Thus the stocks of financial assets must balance to the stocks of physical capital in an economy but their values are sensitive to market conditions. Mechanism of incentive compatible contracts contained in Maskin, Tirole (1990) and Roth (2008) could be applied to separate normal borrower and lenders from risky ones under asymmetric information to solve moral hazard or adverse selection problems required to efficient equilibrium path EE by minimising gaps in their evaluation as shown above by LL and BB lines in Figure 4. Kiyotaki and Moore (2006) illustrate importance of the bilateral and multilateral commitment in maintaining the efficiency of the financial system (\( \phi \)). How financial crises of 2008 could be explained due to the shocks to these real sides of the financial system is illustrated with standard dynamics contained in simple cash in advance (CIA) and money in utility (MIU) models of the form discussed in Williamson (2008) and Walsh (1998), in the next two sections before presenting more elaborate general equilibrium models of France, Germany and the UK.

### 3 Friedmann Rule with Cash in Advance Constraint

How the financial sector can contribute most to the economic growth when stock of money grows according to the growth rate output can be shown by solutions based on the optimal conditions in a cash in advance monetary economy where households maximise lifetime utility \( U(C_t) \) from consumption \( C_t \) but experience disutility from labour from efforts put in work, \( V(L_t) \). The problem of the economy is to maximize this utility (7) with technology (8), cash in advance (9) and lifetime budget constraints (10) as:

\[
\max \sum_{t=0}^{\infty} \beta^t [U(C_t) - V(L_t)]
\]

(7)

Subject to the technology constraint:

\[
Y_t = z L_t
\]

(8)

and the Cash in advance constraint

\[
P_t C_t + q_t B_{t+1} + P_t s_t X_{t+1} + P_t T_t = M_t + B_t + P_t X_t
\]

(9)

where \( P_t C_t \) is consumption expenditure, \( P_t \) price of goods, \( C_t \) consumption, \( B_{t+1} \) is the amount of nominal bonds, \( q_t \) is the price of nominal bonds, \( X_{t+1} \) real bonds, \( s_t \) prices of real bonds, \( T_t \) lump sum tax payment, \( M_t \) money. Budget constraint of the consumer include income from production and allocation of money for the next period.
\[ P_t C_t + q_t B_{t+1} + P_t s_t X_{t+1} + P_t T_t + M_{t+1} = M_t + B_t + P_t X_t + P_t z L_t \] (10)

Government controls the money supply and engages itself in inflationary tax. Its budget constraint for a particular time \( t \) is:

\[ \overline{M}_{t+1} - \overline{M}_t = -P_t T_t \] (11)

The stock of money grows at a constant rate \( \alpha \), thus \( \overline{M}_{t+1} = (1 + \alpha) \overline{M}_t \). With this provision, \( \alpha \overline{M}_t = -P_t T_t \). Normalising the cash in advance and budget constraints by \( \overline{M}_t \) and denoting the real values in small case letters, the cash in advance constraint and budget constraints become

\[ p_t C_t + q_t b_{t+1} (1 + \alpha) + P_t s_t X_{t+1} + P_t T_t = m_t + b_t + p_t X_t \] (12)

and

\[ p_t C_t + q_t b_{t+1} (1 + \alpha) + P_t s_t X_{t+1} + P_t T_t + m_{t+1} (1 + \alpha) = m_t + b_t + p_t X_t + p_t z L_t \] (13)

The representative agent in the economy chooses \( C_t, L_t, b_{t+1}, X_{t+1}, m_{t+1} \) from \( t = 0, 1, 2, ... \) to \( \infty \). The Bellman value function for this problem is:

\[ v(m_t, b_t, X_t, p_t, q_t, s_t) = \max_{C_t, L_t, b_{t+1}, X_{t+1}, m_{t+1}} [U(c_t) - V(L_t)] + \beta v(m_{t+1}, b_{t+1}, X_{t+1}, p_{t+1}, q_{t+1}, s_{t+1}) \] (14)

With the first order conditions for dynamic optimisation, as given in the appendix A, the steady state levels of prices and quantities are obtained in terms of parameters \( \alpha, \beta \) and \( z \). First simplify the steady state with \( m_t = 1, b_t = 0, X_t = 0 \). Then above equilibrium conditions, the budget constraint becomes:

\[ p_t C_t = 1 + \alpha \] (15)

This shows that in CIA model like this money is held only for consumption which equals total output, \( C_t = z L_t \). Setting steady state variables to constant values, \( C_t = C, L_t = L, p_t = p, q_t = q, s_t = s \) analytical solutions for prices and quantities are then expressed in terms of subjective discount factor \( (\beta) \) and the growth rate of money supply \( (\alpha) \).

Price of nominal bond from (A.10) is given in terms of \( \beta \) and \( \alpha \):

\[ q = \frac{\beta}{1 + \alpha} \] (16)

Price of real bond from (A.11) is:

\[ s = \beta \] (17)

The level of employment is given implicitly by (A.12)

\[ (1 + \alpha) V'(L_t) - \beta z U'(z L) = 0 \] (18)

Given the steady state \( (C) \) the price of commodity is directly proportional to the growth rate of money supply and inversely to the level of output and the productivity of the labour.
Table 3: Parameters of CIA Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$L_0$</th>
<th>$z$</th>
<th>$m$</th>
<th>$b$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIA</td>
<td>0.03</td>
<td>0.99</td>
<td>100</td>
<td>(1, 0.05)</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ p = \frac{1 + \alpha}{C} = \frac{1 + \alpha}{zL} \]  
(19)

Nominal interest rate depends on the price of nominal bonds, directly on the growth rate of money and inversely on the discount factor.

\[ R = \frac{1}{q} - 1 = \frac{1 + \alpha}{\beta} - 1 \]  
(20)

Real interest rate inversely relates to the price of real bond and the subjective rate of time preference:

\[ r = \frac{1}{s} - 1 = \frac{1}{\beta} - 1 \]  
(21)

Inflation rate equals the growth rate of money supply in the steady state:

\[ i = \frac{P_{t+1}}{P_t} - 1 = \frac{p_{t+1}M_{t+1}}{p_tM_t} - 1 = 1 + \alpha - 1 = \alpha \]  
(22)

Fisher equation implies gross real interest rate to be inverse of the discount factor:

\[ 1 + r = \frac{1 + R}{1 + i} = \frac{1 + \alpha}{\beta} \div 1 + \alpha = \frac{1}{\beta} \]  
(23)

Thus the prices $q$, $s$, $p$, $R$, $r$, $i$ and $\lambda$ are all solved in terms of growth rate of money ($\alpha$) and the discount rate ($\beta$). From the equilibrium condition $Y = C = zL = \frac{1 + \alpha}{\beta}$ and $L = \frac{1 + \alpha}{\beta}$. Thus the level of output, consumption and employment increase with $\alpha$ and decline with inflation. While the greater liquidity helps to mobilise resources, the higher rate of inflation distorts the intertemporal decisions. Higher growth rate of money supply lowers the level of employment by causing distortions through inflation.

Now let us perturb this model around this steady state and show how the shocks in growth rate of money supply or the level of technology can impact on the transitional dynamics of the economy. These are shown in a series charts that represent solutions of this model to the shocks in $\alpha$ or $z$ for given values of parameters in Table 3.
This means under the Friedman rule the cash in advance constraint does not bind. There are no distortions between the real and nominal assets; the rate of return in all assets are equal in equilibrium.

With parameter sets in Table 4, a simple three country version of this model is solved subject to idiosyncratic technological shocks for 15 years to generate time profiles of capital, output, prices, money, consumption, investment, labour supply and lifetime utilities of households as shown by multiple bars for three interdependent economies, $i = 1, 2, 3$. The fluctuations in these economies are originate in financial sector and can have significant consequences in the level of welfare in the economy.

Main lessons that can be drawn from above model is that the financial crises occur because of shift in the investor and consumer confidences, changes in perceptions and beliefs and technological

![Utility of household in CIA Model](chart1.png)

**Fig. 8**

![Distribution of Welfare by Countries](chart2.png)

**Fig. 10**
shocks that hit the system. Impacts of such changes can be very sudden which affects the velocity of circulation of money, technological progress, discount factors or the beliefs in the underlying growth rates of the economy. These factors impact on prices, trend of output, prices and other features of the economy as shown by the path of model variables and welfare solutions as presented in above figures. It is clear that a balanced path of financial depth enhances welfare of households but this depends on the attitude of the consumers toward the future of the economy. By raising discount rate for future economic activities, financial crises, this will have adverse consequences in capital formation, output, consumption and welfare of households (see Chada and Nolan (2002) for new Keynesian perspective on such models). These features are not typical of an economy with exogenous money but can persist even with the endogenous growth rate of money. This is shown using a solution of the money in utility function model in the next section.

4 Financing with Money in the Utility Function

Role of money was for pure exchange in the cash in advance model and the growth rate of money $\alpha$ was exogenous. There are circumstances when household prefer to store cash, particularly during the financial crises, stock of money is endogenous to household decision. This feature is captured by the money in the utility function model. When this desire is excessive it causes a crisis in the system as observed during the recession that started in 2008. The problem of household as in the CIA is to maximise the lifetime welfare that is obtained by consumption and money.

$$\max W = \sum_{t=0}^{\infty} \left[ \beta^t U(c_t, m_t) \right]$$

Subject to the technology constraint:

$$Y_t = z F(K_t, L_t)$$

Under constant returns to scale $y_t = f(k_t)$ where $y_t = \frac{Y_t}{K_t}$ and $k_t = \frac{K_t}{L_t}$. Economy wide budget constraint is given by

$$Y_t + \tau_t L_t + (1 - \delta) K_{t-1} + \frac{M_{t-1}}{P_{t-1}} = C_t + K_t + \frac{M_t}{P_t}$$

where $Y_t$ is output, $P_t$ price of goods, $C_t$ consumption, $K_{t+1}$ is capital stock, $\tau_t$ is net transfer for each individual, $M_t$ money, $L_t$ employment and $\delta$ is the rate of depreciation of capital. In per capita terms

$$\omega_t = f(k_{t-1}) + \tau_t + \left( \frac{1 - \delta}{1 + n} \right) k_{t-1} + \frac{m_{t-1}}{(1 + \pi_{t-1})(1 + n)} = c_t + k_t + m_t$$

The dynamic program of this problem is:

$$V(\omega_t) = u(c_t, m_t) + \beta V(\omega_{t+1})$$

$$V(\omega_t) = \max \left\{ u(c_t, m_t) + \beta V \left[ f(\omega_t - c_t - m_t) + \tau_{t+1} + \left( \frac{1 - \delta}{1 + n} \right) (\omega_t - c_t - m_t) + \frac{m_t}{(1 + \pi_{t+1})(1 + n)} \right] \right\}$$
Table 5: Parameters of MIU Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIU</td>
<td>0.3</td>
<td>0.99</td>
<td>0.05</td>
<td>(1, 0.05)</td>
</tr>
</tbody>
</table>

Dynamic optimisation with the first order conditions presented in Appendix B results into:

\[
u_m(c_t, m_t) + \frac{\beta u_c(c_{t+1}, m_{t+1})}{(1 + \pi_{t+1})(1 + \nu)} = u_c(c_t, m_t) (30)\]

Left hand side gives the total marginal benefit of holding money; the first term in it is the direct utility of money and the second term denotes the real balance effect of holding money $m_t$ at $t$ for $t+1$. Thus the marginal utility of holding money should equal to marginal utility of consumption. By constant returns to scale assumption $r^k k + w = f_k k + (f(k) - f_k k) = f(k)$. Consider a steady state with $n = 0$ and $V_\omega(\omega_t) = V_\omega(\omega_{t+1}) = V_\omega(\omega^{ss})$. From the first first order conditions $1 - \beta [f_k (k^{ss}) + (1 - \delta)] = 0$

\[zf_k (k^{ss}) + (1 - \delta) = \frac{1}{\beta}\]

Assuming a Cobb-Douglas production function $f(k) = z k^\alpha$ this condition converts to $\alpha z k^{\alpha - 1} + (1 - \delta) = \frac{1}{\beta}$

\[k^{ss} = \left[\frac{\alpha z \beta}{1 + \beta (\delta - 1)}\right]^{\frac{1}{1 - \alpha}}\]  
(31)

Consumption in the steady state:

\[c^{ss} = zf (k^{ss}) - \delta k^{ss} = \left[\frac{\alpha z \beta}{1 + \beta (\delta - 1)}\right]^{\frac{1}{1 - \alpha}} - \delta \left[\frac{\alpha z \beta}{1 + \beta (\delta - 1)}\right]^{\frac{1}{1 - \alpha}}\]  
(32)

Steady state inflation rate equals growth rate of money supply:

\[
\frac{\Delta m^{ss}}{m^{ss}} = \theta^{ss} - \pi^{ss} = 0
\]

where $\Delta m^{ss} = 0$ implies growth rate of money supply, $\theta^{ss} = \frac{\Delta M^{ss}}{M^{ss}}$, and inflation are equal $\theta^{ss} = \pi^{ss}$. As in the CIA model the transitional dynamics of the MIU model is found numerically for the set of parameters in Table 5. The response of the economy to shocks are represented in Figures 11 to 15.
Fig. 11

Fig. 12

Fig. 13
Table 6: Parameters of MIU Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\delta$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$g_y$</th>
<th>$L_0$</th>
<th>$v$</th>
<th>$g_m$</th>
<th>$\ln(z)$</th>
<th>$M_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy 1</td>
<td>0.05</td>
<td>0.5</td>
<td>0.95</td>
<td>0.01</td>
<td>100</td>
<td>1</td>
<td>0.01</td>
<td>(1, 0.05)</td>
<td>100</td>
</tr>
<tr>
<td>Economy 2</td>
<td>0.05</td>
<td>0.45</td>
<td>0.99</td>
<td>0.02</td>
<td>100</td>
<td>2</td>
<td>0.02</td>
<td>(1, 0.05)</td>
<td>100</td>
</tr>
<tr>
<td>Economy 3</td>
<td>0.05</td>
<td>0.45</td>
<td>0.98</td>
<td>0.015</td>
<td>100</td>
<td>1.5</td>
<td>0.015</td>
<td>(1, 0.05)</td>
<td>100</td>
</tr>
</tbody>
</table>

As suggested by Farmer (2013) and Miller and Stiglitz (2010) above underlying factors for such fluctuations are the moral hazard and adverse selection between lenders and borrowers or the public or the private sectors or the shocks to the technology or sudden shift in the belief. Analysis of results of all economic activities is the welfare of the households from the consumption of goods and services reveals how financial crises impacts their prospects (Stiglitz and Weiss (1981)). Producers face lower stock of capital and to combine labour and can supply less of those goods and have to pay more for factors. Direct and indirect taxes distort choices of households (Townsend and Euda (2006)). Again scenarios are derived from three economies with a set of plausible parameters as given in Table 6. The time path of variables $y_t, k_t, c_t, m_t, u_t$ are easily computed based on model solutions, (Fig 16 and 17).
The CIA and MIU models provide intuition about the nature of fluctuations that affect interdependent economies and allocation of welfare. Policy analyses should be based in more detailed assessment of the structural features of the economy as found in the microconsistent dataset for consumption, production and trade. Therefore an attempt is made below in describing an a dynamic general equilibrium model of financial deepening with realistic microfoundation for analysis of efficiency, growth and redistribution refining Bhattacharai (1997 and 2005) and is illustrated in the next section.

5 Finance in a Dynamic General Equilibrium Model

A dynamic general equilibrium model properly accounts for the intertemporal preferences of households between the current and future consumption (and saving), long run decision of investors in accumulating capital and the policies of government that often distorts positively or negatively
and affect on choices of firms and households. With the increasing level of globalisation, capital now flows more swiftly from one country to another causing volatility in the values of financial assets, causing bubbles as shown by Miller and Stiglitz (2010) and it does not settle down until the investors find the best return from their investment. It frequently results in runs, panics or exuberance as shown in Figures 1, 2 and 4 above. Theoretical works to analyse this issue as found in Greenwood and Boyan (1990), Fama (1980), Levine (1997), Boyd and Prescott (1986), Epstein and Zin (1989), Townsend (1983), econometric studies in Arestis, Demtriades and Luintel (2001), Hansen, Sargent and Tallarini (1999), Chari, Kehoe and McGrattan (2000) Raghuram and Zingales (1998), Benarji and Basu (2009), Pilbeam et al. (2011) but very few on applied work in the framework of a dynamic general equilibrium context (Mercenier and Srinivasan (1994)). Therefore it is pertinent to present the generic structure of a dynamic general equilibrium model here and to apply it to France, Germany and the UK to study the long run impacts of financial deepening in these economies.

5.1 Consumers

Consumers are forward looking in the model. They are interested in smoothing out their life time consumption in order to guarantee a certain level of utility or standard of life for each period in their life. This requires intertemporal optimisation over the life time, maximising lifetime utility \( U^h_0 \) given the life-time income and budget constraints. Each consumer starting from initial endowment of physical capital \( K^h_0 \) and labour time \( L^h_0 \) makes decision to consume \( C^h_i \) and work \( LS^h_i = L^h_i - L^h_t \) and save from its full income \( I^h_t \) in each period leaving it to the banking system to channel those savings to the potential investors.

\[
U^h_0 = \sum_{t=1}^{\infty} U^h_t \\
U^h_t = U \left( C^h_i, L^h_t; \sigma_c \right)
\]

\[
I^h_0 = \sum_{t=0}^{\infty} e^{-pt} \sum_{i=1}^{N} \left\{ P_{i,t} \left( 1 + t_i \right) C^h_{i,t} \right\} + w^h_t \left( 1 - t_i \right) L^h_t \\
= \sum_{t=0}^{\infty} e^{-pt} I^h_t = \left[ \sum_{t=0}^{\infty} w^h_t \left( 1 - t_i \right) L^h_t + r_t \left( 1 - t_k \right) K^h_t \right] \\
= \sum_{t=0}^{T} \sum_{i=1}^{N} P_{i,t} \left( 1 + t^h_{i,t} \right) C^h_{i,t} = \sum_{t=0}^{T} \left[ r_t \left( 1 - t_k \right) K^h_t + R^h_t + w^h_t \left( 1 - t_l \right) LS^h_t \right]
\]

Households supply factors of production, \( K^h_t \) and \( LS^h_t \), to firms. They receive net of tax wage income in return to labour supply \( w^h_t \left( 1 - t_l \right) L^h_t \) and capital income \( r_t \left( 1 - t_k \right) K^h_t \) in return to their investment. They pay taxes on their capital and labour incomes and may receive transfer payments from the government on the mean tested basis.
5.2 Firms

Firms are central to the supply of goods and services. Given the production technology optimal choices of inputs are made to maximise profit in each period and over the model horizon. Entry and exit is allowed with regulations to maintain a competitive economy. Therefore in each period, firms compare prices of inputs \( r_{i,t}, w^h_{i,t}, p_{i,t} \) and products and determine the optimum level of output that would maximise inputs. Implicitly the level of output depends on relative prices of inputs and outputs as:

\[
Y_{i,t} = F_i \left[ K_{i,t} \left( r_{i,t}, w^h_{i,t}, p_{i,t} \right), L_i \left( w^h_{i,t}, p_{i,t} \right), A_i, \sigma_c \right] \tag{37}
\]

\[
\sum_{t=0}^{T} P_{i,t} Y_{i,t} = \sum_{t=0}^{T} \left[ r_{i,t} (1 + t_k) K_{i,t} + \sum_{h=i}^{H} w^h_{i,t} (1 + tl) L^h_{i,t} \right] \tag{38}
\]

The structure of inputs and levels of technology may differ for firms operating in different sectors - agriculture, manufacturing, services, but all of them are interested to maximise total profit given the process of capital accumulation, \( K_{i,t} = (1 - \delta_{i,t}) K_{i,t-1} + I_{i,t} \).

5.3 Trade

Economies modelled here are price takers in the global market except that they need to balance their trade over time. Adjustment in the real exchange rates brings such balance in the value of imports \( \sum_{t=0}^{T} \sum_{i=1}^{N} PM_{i,t} L_{i,t} \) and exports \( \sum_{t=0}^{T} \sum_{i=1}^{N} PE_{i,t} E_{i,t} \) and net flows of capital \( \pm FL_t \).

\[
\sum_{t=0}^{T} \sum_{i=1}^{N} PE_{i,t} E_{i,t} = \sum_{t=0}^{T} \sum_{i=1}^{N} PM_{i,t} M_{i,t} \tag{39}
\]

\[
\sum_{i=1}^{N} PE_{i,t} E_{i,t} - \sum_{i=1}^{N} PM_{i,t} M_{i,t} = \pm FL_t \tag{40}
\]

5.4 Government

Government provides public services like law and order, education and health, social security and pension and protection of environment to households and firms and adds to the public capital by investing economic infrastructure, health and education. These expenditures enhance productivity and make these economies more competitive in the global market. In a dynamic economy the public spending should balance to the public revenue as shown in (41).

\[
\sum_{t=0}^{\infty} e^{-\rho t} RV_t \leq \sum_{t=0}^{\infty} e^{-\rho t} \left( G_t + R^h_t \right) \tag{41}
\]

Government collects revenue through direct taxes on income of households and firms and indirect taxes on their consumption. The optimal level of public expenditure and revenues is set when the benefits from the public spending equal the costs of public funds in equilibrium (see Mirrlee’s et at. (2010)).
5.5 Financial Deepening

Financial deepening \((F_t)\) is the result of the growth process in the economy and varies across production sectors \((F_{i,t})\) through investment and saving activities. Banks channel funds saved by households or enterprises for investment by firms at the real interest rate that matches cost and productivity of funds to the firms. The degree of real financial deepening is then indicated by the ratio of capital stocks to the GDP.

\[
F_t = \frac{K_t}{Y_t}, \quad F_{i,t} = \frac{K_{i,t}}{Y_{i,t}} \quad (42)
\]

This real measure of optimal financial deepening, resulting from the optimisation behaviour of consumers and firms in the economy, should equal to the ratio of financial assets to GDP in the financial market in an ideal world. Such intertemporal equilibria is guaranteed by the flexibility of prices, wages and interest rates in the economy. Imbalances either due to the rigid or inflexible prices cause market imperfections or crises.

Good financial policies result in right set of accumulation process and higher growth rate of the economy over periods. Wrong financial sector policies lead to mismatch between the volume borrowed and lent, that often manifests in terms of bail outs or subsidies or preferential treatment of one sector against another, which distorts the accumulation process ultimately reducing the prospects of the economy in the long run.

5.6 Markets

This dynamic economy is run efficiently by the market clearing relative price system. Prices of commodities and services and factors of production continue to adjust until demands are balanced to supplies in each market. The applied general equilibrium model that was used to assess prospects of financial development in three economies consisted of eleven sectors of goods and services, capital assets differentiated by sectors and labour differentiated by skills. The real exchange rate links between the domestic and foreign sectors were results of the flow of imports and exports.

6 Analysis of Dynamic GE Model of Financial Deepening

The micro-consistent data for this model is taken from the input output table published by the OECD in 2006 for Germany, France and UK (Appendix Tables C1 - C3 available upon request). This data set provide information on the actual values for demand supply balances of firms, revenue and expenditure of the government, saving and investment balance for the private sector and the export-import balance for the economy.

A number of assumptions are made regarding the nature of the steady states among these economies. First, benchmark rate of return on capital stock is chosen to be the natural rate of interest \((r)\) for each country. Information about the rate of depreciation of capital \((\delta_i)\) in each sector is obtained from the historical data and tested with sensitivity analyses. The steady state growth rates \((g_i)\) are made consistent with the historical growth rates for each sector. The parametric values of \(r, \delta_i\) and \(g_i\) define the reference path of the economy. Elasticities of substitution in consumption \((\sigma_c)\) and production \((\sigma_p)\) are based on the literature. Fundamentals to all these is the optimising behavior of households regarding the division of labour between leisure \((L^h_t)\) and work and division of income between consumption \((C^h_t)\) and saving \((S^h_t)\). Tax rates \(\{t_c, t_w, t_k, R^h_t\}\) are retained for
Table 7: Optimal and actual financial deepening in France, Germany and the UK

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimal Financial Deepening</th>
<th>Actual Financial Deepening</th>
<th>Over Financing</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>3.16</td>
<td>10.98</td>
<td>3.5</td>
</tr>
<tr>
<td>Germany</td>
<td>3.31</td>
<td>8.02</td>
<td>2.4</td>
</tr>
<tr>
<td>UK</td>
<td>3.24</td>
<td>19.12</td>
<td>5.1</td>
</tr>
</tbody>
</table>

all sectors except for the financial and real estate sectors in the counter factual analyses. Model is applied for policy analysis only after the calibration of the benchmark economies.

6.1 Optimal and actual financial deepening

The general equilibrium theory provides a very solid framework for analysis of results obtained by solving more than 14 thousand equations simultaneously for France, Germany and UK. Results on optimal and actual financial deepening, the ratios of financial assets of GDP, relevant for this paper are summarised in Table 7 below. The overall optimal real financial deepening ratios from the general equilibrium models are consistent across countries; these are found to be around 3.31 in Germany, 3.16 in France and 3.24 for the UK. These are sensible results and consistent to the converging patterns of economic growth across these countries. The actual ratios of financial deepening were 10.98, 8.02 and 19.12 given in Table were 3.5, 2.4 and 5.1 time higher than the optimal ratios computed from the solutions of the general equilibrium models for three economies as shown in Table 7. The discrepancy between the real and the nominal magnitudes of financial deepening gives credibility to the hypothesis that UK economy is more vulnerable to financial crises as it has more assets originating from the financial derivatives and more subject to the problems caused by asymmetric information. Sectoral impacts of financial sector reforms are different for each of the three countries. Despite this economic growth rates in these models are driven by fundamentals of the financial markets based on the net present value calculation, portfolio selection satisfying the arbitrage across market, risk-return analysis to minimise risks and maximise returns and insurances to cover unforeseen contingencies. Supply of funds arises from inter-temporal utility maximising consumers and demand for funds for investment originates from profit maximising producers. Subjective discount factors of consumers and depreciation rates of capital is balanced by the real interest rates so that funds are allocated according to the marginal utilities of households or productivities across various sectors leaving regulatory roles to the government for maintaining law and order to creates opportunities for the participants from the private sector.

On-going financial sector reforms can be expected to make these economies more efficient so that the costs of funds decline in the counter factual experiments, where the taxes on the financial sectors are set to minimise distortions relative to the benchmark. Such measures will then result in the higher rate of growth of output, employment and capital stock in almost all sectors even with lower capita output ratios. The financial liberalisation is paying for itself, welfare of consumers improves with reforms rather than without it.

\(^2\)Detailed solutions of these models to be available upon request.
The proper reforms of financial markets improve efficiency of financial intermediation and brings speedier rate of economic growth by linking the lending and borrowing rates to the fundamentals of demand and supply of funds; removing controls on credits; by creating right structure of incentives for investors and depositors and freeing up the foreign exchange market from arbitrary decisions and by making it subject to fundamentals of domestic and foreign markets. These mechanism remove repressory regimes with non inflationary public finance for smooth process of capital accumulation, increased liquidity, technical advancement and economic growth, elimination of parallel markets and reducing the proportion of toxic non-performing assets. Liberation and reform mechanisms thus are instrumental in reversing repressory financial regimes towards more classical free enterprise economy that would promote accumulation and growth in these model economies.

The general equilibrium model results presented above rely on classical economic principles in which the self-adjusting mechanism of the real interest rates would balance demand for and supply of financial assets in a market driven economy and do not contain liquidity trap and credit crunch situations as imagined by Keynes (1936). These results are consistent to literature that has emerged since the late 1960s on harmful impacts of financial repressions in works of McKinnon (1968), Shaw (1968), Roubini and Sala-i-Martin (1992) and Stiglitz and Weiss (1981) and more recently in Boyd and Jalal (2012).

Competitive financial markets are perfect in allocating assets as all agents that have complete information and are efficient in processing such information. This assumption, however, is far from perfect. Financial markets are full of asymmetric information, activities of one set of players depend on actions taken by another set of players and the amount of information they have impacts on the likely choices of others. This requires state contingent incentive compatible mechanisms in this general equilibrium system and is an issue for further investigation.

7 Conclusion

A Venn diagram is used to show the efficient allocation of resources in terms of the core of Shapley-Shubik games and general equilibrium models. These concepts are applied to study the role of real finance in growth with and endogenous, cash in advance, money in utility function and applied dynamic general equilibrium model of Germany, France and UK. Computations of the general equilibrium model confirms the over financing hypothesis. The actual financial deepening was 3.5, 2.4 and 5.1 times more than optimal financial deepening for France, Germany and the UK respectively. This explains the wide-spread impacts of financial crises on growth and employment in these economies that was observed after the 2008 recessions. Shocks in financial deepening ratio cause massive macroeconomic fluctuations. Smooth and sustainable growth of the economy requires adoptions of the separating equilibrium in line of Miller-Stiglitz-Roth mechanisms to avoid the problem of asymmetric information in process of financial intermediation.

References


22


24


A Cash in Advance Model

It is easier to solve this problem if it is written in a Lagrangian constrained optimisation problem as:

$$\mathcal{L}(C_t, L_t, b_{t+1}, X_{t+1}, m_{t+1}, \lambda_t, \mu_t) = \sum_{i=0}^{\infty} \beta^i [U(C_t) - V(L_t)]$$

$$+ \lambda_t [m_t + b_t + p_t X_t - p_t C_t - q_t b_{t+1} (1 + \alpha) - p_t s_t X_{t+1} - p_t T_t]$$

$$+ \mu_t [m_t + b_t + p_t X_t + p_t z L_t - p_t C_t - q_t b_{t+1} (1 + \alpha) - p_t s_t X_{t+1} - p_t T_t - m_{t+1} (1 + \alpha)]$$

This CIA model is solved analytically with the first order conditions for optimisations as:

$$C_t : U'(C_t) - (\lambda_t + \mu_t) p_t = 0$$
\[ L_t : -V'(L_t) + \mu_t p_t z = 0 \]  
(A.3)

\[ b_{t+1} : -q_t (1 + \alpha) (\lambda_t + \mu_t) + \beta \frac{\partial v}{\partial b_{t+1}} = 0 \]  
(A.4)

\[ X_{t+1} : -p_t s_t (\lambda_t + \mu_t) + \beta \frac{\partial v}{\partial X_{t+1}} = 0 \]  
(A.5)

\[ m_{t+1} : -(1 + \alpha) \mu_t + \beta \frac{\partial v}{\partial m_{t+1}} = 0 \]  
(A.6)

By the envelop theorem on differentiating the Bellman equation:

\[ \frac{\partial v}{\partial b_t} = (\lambda_t + \mu_t) \]  
(A.7)

\[ \frac{\partial v}{\partial X_t} = p_t (\lambda_t + \mu_t) \]  
(A.8)

\[ \frac{\partial v}{\partial m_t} = (\lambda_t + \mu_t) \]  
(A.9)

Combining above last three and the first two first order conditions, the middle three FOCs can be expressed as:

\[ -q_t (1 + \alpha) \frac{U'(C_t)}{p_t} + \beta \frac{U'(C_{t+1})}{p_{t+1}} = 0 \]  
(A.10)

\[ -sU'(C_t) + \beta U'(C_{t+1}) = 0 \]  
(A.11)

\[ -\frac{(1 + \alpha) V'(L_t)}{p_t z} + \beta \frac{U'(C_{t+1})}{p_{t+1}} = 0 \]  
(A.12)

Higher productivity lowers the level of employment:

\[ \frac{dL}{d\alpha} = \frac{-V''}{(1 + \alpha) V'' - \beta z^2 U''} < 0 \]  
(A.13)

Here \( \alpha \) can be set to achieve the optimal inflation in inflation targeting regimes to maximize the level of welfare in the economy, \( \max_{\{C_t, L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [U(C_t) - V(L_t)] \). The optimal employment \( (L^*) \) is obtained implicitly

\[ zU'(zL^*) - V'(L^*) = 0 \]  
(A.14)

The optimal growth rate of money supply is given by the Friedman rule is \( \alpha = \beta - 1 \) where the nominal interest rate is zero \( R = \frac{1}{q} - 1 = 0 \Rightarrow q = 1 \), the real interest rate is \( r = \frac{1}{\beta} - 1 \), cash in advance constraint does not bind \( \alpha = \beta - 1 \) because \( \lambda = 0 \).
\[
\lambda = \frac{U'(C)}{p} - \mu = C.U'(C) \frac{V'(L)}{1 + \alpha} - \frac{\beta}{1 + \alpha} \frac{U'(C)}{p} = C.U'(C) \frac{1}{1 + \alpha} - \frac{\beta}{1 + \alpha} \frac{U'(C)}{p} = C.U'(C) \frac{1}{1 + \alpha} (1 - q) \quad (A.15)
\]

B Money in Utility Function Model

Again using the Lagrange multiplier \((\lambda_t)\) to simplify this constrained optimisation problem:

\[
\mathcal{L}(c_t, m_t, \lambda_t) = \sum_{t=0}^{\infty} \left[ \beta^t u(c_t, m_t) \right] + \lambda_t \left[ f(\omega_t - c_t - m_t) + \tau_{t+1} + \left( \frac{1 - \delta}{1 + n} \right) (\omega_t - c_t - m_t) + \frac{m_t}{(1 + \pi_{t+1})(1 + n)} \right] \quad (B.16)
\]

As before solving MIU model explicitly means expressing the prices and quantities like \(y_t, k_t, c_t, m_t\) in terms of the preference and technology parameters as \(\beta, \delta, \alpha\) and \(n\). In other words the optimal values of variables are determined by subjective discount factor \((\beta)\), depreciation \((\delta)\), productivity of capital \((\alpha)\) and growth rate of population \((n)\). This is done using the first order conditions:

\[
c_t : u_c(c_t, m_t) - \beta \left[ f_k(k_t) + \left( \frac{1 - \delta}{1 + n} \right) \right] V_\omega(\omega_{t+1}) = 0 \quad (B.17)
\]

Here marginal utility of holding capital \(c_t : u_c(c_t, m_t) - \beta \left[ f_k(k_t) + \left( \frac{1 - \delta}{1 + n} \right) \right] V_\omega(\omega_{t+1})\) should equal the marginal utility of consumption \(u_c(c_t, m_t)\).

\[
m_t : u_m(c_t, m_t) - \beta \left[ f_k(k_t) + \left( \frac{1 - \delta}{1 + n} \right) \right] V_\omega(\omega_{t+1}) + \frac{\beta V_\omega(\omega_{t+1})}{(1 + \pi_{t+1})(1 + n)} = 0 \quad (B.18)
\]

Transversality conditions

\[
\lim_{t \to \infty} \beta^t \lambda_t k_t = 0; \lim_{t \to \infty} \beta^t \lambda_t m_t = 0 \quad (B.19)
\]

By envelope theorem:

\[
\lambda_t = V_\omega(\omega_t) = u_c(c_t, m_t) \quad (B.20)
\]