The Sustainable Rate of Return of Defined-Contribution Pension Schemes

Gronchi, Sandro and Nisticò, Sergio

Department of Economics and Law, University of Cassino and Southern Lazio

June 2012

The sustainable rate of return of defined-contribution pension schemes

Sandro Gronchi

(Sapienza University of Rome)

Sergio Nisticò

(University of Cassino and Southern Lazio)

Introduction

Pension schemes can be classified along various dimensions, one of which is their degree of funding (Lindbeck and Persson 2003, Valdés-Prìeto 2006) - that is the ratio *k* between the value of the reserve fund and the liabilities towards the members. Along such a dimension, pay-as-you-go (PAYG) schemes and fully funded (FF) schemes can be represented as the two extremes of the [0-1] real interval.

For the schemes where $k>0$, the return on the reserve fund represents a second source of revenue that can be used either to pay more generous pensions than the contribution revenue would allow or to increase the fund itself.

It is well known that Defined-Contribution (DC) schemes provide all individuals with a personal, interest-bearing account that registers all individual contributions paid in during active life. At retirement, the balance of the account is transformed into a financially equivalent stream of pension annuities according to the life expectancy of the individual's cohort. Contrary to what happens in defined-benefit schemes, wherein the contribution rate is the key variable ensuring their financial stability, the sustainability of defined-contribution schemes is ensured by the appropriate choice of the rate of return yearly credited on all account balances.

The aim of this note is to identify a notion of sustainable rate of return, which is sufficiently general to be applicable to whatever DC scheme, regardless of its degree of funding. In particular, for a DC pension scheme whose degree of funding in a given period of time coincides with its desired level, financial sustainability obtains when the flows of yearly contributions and interests accruing to the fund allow both to finance the disbursement for pensions and to let assets grow in line with liabilities, so that the degree of funding remains at its desired level. For the sake of simplicity we will refer to a stationary state in which the age structure of the population and the growth rates of the new plan members and of the average earnings are constant through time [1]

For PAYG DC schemes, whose degree of funding is nil, it has been shown (Samuelson 1958, Aaron 1966, Valdés-Prieto 2000, Gronchi and Nisticò 2006, 2008) that the sustainable rate of return to be credited on all pension accounts is equal to the growth rate of the total earnings of the active members.[2] In fact, this rate of return ensures that the contribution revenue exactly matches the pension expenditure in each year, thus ensuring also that the system remains purely pay-as-you-go, namely that its degree of funding does not become either negative or positive.

When the DC scheme is endowed with a reserve fund, whose management yields the market interest rate, those interests should be taken into account for the identification of the sustainable rate of return. A general rule for the identification of the sustainable rate of return for the DC schemes with a positive degree of funding can be developed starting from the consideration that, in a stationary state, the liabilities of the scheme grow, *ceteris paribus*, at the same rate of the total earnings of the active population. Therefore, if one wants that the degree of funding remains constant, the value of the fund must also grow at that rate, which requires that an appropriate share of the interests be yearly 'allocated' for that purpose; and only the remaining share can be 'promised' to the members and credited on their accounts. More specifically, the flow of interests that can be used to 'correct' the sustainable rate of return of pure PAYG schemes is equal to the difference (either positive or negative) between the market interest rate and the growth rate of the total earnings of active population times the value of the fund; and, consequently, the extra rate of return (positive or

---

[1] As it is well known, the Swedish pension system has developed an accounting mechanism that monitors each year the appropriateness of the conventional rate of return according to the yearly contribution flow and the age structure of the insured population. See Settergren and Mikula (2006).

[2] The proof in Gronchi and Nisticò (2006 and 2008) is given with reference to a four-overlapping generations model and it is unique in removing the assumption of a constant growth rate of wages.

negative), with respect to the sustainable rate of pure PAYG systems, is equal to such a flow divided by the value of the liabilities that, in a steady state with constant mortality patterns, is equal to the sum of all members' account balances.

In fact, one can prove the validity of the following

PROPOSITION:

If a DC scheme aims to maintain its degree of funding unaltered through time, it must credit on all account balances a rate of return equal to the growth rate of total earnings plus a share, equal to its degree of funding, of the difference between the market interest rate earned by the fund manager and the growth rate of total earnings.

PROOF

The proof can be given with reference to a very simple two-overlapping-generations model representing a hypothetical DC pension scheme that, when started, saves in a fund a share k (with $0 \leq k \leq 1$) of the contributions paid by the first generation of active workers and 'donates' the remaining share $(1-k)$ to first generation of retirees. As mentioned above, the steady state with a constant age-distribution of the insured population will also be assumed.

Given that the liabilities of the scheme at its birth in period *t* coincide with the contributions levied on the first generation of active workers, if a share k of this revenue is saved in a fund, the scheme's initial degree of funding is precisely equal to k.

The following other notations are adopted:

W_t : Total earnings of the active members in period *t*;

F_t : Value of the reserve fund in period *t;*

L_t : Liabilities in period *t;*

r : Market interest rate;

$\widehat{w}$: Growth rate of total earnings;

π : Conventional rate of return credited on all account balances;

c : Contribution rate.

We will prove that the rate of return to be credited on all account balances for the degree of funding to remain constant through time is

$$\pi^* = \widehat{w} + \frac{F_t}{L_t} \cdot \left(r - \widehat{w}\right).$$

Let us start by reiterating that the liabilities of the scheme in period t are

(1) $$L_t = c \cdot W_t ,$$

whereas the value of the fund in the same period is a share k of the contribution revenue on the right hand side of (1), so that the degree of funding in period t is

$$\frac{F_t}{L_t} = \frac{k \cdot c \cdot W_t}{c \cdot W_t} = k .$$

For the degree of funding to be equal to k also in period t+1, the rate of return credited on the account balances must ensure that

(2) $$\frac{F_{t+1}}{L_{t+1}} = \frac{k \cdot c \cdot W_t \cdot \left(1+r\right) + c \cdot W_t \cdot \left(1+\widehat{w}\right) - c \cdot W_t \cdot \left(1+\pi\right)}{c \cdot W_t \cdot \left(1+\widehat{w}\right)} = k ,$$

where the numerator of (2) expresses the value of the fund in t+1 as the sum of the value of the fund in period t gross of the interests matured at the market interest rate r, and the difference between the contribution revenue and the pension expenditure in period t+1, the latter being equal to the contributions paid in period t gross of the interests, computed at the rate π, credited on the account balances of the first generation of active workers. The liabilities in t+1, at the denominator of (2), are simply given by the contributions paid by the active workers in period t+1, the previous debt having been fully redeemed with the payment of the pensions.
Simplifying and reordering, (2) becomes:

$$\frac{\left(1+\pi\right)}{\left(1+\widehat{w}\right)} = \frac{k \cdot \left(1+r\right) + \left(1+\widehat{w}\right)}{\left(1+\widehat{w}\right)} - k ,$$

from which:

(3) $$\begin{aligned} \pi &= \left(1+\widehat{w}\right) + k \cdot \left[\left(1+r\right) - \left(1+\widehat{w}\right)\right] - 1 \\ &= \widehat{w} + k\left(r - \widehat{w}\right) \end{aligned}$$

Q.E.D.

COMMENTS

1. Condition (3) applies to any DC scheme, regardless of its degree of funding. In fact: (i) for $k = 0$, i.e. for pure PAYG schemes, the sustainable return is equal to the growth rate of the total earnings of active workers; (ii) for $k = 1$, i.e. for fully funded schemes, the sustainable rate of return equals the market interest rate.
2. Choosing a rate of return equal to (3) implies the equality between contribution revenue and pension disbursement only if the market interest rate is equal to the growth rate of the total earnings of the active population.
3. Choosing a rate of return equal to the growth rate of the total earnings of the active population ensures in any case the equality between the contribution revenue and the pension expenditure in each year. In this case, the value of *k* will increase, decrease or remain stationary according to whether the market interest rate exceeds, falls short of or equals the growth rate of the total earnings of the active population.
4. In reality, i.e. outside the steady state and with a varying age-structure of the population, some DC systems use their reserves as a buffer fund whose value goes up and down according to the economic and demographic cycles without any commitment to fund their liabilities to a pre-determined extent; in such cases a sort of automatic balance mechanism *à la suédoise* (3) might be needed for the buffer fund to remain always positive.

References

Aaron, H. (1966): 'The social insurance paradox'. *Canadian Journal of Economics and Political Science*, 32 (3), pp. 371–4.

Gronchi, S and S. Nisticò (2006): Implementing the NDC theoretical model. A comparison of Italy and Sweden. In Holzmann R. e E. Palmer (2006) (edited by) *Pension reform. Issues and prospects for non-financial defined contribution (NDC) schemes*, Washington: World Bank.

Gronchi, S and S. Nisticò (2008): Theoretical Foundations of Pay-as-you-go Defined-Contribution Pension Schemes. *Metroeconomica*, 58: 2, 131-59.

Lindbeck, A. and M. Persson. (2003): “The gains from pension reform”. *Journal of Economic Literature* 41. No.1. pp. 74-112.

Samuelson, P. (1958): ‘An exact consumption loan model of interest with or without the social contrivance of money’. *Journal of Political Economy*, 68 (6), pp. 467-82.

Settergren O., and B.D. Mikula (2006): The rate of return of pay-as-you-go pension systems: a more exact consumption-loan model of interest. In Holzmann R. and E. Palmer (eds), *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, Washington: World Bank.

Valdés-Prieto, S. (2000): ‘The financial stability of notional account pensions’, *Scandinavian Journal of Economics*, 102 (3), pp. 395–417.

ValdésPrieto, S. (2006). A Market Method to Endow NDC Systems with Automatic Financial Stability. In Holzmann, R., Palmer, E. (eds), *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, World Bank, Washington, DC.