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Is the Futures Market for Treasury Bills Efficient?

Anthony J. Vignola and Charles J. Dale†

In a recent article in this Journal, p. 1, Puglisi developed and tested a model for evaluating the efficiency of the Treasury bill futures market. His model was based on the expectations hypothesis of the term structure of interest rates whereby the futures market discount rate is related to the expected short-term, or forward, rate implied by the term structure. He found that the market for Treasury bill futures was not efficient because arbitrage opportunities existed involving transactions in futures and outstanding Treasury bills. He concluded, however, that such opportunities have “ebbed as the market has continued to mature.”

These conclusions are based upon a comparison of the returns from investment strategies that involve only bills and a combination of bills and futures. The differences between the returns from these two strategies for equal holding periods were evaluated; it was shown that, although the mean difference was not significantly different from zero, except for the first futures contract (March, 1976), a sign test indicated that the number of times that the differences were significantly different from zero held for six of the seven futures contracts examined.

†The authors wish to acknowledge the assistance of Rosemarie Workman, without whose programming aid this work would not have been possible, and Jeanne Rickey of the Chicago Mercantile Exchange for providing data used on this paper. The views expressed here are those of the authors and do not necessarily reflect the views of the Treasury Department.

The purpose of this comment is to show that the summary statistics reported by Puglisi are misleading and may be misinterpreted, that the Treasury bill futures market may be used to increase returns, and that the spot and futures markets must be evaluated for such purposes on a daily basis. The time series of the differences in returns from bills-only and bills-futures transactions clearly shows that, although the mean difference between these returns on average are small and their standard deviations large, there are distinct arbitrage returns from using the futures market. Moreover, these returns may be substantial on a given day, even though the mean return for all days of a particular contract is zero. These returns and the times series trends in these returns that may assist the portfolio manager in his use of the futures market become clearer when arbitrage returns are evaluated on a daily basis. Furthermore, our data show results substantially at odds with those reported by Puglisi, and we have found that, according to more recent data, the futures market has remained inefficient.\(^2\)

Following the model developed by Puglisi, we have computed the annualized returns from a short-term bill that matures on the same date as the expiration of the futures contract, and the annualized return from a longer-term bill (one with three more months to maturity) that may be delivered against the short sale of the futures contract.\(^3\) The annualized return from these two investments is given by equations 1 and 2, respectively.

\[
R_s = \frac{D_s}{S} \times \frac{365}{N_s}
\]

\[
R_L = \frac{F-L-T}{L} \times \frac{365}{N_s}
\]

where \(R_s\) = annualized return from a bills-only strategy, \(R_L\) = annualized return from a bills-futures strategy, \(D_s\) = discount on the short-term bill, \(S\) = price of the short-term bill, \(N_s\) = number of days to maturity, i.e., the number of days from the transaction date to the delivery date of the futures contract, \(F\) = price of the futures contract, as specified \(N_s\) days before delivery, \(L\) = price of the long-term bill, and \(T\) = minimum transaction costs on a futures contract ($60).\(^4\)

\(^2\)The market is assumed inefficient if there are arbitrage opportunities involving futures and outstanding Treasury bills.

\(^3\)For the period three months prior to the contract delivery date, the outstanding spot market bills have the exact maturity date as the futures contract expiration and the maturity of the deliverable bill. For periods when the corresponding bills do not exist, the nearest one-year bill is used as an approximation.

\(^4\)It is noteworthy to realize that, while we only consider the alternative for the short seller of the futures contract, symmetrical results hold for the buyer of a futures contract. That is, when the short-term investor increases his returns from buying a longer-term bill and selling a
The source of our spot market data is the Federal Reserve Bank of New York, which compiles composite quotes on all Treasury securities. The Chicago Mercantile Exchange provided futures market quotes. Daily settlement prices for futures contracts and closing bid and ask prices for spot market Treasury bills were used. We report results for bid and ask spot prices, because there are different conclusions depending on which prices are used and they confirm our findings that the market has remained inefficient. Furthermore, it has been conjectured that the Treasury bill market is a dealer-dominated market, indicating that the bid side quotes for the spot market should determine the trading price. We offer no conclusion on this conjecture. However, the results using bid and ask prices are different enough to warrant reporting both results. On one half of the eight contracts evaluated, the spread is enough to change the sign of the arbitrage profits.

Tables 1 and 2 report our results for ask and bid spot prices, respectively. For each contract, the daily return from each investment strategy is computed for approximately nine months using the six-month bill for the last 91 days of the contract and the nearest one-year bill for all other days. Row 1 of each table gives the mean return from an outright bill purchase (equation 1), and row 2 gives the mean returns from a bill purchase in combination with a futures market short sale (equation 2). Row 3 is the bills-only return minus the bills-future return. The table also gives the standard errors for each of these returns, along with the number of observations used for each contract and the results of a sign test.

Our results show that there were positive arbitrage profits from the covered short sale of a futures contract on only one contract out of the first eight, the December, 1977 contract, when using ask spot market prices. These results differ substantially from Puglisi's findings that there were positive returns from the March, June, December, 1976 and the June, 1977 contracts. Our findings indicate that positive arbitrage profits accrued to the long purchase of a futures contract on the first seven contracts. Using bid spot prices, there were positive arbitrage gains on the short of the futures market on five of the eight contracts. However, these conclusions are misleading, since the “t” statistics for all eight contracts indicate that none of the mean differences using bid or ask prices is significantly different from zero at any acceptable level of confidence.

We have also computed sign tests on the number of times that the returns from the bills-only strategy were less than the returns on the
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</thead>
<tbody>
<tr>
<td>R1</td>
<td>4.576%</td>
<td>4.994%</td>
<td>5.316%</td>
<td>5.335%</td>
<td>5.155%</td>
<td>4.863%</td>
<td>5.135%</td>
<td>5.608%</td>
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<tr>
<td>SD(R1)</td>
<td>(.142)</td>
<td>(.212)</td>
<td>(.283)</td>
<td>(.550)</td>
<td>(.676)</td>
<td>(.318)</td>
<td>(.229)</td>
<td>(.292)</td>
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<tr>
<td>R2</td>
<td>4.402%</td>
<td>4.940%</td>
<td>5.166%</td>
<td>5.274%</td>
<td>5.043%</td>
<td>4.813%</td>
<td>4.971%</td>
<td>5.714%</td>
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<tr>
<td>SD(R2)</td>
<td>(.540)</td>
<td>(.468)</td>
<td>(.270)</td>
<td>(.578)</td>
<td>(.666)</td>
<td>(.342)</td>
<td>(.408)</td>
<td>(.497)</td>
</tr>
<tr>
<td>R3</td>
<td>+.174%</td>
<td>+.054%</td>
<td>+.150%</td>
<td>+.061%</td>
<td>+.112%</td>
<td>+.048%</td>
<td>+.164%</td>
<td>-.106%</td>
</tr>
<tr>
<td>SD(R3)</td>
<td>(.489)</td>
<td>(.459)</td>
<td>(.213)</td>
<td>(.254)</td>
<td>(.191)</td>
<td>(.226)</td>
<td>(.437)</td>
<td>(.294)</td>
</tr>
<tr>
<td>t (R3)</td>
<td>0.356</td>
<td>0.118</td>
<td>0.704</td>
<td>0.240</td>
<td>0.586</td>
<td>0.212</td>
<td>0.375</td>
<td>0.361</td>
</tr>
<tr>
<td>N</td>
<td>48</td>
<td>113</td>
<td>175</td>
<td>198</td>
<td>200</td>
<td>191</td>
<td>197</td>
<td>184</td>
</tr>
<tr>
<td>N(R1) &lt; N(R2)</td>
<td>20</td>
<td>67</td>
<td>73*</td>
<td>81*</td>
<td>45**</td>
<td>63**</td>
<td>47**</td>
<td>106*</td>
</tr>
</tbody>
</table>

R1 is annualized return from bills-only strategy, R2 is annualized return from bills-futures strategy, R3 is R1 less R2, SD is standard deviation, t is t-statistic, N is the number of observations, N(R1) < N(R2) is number of times bills-only strategy returned less than bills-futures strategy, MAE is mean absolute error, *significant at the $\alpha = 0.01$, and **significant at the $\alpha = 0.05$. 
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</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>4.866%</td>
<td>5.197%</td>
<td>5.452%</td>
<td>5.452%</td>
<td>5.265%</td>
<td>5.005%</td>
<td>5.260%</td>
<td>5.730%</td>
</tr>
<tr>
<td>SD (R1)</td>
<td>(.157)</td>
<td>(.235)</td>
<td>(.265)</td>
<td>(.550)</td>
<td>(.664)</td>
<td>(.319)</td>
<td>(.310)</td>
<td>(.318)</td>
</tr>
<tr>
<td>R2</td>
<td>5.078%</td>
<td>5.410%</td>
<td>5.431%</td>
<td>5.490%</td>
<td>5.244%</td>
<td>5.015%</td>
<td>5.189%</td>
<td>5.907%</td>
</tr>
<tr>
<td>SD (R2)</td>
<td>(.262)</td>
<td>(.474)</td>
<td>(.231)</td>
<td>(.405)</td>
<td>(.535)</td>
<td>(.341)</td>
<td>(.376)</td>
<td>(.554)</td>
</tr>
<tr>
<td>R3</td>
<td>-.212%</td>
<td>-.217%</td>
<td>.021%</td>
<td>-.037%</td>
<td>.022</td>
<td>-.010</td>
<td>.071</td>
<td>-.177</td>
</tr>
<tr>
<td>SD (R3)</td>
<td>(.184)</td>
<td>(.329)</td>
<td>(.161)</td>
<td>(.220)</td>
<td>(.179)</td>
<td>(.187)</td>
<td>(.186)</td>
<td>(.283)</td>
</tr>
<tr>
<td>t (R3)</td>
<td>1.512</td>
<td>0.660</td>
<td>0.160</td>
<td>0.1681</td>
<td>0.123</td>
<td>0.053</td>
<td>0.382</td>
<td>0.625</td>
</tr>
<tr>
<td>N</td>
<td>48</td>
<td>113</td>
<td>175</td>
<td>198</td>
<td>200</td>
<td>191</td>
<td>197</td>
<td>184</td>
</tr>
<tr>
<td>N(R1) &lt; N(R2)</td>
<td>44**</td>
<td>102**</td>
<td>82</td>
<td>112</td>
<td>98</td>
<td>83</td>
<td>63**</td>
<td>123**</td>
</tr>
</tbody>
</table>

R1 is annualized return from bills-only strategy, R2 is annualized return from bills-futures strategy, R3 is R1 less R2, SD is standard deviation, t is t-statistic, N is the number of observations, N(R1) < N(R2) is number of times bills-only strategy returned less than bills-futures strategy, MAE is mean absolute error, *significant at the $\alpha = 0.01$, and **significant at the $\alpha = 0.05$. 

bills-futures strategy. The sign tests indicate that the difference in returns was significantly different from zero at the .05 level of significance, or better, for six of the eight contracts on the ask side and for four contracts on the bid side. Our sign test results also differ from those reported by Puglisi. The sign tests indicate that the market has remained inefficient and that the arbitrage returns on Treasury bill futures has been increasing, not decreasing. This is further confirmed by the fact that the last two contracts analyzed, the September and December, 1977 contracts, are the only ones where the bid-ask spread does not change the sign of arbitrage returns and the sign tests are significant at the .01 level of confidence for both bid and ask spot market prices.

Our major concern is not that our results differ from those reported by Puglisi, since such differences may be because of our different data source, the use of different one-year bills, and the use of true bond yield equivalent rates. Instead, we are concerned that the distribution of the difference in returns from bills-only and bills-futures (R3) must be examined on a daily basis for the portfolio manager to assess adequately the profit possibilities or arbitrage. It is our conclusion that one can only obtain an adequate picture of the futures market by examining the distribution and time series properties of R3. For this reason, we have provided charts of the arbitrage returns for each contract.

The charts show the bills-futures returns minus the bills-only returns for each contract. The horizontal axis represents the number of days to the delivery of the futures contract, with time moving from the right to the left. Each chart begins roughly nine months prior to the contract delivery date. It should be recalled that, while positive numbers on these charts represent gains from arbitrage to the seller of a futures contract, the opposite is true for the purchaser of a futures contract. The charts clearly indicate that summary statistics are misleading and that there are time series trends in arbitrage returns. We have examined this trend and found that there is significant autocorrelation in arbitrage returns for each contract, confirming that the futures market is inefficient, not only in an arbitrage sense but also in the sense that the arbitrage returns are not distributed randomly over time.\footnote{It is not our purpose to pass judgment on the appropriate test for market inefficiency, but to replicate and update the work of Puglisi. Others, however, have indicated that the sign test and the existence of autocorrelation are not strong enough tests to conclude that market inefficiency exists. See E. Fama, "Efficient Capital Markets: A Review of Theory and Empirical Work," \textit{Journal of Finance} (May, 1970), pp. 383-420, and W. Cornell and J. Dietrick, "The Efficiency of the Market for Foreign Exchange Under Floating Exchange Rates," \textit{Review of Economics and Statistics} (February, 1978), pp. 111-120.}

The time series of differences in returns for each contract present a more revealing overview of arbitrage and inefficiency in the Treasury bill futures market. Moreover, there is a great deal of similarity in the
ALL EIGHT GRAPHS ARE PUBLISHED ON ONE PAGE
ARBTRAGE RETURNS 1977
Bill-Future Returns Minus Bills-Only Returns
(Arbitrage Spot Prices)

MARCH 1977 CONTRACT

JUNE 1977 CONTRACT

SEPTEMBER 1977 CONTRACT

DECEMBER 1977 CONTRACT
trend of these differences in all contracts since September, 1976. For
the June, September, and December, 1977 contracts, in particular,
there is an upward trend in arbitrage returns for the short seller. Such
a situation implies that, for the more recent contracts, there has been
an excess demand for futures contracts, bidding up futures prices, espe-
cially in the period when the underlying deliverable security is out-
standing, roughly 91 days prior to the expiration of the futures contract.
The charts support the findings that the Treasury bill-futures market
is inefficient. In an efficient market, arbitrage gains should rarely ap-
pear and when they do, they should quickly disappear. The diagrams
not only show that arbitrage profits do not vanish, but also show that
discrepancies tend to persist with the same sign for long periods. They
refute, as do our summary statistics, that the inefficiencies only occurred
early in the life of the futures market. Inefficiency has not diminished
with the maturation of this market. The charts point out the reason for
the small mean differences and the large standard errors reported in our
results. The small means are the net result of wide swings above and
below zero. Since investors buy and sell on individual days, not at the
mean return, it is the daily returns that are important to the portfolio
manager.

Finally we wish to emphasize that we have considered only the case
of arbitrage as a measure of market inefficiency. Other measures of
the futures market's impact, such as its impact on bid-ask spreads and
price volatility, are not at issue in this paper. The net effect of the fu-
tures market on the spot market, and its price relationship to the spot
market, is a complex topic.6

References

111-120.
3. Froewiss, Kenneth, “GNMA Futures: Stabilizing or Destabilizing,” San Francisco Federal
5. Sandor, Richard, “Comment (on a paper by L. Ederington and L. Plumly),” in Futures

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6 See R. Sandor, “Comment (on a paper by L. Ederington and L. Plumly),” in Futures Trad-
20-29.