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The Efficiency of the Treasury Bill Futures Market: An Analysis of Alternative Specifications

Anthony J. Vignola and Charles Dale†

Treasury-bill futures began trading on January 6, 1976, on the International Monetary Market (*IMM*) of the Chicago Mercantile Exchange. The *IMM* trades Treasury Bill futures contracts for delivery in March, June, September, and December. Trading in each contract ends on the second business day following the Treasury Bill auction of the third week of the delivery month. Contracts call for the delivery of $1 million of Treasury Bills to mature in 90 days. Until the existence of financial futures, testing the determinants and the informational content of futures market prices has been difficult because of the vagaries associated with commodity markets. In the case of financial futures, and in particular, Treasury Bill futures, the existence of an active secondary market and the resulting term structure of interest rates enables one to test alternative hypotheses about the prices of future contracts. The merits of futures markets as a mechanism for price discovery have been debated for a considerable length of time.

In the present study, the pricing of Treasury Bill futures contracts is examined. Actual futures prices are compared with two alternative specifications of equilibrium futures prices, *i.e.*, those implied by carrying charges and those derived from the unbiased expectations hypothesis of the theory of the term structure of interest rates. Specifically, we

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present and empirically test two alternative specifications of Treasury Bill futures market efficiency.[1] These are: (1) the determination of pure-arbitrage opportunities between the futures and the cash market and (2) the determination of quasi-arbitrage opportunities between the futures and the cash market.[2] The pure-arbitrage model is based upon Working's theory of storage costs.[3] The quasi-arbitrage model is based on the expectations hypothesis of forward rates.

I. RECENT EMPIRICAL ANALYSES

The secondary market for Treasury securities gives rise to a well defined term structure of interest rates, or yield curve.[4] Since theories and tests of the term structure abound in the economic and financial literature, it is natural for economists to turn to term structure theories for the analysis of Treasury Bill futures, since implied forward rates embodied in the term structure are the spot market's assessment of future rates of interest.[5] Several recent papers have been written that empirically evaluate the futures market using the expectations hypothesis of the term structure as a basis for establishing the equilibrium price of a futures contract. These papers have been concerned with verifying various term structure hypotheses and testing the efficiency of the Trea-

[1] In recent studies of the futures market, inefficiency has been assumed to exist when there are profitable arbitrage opportunities. In this paper, the writers adopt the terminology of market inefficiency in the current context to mean that arbitrage possibilities exist between the cash and futures market.

[2] Pure arbitrage refers to arbitrage where the position must be financed from borrowed funds, as in the case of a fully leveraged firm. Quasi-arbitrage refers to situations involving existing portfolios. For example, in the case of quasi-arbitrage a holder of Treasury Bills can alter his his portfolio mix and obtain arbitrage profits by selling a three-month bill, buying a six-month bill and shorting a futures contract for delivery in three months. The terms of the portfolio will be unchanged, but the composition will be altered.

[3] According to Working, any divergence between spot prices and futures prices can be explained by carrying charges, that is, the cost of storing the commodity until future delivery. For futures in tangible commodities, these costs have traditionally included insurance, warehousing, interest and transportation. See Working [22, 23]. Of course, a necessary condition is that a commodity be storable. Technically, deliverable Treasury bills are only storable for approximately 91 days prior to delivery. Prior to that period, only proxies for storable bills exist.

[4] For a thorough discussion of yield curves, forward rates and the term structure, see Malkiel [11], especially Chapters I and II, Nelson [13] and Roll [18]. See Cox, Ingersoll and Ross [5] for explicit specification of the differences among alternative theories of the term structure.

[5] Since Working postulated that spot and futures prices are intimately connected, his theories are more general but consistent with term structure theories since they also embody an implied cost of storage. In fact, the Hicksian development of the theory of the term structure is based on concepts derived from commodity futures markets. Hicks viewed all markets, spot, futures and forward as mutually interdependent. See Hicks [8], especially Chapter 11.

sury *Bill futures markets.*[6] *Using the traditional approach to the term* structure for evaluating the efficiency of the futures market poses difficulties, for it assumes that the same term structure theories that apply to the spot market hold for the futures market.

Poole [15] examined the case of arbitrage between the cash market returns to a bill holder and the returns to a combination of bills and futures transactions. He found that profitable arbitrage opportunities rarely exist, and are small when they do exist, implying that the markets are efficiently priced. Lang and Rasche [9] rejected the hypothesis that Poole's findings can be extrapolated to more distant contracts, and also rejected his findings for the case of arbitrage for more recent periods. Branch [2], Chow and Brophy [4] and Morgan [12] obtained similar results. However, Morgan showed that the characteristics of futures contracts may result in differences between futures prices and implied forward prices, even in an efficient market.

Puglisi [16] and Vignola and Dale [21] also found market inefficiencies that present opportunities for portfolio investors in Treasury Bills. Capozza and Cornell [3] concluded that for approximately 18 weeks prior to contract delivery, futures prices (rates) were too high (low) and that they were too low (high) for more distant periods when forward rates were compared to futures rates. They attributed these differences to transactions costs, but they were unable to explain the differences for the contract nearest delivery. On the other hand, Oldfield [14] concluded that profitable arbitrage opportunities do not exist.

Rendleman and Carabini [17] compared actual futures prices with estimated equilibrium prices treating futures contracts as forward contracts. They found significant price differences between actual and hypothetical equilibrium prices, when no transaction costs were included. Nearby futures contracts were generally overpriced and longer term contracts were generally underpriced. When transaction costs were introduced, the number of arbitrage possibilities declined. However, they found that a significant number of arbitrage opportunities remained and that the tendency for such opportunities to exist was increasing, particularly for the nearby contract.

Despite the conflicting evidence, in general, futures prices (rates) are found to differ from prices implied by the term structure, with nearby contracts showing futures prices too high and more distant contracts having prices too low. Some of the confusion seems to be due to differences in annualization periods, compounding assumptions, and rate approximation formulas. Some price differences are annualized over the number of days to the delivery of the futures contract, some to the ma-

[6] Studies that use the expectations hypothesis to evaluate the efficiency of the futures are *performing joint tests of the forward rate and the efficiency of the futures market.*

turity of the deliverable security, others over a constant 91-day period.[7]

The findings that actual futures prices do not equal equilibrium futures price have resulted in a myriad of explanations for this apparent market inefficiency. Nearly all authors revert to some form of transactions cost for an explanation, ranging from the cost of borrowing a security (50 basis points) to the implied transactions costs in the spot market as represented by bid and ask spreads.[8] Other explanations include the riskiness of futures as compared to spot bills, the effects of marking to the market, the newness of the markets, the lack of use by institutional investors, and where these reasons are unable to explain the differences, the conclusion is reached that the futures market is inefficient.[9]

The conflicting evidence regarding the pricing and efficiency of the Treasury Bill futures markets makes it clear that a number of unresolved issues remain. The extant literature has centered on quasi-arbitrage tests of the efficiency of the Treasury Bill futures market. Only Rendleman and Carabini raise the issue of pure arbitrage versus quasi-arbitrage. By examining both pure and quasi-arbitrage, the present paper offers a reconciliation of the conflicting findings with regard to pricing and efficiency of the Treasury Bill futures market and offers a valuable approach for analyzing other markets of financial futures.

II. DEVELOPMENT OF MODEL

This section describes two specifications of equilibrium Treasury Bill futures prices. A later section evaluates them using a consistent set of data. Hypothetical equilibrium futures prices are derived from (1) the theory of storage costs and (2) the expectations hypothesis of forward rates implied by the term structure. These hypothetical equilibrium prices are then compared to actual futures prices.

[7] Only Rendleman and Carabini [17] and Vignola and Dale [21] specifically note this effect. Vignola and Dale [21] show that when only bid or ask prices are used, the dollar difference for the long and the short of a futures contract are the same, but the annualized basis point differential is dramatically affected depending on whether the time period of arbitrage profit is to the delivery of a futures contract or to the maturity of the deliverable bill.

[8] Explanations of differences between prices that are caused by bid-ask spreads in the spot market are not without shortcomings. Bid-ask spreads in the spot market are a function of many factors, particularly the time since issue and the dollar amount of a basis point. (See Baker and Vignola [1] for a discussion of this issue). A further complication in the present context arises from the discontinuous nature of the spread as one moves from one-year bills as proxies for deliverable bills to six- and three-month bills.

[9] The conclusion that the futures market is inefficient may merely be the result of the use of inappropriate carrying charges. Dusak [6] shows that the appropriate discount for cash or, equivalently, the financing charge is the only issue in the pricing of futures contracts.

For simplification, the following notations are adapted: time is assumed to flow from right to left, t represents the present time, m is the delivery date of the futures contract, and n is the maturity date of the deliverable instrument.

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n $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ m $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ t

$<$________ 91 days ________$>$ $|$ $<$________ SD days ________$>$

The time from t to m is referred to as SD days, and the time from m to n is always 91 days. Therefore, from time t to time n, the maturity of the deliverable bill is $SD + 91$ days. The price of a futures contract, PF, is that price which is established at time t and is to be paid at m, SD days later. The prices of spot market bills are subscripted with the time period to which they apply.

The relation between the quoted price of a futures contract and the actual price of a futures contract is easily established:

$$PF = 100 - (100 \times Df \times 90) / 360 \qquad (1)$$

where PF = actual price of futures contract, and Df = discount rate at time t on the futures contract (100-futures price index).

Since the Treasury issues 91- and 182-day bills, which are perfectly interchangeable, there is a period of 91 days when the futures contract trades for the delivery of an existing commodity which may be stored for future delivery. As a result, during the three-month period before the expiration of a futures contract, perfect arbitrage possibilities may exist. For periods greater than 91 days prior to the expiration of the futures contract, arbitrage conditions may be explored by using the one-year bill, as a proxy for the deliverable security.

A. Cost of Carry Equilibrium

According to Working's theory of carrying charges, arbitrage possibilities arise when the price of the commodity in the spot market plus the cost of storage differ from the price of the futures contract.[10] In the case of Treasury Bills, the relevant cost of the spot commodity is the price of the deliverable bill. The spot price of a bill is given by:

$$PBn = 100 - (100 \times Rn \times (SD + 91)) / 360 \qquad (2)$$

[10] Working [23] defines the price of storage as the difference between the price of a futures contract and the current cash price which may be positive or negative. Therefore, one could solve for the equilibrium financing charge and evaluate the futures market on the basis of whether that financing charge were available. In order to establish the equilibrium price of a futures contract and compute prices for forward rates, take the cost of storage as given and compute equilibrium prices.

where PBn = actual price of a bill with n days to maturity, Rn = discount rate on a bill with n days to maturity, and SD = number of days to the futures contract expiration.

The cost of storage is the financing cost necessary to store the commodity (which can be purchased for a price of PB) until the delivery of the futures contract, SD days from the present. The cost of storage is therefore given by:

$$C = PBn \times Rs \times SD / 365 \tag{3}$$

where C = cost of storage, PBn = amount borrowed—the cost of the deliverable bill, and Rs = the annualized storage rate of interest.

Therefore, according to the cost of carry theory of futures prices, the price of a futures contract, $ARBc$, in dollar terms should be:

$$ARBc = PBn + C. \tag{4}$$

Several problems arise when attempting to determine the relevant cost of storage for Treasury Bills. Initially, one may conclude that the financing rate, rm, given by a Treasury Bill with SD days to maturity, represents the proper financing cost. Such an assumption, however, does not necessarily reflect either profit-maximizing behavior or the institutional characteristics of the Treasury Bill spot market or the futures market. The rate, rm, does not take into account the opportunity cost of money to the holder of a bill or to the seller of a bill. Such a rate is appropriate for "quasi-arbitrage," in which an investor already holds Treasury Bills and uses futures to increase his net return. However, for the case of "pure arbitrage," which may more appropriately typify firms engaging in futures markets arbitrage, the additional cost of shorting a Treasury Bill would have to be included.[11]

However, even the rate for establishing a short position in an inappropriate financing cost, due to the fact that institutional developments in repurchase agreements (*RP's*) have resulted in the near elimination of short selling. For example, an effective short position can be established by simultaneously purchasing a security under a reverse-*RP*, and selling the security. The current popularity of this technique means that the *RP* rate is the most representative rate for financing Treasury Bills.[12]

[11] In a short-sale, an investor borrows a security and immediately sells it, anticipating that the security can be purchased at a lower price and returned to the original owner. The borrower must pay interest accruing on the security plus the borrowing fee. A 1979 survey of futures market participants by the Commodity Future Trading Commission showed commercial traders which include security dealers, held 34% of the outstanding bill contracts. These firms are marginal net borrowers of funds for which pure arbitrage would apply.

[12] A repurchase agreement involves the sale of a security with the simultaneous agreement for the repurchase at some later date. A reverse *RP* refers to the purchase of a security or the opposite side of a repurchase agreement. For a review of developments in the *RP* market, see

In order to have riskless arbitrage, a holding period, or term *RP* rate is warranted. This presents a two-fold dilemma; the term *RP* market is thin and not uniformly priced and the holding period *RP* rate involving a particular bill, the "special" *RP* rate, entails significant premiums. Therefore, one must resort to proxies of the appropriate financing rate. The most representative rate characterizing the marginal financing costs of firms engaged in government securities arbitrage is the overnight rate. Since these firms are also major arbitrage participants in the futures market, the overnight *RP* rate may also reflect their marginal borrowing needs to finance arbitrage as it is often referred to and carried out in the futures market. If an overnight rate is used, the necessary economic assumption is that the expected value of that rate over the holding period will be equal to the current rate.[13] This specification is consistent with a random walk model of short term rates. In a practical sense, such an assumption is not unrealistic given the uncertainties of market forecasts of short-term rates. However, arbitrage using an overnight borrowing rate is not riskless and requires equation (3) to be specified in the form:

$$C = PBn\,[\,(1 + Rs)^{SD/365} - 1]. \qquad (5)$$

Unfortunately, no representative overnight *RP* rate is either published or readily available. The closest proxy for the rate is the federal funds rate. The use of the federal funds rate as a proxy for the overnight *RP* rate introduces a negative bias in the calculation of the cost of carry since the federal funds rate is generally higher than the *RP* rate.[14] The writers, therefore, test the pure arbitrage model with the federal funds rate, recognizing as Dusak [6] does that the empirical question in pric-

Lucas, Jones and Thurston [10], Simpson [19] and Smith [20]. The reverse *RP* for acquiring a specific bill is called a "special." The market for such transactions is very thin and can entail premiums ranging from 50 to 100 basis points.

[13] Other studies of short-term rates, for example Hamburger and Platt [7], show that short-term rates are consistent with the efficient market model and that "investors appear to behave as if they expect rates of interest . . . to be the same as the current rates." Morgan [12], shows that the price of a futures contract not only depends upon the interest rate expected to prevail on the delivery date but also depends on the expected course of interest rates between the date of agreement and the delivery date.

[14] The *RP* rate is a secured loan rate and is generally lower than federal funds rate by 1/8 to 1/4 percent. The federal funds rate is the rate on unsecured overnight loans. See Simpson [19]. For the more aggressive banks and government securities dealers who participate in the futures market, the federal funds rate may properly reflect marginal borrowing costs for Treasury Bill futures. Daily observations on the federal funds rate are sometimes greatly influenced by bank reserve excesses and shortages and are frequently not representative on Wednesday as a result of the end of the statement week for reserve requirement purposes. This results in many outlier observations for Wednesday. *Therefore, when this rate is used the model is estimated without Wednesday observations.*

ing futures contracts reduces to the determination of the appropriate financing costs.[15]

B. Implied Forward Rate Equilibrium Prices

According to the expectations theory of the term structure of interest rates, the relationship between the futures market and the cash market is determined by the fact that the purchase and sale of securities in the spot market can result in the same position as can be established in the futures market. The costs of undergoing such transactions, however, generally preclude carrying them out. Nonetheless, such transactions should be expected to yield a return equivalent to the forward rate implied by the expectations hypothesis of the term structure of interest rates. For example, the expectations hypothesis holds that an investor should be indifferent between an investment in a six-month Treasury Bill and two consecutive purchases of three-month bills. The expected forward rate to prevail three months from today, therefore, is a function of the relationship between the yields on three-month bills and six-month bills. The expected three-month rate Re, is frequently given by the expression:

$$Re = 1 - [\,(1 - Rn)^{(\,(SD + 91\,)/360)} \times (1 - Rm)^{-(SD/360)}\,]^{(360/91)} \quad (6)$$

where Rn = discount bill rate for n days, Rm = discount bill rate for m days, Re = expected three-month discount rate, and SD = number of days to maturity of Rm, the delivery date of a futures contract.

Equation (6) gives the futures market equilibrium rate according to the expectations hypothesis. In dollar terms, the price of a bill ($ARBe$) with the expected discount rate, Re, to prevail in m days follows from the formula for the price of a bill:

$$ARBe = 100 - 100 \times Re \times 91/360. \quad (7)$$

Equations (6) and (7) are approximations to the arbitrage price of a futures contract since they assume interest rates are determinate. Since Treasury Bills are discount instruments and are traded in terms of price, the arbitrage price, $ARBe$, can be solved for directly. Letting Pm represent the price of a discount instrument that is worth 100 at maturity, $100/Pm$ defines the closed form pure rate of interest over the period m, that is, $Pm \times (1 - Rm)^{-m} = 100$. $100/Pm$ defines the rate of interest without assumptions about compounding or linear approximations.

[15] Since the October 6, 1979, policy changes by the Federal Reserve in conducting monetary policy, the historical relationship between the federal funds rate and the *RP* rate has broken down, making updated analysis based on this relationship impossible.

By substitution, equations (6) and (7) reduce to:

$$ARBe = (100/Pm) \times Pn. \tag{7'}$$

In this form, it becomes clear that the relationship between the expectations hypothesis and the cost of carry formulation reduces to a difference in the appropriate discounting rate since $100/Pm = (1 - Rm)^{-m}$. The only difference is the implied financing charge.

Equations (5) and (7') are arbitrage conditions: equation (5) appropriate for pure arbitrage; equation (7') for quasi-arbitrage. Arbitrage will take place if the arbitrage price does not equal the futures price, that is, if $ARB \neq FP$.[16] If $FP > ARB$, indicating that the price (rate) of a futures contract is too high (low) relative to the spot market, the arbitrager would purchase the long bill (the deliverable bill) and short the futures contract. His profit would be:

$$DIFF = FP - ARB. \tag{8}$$

On the other hand, if $ARB > FP$, indicating that the futures price (rate) is too low (high) relative to the spot market, the arbitrager should buy a shorter bill, one with *SD* days to maturity, and go long the futures contract rather than hold a longer term bill.

To calculate the arbitrage price, *ARB*, the writers use the bid-ask mean for all trades, rather than the separate bid or ask prices for selling or buying. The reasons for using bid/ask means follow from theoretical and institutional considerations. Neither buyers no sellers of Treasury Bills or futures contracts should inordinately influence either of these markets, implying equilibrium should approach the mean. Furthermore, bills have different spreads depending on their characteristics. Since three- and six-month bills are used, as well as one-year bills as a proxy until the six-month bills are issued, this eliminates discontinuities.[17] Using bid-ask means further simplifies the analysis. When bid-ask means are used the dollar gain (loss) from the combination of a long bill purchase and a short sale of a futures equals the dollar loss (gain) to the buyer of a short bill and the purchaser of a futures contract.

[16] Futures market transactions costs are ignored. Such costs are minimal. Initially round-trip commissions were $60. Since March 1978, they have been negotiable. Margin requirements are also small ($800), and their effective cost is the earnings on such funds. Furthermore, they may be satisfied with no initial cash outlay by letters of credit or marketable securities.

[17] Spot market transactions for Treasury Bills rarely take place at the bid or ask price. The quoted bid price is a lower bound for selling a security and the ask price an upper bound for buying a security. Both are paid by only the less frequent participants in this market. In addition, spreads are a function of many factors. Amont these are the time since issue and the dollar value of a basis point for a given term to maturity. Bid-ask spreads in the spot market are frequently as much as 50 basis points for bills with less than 13 weeks to delivery. If an arbitrage band is calculated using such spreads, as many authors have done, it is unlikely that any profitable arbitrage situations will be found.

Two daily series of differences between the arbitrage or equilibrium price of a futures contract and the actual price of the contract were computed. One series used the theory of carrying charges (*ARBc*) while the other series used the expectations hypothesis (*ARBe*). The data sample included all contracts since the beginning of trading in Treasury Bill futures, January 6, 1976, through the December 1978 contract. For both cases, a positive difference represents a price which benefits the short seller of a futures contract, and a negative difference represents a loss to the short seller but a gain to the purchaser of a short bill in conjunction with a futures contract.

III. RESULTS

Tables 1 and 2 contain summary statistics for the dollar amounts of the differences between actual futures prices and computed equilibrium prices for the cost-of-carry model and the forward-rate model, respectively. Tables 3 and 4 give the annualized rates of return for these differences. Each table separates the results into three quarterly subperiods. The nearby period for each contract refers to the 91-day period immediately preceding the futures contract maturity, the period when the exact deliverable security exists. The second and third quarters refer to the subsequent 91-day periods. Thus, summary statistics for a cross section of contracts at the same time relative to their maturity are arrayed horizontally and a time series over the last nine months of any one contract is listed vertically.

As noted above, summary statistics about contracts must be viewed cautiously and may be misleading because of the distribution and time series properties evident in the data. Therefore, charts of the daily results are given in Figures I-IV. The figures show the dollar difference and annualized percent differences for all contracts, grouping the data according to the year in which the contract matured. The horizontal axis of the figures represents the number of days until the delivery of the futures contract, with time flowing from the right to the left. The vertical axis in Figures I and II give the dollar difference between actual futures prices and calculated equilibrium prices. Figures III and IV give the annualized percentage rates of return represented by these dollar differences.

A positive difference between the actual futures price and the equilibrium price means that returns can be increased over the period *SD*, the time to maturity of the futures contract, by purchasing a security with (SD + 91) days to maturity and selling a futures contract instead of buying a security with *SD* days to maturity. A negative difference indicates that returns can be increased over the period SD + 91, the peri-

TABLE 1. Cost of Carry Model Summary Statistics for Dollar Differences: Futures Price Less Equilibrium Price**

		1976				1977				1978				
		Mar.	Jun.	Sep.	Dec.	Mar.	Jun.	Sep.	Dec.	Mar.	Jun.	Sep.	Dec.	All (contracts)
Nearby Contract	μ	134	110	−32	66	−86	−120	−171	61	24	−320	−519	−212	−85
		(27)*	(32)	(17)	(14)	(23)	(37)	(26)	(16)	(36)	(38)	(38)	(57)	(433)
	$\|\mu\|$	194	242	124	124	144	210	187	128	198	330	532	429	268
		(39)	(50)	(44)	(28)	(55)	(76)	(75)	(30)	(58)	(120)	(176)	(137)	(351)
	+	27	26	17	32	14	17	8	25	19	9	3	18	215
	−	12	22	33	16	34	33	41	26	29	40	46	30	462
Second Quarter	μ		1839	691	893	315	382	−118	306	466	637	281	519	550
			(93)	(91)	(68)	(48)	(67)	(75)	(32)	(050)	(080)	(042)	(057)	(753)
	$\|\mu\|$		1839	734	921	469	536	549	383	538	765	362	729	704
			(93)	(94)	(68)	(66)	(87)	(149)	(47)	(59)	(098)	(052)	(079)	(600)
	+		43	40	49	36	40	16	41	39	43	35	41	423
	−		0	7	0	14	9	34	8	8	4	13	21	118
Third Quarter	μ			4049	4001	2561	984	1146	865	952	1609	1627	1973	2031
				(156)	(235)	(151)	(95)	(108)	(166)	(183)	(140)	(120)	(100)	(1647)
	$\|\mu\|$			4049	4101	2561	1039	1295	1071	952	1611	1763	1973	2086
				(156)	(235)	(151)	(99)	(124)	(181)	(183)	(140)	(157)	(100)	(1572)
	+			43	61	62	45	50	36	40	49	45	48	479
	−			0	0	0	7	7	10	1	1	2	0	28

μ = mean
$|\mu|$ = mean absolute value
+ = number of + observations
− = number of − observations
* = numbers in parentheses are standard errors
** = Differences are in dollars for $1,000,000 contract.

TABLE 2. Forward Rate Model Summary Statistics for Dollar Differences: Futures Price Less Equilibrium Price**

		1976				1977				1978				
		Mar.	June	Sep.	Dec.	Mar.	Jun.	Sep.	Dec.	Mar.	Jun.	Sep.	Dec.	All (contracts)
Nearby Contract	μ	175	119	81	265	76	165	166	552	401	345	371	612	282
		(18)*	(14)	(9)	(17)	(15)	(17)	(12)	(30)	(29)	(28)	(41)	(65)	(379)
	$\|\mu\|$	178	151	93	273	121	208	176	549	413	413	517	802	342
		(22)	(22)	(16)	(19)	(24)	(25)	(16)	(30)	(30)	(38)	(57)	(83)	(317)
	+	39	44	46	54	41	54	50	64	56	45	44	47	584
	−	9	15	15	5	19	9	12	0	5	15	18	13	135
Second Quarter	μ		540	655	247	114	−255	−316	385	351	367	755	1417	269
			(39)	(48)	(27)	(19)	(30)	(42)	(35)	(32)	(32)	(55)	(54)	(664)
	$\|\mu\|$		547	664	344	259	308	373	440	498	399	801	1417	555
			(40)	(198)	(43)	(43)	(96)	(117)	(42)	(55)	(36)	(59)	(54)	(444)
	+		51	4	50	39	14	16	49	45	54	53	62	437
	−		3	55	11	23	48	47	13	15	6	8	0	229
Third Quarter	μ			−162	−898	−1086	−233	−1227	−583	−738	88	−333	303	−500
				(47)	(62)	(70)	(36)	(50)	(65)	(55)	(25)	(41)	(36)	(727)
	$\|\mu\|$			846	898	1086	416	1227	606	738	233	361	530	713
				(193)	(229)	(270)	(112)	(310)	(189)	(230)	(48)	(715)	(62)	(539)
	+			22	0	1	23	0	7	0	49	8	51	161
	−			33	77	77	43	72	51	52	15	53	26	499

μ = mean
$|\mu|$ = mean absolute value
+ = number of + observations
− = number of − observations
* = numbers in parentheses are standard errors
** = Differences are in dollars for $1,000,000 contract.

TABLE 3. Cost of Carry Model Summary Statistics for Annualized Dollar Differences: Futures Price Less Equilibrium Price (Percent)

		1976				1977				1978				
		Mar.	Jun.	Sep.	Dec.	Mar.	Jun.	Sep.	Dec.	Mar.	Jun.	Sep.	Dec.	All (contracts)
Nearby Contract	μ	.136	.125	.006	.068	.015	.032	−.031	.051	.035	−.067	−.148	−.029	.020
		(.019)	(.026)	(.006)	(.011)	(.012)	(.030)	(.009)	(.009)	(.017)	(.015)	(.011)	(.017)	(.159)
	$\|\mu\|$	.160	.162	.050	.089	.069	.118	.063	.072	.085	.112	.154	.157	.105
		(.021)	(.027)	(.010)	(.013)	(.015)	(.034)	(.017)	(.010)	(.020)	(.032)	(.046)	(.038)	(.121)
	+	27	26	17	32	14	17	8	25	19	9	3	18	215
	−	12	22	33	16	34	33	41	26	29	40	46	30	462
Second Quarter	μ		.515	.172	.240	.076	.108	−.005	.094	.130	.173	.072	.121	.149
			(.016)	(.020)	(.014)	(.011)	(.015)	(.017)	(.008)	(.012)	(.017)	(.009)	(.011)	(.177)
	$\|\mu\|$		.515	.179	.240	.106	.133	.107	.105	.141	.193	.087	.158	.174
			(.016)	(.020)	(.014)	(.014)	(.018)	(.026)	(.009)	(.013)	(.020)	(.010)	(.014)	(.152)
	+		43	40	49	36	40	16	41	39	43	35	41	423
	−		0	7	0	14	9	34	8	8	4	13	21	118
Third Quarter	μ			.671	.611	.381	.148	.192	.139	.155	.260	.267	.292	.317
				(.018)	(.026)	(.017)	(.014)	(.016)	(.025)	(.027)	(,024)	(.017)	(.013)	(.236)
	$\|\mu\|$			.671	.611	.381	.156	.208	.163	.155	.261	.281	.292	.323
				(.018)	(.026)	(.017)	(.014)	(.017)	(.026)	(.027)	(.024)	(.020)	(.013)	(.228)
	+			43	61	62	45	50	36	40	49	45	48	479
	−			0	0	0	7	7	10	1	1	2	0	28

μ = mean
$|\mu|$ = mean absolute value
+ = number of + observations
− = number of − observations
* = numbers in parentheses are standard errors

TABLE 4. Forward Rate Model Summary Statistics for Annualized Dollar Differences: Futures Price Less Equilibrium Price (Percent)

		1976				1977				1978				
		Mar.	Jun.	Sep.	Dec.	Mar.	Jun.	Sep.	Dec.	Mar.	Jun.	Sep.	Dec.	All (contracts)
Nearby Contract	μ	.163	.164	.097	.223	.089	.220	.146	.529	.349	.256	.247	.458	.248
		(.018)	(.026)	(.012)	(.016)	(.012)	(.029)	(.016)	(.045)	(.043)	(.020)	(.023)	(.043)	(.291)
	$\|\mu\|$	.170	.176	.104	.227	.103	.221	.152	.529	.353	.279	.300	.513	.264
		(.018)	(.026)	(.012)	(.016)	(.012)	(.030)	(.016)	(.045)	(.043)	(.021)	(.026)	(.045)	(.276)
	+	39	44	46	54	41	54	50	64	56	45	44	47	584
	−	9	15	15	5	19	9	12	0	5	15	18	13	135
Second Quarter	μ		.150	−.099	.087	.043	−.038	−.041	.126	.126	.122	.241	.389	.100
			(.009)	(.007)	(.009)	(.006)	(.004)	(.006)	(.013)	(.012)	(.013)	(.021)	(.014)	(.169)
	$\|\mu\|$		.152	.107	.101	.067	.052	.063	.134	.147	.127	.248	.389	.144
			(.009)	(.029)	(.010)	(.008)	(.014)	(.016)	(.013)	(.013)	(.013)	(.021)	(.014)	(.135)
	+		51	4	50	39	14	16	49	45	54	53	62	437
	−		3	55	11	23	48	47	13	15	6	8	0	229
Third Quarter	μ			.013	−.098	−.117	−.021	−.141	−.065	−.085	.020	−.036	.064	−.056
				(.007)	(.006)	(.007)	(.004)	(.006)	(.007)	(.006)	(.003)	(.004)	(.007)	(.073)
	$\|\mu\|$			.115	.098	.117	.052	.141	.070	.085	.035	.042	.087	.074
				(.023)	(.023)	(.028)	(.012)	(.034)	(.020)	(.024)	(.005)	(.012)	(.009)	(.054)
	+			22	0	1	23	0	7	0	49	8	51	161
	−			33	77	77	43	72	51	52	15	53	26	499

μ = mean
$|\mu|$ = mean absolute value
\+ = number of + observations
− = number of − observations
* = numbers in parentheses are standard errors

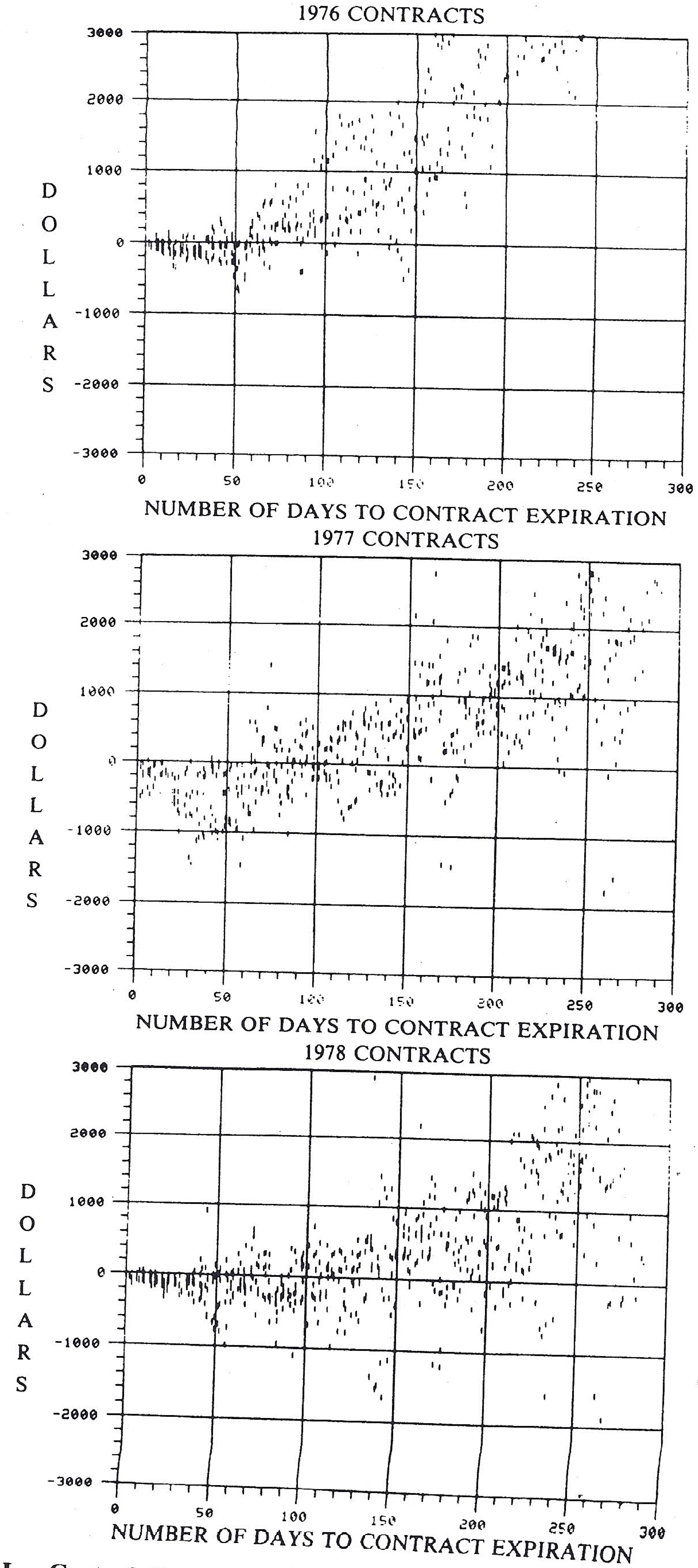

FIGURE I. Cost-of-Carry Model Dollar Differences: Futures Price Less Equilibrium Price

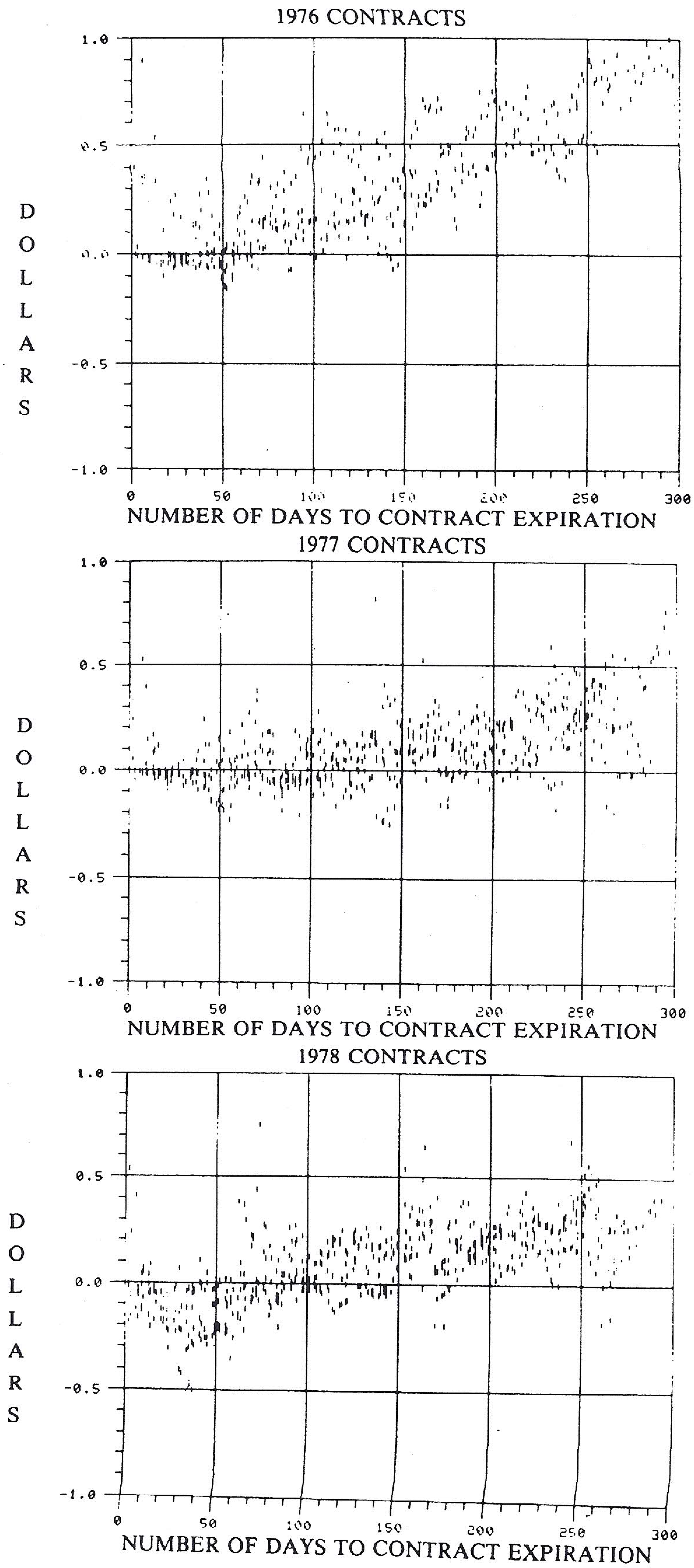

FIGURE II. Forward Rate Model Dollar Differences: Futures Price Less Equilibrium Price

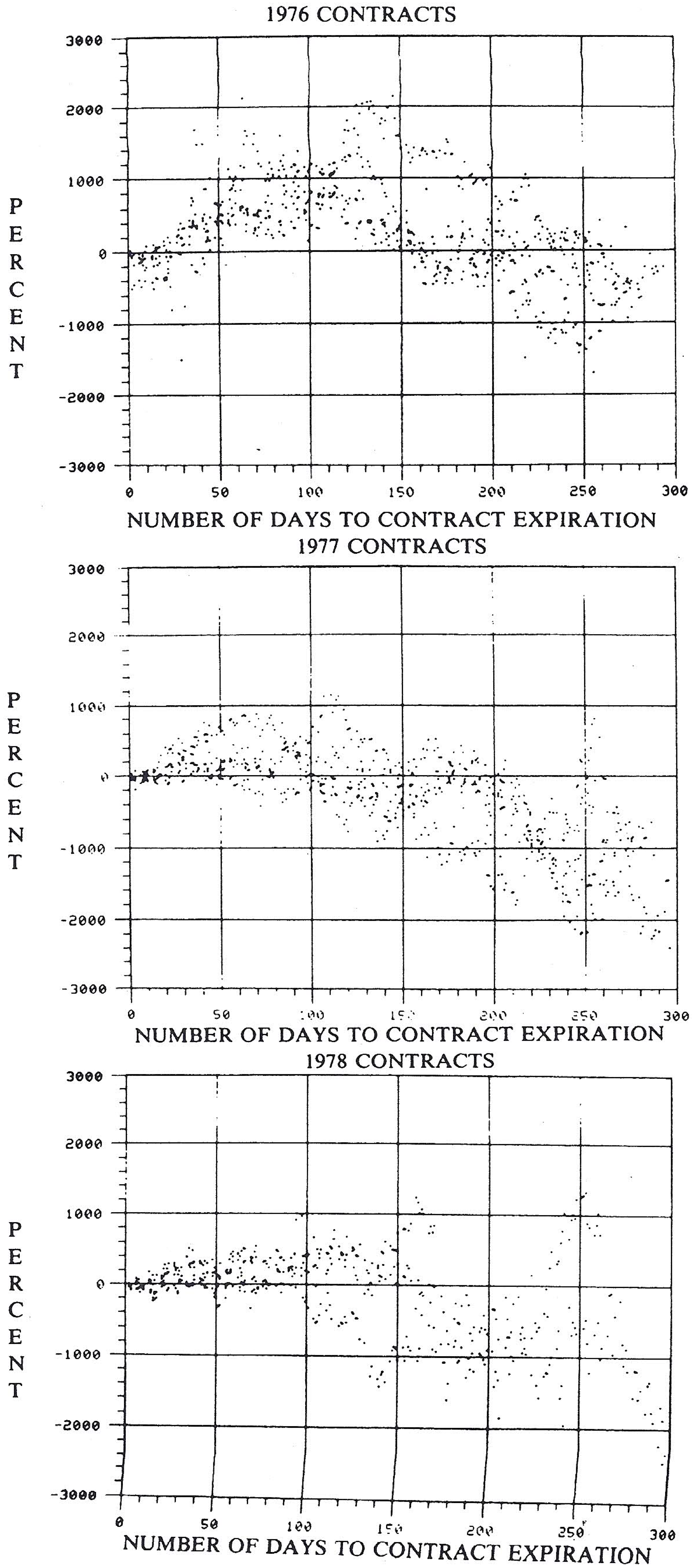

FIGURE III. Cost of Carry Model Annualized Dollar Differences: Futures Price Less Equilibrium Price

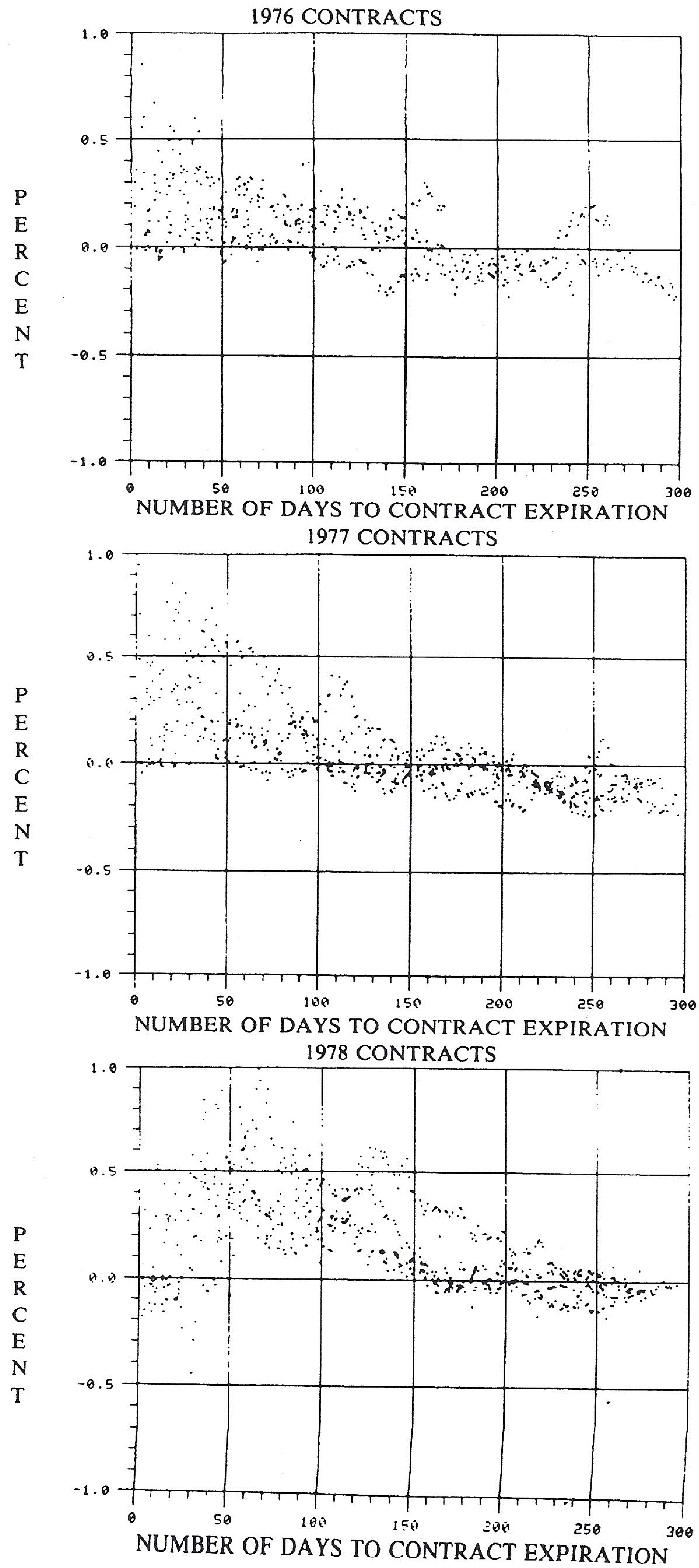

FIGURE IV. Forward Rate Model Annualized Dollar Differences: Futures Price Less Equilibrium Price

od to the maturity of the security deliverable on the futures contract, by purchasing a security with SD day to maturity and buying a futures contract instead of buying a security with $(SD + 91)$ days to maturity. Because these arbitrage conditions derive from positive and negative differences between the actual and theoretical price, they are referred to as overpriced and underpriced situations. For the forward rate model, overpriced and underpriced situations are applicable to the cash investor in Treasury Bill, the case of quasi-arbitrage. The cost-of-carry results more appropriately apply to the opportunity cost of funds, the case of pure arbitrage.

The data show that there are different conclusions to be drawn as to the pricing of futures contracts and its implications for market efficiency depending upon the implied financing charge or, equivalently, on whether equilibrium prices are derived from the cost-of-carry specification or the forward-rate specification of the model.

In terms of the summary statistics, the dollar differences (per $1,000,000 contract, Tables 1 and 2) show that the mean price difference for all contracts is $282 for the nearby contract for forward rates and only $85 for the cost-of-carry model. For both theories the sign of the price difference tends to reverse as the time from contract maturity increases. The cost-of-carry differences tend to increase as the time from maturity increases, and the forward-rate differences decrease. These trends are highlighted in Figures I and II.

Since the inception of futures trading, the price differences derived from forward rates for the nearby contracts have been increasing,[18] and those for the cost of carry have fluctuated around zero. For the more distant contracts, the cost-of-carry price differences have declined over time from large positive values, while those derived from forward rates have been persistently negative. The forward rate data in Table 2 and Figure II show that roughly 150 days from contract maturity there is a switching from overpricing to underpricing of the futures contract. The mean absolute differences confirm these tendencies. For both the nearby and the third-quarter contracts, the mean absolute difference is nearly equal to the mean difference, but in the second quarter it is nearly double the mean difference. These findings for forward rates are consistent with those of other researchers who have examined futures prices, and have led some to conclude that theories of backwardation hold for longer-term contracts, or that futures contract risk exists which requires excess returns. The dollar differences for the cost-of-carry model (Table 1 and Figure I) are positive for distant contracts and move toward zero for nearby contracts.

The annualized data present a much more striking indication of the differences between the two models and the trends within these models.

[18] Rendleman and Carabini [17] found a similar trend in differences that were based on forward rates.

~~differences between the two models and the trends within these models.~~ ę Since the overpricing in the cost-of-carry model occurs in the third quarter, it is much less significant in increasing annual rates of return than the overpricing in the forward rate model, where the overpricing occurs in the nearby contracts and the implicit annualization assumption is that it can be replicated many times throughout the year.[19] Tables 3 and 4 give the summary statistics for the annualized basis point difference for the two models, and Figures III and IV plot the daily data segmented by yearly periods.

Unlike the dollar data, the annualized basis point data show that the two theories differ considerably with respect to the nearby contract and the more distant, third-quarter contract. The mean annualized basis point differential for the nearby contract using forward rates is 25 basis points. In the contract month of December 1977, it reached a high of 53 basis points (Table 4).[20] Confirmation of the consistency of this overpricing is given by the mean absolute difference which is nearly equal to the mean in every contract. On the other hand, the annualized cost-of-carry differences for the nearby contract were only 2 basis points. Furthermore, unlike the forward-price differences, the mean absolute of the cost-of-carry differences is far greater than the mean, indicating fluctuations around zero are common.

The second quarter shows only slight overpricing for both theories. The third quarter, however, supports the earlier conclusions that the forward-rate model results in underpricing and the cost-of-carry model in overpricing. Figures III and IV show the different conclusions concerning futures market pricing and market efficiency that result from alternative assumptions about financing charges. For the nearby contract, it is evident that the financing cost plays an important role in influencing arbitrage decisions. For quasi-arbitrage, returns can be en-

[19] For example, a difference of $1,000 on an investment of $1,000,000, if obtained over a period of 91 days, is at an annual rate of 0.40 percent (40 basis points); if obtained over 30 days or 182 days, the annual rate would be approximately 1.20 or 0.20 percent, respectively. In recent work on this subject the annualization period seems to have been a major reason for conflicting findings. For example, Puglisi [16] and Vignola and Dale [21] annualized all dollar differences over *SD* days, consistent with their narrow approach aimed at portfolio holders. Others [17] appear to use an annualization period of 91 days for all their differences. Using forward rates generally implicitly assumes a 91-day period for annualization. Because of these difficulties, it is preferable first to calculate the price differences of arbitrage, then explicity to take annualization into account.

[20] This study uses closing prices for futures and spot markets. Since the futures market closes earlier than the cash markets, our results are affected by spot market price changes that occur late in the day. The spot market closes roughly one hour later than the futures market. Thus, the existence or nonexistence of arbitrage possibilities may be due to data reporting. For example, for the December 1978 contract, when the money market was highly volatile due to Federal Reserve policy changes and the November 1, 1978, actions to support the dollar in foreign exchange markets, the closing price data resulted in a number of outlier observations. Examination of these observations, however, did not indicate any unidirectional bias.

hanced by using the futures markets, since overpricing is generally present (Figure IV). For pure arbitrage, however, there is a slight underpricing of nearby contracts (Figure III).

The pure arbitrage findings must be viewed cautiously since the results are sensitive to the storage-cost rate. The writers have used the overnight federal funds rate throughout the paper. Had an *RP* rate been used, the financing costs would have been lowered by 1/8 to 1/4 percent, resulting in a lower equilibrium price.[21] Even though the overnight *RP* rate is lower than the federal funds rate, the federal funds rate may give more realistic results since the term structure of *RP* rates generally reflects the upward sloping term structure found in Treasury Bills. Unfortunately, the necessary *RP* series or proxies for such series do not exist. Factoring in these considerations, the futures market appears to have been priced efficiently with respect to pure arbitrage opportunities.

IV. CONCLUSIONS

The effects of alternative financing costs on the pricing of Treasury Bill futures are clearly demonstrated in this paper. In general, the writers' results show that the overnight cost-of-carry model is better for explaining futures prices than are forward rates derived from the yield curve. The conclusion is that the cost-of-carry and pure arbitrage dominate futures prices, although there are cases of pure arbitrage. However, there are more significant opportunities for quasi-arbitrage. If one uses the existence or nonexistence of arbitrage profits to draw conclusions about market efficiency, as many authors have, one will reach a different conclusion depending on which price model is adopted and what financing costs are implicity assumed. The fact that forward rates are inappropriate for determining the equilibrium price of futures contracts has been responsible for the often-cited underpricing and overpricing of the futures market and the implication that the market is inefficient. However, these findings are appropriate for portfolio holders of Treasury Bills who may increase their returns by using Treasury Bill futures.

The use of the cost-of-carry model removes the need to resort to the explanations that have been forthcoming to rationalize the findings when forward rates are used as the basis for judging futures markets pricing and efficiency. It is reasonable for economists to have turned to traditional theories of the term structure in exploring futures markets. However, in doing so they have explicitly considered only a narrow

[21] *For the average term of 45 days for the nearby contract, 1/8 of a percent results in a financing cost difference of approximately $150.*

framework for drawing their conclusions and have had to explain their findings by referring to transactions costs, institutional practices, risk premiums and other market imperfections. A more flexible model based on the cost-of-carry does not need to revert to such *ad hoc* explanations. The conclusion is, therefore, the question of futures market efficiency and pricing reduces to the question of the use of the appropriate financing charges.

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