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Abstract

This paper examines the effects of asset bubbles in an overlapping generations model with endogenous labor supply. We derive a set of conditions under which asset bubbles will lead to an expansion in steady-state capital, investment, employment and output. We also provide a specific numerical example to illustrate these results.

Keywords: Asset Bubbles, Overlapping Generations, Endogenous Labor.

JEL classification: E22, E44.

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1 Introduction

The existence and consequences of asset bubbles have long been a subject of interest to economists. In a seminal paper, Tirole (1985) showed that asset bubbles can exist in an overlapping generations economy with rational consumers and exogenous labor supply. A central implication of Tirole's model is that asset bubbles will always crowd out investment in productive capital and reduce capital accumulation. Since labor supply is inelastic, this will also lead to a reduction in aggregate output. This negative relationship between asset bubbles and aggregate economic activities, however, is in contrast with empirical evidence. As pointed out by Martin and Ventura (2012), episodes of asset bubbles in the U.S. and Japan are typically associated with periods of robust economic expansions. In the present study, we show that this conflict between theory and evidence can be resolved when labor supply is endogenous.[1] More specifically, we show that asset bubbles can induce an expansion in steady-state capital, investment, employment and output if labor supply responds strongly and positively to changes in interest rate. This type of response is possible when the intertemporal elasticity of substitution (IES) in consumption is small and the Frisch elasticity of labor supply is large. The intuition of this result will be explained later. We also provide a specific numerical example to illustrate our findings.

2 The Model

The model economy under study is essentially the one considered in Tirole (1985, Section 2), except that labor supply is now endogenously determined. Specifically, consider an overlapping generations model in which each consumer lives two periods: young and old. In each period $t \geq 0$, a new generation of identical consumers is born. The size of generation t is given by $N_t = (1+n)^t$, with $n > 0$. All consumers have one unit of time endowment which can be allocated between work and leisure. Retirement is mandatory in the second period of life, so the labor supply of old consumers is zero.

Consider a consumer who is born at time $t \geq 0$. Let $c_{y,t}$ and $c_{o,t+1}$ denote his consumption when young and old, respectively, and let l_t denote his labor supply when young. The consumer's preferences are represented by

$$\mathcal{U}(c_{y,t}, l_t, c_{o,t+1}) = \frac{c_{y,t}^{1-\sigma}}{1-\sigma} - A\frac{l_t^{1+\psi}}{1+\psi} + \beta\frac{c_{o,t+1}^{1-\sigma}}{1-\sigma}, \tag{1}$$

[1] Olivier (2000), Farhi and Tirole (2012) and Martin and Ventura (2012) have explored other channels through which asset bubbles can crowd in productive investment and foster economic growth in overlapping generations models. Miao and Wang (2012) have developed an infinite-horizon model in which asset bubbles can promote total factor productivity. None of these studies have examined the connections between endogenous labor supply and asset bubbles.

where $\sigma > 0$ is the inverse of the IES in consumption, $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply, $\beta \in (0,1)$ is the subjective discount factor and A is a positive constant. Let w_t be the market wage rate at time t. Then the consumer's labor income when young is $w_t l_t$. The consumer can save in two types of assets: physical capital and an intrinsically worthless asset.[2] The total supply of the intrinsically worthless asset is constant over time and is denoted by $M \geq 0$.[3] Denote savings in the form of physical capital by s_t, and savings in the form of intrinsically worthless asset by m_t. The gross return from physical capital between time t and $t+1$ is given by R_{t+1}. The price of the intrinsically worthless asset at time t is p_t. No-arbitrage means that these two types of assets must yield the same return in every period, so that $R_{t+1} = p_{t+1}/p_t$ for all $t \geq 0$. Taking $\{w_t, p_t, p_{t+1}, R_{t+1}\}$ as given, the consumer's problem is to choose an allocation $\{c_{y,t}, l_t, c_{o,t+1}, s_t, m_t\}$ so as to maximize his lifetime utility in (1), subject to the budget constraints:

$$c_{y,t} + s_t + p_t m_t = w_t l_t, \qquad \text{and} \qquad c_{o,t+1} = R_{t+1} s_t + p_{t+1} m_t.$$

The first-order conditions for this problem are given by

$$w_t c_{y,t}^{-\sigma} = A l_t^{\psi}, \qquad \text{and} \qquad c_{y,t}^{-\sigma} = \beta R_{t+1} c_{o,t+1}^{-\sigma}. \tag{2}$$

Using these equations, we can obtain

$$c_{y,t} = \frac{c_{o,t+1}}{(\beta R_{t+1})^{\frac{1}{\sigma}}} = \frac{w_t l_t}{1 + \beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}},$$

$$l_t = A^{-\frac{1}{\sigma+\psi}} \left(1 + \beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}\right)^{\frac{\sigma}{\sigma+\psi}} w_t^{\frac{1-\sigma}{\sigma+\psi}},$$

$$s_t + p_t m_t = \Sigma(R_{t+1}) w_t l_t, \qquad \text{where} \qquad \Sigma(R_{t+1}) \equiv \frac{\beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}. \tag{3}$$

An increase in R_{t+1} has two opposing effects on saving. These effects are captured by the function $\Sigma : \mathbb{R}_+ \rightarrow [0,1]$ defined in (3). First, an increase in R_{t+1} means that for the same level of total savings, the consumer will receive more interest income when old. This creates an income effect which

[2] The second type of asset is called "intrinsically worthless" because it has no consumption value and cannot be used for production. The only motivation for holding this type of asset is to resell it at a higher price in the next period.

[3] At time 0, all assets are owned by a group of "initial-old" consumers. The decision problem of these consumers is trivial and does not play any role in the following analysis.

encourages consumption when young and discourages saving. Second, an increase in interest rate also lowers the relative price of future consumption. This creates an intertemporal substitution effect which discourages consumption when young and promotes saving. The relative strength of these two effects depends on the value of σ. In particular, the intertemporal substitution effect dominates when $\sigma < 1$. In this case, an increase in R_{t+1} will always increase the savings rate so that $\Sigma(\cdot)$ is a strictly increasing function. When $\sigma > 1$, the income effect dominates so that $\Sigma(\cdot)$ is strictly decreasing. The two effects exactly cancel out when $\sigma = 1$. In this case, $\Sigma(\cdot)$ is a constant.

On the supply side of the economy, there is a large number of identical firms. In each period, each firm hires labor and physical capital from the competitive factor markets, and produces output according to

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \qquad \text{with } \alpha \in (0,1),$$

where Y_t denotes output produced at time t, K_t and L_t denote capital input and labor input, respectively. Since the production function exhibits constant returns to scale, we can focus on the choices made by a single price-taking firm. We assume that physical capital is fully depreciated after one period, so that R_t coincides with the rental price of physical capital at time $t \geq 0$. The representative firm's problem is given by

$$\max_{K_t, L_t} \left\{ K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t \right\},$$

and the first-order conditions are $R_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}$, and $w_t = (1-\alpha) K_t^{\alpha} L_t^{-\alpha}$.

Given $M \geq 0$, a competitive equilibrium of this economy consists of sequences of allocations $\{c_{y,t}, l_t, c_{o,t+1}, s_t, m_t\}_{t=0}^{\infty}$, aggregate inputs $\{K_t, L_t\}_{t=0}^{\infty}$, and prices $\{w_t, p_t, R_t\}_{t=0}^{\infty}$ such that (i) given $\{w_t, p_t, p_{t+1}, R_{t+1}\}$, the allocation $\{c_{y,t}, l_t, c_{o,t+1}, s_t, m_t\}$ is optimal for the consumers in generation $t \geq 0$, (ii) given $\{w_t, R_t\}$, the aggregate inputs $\{K_t, L_t\}$ solve the representative firm's problem at time $t \geq 0$, and (iii) all markets clear in every period, so that $L_t = N_t l_t$, $N_t m_t = M$ and

$$K_{t+1} = N_t s_t = \left[\frac{\beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}} \right] w_t l_t - p_t m_t, \qquad \text{for all } t \geq 0. \tag{4}$$

Let $k_t \equiv K_t / N_t$ be the quantity of physical capital per worker at time t, and let $a_t \equiv p_t m_t$ be the quantity of unproductive savings per young consumer. Then the equilibrium wage rate can be expressed as $w_t = (1-\alpha) k_t^{\alpha} l_t^{-\alpha}$, and (4) can be rewritten as

$$(1+n) k_{t+1} = (1-\alpha) \left[\frac{\beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}{1+\beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}} \right] \left(\frac{k_t}{l_t} \right)^{\alpha} l_t - a_t. \tag{5}$$

The dynamics of a_t is determined by

$$a_{t+1} = p_{t+1} m_{t+1} = \frac{p_{t+1} m_{t+1}}{p_t m_t} a_t = \frac{R_{t+1}}{1+n} a_t.$$

3 Stationary Equilibrium

3.1 Economy without Intrinsically Worthless Assets

Before analyzing the effects of asset bubbles, we first characterize the stationary equilibrium of an economy with zero supply of intrinsically worthless asset, i.e., $M = 0$ and $a_t = 0$ for all $t \geq 0$. A stationary equilibrium is a competitive equilibrium in which $k_t = k^*$, $l_t = l^*$ and $R_t = R^*$ for all $t \geq 0$. Substituting these conditions into (5) gives

$$\frac{\beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}}{1+\beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}} \left(\frac{k^*}{l^*} \right)^{\alpha-1} = \frac{1+n}{1-\alpha}$$

$$\Rightarrow \Lambda(R^*) \equiv \frac{\beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}}}{1+\beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}} = \frac{(1+n)\alpha}{1-\alpha}. \tag{6}$$

Equation (6) follows from the fact that $R^* = \alpha (k^*/l^*)^{\alpha-1}$. For any $\sigma > 0$, the function $\Lambda : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing with $\Lambda(0) = 0$ and $\lim_{R\to\infty} \Lambda(R) = \infty$. Hence, there exists a unique $R^* > 0$ that solves (6). The steady-state value of all other variables can be uniquely determined by

$$w^* = (1-\alpha) \left(\frac{\alpha}{R^*} \right)^{\frac{\alpha}{1-\alpha}}, \tag{7}$$

$$l^* = A^{-\frac{1}{\sigma+\psi}} \left[1+\beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1} \right]^{\frac{\sigma}{\sigma+\psi}} (w^*)^{\frac{1-\sigma}{\sigma+\psi}}, \tag{8}$$

$$k^* = l^* \left(\frac{\alpha}{R^*} \right)^{\frac{1}{1-\alpha}}, \qquad \text{and} \qquad c_y^* = \frac{c_o^*}{(\beta R^*)^{\frac{1}{\sigma}}} = \frac{w^* l^*}{1+\beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}}. \tag{9}$$

This establishes the following result.

Proposition 1 *A unique bubbleless steady state exists for any $\sigma > 0$. The steady-state values*

$\{R^*, w^*, k^*, l^*, c_y^*, c_o^*\}$ *are determined by (6)-(9).*

3.2 Economy with Intrinsically Worthless Assets

Suppose now the economy has a strictly positive supply of intrinsically worthless assets, i.e., $M > 0$. In the following analysis, we focus on stationary equilibria in which the price of these assets exceeds their fundamental value, i.e., $p_t = p^* > 0$. Formally, a "bubbly" steady state is a set of values $\left\{\widetilde{a}^*, \widetilde{R}^*, \widetilde{w}^*, \widetilde{k}^*, \widetilde{l}^*, \widetilde{c}_y^*, \widetilde{c}_o^*\right\}$ that satisfies the following conditions: $\widetilde{a}^* > 0$, $\widetilde{R}^* = 1 + n$,

$$\widetilde{a}^* + (1+n)\,\widetilde{k}^* = (1-\alpha)\left[\frac{\beta^{\frac{1}{\sigma}}\left(\widetilde{R}^*\right)^{\frac{1}{\sigma}-1}}{1+\beta^{\frac{1}{\sigma}}\left(\widetilde{R}^*\right)^{\frac{1}{\sigma}-1}}\right]\left(\frac{\widetilde{k}^*}{\widetilde{l}^*}\right)^{\alpha}\widetilde{l}^*, \tag{10}$$

and (7)-(9).[4] Substituting $\widetilde{a}^* > 0$ and $\widetilde{R}^* = 1 + n$ into (10) gives

$$\frac{1+n}{1-\alpha} < \left[\frac{\beta^{\frac{1}{\sigma}}(1+n)^{\frac{1}{\sigma}-1}}{1+\beta^{\frac{1}{\sigma}}(1+n)^{\frac{1}{\sigma}-1}}\right]\left(\frac{\widetilde{k}^*}{\widetilde{l}^*}\right)^{\alpha-1} \Rightarrow \frac{(1+n)\,\alpha}{1-\alpha} < \Lambda(1+n)\,. \tag{11}$$

Since $\Lambda(\cdot)$ is strictly increasing, (6) and (11) together imply that $R^* < 1+n$. This shows that $R^* < 1+n$ is a necessary condition for the existence of bubbly steady state. Suppose this condition is satisfied. Then substituting $\widetilde{R}^* = 1 + n$ into (7)-(9) yields a unique set of values for $\left\{\widetilde{w}^*, \widetilde{k}^*, \widetilde{l}^*, \widetilde{c}_y^*, \widetilde{c}_o^*\right\}$. Using (10), we can obtain a unique value of a^*, which is strictly positive as $R^* < 1 + n$ and $\Lambda(\cdot)$ is strictly increasing. Hence, a unique bubbly steady state exists. This proves the following result.

Proposition 2 *A unique bubbly steady state exists if and only if $R^* < 1 + n$.*

Similar to Tirole (1985), our model predicts that equilibrium interest rate will increase in the presence of asset bubbles. When labor supply is exogenous, the steady-state value of per-worker capital is determined by $k^* = (\alpha/R^*)^{\frac{1}{1-\alpha}}$. Thus, a higher interest rate in the bubbly steady state means that there is fewer per-worker capital than in the bubbleless steady state, i.e., $\widetilde{k}^* < k^*$. When labor supply is endogenous, the value of k^* is jointly determined by l^* and R^* as shown in (9). If the existence of asset bubbles can induce young consumers to work more (i.e., $\widetilde{l}^* > l^*$), and if this effect is strong enough to overcome the increase in interest rate, then more capital will be accumulated in the bubbly steady state than in the bubbleless one, i.e., $\widetilde{k}^* > k^*$. The rest of this paper is intended to formalize

[4] Note that equations (7)-(9) must be satisfied in any steady state, regardless of the existence of asset bubbles.

this idea. Two remarks are in order before we proceed. First, the above description makes clear that $\widetilde{k}^* > k^*$ can happen *only if* labor supply is adjustable. This highlights the importance of introducing endogenous labor into Tirole's model. Second, suppose $\widetilde{l}^* > l^*$ and $\widetilde{k}^* > k^*$ are true. Then per-worker output in the bubbly steady state must also be higher than in the bubbleless steady state.

Suppose $R^* < 1 + n$. Then using (7)-(9), which are valid in both bubbleless and bubbly steady states, we can obtain

$$k^* = (1-\alpha)^{\frac{1-\sigma}{\sigma+\psi}} A^{-\frac{1}{\sigma+\psi}} \left[1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}\right]^{\frac{\sigma}{\sigma+\psi}} \left(\frac{\alpha}{R^*}\right)^{\phi},$$

$$\widetilde{k}^* = (1-\alpha)^{\frac{1-\sigma}{\sigma+\psi}} A^{-\frac{1}{\sigma+\psi}} \left[1 + \beta^{\frac{1}{\sigma}} (1+n)^{\frac{1}{\sigma}-1}\right]^{\frac{\sigma}{\sigma+\psi}} \left(\frac{\alpha}{1+n}\right)^{\phi},$$

where $\phi \equiv \frac{1}{1-\alpha}\left[1 + \frac{\alpha(1-\sigma)}{\sigma+\psi}\right] > 0$ for any $\sigma > 0$. Hence, $\widetilde{k}^* > k^*$ if and only if

$$\left[1 + \beta^{\frac{1}{\sigma}} (1+n)^{\frac{1}{\sigma}-1}\right]^{\frac{\sigma}{\sigma+\psi}} \left(\frac{\alpha}{1+n}\right)^{\phi} > \left[1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}\right]^{\frac{\sigma}{\sigma+\psi}} \left(\frac{\alpha}{R^*}\right)^{\phi}$$

$$\Leftrightarrow \left(\frac{R^*}{1+n}\right)^{\phi} > \left[\frac{1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} (1+n)^{\frac{1}{\sigma}-1}}\right]^{\frac{\sigma}{\sigma+\psi}}. \tag{12}$$

Note that this condition *cannot* be satisfied if $\sigma \geq 1$. Since $R^* < 1+n$, we have $(R^*)^{\frac{1}{\sigma}-1} \geq (1+n)^{\frac{1}{\sigma}-1}$, whenever $\sigma \geq 1$. Condition (12) then implies

$$\left(\frac{R^*}{1+n}\right)^{\phi} > \left[\frac{1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} (1+n)^{\frac{1}{\sigma}-1}}\right]^{\frac{\sigma}{\sigma+\psi}} \geq 1,$$

which contradicts $R^* < 1+n$. Thus, a necessary condition for $\widetilde{k}^* > k^*$ is $\sigma < 1$. The intuition underlying this result is straightforward: In the presence of asset bubbles, equilibrium interest rate rises from R^* to $\widetilde{R}^* = 1+n$. Such an increase will create an income effect and an intertemporal substitution effect on the young's consumption. Since consumption and labor supply is inversely related, the income effect will discourage young consumers from working, whereas the intertemporal substitution effect will induce them to work more.[5] Since $\widetilde{k}^* > k^*$ can happen only if $\widetilde{l}^* > l^*$, it is necessary to have the intertemporal substitution effect dominates the income effect, i.e., $\sigma < 1$.[6]

[5] The inverse relationship between $c_{y,t}$ and l_t can be seen by combining the first-order condition $w_t c_{y,t}^{-\sigma} = A l_t^{\psi}$ with the expression for the equilibrium wage rate $w_t = (1-\alpha)(k_t/l_t)^{\alpha}$.

[6] In infinite-horizon models, it is typical to assume that σ is greater than or equal to one. However, in overlapping generations model, it is typical to assume that the intertemporal substitution effect is greater than the income effect.

We now derive a sufficient condition for $\widetilde{k}^* > k^*$. Suppose $R^* < 1+n$ and $\sigma < 1$ are satisfied. Using (6), we can get

$$1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1} = \frac{(1-\alpha)\,\beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}}}{\alpha\,(1+n)}.$$

Substituting this into (12) and rearranging terms gives

$$\underbrace{\left(\frac{R^*}{1+n}\right)^{\phi(\sigma+\psi)-1}}_{\Upsilon} > (1+n)^{1-\sigma} \left\{ \frac{(1-\alpha)\,\beta^{\frac{1}{\sigma}}}{\alpha\left[1+\beta^{\frac{1}{\sigma}} (1+n)^{\frac{1}{\sigma}-1}\right]} \right\}^{\sigma}, \tag{13}$$

where $\phi(\sigma+\psi) - 1 = \frac{\psi+\alpha}{1-\alpha} - (1-\sigma)$. Note that the parameter ψ does not affect the value of R^* nor the expression on the right-hand side of (13). Since $R^* < 1+n$, lowering the value of ψ will raise the value of Υ. Thus, holding other parameters constant, $\widetilde{k}^* > k^*$ is more likely to occur when the value of ψ is low (i.e., close to zero). A low value of ψ means that the Frisch elasticity of labor supply is large. This, together with $\sigma < 1$, ensures that young consumers will significantly increase their labor supply when interest rate rises. A low value of ψ is not uncommon in macroeconomic studies. In the extreme case when $\psi = 0$, the preferences in (1) become quasi-linear in labor. Hansen (1985) shows that this type of utility function can arise in a model with indivisible labor. Quasi-linear utility function is now commonly used in business cycle models and monetary-search models.

The main results of this paper are summarized in Proposition 3.[7]

Proposition 3 *(i) Suppose $R^* < 1+n$. Then a necessary condition for $\widetilde{k}^* > k^*$ is $\sigma < 1$. (ii) Suppose $R^* < 1+n$ and $\sigma < 1$ are satisfied. Then $\widetilde{k}^* > k^*$ if (13) is satisfied.*

3.3 Numerical Example

We now provide a specific numerical example to illustrate the results in Proposition 3. Suppose one model period takes 30 years. Set the annual subjective discount rate to 0.9950 and the annual employ-

Galor and Ryder (1989) shows that this assumption plays an important role in establishing the existence, uniqueness and stability of both stationary and non-stationary equilibria in an overlapping generations model with exogenous labor supply. Nourry (2001) uses similar conditions to examine the local stability of stationary equilibria in a model with endogenous labor supply. In a well-known study on stochastic bubbles, Weil (1987) focuses on the case when the interest elasticity of savings is positive. Under a constant-relative-risk-aversion utility function, this assumption is equivalent to $\sigma < 1$. There is also some empirical support for $\sigma < 1$. See, for instance, the results in Table III and Table IV of Gourinchas and Parker (2002).

[7] Following Tirole (1985) and Weil (1987), we state our main results in terms of R^*, which is an endogenous variable. In the next subsection, we provide a set of parameter values under which the conditions in Proposition 3 are satisfied.

ment growth rate to 1.6%.[8] Then we have $\beta = (0.9950)^{30} = 0.8604$ and $n = (1.0160)^{30} - 1 = 0.6099$. We also set $\alpha = 0.30$ and $\psi = 0$. The value of A is calibrated so that l^* is about one-third. Under this calibration procedure, $\widetilde{k}^*$ is greater than k^* for any $\sigma \in [0, 0.16]$. In Table 1, we report the results obtained under $\sigma = 0.15$ and $A = 0.5862$. Under these parameter values, the bubbly steady state has a higher level of employment, per-worker capital and per-worker output than the bubbleless steady state.[9]

Table 1: Numerical Results

	Bubbleless Steady State	Bubbly Steady State
R	1.2416	1.6099
k	0.0438	0.0461
a	0	0.0721
l	0.3333	0.5084
w	0.3808	0.3407
y	0.1813	0.2474

Note: The notation y denotes per-worker capital, i.e., $y = k^{\alpha} l^{1-\alpha}$.

4 Conclusions

In this paper, we show that a simple modification of the Tirole (1985) model can lead to a drastically different conclusion. Specifically, we show that when labor supply is elastic, deterministic rational bubbles can induce an expansion in aggregate economic activities under certain conditions. In the present study, specific forms of utility function and production function have been used . This allows us to deliver our main results in a clear and concise manner. One direction for future research is to extend our results to general utility functions and production technologies. Another possibility is to extend the model to allow for financial market frictions and agency costs as in Azariadis and Chakraborty (1998).

[8] The latter coincides with the average annual growth rate of employed workers (over age 16) in the United States over the period 1953-2008.

[9] Similar results can be obtained for other values of $\{\alpha, \beta, n\}$ and some non-zero values of ψ. In general, one can extend the range of σ under which $\widetilde{k}^* > k^*$ by either raising the value of β or lowering the value of α. On the other hand, changing the value of A has no effect on the relative magnitude between $\widetilde{k}^*$ and k^*.

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