Analogy Making, Option Prices, and Implied Volatility

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1 July 2013

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Working Paper

This version: July 2013

Previous versions circulated as "Coarse Thinking, Implied Volatility, and the Price of Call and Put Options" and "Thinking by Analogy and Option Prices"

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Abstract

We put forward a new option pricing formula based on the notion that people tend to think by analogies and comparisons. The new formula differs from the Black Scholes formula due to the appearance of a parameter in the formula that captures the risk premium on the underlying. The new formula, called the analogy option pricing formula, provides an explanation for the implied volatility skew puzzle in equity options. We also discuss the key empirical predictions of the analogy formula.

Keywords: *Analogy Making, Implied Volatility, Implied Volatility Skew, Option Prices, Risk Premium*

JEL Classifications: G13; G12

[1] I am grateful to Hersh Shefrin (Santa Clara University), Don Chance (Louisiana State University), and seminar participants at Lahore University of Management Sciences for helpful comments and suggestions.

Analogy Making, Option Prices, and Implied Volatility

People tend to think by analogies and comparisons. It has been argued in cognitive science and psychology literature that analogy making forms the core of human cognition and it is the fuel and fire of thinking (see Hofstadter and Sander (2013)). Hofstadter and Sander (2013) write, *"[…] each concept in our mind owes its existence to a long succession of analogies made unconsciously over many years, initially giving birth to the concept and continuing to enrich it over the course of our lifetime. Furthermore, at every moment of our lives, our concepts are selectively triggered by analogies that our brain makes without letup, in an effort to make sense of the new and unknown in terms of the old and known."*
(Hofstadter and Sander (2013), Prologue page1).

According to Hoftsadter and Sander (2013), we engage in analogy making when we spot a link between what we are just experiencing and what we have experienced before. For example, if we see a pencil, we are able to recognize it as such because we have seen similar objects before and such objects are called pencils. Furthermore, spotting of this link is not only useful in assigning correct labels, it also gives us access to a whole repertoire of stored information regarding pencils including how they are used and how much a typical pencil costs. Hofstadter and Sander (2013) argue that analogy making not only allows us to carry out mundane tasks such as using a pencil, toothbrush or an elevator in a hotel but is also the spark behind all of the major discoveries in mathematics and the sciences. They argue that analogy making is responsible for all our thinking, from the most trivial to the most profound. Of course, there is always a danger of making wrong analogies. When a small private plane flew into a building in New York on October 11, 2006, the analogy with the events of September 11, 2001 was irrepressible and the Dow Jones Index fell sharply in response.

Recent experimental evidence suggests that analogy making plays an important role in financial markets. Siddiqi (2012) and Siddiqi (2011) show through controlled laboratory experiments that analogy making strongly influences how much people are willing to pay for call options. These experiments exploit an analogy between in-the-money call options and their underlying stocks and show that in-the-money calls are overpriced due to participants forming an analogy between riskier options and comparatively less risky underlying stocks.

In this article, we investigate the theoretical implications of analogy making for option pricing. If investors are pricing a call option in analogy with the underlying stock, then a new option pricing formula (alternative to the Black Scholes formula) is obtained. The new formula, which we call the analogy option pricing formula, provides an explanation for the implied volatility skew puzzle. Interestingly, the new formula differs only slightly from the Black-Scholes formula by incorporating the risk premium on the underlying in option price. This additional parameter is sufficient to explain implied volatility. We also provide testable predictions of the analogy option pricing formula.

This paper adds to the literature in several ways. 1) We show how a particular bias, called analogy making, leads to a new option pricing formula and explains the implied volatility puzzle. In particular, we show that the implied volatility skew is generated if actual price dynamics are determined according to the analogy formula and the Black-Scholes formula is used to back-out implied volatility. Our approach is broadly consistent with Shefrin (2008) who provides a systematic treatment of how behavioral assumptions impact the pricing kernel at the heart of modern asset pricing theory. However, the treatment here differs from Shefrin (2008) as we focus on one particular bias and explore its implications. 2) We provide a number of testable predictions of the model and summarize existing evidence. The existing evidence is strongly favorable to the analogy approach. Robust empirical testing of these predictions is the subject of future research. 3) Our approach relates to Bollen and Whaley (2004). They argue that, in the presence of limits to arbitrage, net demand pressure could determine the level and the slope of the implied volatility curve. In our approach, the source of demand pressure behind the skew is analogies that investors make between in-the-money calls and the underlying stocks. Such analogies lead them to consider in-the-money calls as replacements of the underlying stocks.[2] 4) Duan and Wei (2009) use daily option quotes on the S&P 100 index and its 30 largest component stocks, to show that, after controlling for the underlying asset's total risk, a higher amount of systematic risk leads to a higher level of implied volatility and a steeper slope of the implied volatility curve. In the analogy option pricing model, higher risk premium for a given level of historical volatility generates this result. As risk premium is related to systematic risk, this prediction of the analogy model is quite intriguing. 5) Our approach is an example of behavioralization of finance. Shefrin (2010) argues that finance is in the midst of a

[2] Option traders and investment professionals often advise people to buy in-the-money calls rather than the underlying stocks. As illustrative examples, see the following:
http://ezinearticles.com/?Call-Options-As-an-Alternative-to-Buying-the-Underlying-Security&id=4274772,
http://www.investingblog.org/archives/194/deep-in-the-money-options/,
http://www.triplescreenmethod.com/TradersCorner/TC052705.asp,
http://daytrading.about.com/od/stocks/a/OptionsInvest.htm

paradigm shift, from a neoclassical based framework to a psychologically based framework. Behavioralizing finance is the process of replacing neoclassical assumptions with behavioral counterparts.

In general, pricing models that have been proposed to explain the implied volatility skew can be classified into three broad categories: 1) Stochastic volatility and GARCH models (Heston and Nandi (2000), Duan (1995), Heston (1993), Melino and Turnbull (1990), Wiggins (1987), and Hull and White (1987)). 2) Models with jumps in the underlying price process (Amin (1993), Ball and Torous (1985)). 3) Models with stochastic volatility as well as random jumps. See Bakshi, Cao, and Chen (1997) for a discussion of their empirical performance (mixed). Most of these models modify the price process of the underlying. Hence, the focus of these models is on finding the right distributional assumptions that could explain the implied volatility puzzles. Our approach differs from them fundamentally as we do not modify the underlying price process. That is, as in the Black Scholes model, we continue to assume that the underlying's stochastic process is a constant coefficient geometric Brownian motion. To our knowledge, ours is the only model that explains the implied volatility skew without modifying the geometric Brownian motion assumption of the Black-Scholes model. Hence, contrary to popular belief, explaining implied volatility does not require that the assumption of geometric Brownian motion for the underlying is modified.

Analogy making is likely to play an important role in understanding financial market behavior. Many researchers have pointed out that there appears to be clear departures from Bayesian thinking (Babcock & Loewenstein (1997), Babcock, Wang, & Loewenstein (1996), Hogarth & Einhorn (1992), Kahneman & Frederick (2002), Kahneman, Slovic, & Tversky (1982)). Such departures from rational thinking have been measured both at the individual as well as the market level (Siddiqi (2009), Kluger & Wyatt (2004)). However, the question of what type of behavior to allow for if non-Bayesian behavior is admitted is a difficult one to address in the absence of an alternative which is amenable to systematic analysis. Analogy making may provide such an alternative especially when its intuitive appeal is undeniable.

This paper is organized as follows. Section 1 illustrates the difference between the principle of no arbitrage and the principle of analogy making through a simple example. Section 2 develops the key ideas in this paper in the context of a one period binomial model. Section 3 puts forward the analogy option pricing formula. Section 4 shows if prices are determined in accordance with the analogy formula and the Black Scholes formula is used to infer implied volatilities then the implied volatility skew is observed. Section 5 puts forward the key empirical predictions of the model. Section 6 concludes.

1. An Example: Principle of No-Arbitrage vs. Principle of Analogy Making

Consider an investor who has initially put his money in two assets: A stock (S) and a risk free bond (B). The stock has a price of $140 today. In the next period, the stock could either go up to $200 (the red state) or go down to $90 (the blue state). Each state has a 50% chance of occurring. The bond costs $100 today and it also pays $100 in the next period implying a continuously compounded interest rate of zero. Suppose a new asset "A" is introduced to him. The asset "A" pays $140 in the red state and $30 in the blue state. How much should he be willing to pay for it?

Conventional finance theory provides an answer by appealing to the principle of no-arbitrage: *identical assets should offer the same returns*. An asset identical to "A" is a portfolio consisting of a long position in S and a short position in 0.60 of B. In the red state, S pays $200 and one has to pay $60 due to shorting 0.60 of B resulting in a net payoff of $140. In the blue state, S pays $90 and one has to pay $60 on account of shorting 0.60 of B resulting in a net payoff of $30. That is, payoffs from S-0.60B are identical to payoffs from "A". Hence, according to the no-arbitrage principle, "A" should be priced in such a way that its expected return is equal to the expected return from (S-0.60B). It follows that the no-arbitrage price for "A" is $80.

In practice, finding an asset identical to A is no easy task. When simple tasks such as the one described above are presented to participants in a series of experiments, they seem to rely on analogy-making to figure out their willingness to pay. See Siddiqi (2012) and Siddiqi (2011). So, instead of trying to construct a hypothetical asset which is identical to asset "A", people find an actual asset similar to "A" and price "A" in analogy with that asset. That is, they rely on the principle of analogy: *similar assets should offer the same returns* rather than on the principle of no-arbitrage: *identical assets should offer the same returns.*

Asset "A" is similar to asset S. It pays more when asset S pays more and it pays less when asset S pays less. Expected return from S is 1.0357 $\left(\frac{0.5\times200+0.5\times90}{140}\right)$. According to the principle of analogy, A's price should be such that it offers the same expected return as S. That is, the right price for A is $82.07.

In the above example, there is a gap of $2.07 between the no-arbitrage price and the analogy price. Rational investors should short "A" and buy "S-0.60B". However, if we introduce a small transaction cost of 1%, then the total transaction cost of the proposed scheme exceeds $2.07, preventing arbitrage. The transaction cost of shorting "A" is $0.8207 whereas the transaction cost of buying "S-0.60B" is $1.6 so the total transaction cost is $2.4207. Hence, in principle, the deviation between the no-arbitrage price and the analogy price may not be corrected due to transaction costs. Next, we illustrate the key ideas in a binomial context.

2. Analogy Making: The Binomial Case

Consider a simple two state world. The equally likely states are Red, and Blue. There is a stock with payoffs $X_1, and\ X_2$ corresponding to states Red, and Blue respectively. The state realization takes place at time T. The current time is time t. We denote the risk free discount rate by r. The current price of the stock is S. There is another asset, which is a call option on the stock. By definition, the payoffs from the call option in the two states are:

$$C_1 = max\{(X_1 - K), 0\}, C_2 = max\{(X_2 - K), 0\} \quad (1)$$

Where K is the striking price, and $C_1, and\ C_2,$ are the payoffs from the call option corresponding to Red, and Blue states respectively.

As can be seen, the payoffs in the two states depend on the payoffs from the stock in corresponding states. Furthermore, by appropriately changing the striking price, the call option can be made more or less similar to the underlying stock with the similarity becoming exact as K approaches zero (all payoffs are constrained to be non-negative). As our focus is on in-the-money call options, we assume:

$$X_1 - K > 0, and\ X_2 - K > 0.$$

How much is an analogy maker willing to pay for this call option?

An analogy maker co-categorizes this call option with the underlying and values it in *transference* with the underlying stock. In other words, an analogy maker relies on the principle of analogy: *similar assets should offer the same return*. In contrast, a rational investor relies on the principle of no-arbitrage: *identical assets should offer the same return.*

We denote the return on an asset by $q \in Q$, where Q is some subset of $\Re$ (the set of real numbers). In calculating the return of the call option, an analogy maker faces two similar, but not identical, observable situations, $s \in \{0,1\}$. In $s = 0$, "return demanded on the call option" is the attribute of interest and in $s = 1$, "actual return available on the underlying stock" is the attribute of interest. The analogy maker has access to all the information described above. We denote this public information by I.

The actual expected return available on the underlying stock is given by,

$$E[q|I, s = 1] = \frac{(X_1 - S) + (X_2 - S)}{2S} \tag{2}$$

For the analogy maker, the expected return demanded on the call option is:

$$\begin{aligned} E[q|I, s = 0] &= E[q|I, s = 1] \\ &= \frac{\{X_1 - S\} + \{X_2 - S\}}{2 \times S} \end{aligned} \tag{3}$$

So, the analogy maker infers the price of the call option, P_c, from:

$$\frac{\{C_1 - P_c\} + \{C_2 - P_c\}}{2 \times P_c} = \frac{\{X_1 - S\} + \{X_2 - S\}}{2 \times S} \tag{4}$$

It follows,

$$P_c = \frac{C_1 + C_2}{X_1 + X_2} \times S$$

$$=> P_c = \left(1 - \frac{2K}{X_1 + X_2}\right) S \tag{5}$$

We know,

$$S = e^{-(r+\delta)(T-t)} \times \frac{X_1 + X_2}{2} \tag{6}$$

where δ is the risk premium on the underlying.

Substituting (6) into (5):

$$P_c = S - Ke^{-(r+\delta)(T-t)} \tag{7}$$

The above equation is the one period analogy option pricing formula for in-the-money binomial case.

The rational price P_r is (from the principle of no-arbitrage):

$$P_r = S - Ke^{-r(T-t)} \tag{8}$$

If limits to arbitrage prevent rational arbitrageurs from making riskless profits at the expense of analogy makers, both types will survive in the market. If α is the weight of rational investors in the market price and $(1-\alpha)$ is the weight of analogy makers, then the market price becomes:[3]

$$P_c^M = \alpha\left(S - Ke^{-r(T-t)}\right) + (1-\alpha)\left(S - Ke^{-(r+\delta)(T-t)}\right) \tag{9}$$

Proposition 1 ***The price of a call option in the presence of analogy makers*** $(\alpha < 1)$ ***is always larger than the price in the absence of analogy makers*** $(\alpha = 1)$ ***as long as the underlying stock price reflects a positive risk premium. Specifically, the difference between the two prices is*** $(1-\alpha)\left(Ke^{-r(T-t)} - Ke^{-(r+\delta)(T-t)}\right)$ ***where*** δ ***is the risk premium reflected in the price of the underlying stock.***

Proof.

Subtracting equation (8) from equation (9) yields the desired expression which is greater than zero as long as $\delta > 0$.

■

Proposition 2 shows that the presence of analogy makers does not change the Sharpe-ratio of options. This shows that the mispricing of options with respect to the underlying due to the presence of analogy makers is not reflected in the Sharpe-ratio as the change in expected excess return is offset by the change in standard deviation.

[3] In general, alpha weighted average is obtained if we assume log utilities.

Proposition 2 ***The Sharpe-ratio of a call option remains unchanged regardless of whether the analogy makers are present or not. Specifically, the Sharpe-ratio remains equal to the Sharpe-ratio of the underlying regardless of the presence of analogy makers.***

Proof.

Initially, assume that analogy makers are not present. Let x be the number of units of the underlying stock needed and let B be the dollar amount invested in a risk-free bond to create a portfolio that replicates the call option. It follows,

$$P_c = Sx + B \tag{I}$$

$$X_1 - K = xX_1 + B(1 + r) \tag{II}$$

Substitute B from (I) into (II) and re-arrange to get:

$$X_1 - K - P_c = x(X_1 - S) - xSr + P_c r$$

$$\Rightarrow \Delta C = x\Delta S - xSr + P_c r$$

$$\Rightarrow \Delta C = xS\left(\frac{\Delta S}{S} - r\right) + P_c r$$

$$\Rightarrow \frac{\Delta C}{P_c} = x\frac{S}{P_c}\left(\frac{\Delta S}{S} - r\right) + r$$

$$\Rightarrow R_c = \Omega(R_s - r) + r$$

Where Ω is the elasticity of option's price with respect to the stock price, R_c and R_s are returns on call and stock respectively. It follows,

$$E[R_c] = \Omega(E[R_s - r]) + r \tag{III}$$

$$Var[R_c] = \Omega^2 Var[R_s]$$

$$\Rightarrow \sigma_c^2 = \Omega^2 \sigma_s^2 \tag{IV}$$

$$Sharpe - ratio\ of\ Call\ without\ analogy\ makers = \frac{E[R_c] - r}{\sigma_c}$$

$$= \frac{\Omega(E[R_S] - r) + r - r}{\Omega\sigma_S} = \frac{E[R_S] - r}{\sigma_S} = Sharpe - ratio\ of\ the\ underlying\ stock$$

If analogy makers are also present, it follows,

$$E[R_c] = \alpha\{\Omega(E[R_s - r]) + r\} + (1 - \alpha)E[R_s] \qquad (V)$$

$$\sigma_c^2 = \{\alpha\Omega + (1 - \alpha)\}^2\sigma_s^2 \qquad (VI)$$

$$\text{Sharpe} - \text{ratio of Call with analogy makers} = \frac{\alpha\{\Omega(E[R_s - r] + r)\} + (1 - \alpha)E[R_s] - r}{\{\alpha\Omega + (1 - \alpha)\}\sigma_s}$$

$$= \frac{\{\alpha\Omega + (1 - \alpha)\}E[R_S] - r\{\alpha\Omega + (1 - \alpha)\}}{\{\alpha\Omega + (1 - \alpha)\}\sigma_S} = \frac{E[R_S] - r}{\sigma_S} = Sharpe - ratio\ of\ the\ underlying\ stock$$

■

Proposition 2 shows that the mispricing caused by the presence of analogy makers does not change the Sharpe-ratio. It remains equal to the Sharpe-ratio of the underlying. Hence, the fact that the empirical Sharpe-ratios of call options and underlying stocks do not differ cannot be used to argue that there is no mispricing in options with respect to the underlying.

Proposition 3 shows the condition under which rational arbitrageurs cannot make arbitrage profits at the expense of analogy makers. Consequently, both types may co-exist in the market.

Proposition 3 ***Analogy makers cannot be arbitraged out of the market if***

$(\mathbf{1} - \boldsymbol{\alpha})\left\{\boldsymbol{K}e^{-r(T-t)} - \boldsymbol{K}e^{-(r+\delta)(T-t)}\right\} < c$ ***where c is the transaction cost involved in the arbitrage scheme and δ>0***

Proof.

The presence of coarse thinkers increases the price of an in-the-money call option beyond its rational price. A rational arbitrageur interested in profiting from this situation should do the following: Write a call option and create a replicating portfolio. If there are no transaction costs involved then he would pocket the difference between the rational price and the market price without creating any liability for him when the option expires. As proposition 1 shows, the difference is $(1-\alpha)\{\mathrm{K}e^{-r(T-t)} - \mathrm{K}e^{-(r+\delta)(T-t)}\}$. However, if there are transactions costs involved then he would follow the strategy only if the benefit is greater than the cost. Otherwise, arbitrage profits cannot be made.

■

Analogy makers overprice a call option. When such overpriced calls are added to portfolios then the dynamics of such portfolios would be different from the dynamics without overpricing. Proposition 4 considers the case of covered call writing and shows that the two portfolios grow with different rates with time.

Proposition 4 ***If analogy makers set the price of an in-the-money call option then the covered call writing position (long stock+short call) grows in value at the rate of $r+\delta$. If rational investors set the price of an in-the-money call option then the covered call writing position grows in value at the risk free rate r.***

Proof.

Re-arranging equation (8):

$$Ke^{-r(T-t)} = S - P_r$$

The right hand side of the above equation is the covered call writing position with rational pricing. Hence, it follows that the covered call portfolio grows in value at the risk free rate with time if investors are rational.

For analogy makers, re-arrange equation (7):

$$Ke^{-(r+\delta)(T-t)} = S - P_c$$

The right hand side of the above equation is the covered call writing position when analogy makers price the call option. As the left hand side shows, this portfolio grows at a rate of $r + \delta$ with time.

■

Proposition 4 is useful in developing intuition regarding the derivation of the option pricing formula when we consider the general case, which we turn to next.

3. The Option Pricing Formula

In this section, we derive a new option pricing formula by allowing the underlying to follow a geometric Brownian motion instead of the restrictive binomial process assumed in the previous section. By exploring the implications of analogy making in the binomial case, the previous section develops intuition which carries over to the general case discussed here. Two results from the previous section stand out in this respect. Firstly, the rational and the analogy price differ in one parameter only, δ, which is the risk premium on the underlying stock. The analogy price is larger than the rational price if risk premium is positive (proposition 1). Secondly, for in-the-money call options, the covered call portfolio grows at the rate $r + \delta$ in the analogy case, rather than the rate r, which holds in the rational case (proposition 4).

To simplify notation, in this section, we are denoting the price of a call option by C and the price of a put option by P with the context making clear whether we are talking about the analogy price or the rational price. All other notations carry over from the previous section. Furthermore, we are only discussing European options.

In the binomial world, the portfolio $Sx - C$ grows at the risk free rate r with time under rational pricing whereas it grows at the rate $r + \delta$ with time under analogy pricing (x is the delta of call option, which is equal to 1 for an in-the-money binomial situation). Proposition 5 considers the corresponding case when the underlying follows geometric Brownian motion and derives the analogy option pricing partial differential equation (PDE) for call options.

Proposition 5 ***The Analogy Option Pricing Partial Differential Equation (PDE) is***

$$(r+\delta)C = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}(r+\delta)S + \frac{\partial^2 C^2}{\partial S^2}\frac{\sigma^2 S^2}{2}$$

Proof.

See Appendix A

■

The analogy option pricing PDE can be solved by transforming it into the heat equation. Proposition 6 shows the resulting call option pricing formula for European options.

Proposition 6 ***The formula for the price of a European call is obtained by solving the analogy based PDE. The formula is*** $C = SN(d_1) - Ke^{-(r+\delta)(T-t)}N(d_2)$ ***where*** $d_1 = \frac{ln(S/K)+(r+\delta+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$ ***and*** $d_2 = \frac{ln\left(\frac{S}{K}\right)+\left(r+\delta-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$

Proof.

See Appendix B.

■

Corollary 6.1 ***The formula for the analogy based price of a European put option is*** $Ke^{-(r+\delta)(T-t)}N(-d_2) - SN(-d_1)$

The analogy option pricing formula is different from the Black-Scholes formula due to the appearance of risk premium on the underlying in the analogy formula. It suggests that the risk premium on the underlying stock does matter for option pricing. The analogy formula is derived by keeping all the assumptions behind the Black-Scholes formula except one: the no-arbitrage pricing. That is, the analogy price deviates from the no-arbitrage price, however, as the next section illustrates, the deviation may not be large enough to justify an arbitrage scheme in the presence of transaction costs.

4. The Implied Volatility Skew

All the parameters and variables in the Black Scholes formula are directly observable except for the standard deviation of the underlying's returns. So, by plugging in the values of observables, the value of standard deviation can be inferred from market prices. This is called implied volatility. If the Black Scholes formula is correct, then the implied volatility values from options that are equivalent except for the strike prices should be equal. However, in practice, a skew is observed in which in-the-money call options' (out-of-the money puts) implied volatilities are higher than the implied volatilities from at-the-money and out-of-the-money call options (in-the-money puts).

The behavioral approach developed here provides an explanation for the skew. If the analogy formula is correct, and the Black Scholes model is used to infer implied volatility then skew arises as table 1 shows.

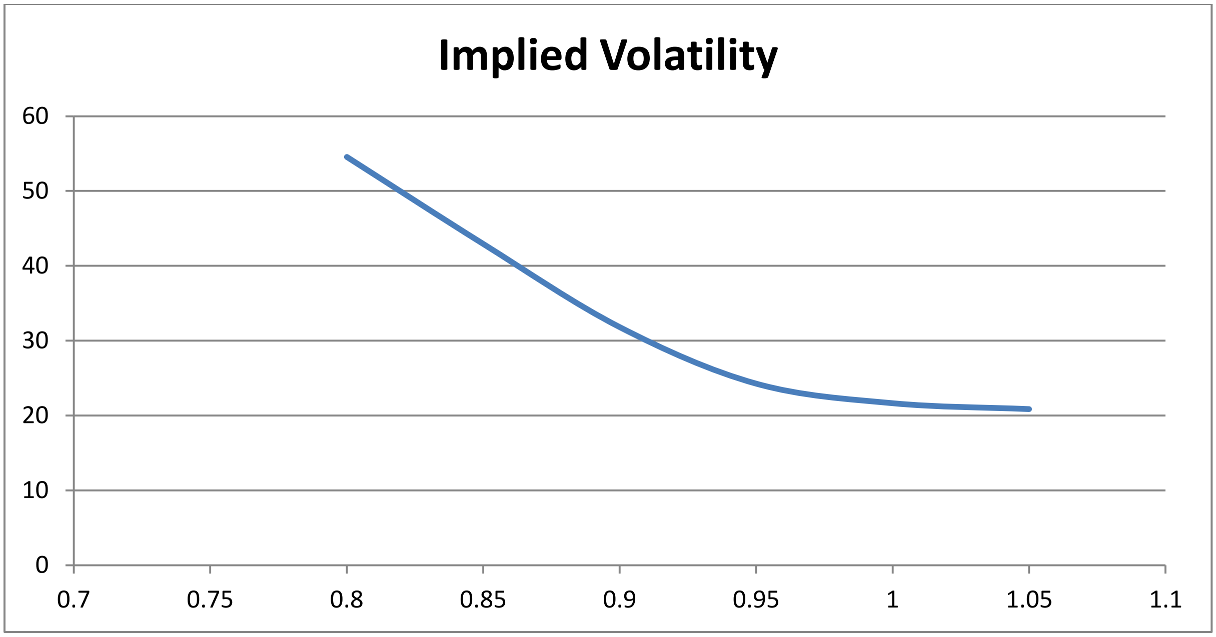
Table 1

Implied Volatility Skew

Underlying's Price=100, Historical Volatility=20%, Risk Premium on the Underlying=5%, Time to Expiry=0.06 year

K	**Black Scholes Price**	**Analogy Price**	**Difference**	**Implied Volatility**
105	0.5072	0.5672	0.06	20.87
100	2.160753	2.326171	0.165417	21.6570
95	5.644475	5.901344	0.25687	24.2740
90	10.30903	10.58699	0.277961	31.8250
85	15.26798	15.53439	0.266419	42.9400
80	20.25166	20.50253	0.250866	54.5700

As table 1 shows, implied volatility skew is seen if the analogy formula is correct, and the Black Scholes formula is used to infer implied volatility. Notice that in the example considered, difference between the Black Scholes price and the analogy price is quite small suggesting that transaction costs could easily prevent arbitrage profits from being realized.



K/S

Figure 1

Figure 1 is the graphical illustration of table 1. It is striking to observe from table 1 and figure 1 that the implied volatility skew is quite steep even when the price difference between the Black Scholes price and the analogy price is small. In the next section, we outline a number of key empirical predictions that follow from the analogy making model.

5. Key Predictions of the Analogy Model

Prediction#1 ***After controlling for the underlying asset's total volatility, a higher amount of risk premium on the underlying leads to a higher level of implied volatility and a steeper slope of the implied volatility curve.***

Risk premium on the underlying plays a key role in analogy option pricing formula. Higher the level of risk premium for a given level of volatility; higher is the extent of overpricing. So, higher risk premium leads to higher implied volatility levels at all values of moneyness. Figure 2 illustrates this. In the figure, implied volatility skews for two different values of risk premia are plotted. Other parameters are the same as in table 1. Duan and Wei (2009) use daily option quotes on the S&P 100 index and its 30 largest component stocks, to show that, after controlling for the underlying asset's total risk, a higher amount of systematic risk leads to a higher level of implied volatility and a steeper slope of the implied volatility curve. As risk premium is related to systematic risk, the prediction of the analogy model is quite intriguing.

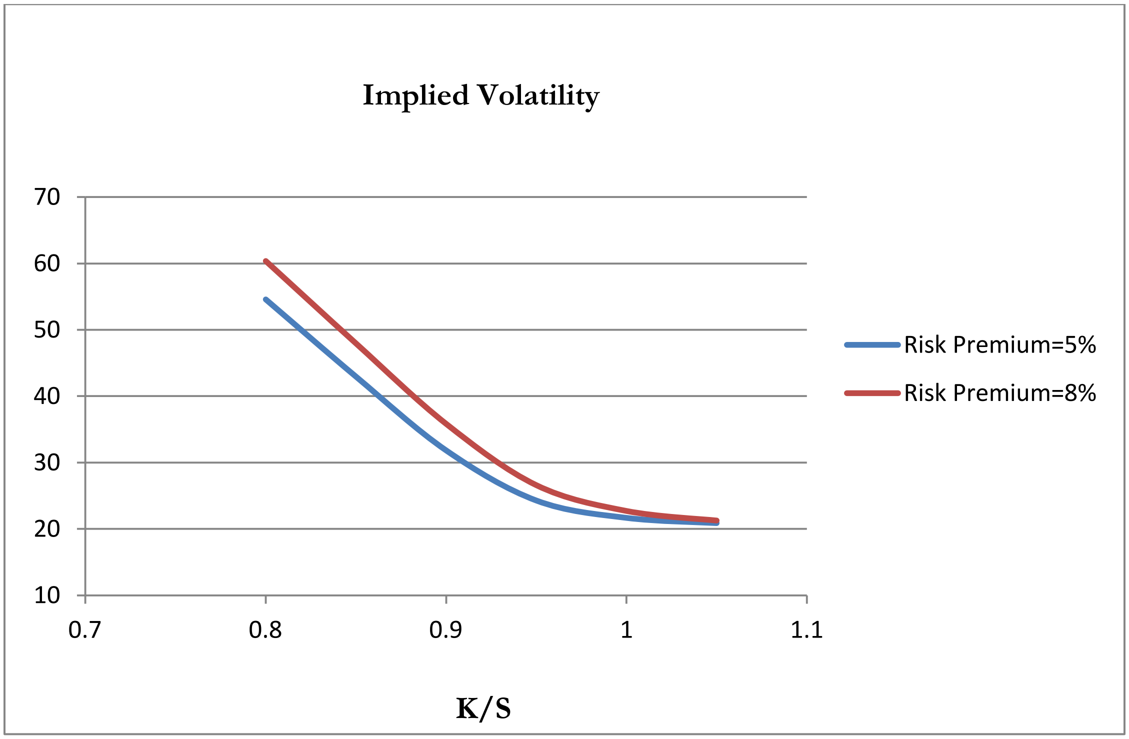


Figure 2

Prediction#2 ***Implied volatility should typically be higher than realized/historical volatility***

It follows directly from the analogy formula that as long as the risk premium on the underlying is positive, implied volatility should be higher than actual volatility. Anecdotal evidence is strongly in favor of this prediction. Rennison and Pederson (2012) calculate implied volatilities from at-the-money options in 14 different options markets over a period ranging from 1994 to 2012. They show that implied volatilities are typically higher than realized volatilities.

Prediction#3 ***Implied volatility curve should flatten out with expiry***

Figure 3 plots implied volatility curves for two different expiries. All other parameters are the same as in table 1. It is clear from the figure that as expiry increases, the implied volatility curve flattens out.

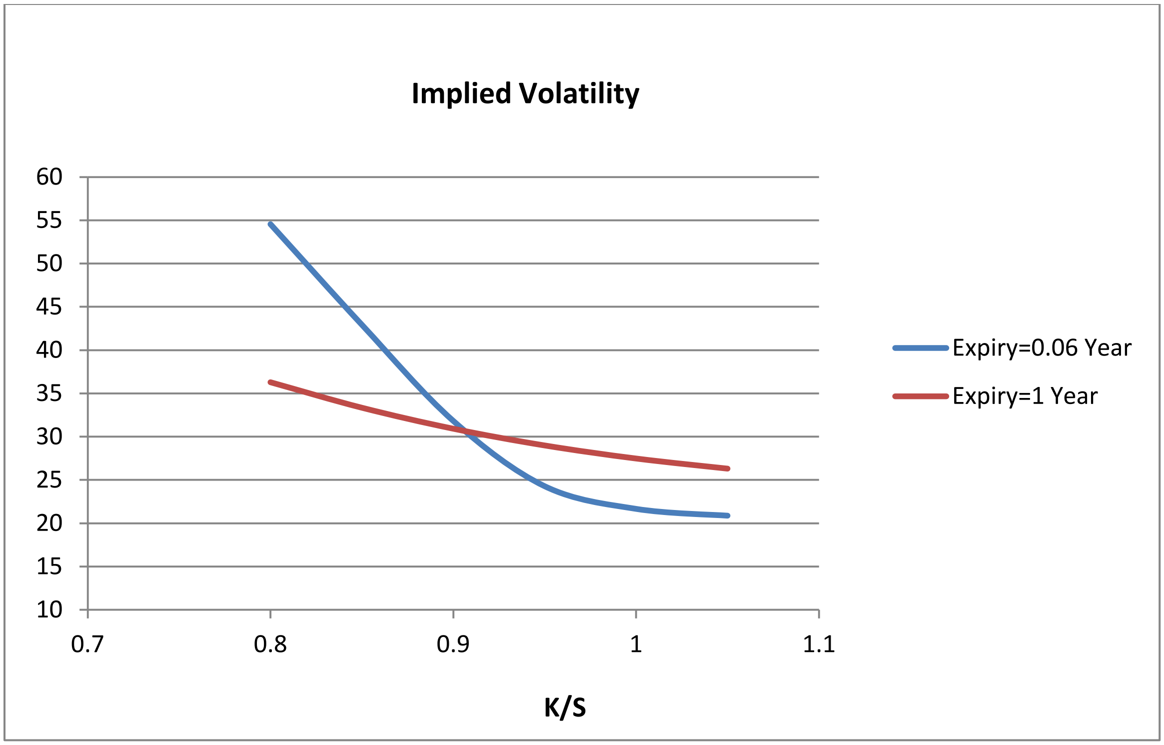


Figure 3

Empirically, implied volatility curve typically flattens out with expiry (see Greiner (2013) as one example). Hence, this match between a key prediction of the analogy model and empirical evidence is quite intriguing.

6. Conclusion

Analogy making appears to be the key to the way we think. In this article, we investigate the implications of analogy making for option pricing. We put forward a new option pricing formula that we call the analogy option pricing formula. The new formula differs with the Black Scholes formula due to the introduction of a new parameter capturing risk premium on the underlying stock. The new formula provides an explanation for the implied volatility skew puzzle. Three testable predictions of the model are also discussed. Rigorous testing of these predictions is the subject of future research.

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