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The perception of distributive fairness and optimal taxation under uncertainty*

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Abstract

This paper incorporates a preference for distributive fairness (inequity aversion) into the analysis on optimal redistributive taxation under uncertainty. We can show that introducing or strengthening the taste for distributive fairness does not affect the socially optimal tax rate (social insurance) directly. This merely works through a reduction in individual risk taking (increase in self-insurance) induced by inequity aversion. If the efficacy of self-insurance is sufficiently small, this renders taxation more desirable and therefore enhances the socially optimal tax rate. In other words, self-insurance should be complemented by social insurance in order to impair the psychic disutility stemming from income inequality. Turning to the case of moral hazard it can be shown that optimal self-insurance efforts are again increasing with the strength of inequity aversion while the effect on the optimal tax rate remains unclear.

JEL classification: D63, H21, H53

Key words: distributive fairness; inequity aversion
optimal taxation; redistribution; uncertainty

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1 Introduction

A growing number of empirical works has recently brought the relevance of distributive fairness or income equity for individual decision making into the limelight (see e.g. Amiel et al., 1999; Alesina et al., 2004; Schwarze and Härpfer, 2007 as well as Verme, 2011). Admittedly, a traditional grasp of income inequality restricts its focus on the dispersion of income. In this case, a distribution's standard deviation provides a frequently applied measure of income inequality in the theory of optimal redistributive taxation under uncertainty (see e.g. Sinn, 1995, 1996).¹

However, an increasing number of empirical and experimental studies reveals that in economic decisions people generally exhibit fairness or equity concerns going beyond the plain consideration of income dispersion (e.g. Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000). Famous examples are given by the workhorses of ultimatum and dictator games. Their outcomes suggest that constituting income inequality or fairness on standard deviation proves to be a rather narrow approach.

Constitutional welfare state theories deal with optimal taxation and redistribution under uncertainty (e.g. Domar and Musgrave, 1944; Eaton and Rosen, 1980a, 1980b, 1980c; Varian, 1980).² In a risky setting redistributive taxation can be interpreted as a measure of social insurance. Based on this point of view, Sinn (1995, 1996) analyzes the socially optimal degree of individual risk taking (self-insurance) and the tax rate (social insurance). Behind the veil of ignorance, a representative individual takes mean income and standard deviation into account. Sinn (1995) shows that the confiscatorial tax rate would be socially optimal. By implementing welfare costs Eichner and Wagener (2004) drop the unrealistic case of confiscatorial taxation. It deserves to be mentioned that constitutional theories of redistributive taxation neglect social and psychological aspects by focussing on egoistic, pecuniary motives underlying the preferences for income redistribution.³

Sinn (1995) defines income inequality as the dispersion of income leading to a congruence of income risk (ex ante) and income inequality (ex post). As mentioned above this seems to be a quite restrictive view on inequality or distributive fairness, respectively. Thus, we aim

¹More precisely, a distribution is defined as less equal if it has been generated by a mean-preserving spread from another distribution.

²In a constitutional setting, individuals decide behind the so-called veil of ignorance where the future position in society is uncertain. This concept goes back to the works of Harsanyi (1953, 1955), Buchanan and Tullock (1962) and in particular to Rawls (1971).

³Linde and Sonnemans (2012) have already pointed out that social attitudes are typically ignored in theories of individual decision making under risk or uncertainty.

to interpret income inequality as the degree of income concentration which is characterized by an income distribution's asymmetry. This asymmetry is indicated by income skewness and provides the foundation of how individuals perceive an income distribution as fair. Our approach of distributive fairness could be interpreted as (altruistic) inequity aversion in the vein Fehr and Schmidt (1999). They assume that an individual experiences a psychic disutility if her income exceeds the income of other society members. The level of information could now be reduced to two observations: mean and median income. For the case of right-skewed income distributions mean income exceeds median income. Transferred to the case of uncertainty the representative individual considers the difference between her expected income (mean income) and the income of the society's middle (median income). A raise in skewness would lead to a larger difference between mean and median income. Proxying the mean-median ratio by skewness an income distribution is perceived as less fair if it is more skewed to the right.⁴ A further interpretation relies on a distributional norm which has also been pointed out by Elster (1989). In our setting individuals are presumed to exhibit a preference for symmetric income distributions. Then, deviation from symmetry give rise to a psychic disutility stemming e.g. from sympathy with low-income earners.

In the following analysis we can drop the finding of Sinn (1995, 1996) whereupon confiscatorial taxation is socially optimal. By assuming that income redistribution is costly the social planner would set a tax rate below one (see also Eichner and Wagener, 2004). Further, we illustrate that introducing or strengthening inequity aversion in the vein of Fehr and Schmidt (1999) induces the socially optimal degree of risk taking to decrease. Higher self-insurance efforts enhance distributive fairness and therefore reduce the psychic disutility experienced by the individual. Beyond, we can show that the taste for distributive fairness does not affect the socially optimal tax rate directly. However, introducing or strengthening inequity aversion reduces the degree of individual risk taking which in turn renders taxation more desirable if the efficacy of self-insurance efforts is sufficiently small. As a main result we claim that governments should complement higher self-insurance by enlarging social insurance or income redistribution respectively when faced with inequity aversion.

Turning to the case of moral hazard where the representative individual does not anticipate

⁴This approach is also in line with the politico-economic approach of Meltzer and Richard (1981) where income inequality is assumed to be represented by the ratio of mean and median income. Further, Alesina and Rodrik (1991) or Bénabou (1996) apply similar types of income inequality. Instead of the median voter who considers the deviation from mean income a representative agent decides on mean income and take the deviation to median income into account.

the balanced fiscal budget and therefore take income skewness as exogenously given we can show that self-insurance efforts are again increasing in inequity aversion. However, it remains unclear whether the optimal tax rate is increasing or decreasing with the taste for distributive fairness.

The remainder of this paper is organized as follows. In section 2 we set up the model and introduce the concept of distributive fairness or inequity aversion, respectively. Then, in section 3 the socially optimal allocation regarding self-insurance and social insurance is analyzed. The individual risk-taking decision and the optimal tax rate under moral hazard is considered in section 4. Concluding remarks are offered in section 5.

2 The model

Post-tax income, taxes and transfers. We employ a welfare state model in the vein of Sinn (1995, 1996) where a large number of homogenous individuals decide behind the veil of ignorance.⁵ Since the future position in society is uncertain, a representative individual faces the random post-tax lifetime income⁶

$$Y := m - \lambda(e)\theta - e - T(e, \tau) + p \geq 0, \quad (1)$$

where $m \in \mathbb{R}_{++}$ denotes the exogenous market income and $L(e, \theta) = \lambda(e)\theta$ is the random income loss due to e.g. productivity differences.⁷ We restrict our attention to strictly positive realizations of the income loss, i.e. $\theta > 0$. The representative individual is now able to hedge this income risk by two measures.

First, self-insurance efforts $e \in [0, \bar{e}]$ as first introduced by Ehrlich and Becker (1972) mitigate the income loss (e.g. through investments in human capital) and therefore raise the lifetime income. Efforts reduce the income loss by the twice differentiable efficacy function $\lambda(e) \in [0, 1]$ which is assumed to be decreasing and convex in self-insurance efforts, i.e. $\lambda'(e) < 0 \leq \lambda''(e)$. In turn, self-insurance causes linear costs in terms of foregone lifetime income such that the individual's pre tax income is given by $X = m - \lambda(e)\theta - e$.

Second, the income loss is alleviated through a redistributive tax policy $\Pi = (\tau, p)$ with the

⁵After the veil of ignorance has lifted individuals differ with respect to the income. According to the law of large numbers the probability distribution of Y turns out to be the ex post income distribution.

⁶In contrast to Sinn (1995, 1996) we abstract from non-market income which would not change the results and provide new insights.

⁷We define $\mathbb{R}_{++} = \mathbb{R}_+ \setminus \{0\}$ throughout the paper.

tax rate $\tau \in [0, 1]$. Thus, the individual's income tax liability is given by

$$T := \tau[m - \lambda(e)\theta], \quad (2)$$

which allows to finance the lump sum transfer $p \in \mathbb{R}_+$. Drawing on the optimal income taxation literature (e.g. Mirrless, 1971) individual self-insurance activities are assumed to be private information from the governmental perspective. Subsequently, self-insurance activities are not deductible from the tax base. Further, following Eichner and Wagener (2004) a fraction $c \in [0, 1]$ of the tax revenue, e.g. due to costs of administration or simply waste of public funds, is not available for income redistribution.⁸ Subsequently, the fiscal budget constraint is depicted by

$$p = (1 - c)\mathbf{E}T. \quad (3)$$

According to (3), the lump-sum transfer has to match the expected tax liability net of the welfare state costs.

The model in terms of pre-tax standard deviation. Following Sinn (1995, 1996) we phrase the model in terms of pre-tax standard deviation which is given by

$$\sigma_X = \lambda(e)\sigma_\theta, \quad (4)$$

where σ_θ denotes the standard deviation of the random income loss θ . By using (4) and the inverse of the efficacy function λ^{-1} we can eliminate self-insurance efforts e in X such that we obtain

$$\bar{\mu}(\sigma_X) := m - k\sigma_X - \lambda^{-1}\left(\frac{\sigma_X}{\sigma_\theta}\right), \quad (5)$$

with $k := \frac{\mu_\theta}{\sigma_\theta}$ which is referred to as self-insurance function. The self-insurance function is concave in σ_X due to the convexity of λ in e . Next, the expected tax liability can be written as

$$\mathbf{E}T := \tau[m - \sigma_X k]. \quad (6)$$

⁸Note that we abstract from the case $c = 1$ which connotes that the entire tax yield is sunk within the redistribution process. Per se, such a system of tax collection could not be optimal.

Based on this, the mean μ_Y and standard deviation σ_X of post-tax income are given by:

$$\mu_Y = \bar{\mu}(\sigma_X) - \tau[m - k\sigma_X] + p \quad (7)$$

$$\sigma_Y = (1 - \tau)\sigma_X. \quad (8)$$

Measure of distributive fairness. As outlined in the introduction we constitute distributive fairness on the measure of income skewness which is assumed to capture income inequality. This could be related to the approach of Fehr and Schmidt (1999). An individual which is (altruistic) inequity averse prefers a smaller difference between her income and the income of others.⁹ This concept could be transferred to the constitutional level. Behind the veil of ignorance, the representative individual is mainly interested in her expected income (mean income). Further, we could define the income of the society's middle (median income) as reference level the representative individual considers. By assuming a right-skewed income distribution mean income exceeds median income. In this case, the difference between mean and median income reflects the degree of income inequality experienced by the representative individual. It is worth mentioning that this approach is also in line with the politico-economic framework of Meltzer and Richard (1981).

If the mean-median income-ratio is presumed to be proxied by income skewness distributive fairness or income inequality could be based on this measure.¹⁰ Then, a more right-skewed income distribution reflects a larger difference between mean μ_Y and median income \tilde{Y} (sketched by figure 1) and is thus perceived as less fair.¹¹ In other words, the representative individual exhibits a concern for the middle of the society. Hence, in our approach distributive fairness is interpreted as inequity aversion in the vein of Fehr and Schmidt (1999). Conveyed to the case of uncertainty, individuals perceive a more symmetric income (probability) distribution as rather fair.

The skewness of the lifetime income is then defined as the (unstandardized) third central

⁹It deserves to be mentioned that we abstract from the case of inequity aversion based on envy which has also been introduced by Fehr and Schmidt (1999).

¹⁰Income skewness is also defined as mean-median-ratio by the empirical study of You (2012). A multilevel analysis based on *World Values Survey* and *European Values Survey* data shows that skewness affects income inequality and subsequently social trust significantly.

¹¹Although this relation is not constituted by a rigorous theoretical analysis numerical simulations with the empirically relevant skew normal distribution reveal that increases in (unstandardized) skewness induce a larger difference between mean and median income.

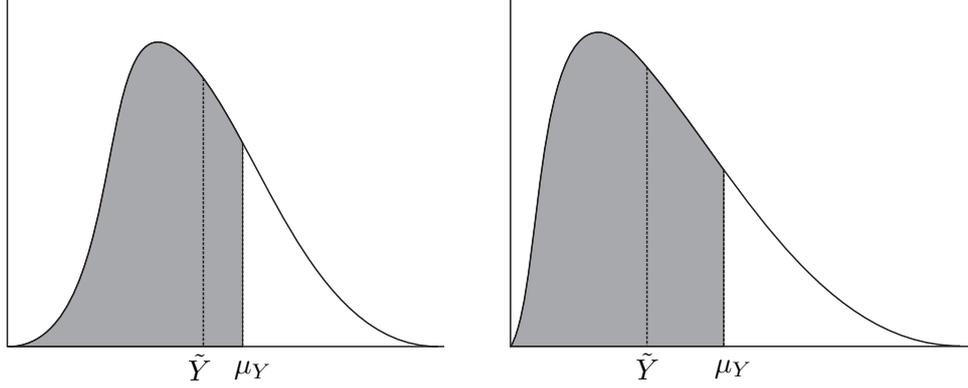


Figure 1: Skewness of income distributions

moment $\mu_3[Y] = \mathbf{E}[(Y - \mathbf{E}Y)^3]$ of the income distribution

$$s_Y(\sigma_X, \tau) = -(1 - \tau)^3 \frac{\mu_3(\theta) \sigma_X^3}{\sigma_\theta^3} \geq 0 \quad (9)$$

which allows to quantify *ceteris paribus* deviations from symmetry. In this case, we are able to separate changes in the concentration of an income distribution from shifts in income dispersion.¹² Recall that we presumed the income loss distribution to be left-skewed ($\mu_3[\theta] < 0$) inducing a right-skewed income distribution. Differentiating (9) with respect to σ_X and τ yields:

$$\frac{\partial s_Y}{\partial \sigma_X} > 0, \quad \frac{\partial s_Y}{\partial \tau} < 0. \quad (10)$$

According to (10), skewness is increasing (decreasing) in the degree of individual risk taking (self-insurance efforts) and decreasing in the tax rate. In other words, both insurance measures are capable to reduce income inequality and therefore enhance distributive fairness.

Preferences. Individual preferences are represented by the continuous, twice differentiable and additively-separable utility function $u : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$,

$$u(\mu_Y, \sigma_Y, s_Y) := U(\mu_Y, \sigma_Y) - aV(s_Y), \quad (11)$$

which consists of two sub-functions. First, the representative individual generates utility $U : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$ from expected post-tax income μ and the associated risk σ . Second, the individual experiences a psychic disutility $V : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ depending on the degree of (expected) distributive

¹²A derivation of income skewness and its partial derivatives is delegated to the appendix. It is worth mentioning that we follow Arditty and Levy (1972), Ingersoll (1975) as well as Diacogiannis (1994) by applying the unstandardized third central moment of the income distribution as skewness measure. Due to the location scale property standardized skewness $\frac{\mu_3[Y]}{\sigma_Y^3}$ is constant in this framework.

fairness which is characterized by unstandardized income skewness s_Y . We assume that the disutility is increasing in skewness as a more right-skewed income distribution is perceived as less fair:

$$V'(s_Y) > 0. \quad (12)$$

Furthermore, the fairness functional is presumed to be convex in skewness, i.e. $V'(s_Y)'' \geq 0$. Finally, preference parameter a in (11) reflects the strength of inequity aversion or the taste for distributive fairness, respectively.

Further, it deserves to be mentioned that the mean-variance framework is taken as an approach which stand on its own.¹³ Let us assume positive but diminishing marginal utility of expected income, i.e. $U_{\mu\mu} \leq 0 < U_\mu$, as well as individual risk aversion, i.e. $U_\sigma < 0$. Then, for the marginal rate of substitution (MRS) between σ_Y and μ_Y follows

$$i(\mu_Y, \sigma_Y) := -\frac{U_\sigma(\mu_Y, \sigma_Y)}{U_\mu(\mu_Y, \sigma_Y)} > 0. \quad (13)$$

According to (13) the marginal willingness to pay for lower risk (or insurance equivalently) in terms of income is positive. Graphically, $i(\mu_Y, \sigma_Y)$ captures the slope of the indifference curves in (μ_Y, σ_Y) -space. In compliance with Sinn (1995, 1996) indifference curves exhibit the following properties:

$$i_\mu(\mu_Y, \sigma_Y) \leq 0, \quad (14)$$

$$\frac{d^2 \mu_Y}{d\sigma_Y^2} = i(\mu_Y, \sigma_Y) \cdot i_\mu(\mu_Y, \sigma_Y) + i_\sigma(\mu_Y, \sigma_Y) > 0. \quad (15)$$

Assumption (14) represents the property of non-increasing absolute risk aversion (Meyer 1987, Lajeri-Chaherli 2003). Beyond, according to (15) indifference curves are assumed to be strictly convex in the (μ_Y, σ_Y) -space. Eichner (2008) shows that $\frac{d^2 \mu_Y}{d\sigma_Y^2} > 0$ holds if agents are presumed to behave risk averse.

3 Social optimum

In this section, we analyze the socially optimal degree of individual risk taking as well as the socially optimal tax rate. This first-best allocation is achieved if the representative individual

¹³In our setting, the mean-variance approach could be considered as equivalent to the maximization of expected utility. However, it is more convenient to compare our results immediately to the results of Sinn (1995). Further, it allows us to express the results directly as a trade-off between expected income and income risk.

anticipates that her choice of self-insurance efforts affects both the balanced fiscal budget as well as the psychic disutility from income inequality. In this case the individual opportunity set is given by

$$\mu_Y = \bar{\mu}(\sigma_X) - c\tau(m - k\sigma_X), \quad (16)$$

which is now referred to as *redistribution function*.¹⁴ Following this, we characterize the constitutional choice of the tax-transfer policy which should also be interpreted as the choice of a social planner.¹⁵

3.1 Self-insurance efforts

The individual decision problem is given by

$$\begin{aligned} \max_{\sigma_X} U(\mu_Y, \sigma_Y) - aV(s_Y) \quad s.t. \quad & \mu_Y = \bar{\mu}(\sigma_X) - c\tau(m - k\sigma_X), \\ & \sigma_Y = (1 - \tau)\sigma_X, \\ & s_Y = s_Y(\sigma_X, \tau). \end{aligned} \quad (17)$$

Restricting our attention to interior solutions, i.e. $\sigma_X > 0$, we obtain the first-order condition

$$\underbrace{-(1 - \tau)i(\mu_Y, \sigma_Y)}_{[1]} - \underbrace{\frac{aV'(s_Y)}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X}}_{[2]} + \underbrace{\bar{\mu}'(\sigma_X) + c\tau k}_{[3]} = 0, \quad (18)$$

which implicitly determines the socially optimal value σ_X^* as a function of the exogenous parameters c , τ and a .¹⁶ In (18), we are able to identify three effects of individual risk taking σ_X on utility. The term [1] is negative and captures the *risk effect of self-insurance*. Increasing the degree of individual risk taking is ceteris paribus associated with a higher post-tax income risk σ_Y . As we assumed risk averse agents ($U_\sigma < 0 \Rightarrow i(\mu_Y, \sigma_Y) > 0$) this marginal effect is negative. Conversely, higher self-insurance efforts (lower pre-tax standard deviation) reduce post-tax risk which is utility-enhancing. Beyond, the term [2] is negative due to $V'(s_Y) > 0$, $U_\mu > 0$ as well as $\frac{\partial s_Y}{\partial \sigma_X} > 0$ and is referred to as *fairness effect of self-insurance*. A higher pre-tax standard deviation ceteris paribus induces skewness and therefore income inequality to increase which then goes along with a higher psychic disutility. Vice versa, raising self-insurance efforts enhances

¹⁴This follows by making use of (3) in (7).

¹⁵To put it in terms of a two-stage game, the tax rate is set on the first stage. Given the tax rate, self-insurance efforts are chosen on the second stage. This game is then solved by backward induction.

¹⁶For details on the second-order conditions we refer to the appendix.

distributive fairness and hence reduces the psychic disutility. Eventually, the term [3] represents the *income effect of self-insurance* which has to be positive as terms [1] and [2] are negative. Thus, increasing pre-tax standard deviation ceteris paribus induces mean post-tax income μ_Y to increase. Vice versa, higher self-insurance efforts decrease mean post-tax income which is utility reducing.¹⁷ In a nutshell, terms [1] and [2] represent the marginal cost (benefit) while term [3] captures the marginal benefit (cost) of individual risk taking (self-insurance). The first-order condition (18) then requires that the marginal benefit and cost of self-insurance efforts balance each other out. In other words, equation (18) describes the trade-off between expected income, income risk and the fairness of an income distribution.

Graphical analysis. The first-order condition (18) is illustrated by figure 2. The concave line depicts the redistribution curve which represents the opportunity frontier in the (μ_Y, σ_Y) -space. In the absence of inequity aversion ($a = 0$), eq. (18) simplifies to $i(\mu_Y, \sigma_Y) = \frac{\bar{\mu}'(\sigma_X) + c\tau k}{1-\tau}$

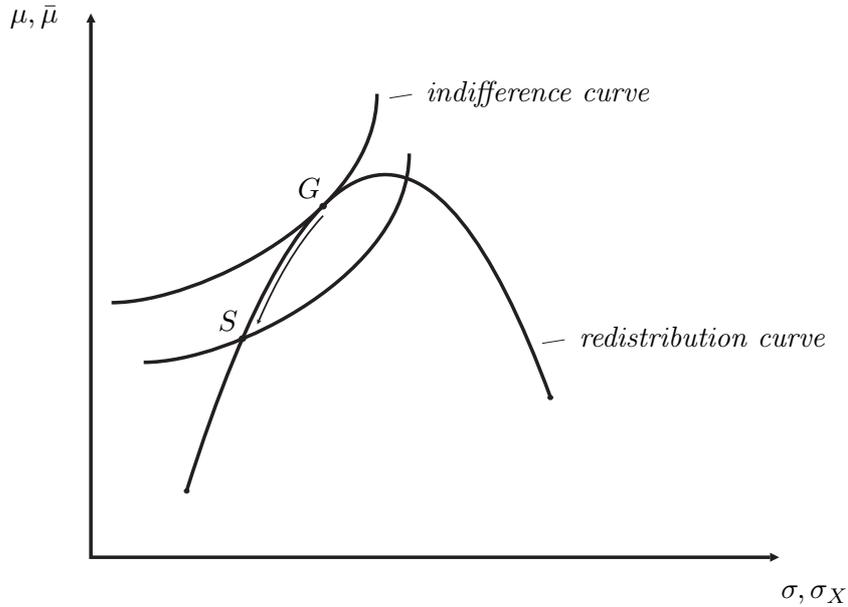


Figure 2: First-order condition

where the slope of the indifference curve equals the slope of the redistribution curve.¹⁸ This condition is satisfied in equilibrium point G . Introducing inequity aversion ($a > 0$) leads to equilibrium point S at which the redistribution curve's slope exceeds the slope of the indifference curve, i.e. $i(\mu_Y, \sigma_Y) < \frac{\bar{\mu}'(\sigma_X) + c\tau k}{1-\tau}$. This relation is satisfied at a point left to the point G and is caused by the negative fairness effect of self-insurance, c.f. term (2) in eq. (18). As moving from the right to the left on the redistribution curve is associated with a lower pre-

¹⁷The sign of $\bar{\mu}'(\sigma_X)$, however, is indeterminate in sign.

¹⁸Differentiation of $\mu_Y = \bar{\mu}(\sigma_X) - c\tau(m - k\sigma_X)$ where $\sigma_X = \frac{\sigma_Y}{1-\tau}$ with respect to σ_Y yields the slope of the redistribution curve $\frac{\bar{\mu}'(\sigma_X) + c\tau k}{1-\tau}$.

tax standard deviation we can state that self-insurance efforts increase due to the existence of inequity aversion.

Self-insurance efforts and inequity aversion. Formally, we differentiate (18) with respect to the pre-tax standard deviation as well as the preference parameter to obtain

$$\text{sgn} \frac{\partial \sigma_X^*}{\partial a} = -\text{sgn} \left[\frac{V'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} \right] < 0, \quad (19)$$

while taking the second-order condition for granted. Hence, pre-tax standard deviation is decreasing in preference parameter a . This result is summarized in the following proposition.

Proposition 1. *The socially optimal degree of self-insurance (individual risk taking) is increasing (decreasing) in the preference intensity of individual inequity aversion. Further, for any preference intensity the society operates at a point on its opportunity frontier where an increase in individual risk taking decreases mean post-tax income.*

More precisely, introducing or increasing the aversion towards income inequity ceteris paribus enhances the psychic disutility. Consequently, the social planner reduces individual risk taking and therefore raises self-insurance in order to impair the disutility. In other words, increasing a is associated with a larger fairness effect of self-insurance which in turn enhances its marginal benefit. The individual is then willing to give up income for a reduction in the income risk and an enlargement of distributive fairness.

Self-insurance and redistributive taxation. Next, we strive to analyze the effect of a tax rate change on the degree of individual risk taking. Implicitly differentiating (18) with respect to σ_X^* and τ leads to

$$\text{sgn} \frac{\partial \sigma_X^*}{\partial \tau} = \text{sgn} \left[ck + i + (1 - \tau)[i_\mu c(m - k\sigma_X) + i_\sigma \sigma_X] - \frac{aV''}{U_\mu} \cdot \frac{\partial s_Y}{\partial \tau} - \frac{aV'}{U_\mu^2} \left(U_\mu \cdot \frac{\partial^2 s_Y}{\partial \sigma_X \partial \tau} + (c[m - k\sigma_X] + U_{\mu\sigma} \sigma_X) \cdot \frac{\partial s_Y}{\partial \sigma_X} \right) \right] > 0, \quad (20)$$

which becomes positively signed for constant absolute risk aversion (CARA), i.e. $i_\mu = 0$, as well as $-U_\mu \cdot \frac{\partial^2 s_Y}{\partial \sigma_X \partial \tau} > (c[m - k\sigma_X] + U_{\mu\sigma} \sigma_X) \cdot \frac{\partial s_Y}{\partial \sigma_X}$.¹⁹ Expression (20) captures the effect of a higher tax rate on the marginal cost and return of self-insurance efforts. Assuming CARA ensures that the income loss in the course of a higher tax rate does not affect the degree of risk

¹⁹These are sufficient but not necessary conditions for a positive sign of (20). Further details on the derivation of (20) could be found in the appendix.

aversion. As a consequence, the individual is willing to increase the degree of risk taking when the effect of taxation on the marginal effect of pre-tax standard deviation on income inequality is sufficiently large. In other words, we get the intuitive result that the socially optimal level of self-insurance efforts shrink in response to a more intense income redistribution. Thus, self-insurance is substituted by social insurance.

3.2 Tax rate

We turn to the choice of the socially optimal tax rate where the social planner takes into account the social optimal level of self-insurance efforts. Thus, the indirect utility function is given by

$$u^* := U(\mu_Y^*, \sigma_Y^*) - aV(s_Y^*), \quad (21)$$

where the asterisks are indicating pre-tax standard deviation to be chosen at the socially optimal level σ_X^* .

We differentiate the indirect utility function (21) with respect to the tax rate τ . Focussing on interior solutions and applying the envelope theorem the first-order condition is given by

$$\frac{\partial u^*}{U_\mu \partial \tau} = \underbrace{-c(m - k\sigma_X^*)}_{[1]} + \underbrace{\sigma_X^* \cdot i(\mu_Y^*, \sigma_Y^*)}_{[2]} - \underbrace{\frac{a \cdot V'(s_Y^*)}{U_\mu} \cdot \frac{\partial s_Y}{\partial \tau}}_{[3]} = 0. \quad (22)$$

According to (22) we can disentangle three contrary marginal effects of the tax rate on optimal utility. The first term [1] represents the *income effect of social insurance*. Increasing the tax rate τ ceteris paribus directly leads to a lower expected income since the share c of tax revenues is not available to finance the lump-sum transfer. Hence, [1] is negative and captures the marginal costs of taxation. In opposition to the marginal costs, the second and third term are positive and capture the marginal benefit of a higher tax rate. The second term [2] depicts the *risk effect of social insurance*. The higher the tax rate is the lower is the post-tax income volatility ceteris paribus. Due to risk aversion the utility enhances. Eventually, the third term [3] maps the *fairness effect of social insurance*. By definition of (9) a higher tax rate ceteris paribus induces a lower skewness ($\frac{\partial s_Y}{\partial \tau} < 0$) equivalent to a lower degree of income inequality. This in turn reduces the psychic disutility ($V'(s_Y) > 0$). However, we can show that the inequity effect of social insurance plays no role for the decision on the optimal tax rate. Evaluating (18) at τ^*

and substituting into (22) the first-order condition for τ^* changes to:

$$-c(m - k\sigma_X^*) + \frac{[\bar{\mu}'(\sigma_X^*) + c\tau k]\sigma_X^*}{1 - \tau} = 0. \quad (23)$$

Equation (23) captures the net marginal cost and benefit of social insurance. There still remains a net income effect, as captured by the first term in (23), caused by the shadow cost of public funds. This net income effect is negative and hence constitutes the net marginal cost of taxation. The second term depicts the marginal net benefit of social insurance. This effect depends on the negative income effect of self-insurance as increasing self-insurance efforts goes along with a reduction in mean income. Obviously, the strength of inequity aversion does not directly affect the socially optimal tax rate since the equity effect of social insurance vanishes. It can be shown that the marginal effects of individual risk taking as well as the tax rate on income inequality and therefore on the psychic disutility compensate each other. Consequently, the net fairness effect of social insurance in (23) gets zero.²⁰

Redistributive taxation and inequity aversion. We now aim to analyze the effect of inequity aversion on the socially optimal tax rate. Taken the second-order condition for granted implicit differentiation of (23) with respect to the tax rate τ^* and the preference parameter a yields

$$\text{sgn} \frac{\partial \tau^*}{\partial a} = \text{sgn} \left(\left[ck + \frac{\bar{\mu}''(\sigma_X^*) \cdot \sigma_X^* + \bar{\mu}'(\sigma_X^*) + c\tau k}{1 - \tau} \right] \cdot \frac{\partial \sigma_X^*}{\partial a} \right) > 0, \quad (24)$$

which is positive for a sufficiently large efficacy loss of self-insurance efforts in terms of foregone mean income.²¹ Expression (24) represents the effect of inequity aversion on the marginal net cost and benefit of social insurance. As outlined above an increase in inequity aversion affects the socially optimal tax rate only indirectly via a change in individual risk taking. We have shown that strengthening the inequity aversion goes along with a lower degree of individual risk taking or higher self-insurance efforts respectively, i.e. $\frac{\partial \sigma_X^*}{\partial a} < 0$. This in turn changes the net marginal cost and benefit of social insurance.

First, a cut in individual risk taking enhances the mean income (by ck) and therefore boosts the shadow cost of public funds. Hence, the negative net income effect of a higher tax rate gets larger which renders redistributive taxation less desirable at the margin. Second, decreasing pre-tax standard deviation affects the net marginal benefit of taxation as captured by the second

²⁰For details we refer to the appendix.

²¹This is stringently satisfied if the second-order condition for the tax rate holds.

term in squared brackets. Ceteris paribus, the induced reduction in expected post-tax income (due to $\bar{\mu}'(\sigma_X^*) + c\tau k > 0$) renders redistributive taxation less desirable at the margin. In contrast, a higher level of self-insurance efforts ceteris paribus results in a lower efficacy of self-insurance (as $\bar{\mu}''(\sigma_X^*) \cdot \sigma_X^* < 0$). This in turn enhances the net marginal benefit of redistributive taxation. Hence, ceteris paribus social insurance gets more desirable at the margin.

As mentioned above, it can be shown that the overall effect of a more intense inequity aversion on the tax rate is positive for a sufficiently large efficacy loss of self-insurance, i.e. if $\bar{\mu}''(\sigma_X^*)$ is sufficiently large. Thus, we can state that

Proposition 2. *The preference for distributive fairness does not affect the tax rate directly. Introducing or strengthening inequity aversion indirectly induces the socially optimal tax rate to increase via a change in individual risk taking if the efficacy of self-insurance is sufficiently small.*

In sum, strengthening inequity aversion enhances the net marginal benefit of taxation which outweighs the larger marginal cost of taxation. We claim that governments should complement the higher degree of self-insurance by enlarging the redistribution of income. Subsequently, in the social optimum self-insurance and social insurance are complements when the preference for income equality becomes larger.

4 Moral hazard

In this section we characterize the second-best allocation where the representative individual does not anticipate the balance of public revenues and expenditures. This gives rise to moral hazard effects as the individual ignores that her self-insurance behavior affects the lumpsum transfer and therefore the fiscal budget. The individual opportunity set is then characterized by (7) which constitutes the *subjective redistribution curve*.

4.1 Self-insurance efforts

Again, we aim to determine the optimal degree of individual risk taking and therefore the optimal level of self-insurance efforts. The individual optimization problem is given by

$$\begin{aligned} \max_{\sigma_X} U(\mu_Y, \sigma_Y) - aV(s_Y) \quad s.t. \quad & \mu_Y = \bar{\mu}(\sigma_X) - \tau[m - k\sigma_X] + p, \\ & \sigma_Y = (1 - \tau)\sigma_X, \\ & s_Y = s_Y(\sigma_X, \tau). \end{aligned} \quad (25)$$

Solving this decision problem and focussing on interior solutions yields the first-order condition

$$\underbrace{-(1 - \tau) \cdot i(\mu_Y, \sigma_Y)}_{[1]} - \underbrace{\frac{aV'(s_Y)}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X}}_{[2]} + \underbrace{\bar{\mu}'(\sigma_X) + \tau k}_{[3]} = 0. \quad (26)$$

which determines the optimal pre-tax standard deviation $\tilde{\sigma}_X$. Again, from equation (26) we can identify three marginal effects of individual risk taking on utility. The effects [1] and [2] correspond to the *risk effect* and the *fairness effect of self-insurance* as already shown in the first-order condition (18) for the socially optimal pre-tax standard deviation. However, the *income effect of self-insurance* [3] differs from that in (18) due to moral hazard. In case of moral hazard the representative individual does not anticipate that her self-insurance decision affects the fiscal budget. Hence, the lumpsum transfer is treated as exogenously given.

Graphical analysis. From the first-order condition (26) follows that the slope of the subjective redistribution curve exceeds the indifference curve's slope. At the redistributive equilibrium the first-order condition (26) together with the fiscal budget constraint (3) has to be satisfied. Subsequently, the equilibrium point lies on the redistribution curve $\mu_Y = \bar{\mu}(\sigma_X) - \tau(m - k\sigma_X)$. The redistributive equilibrium is sketched by figure 3. It can be shown that without inequity aversion ($a = 0$), as characterized by point N , the individual could end up at the descending branch of the redistribution curve where increasing self-insurance efforts leads to a higher expected post-tax income (see Sinn, 1995, 4.2). Introducing a sufficiently strong inequity aversion ($a > 0$) then shifts the equilibrium to the point M which is located at the ascending branch of the redistribution curve. In this case, further increasing self-insurance would reduce mean income. This result is constituted by the following comparative static.

Self-insurance efforts and inequity aversion. Implicitly differentiating (26) with respect

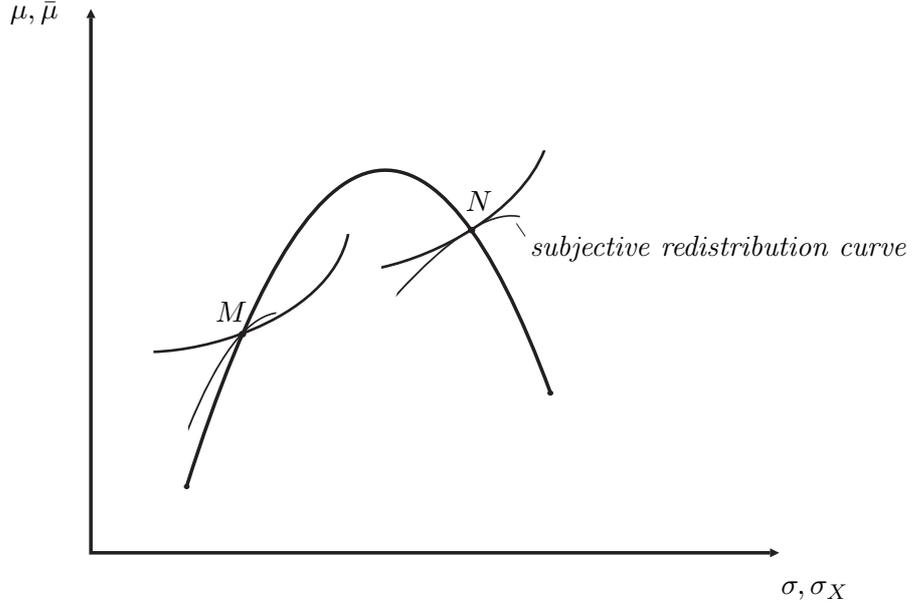


Figure 3: First-order condition

to $\tilde{\sigma}_X$ and a we obtain:

$$\text{sgn} \frac{\partial \tilde{\sigma}_X}{\partial a} = -\text{sgn} \left[\frac{V'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} \right] < 0, \quad (27)$$

which equals expression (19) in the first-best optimum. Hence, self-insurance is again increasing in the preference intensity of inequity aversion such that the first part of proposition 1 also holds under moral hazard. However, we can state that

Proposition 3. *For a sufficiently low fairness preference intensity the society operates at a point on its opportunity frontier where an increase in self-insurance enhances mean post-tax income. Assuming a sufficiently strong preference for distributive fairness it could be shown that the redistributive equilibrium ends up in a situation where increasing self-insurance reduces mean post-tax income.*

This result contrasts the findings of Sinn (1995). Introducing a taste for distributive fairness shifts the equilibrium point to the ascending branch of the redistribution curve. Under moral hazard the individual is again willing to give up income in order to impair the income risk and psychic disutility if the preference for distributive fairness is sufficiently strong.

Self-insurance efforts and taxation. To constitute the optimal taxation scheme in the next section the comparative static effect of the tax rate on self-insurance has to be analyzed.

Implicitly differentiating (26) we obtain:

$$\begin{aligned} \operatorname{sgn} \frac{\partial \tilde{\sigma}_X}{\partial \tau} = \operatorname{sgn} & \left[k + i + (1 - \tau)[c \cdot i_\mu(m - k\tilde{\sigma}_X) + \tilde{\sigma}_X \cdot i_\sigma] \right. \\ & - \frac{a}{U_\mu^2} \left(\left[V'' \cdot \frac{\partial s_Y}{\partial \sigma_X} \cdot \frac{\partial s_Y}{\partial \tau} + V' \cdot \frac{\partial s_Y}{\partial \sigma_X \partial \tau} \right] U_\mu \right. \\ & \left. \left. + V' \cdot \frac{\partial s_Y}{\partial \sigma_X} [cU_{\mu\mu}(m - k\tilde{\sigma}_X) + \tilde{\sigma}_X \cdot U_{\mu\sigma}] \right) \right] > 0. \quad (28) \end{aligned}$$

Again, for CARA as well as a sufficiently strong concavity of the utility function $U(\mu_Y, \sigma_Y)$ in income we can constitute a positive effect of the tax rate on pre-tax standard deviation or individual risk taking, respectively. In other words, a tax increase induces self-insurance efforts to decrease.²² Hence, analogous to the first-best case social insurance and self-insurance are substitutes.

4.2 Tax rate

After analyzing the decision on self-insurance we turn to the choice of the optimal tax rate where the government anticipates the optimal individual self-insurance decision and ensures a balanced fiscal budget. The government's decision problem is then represented by

$$\begin{aligned} \max_{\tau} U(\tilde{\mu}_Y, \tilde{\sigma}_Y) - aV(\tilde{s}_Y) \quad s.t. \quad & \tilde{\mu}_Y = \bar{\mu}(\tilde{\sigma}_X) - c\tau(m - k\tilde{\sigma}_X), \\ & \tilde{\sigma}_Y = (1 - \tau)\tilde{\sigma}_X, \\ & \tilde{s}_Y = s_Y(\tilde{\sigma}_X, \tau). \end{aligned} \quad (29)$$

Solving this problem yields to the first-order condition

$$\begin{aligned} & \underbrace{-c(m - k\tilde{\sigma}_X)}_{[1]} + \underbrace{i(\tilde{\mu}_Y, \tilde{\sigma}_Y) \cdot \tilde{\sigma}_X}_{[2]} - \underbrace{\frac{aV'(\tilde{s}_Y)}{U_\mu} \cdot \frac{\partial \tilde{s}_Y}{\partial \tau}}_{[3]} \\ & + \underbrace{\left[\bar{\mu}'(\tilde{\sigma}_X) + c\tau k - (1 - \tau) \cdot i(\tilde{\mu}_Y, \tilde{\sigma}_Y) - \frac{aV'(\tilde{s}_Y)}{U_\mu} \cdot \frac{\partial \tilde{s}_Y}{\partial \sigma_X} \right] \cdot \frac{\partial \tilde{\sigma}_X}{\partial \tau}}_{[4]} = 0 \quad (30) \end{aligned}$$

which implicitly determines the optimal tax rate $\tilde{\tau}$ set by the government. Equation (30) reveals four marginal utility effects of taxation. The terms [1], [2] and [3] are already known as *income*

²²Note that $i_\mu = 0$ (CARA) and $U_{\mu\mu} < -\frac{\tilde{\sigma}_X \cdot U_{\mu\sigma}}{c(m - k\tilde{\sigma}_X)}$ (strong concavity in income) are sufficient but not necessary conditions for a positive sign of the comparative static in (28).

effect, risk effect and fairness effect of social insurance from equation (22) in the previous section. Additionally, a new effect emerges in (30) under moral hazard which we will refer to as *risk-taking effect of social insurance*. This effect works indirectly through a tax-induced change in self-insurance efforts. As shown above, a higher tax rate increases the degree of individual risk taking ($\frac{\partial \tilde{\sigma}_X}{\partial \tau} > 0$) and therefore curbs self-insurance. A lower self-insurance exhibits three partial welfare effects as captured by the terms in squared brackets. First, a higher degree of individual risk taking ambiguously affects the post-tax income according to $\bar{\mu}'(\tilde{\sigma}_X) + c\tau k$ (indirect income effect). Second, a higher income volatility follows from a higher pre-tax standard deviation as represented by $-(1 - \tau) \cdot i(\tilde{\mu}_Y, \tilde{\sigma}_Y)$ (indirect risk effect) which is welfare-reducing. Eventually, a cut in self-insurance efforts raises income inequality and therefore the psychic disutility which is welfare-reducing (indirect fairness effect). In the aggregate, the risk-taking effect of social insurance is indeterminate in sign.

Analogous to the previous section we can constitute the net marginal effects of social insurance on utility. By taking into account equation (26) evaluated at $\tilde{\tau}$ the first-order condition (30) reads:

$$-c(m - k\tilde{\sigma}_X) + \frac{\tilde{\sigma}_X[\bar{\mu}'(\tilde{\sigma}_X) + \tau k]}{1 - \tau} - \tau k \cdot \frac{\partial \tilde{\sigma}_X}{\partial \tau} = 0. \quad (31)$$

Again, the net fairness effect as well as the indirect net fairness effect of social insurance gets zero.²³ The first and third term of (31) are negative while the sign of the second term remains indeterminate.

Redistributive taxation and inequity aversion. Increasing the preference parameter a affects the marginal benefit and cost of social insurance. Unfortunately, the overall effect of inequity aversion on the optimal tax rate remains indeterminate in sign. Hence, it remains unclear whether the government redistributes more or less income in response to individual inequity aversion.

5 Concluding remarks

The aim of the present paper is to shed some light on the role of inequity aversion or distributive fairness concerning the optimal redistributive tax policy under uncertainty. For that purpose we employ the welfare state theory of Sinn (1995, 1996) extended by welfare costs. In this light, we implement (altruistic) inequity aversion in the vein of Fehr and Schmidt (1999) and take income

²³For details we refer to the appendix.

skewness as a proxy for income inequality.

We can show that introducing or strengthening inequity aversion inevitably induces the socially optimal self-insurance (degree of individual risk taking) to increase (decrease). As (unstandardized) skewness is endogenous in our framework income inequality and therefore the psychic disutility can be reduced by raising self-insurance. Turning to the socially optimal redistributive tax policy it can be clearly shown that the social planner chooses full tax-deductibility of self-insurance efforts when faced with inequity aversion. Further, introducing or strengthening inequity aversion implies a higher socially optimal tax rate. It is worth mentioning that inequity aversion does not affect the socially optimal tax rate directly. The existence of inequity aversion induces the degree of individual risk taking to shrink. Increasing self-insurance efforts renders taxation more desirable at the margin if the efficacy of self-insurance is sufficiently small. As a consequence, it is socially optimal to raise the tax rate. Thus, we claim that governments should complement the higher degree of social insurance by enlarging the redistribution of income.

Turning to the case of moral hazard where the agent does not anticipate that her decision affects the fiscal budget it can be shown that self-insurance efforts are again increasing with the taste for distributive fairness. However, the effect on the optimal tax rate remains unclear.

Finally, we refer to some aspects which could be tackled in future research. First, we consider an exogenous market income and therefore abstract from analyzing work incentives of inequity aversion.²⁴ Second, we presumed preferences concerning inequity aversion to be homogenous in the population. It could be conceivable that individuals differ in their perception of distributive fairness. At last, our model could be augmented by background risks. According to Eichner and Wagener (2004) this kind of (non-insurable) risks becomes more important in a changing environment, e.g. due to ecological or climatical risks.

²⁴Note that self-insurance efforts could be interpreted as lifetime labor. However, endogenizing the market income would allow for a thorough analysis how inequity aversion affects the incentives to work.

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Appendix

Derivation of income skewness s_Y (9)

Substituting (2) into (1) lifetime income is given by:

$$Y = m - \lambda(e)\theta - e - \tau[m - \lambda(e)\theta] + p. \quad (32)$$

Dissolving the bracket yields:

$$Y = m - \lambda(e) \cdot \theta - e - \tau \cdot m + \tau \cdot \lambda(e) \cdot \theta + p. \quad (33)$$

Rearranging and using $\lambda(e) = \frac{\sigma_X}{\sigma_\theta}$ (33) reads:

$$Y = (1 - \tau)m + (\tau - 1) \cdot \frac{\sigma_X}{\sigma_\theta} \cdot \theta - e + p. \quad (34)$$

Based on (34) we can compute the skewness as third central moment of lifetime income Y .

Exploiting the translation-invariance of central moments we obtain:

$$\mu_3(Y) = \mu_3 \left[(\tau - 1) \cdot \frac{\sigma_X}{\sigma_\theta} \cdot \theta \right] \quad (35)$$

Eventually, since the third central moment is homogeneous of degree 3 income skewness is given by

$$\mu_3(Y) = [(\tau - 1) \cdot \frac{\sigma_X}{\sigma_\theta}]^3 \cdot \mu_3(\theta) \quad (36)$$

$$= -(1 - \tau)^3 \frac{\mu_3(\theta) \sigma_X^3}{\sigma_\theta^3} := s_Y(\sigma_X, \tau) \quad (37)$$

which constitutes (9). Differentiating $s(\sigma_X, \tau)$ with respect to σ_X and τ yields:

$$\frac{\partial s_Y}{\partial \sigma_X} = -(1-\tau)^3 \frac{3\mu_3(\theta)\sigma_X^2}{\sigma_\theta^3} > 0, \quad (38)$$

$$\frac{\partial^2 s_Y}{\partial \sigma_X^2} = -(1-\tau)^3 \frac{6\mu_3(\theta)\sigma_X}{\sigma_\theta^3} > 0, \quad (39)$$

$$\frac{\partial s_Y}{\partial \tau} = (1-\tau)^2 \frac{3\mu_3(\theta)\sigma_X^3}{\sigma_\theta^3} < 0, \quad (40)$$

$$\frac{\partial^2 s_Y}{\partial \tau^2} = -(1-\tau) \frac{6\mu_3(\theta)\sigma_X^3}{\sigma_\theta^3} > 0, \quad (41)$$

$$\frac{\partial^2 s_Y}{\partial \tau \partial \sigma_X} = (1-\tau)^2 \frac{9\mu_3(\theta)\sigma_X^2}{\sigma_\theta^3} < 0. \quad (42)$$

Hence, the income skewness raises with σ_X at an increasing rate and shrinks with τ at a decreasing rate. In addition, the cross (partial) derivative of s_Y with respect to σ_X and τ is negative.

Second-order condition for σ_X^* and derivation of (19).

The first-order condition is given by

$$A := -(1-\tau)i - \frac{aV'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} + \bar{\mu}' + c\tau k = 0. \quad (43)$$

Differentiating (43) we yield:

$$\begin{aligned} A_{\sigma_X} &= \bar{\mu}'' - (1-\tau)^2 \frac{d^2 \mu_Y}{d\sigma_Y^2} - \frac{aV'}{U_\mu} \cdot \left[(1-\tau)i_\mu \cdot \frac{\partial s_Y}{\partial \sigma_X} + \frac{\partial^2 s_Y}{\partial \sigma_X^2} \right] \\ &\quad - \frac{aV''}{U_\mu} \cdot \left[\frac{\partial s_Y}{\partial \sigma_X} \right]^2 + \left[\frac{aV'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} \right]^2 \cdot \frac{U_{\mu\mu}}{U_\mu} - \frac{i_\mu \cdot (1-\tau)aV'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} \end{aligned} \quad (44)$$

$$\begin{aligned} A_\tau &= ck + i + c(1-\tau)i_\mu(m - k\sigma_X) + (1-\tau)i_\sigma\sigma_X - \frac{aV''}{U_\mu^2} \cdot \frac{\partial s_Y}{\partial \tau} \\ &\quad - \frac{aV'}{U_\mu^2} \cdot \left[\frac{\partial s_Y}{\partial \sigma_X} (c[m - k\sigma_X] + U_{\mu\sigma}\sigma_X) + U_\mu \cdot \frac{\partial^2 s_Y}{\partial \sigma_X \partial \tau} \right] \end{aligned} \quad (45)$$

$$A_a = -\frac{V'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} < 0. \quad (46)$$

A_{σ_X} follows by making use of the first-order condition (18), $i_\mu \cdot i + i_\sigma = \frac{d^2 \mu_Y}{d\sigma_Y^2}$ and $-\frac{U_{\mu\mu}U_\sigma}{U_\mu} + \frac{U_{\mu\sigma}U_\mu}{U_\mu} = -U_\mu i_\mu$. Since all terms of A_{σ_X} except the second term are negative we can constitute that $i_\mu = 0$ is a sufficient (but not necessary) condition to satisfy the second-order condition, i.e. $A_{\sigma_X} < 0$. Then, expression (19) follows immediately from $\frac{\partial \sigma_X^*}{\partial a} = -\frac{A_a}{A_{\sigma_X}}$.

Derivation of first-order condition (23).

By using (43) in (22) we obtain

$$-c(m - k\sigma_X^*) + \frac{(\bar{\mu}' + c\tau k)\sigma_X^*}{1 - \tau} - \frac{aV'}{U_\mu} \cdot \left[\frac{\sigma_X^*}{1 - \tau} \cdot \frac{\partial s_Y}{\partial \sigma_X} + \frac{\partial s_Y}{\partial \tau} \right] = 0 \quad (47)$$

Considering (38) and (40) we can clearly see that $\frac{\sigma_X^*}{1 - \tau} \cdot \frac{\partial s_Y}{\partial \sigma_X} = \frac{\partial s_Y}{\partial \tau}$. Hence, the terms in the squared brackets cancel each other out such that we can write

$$B := -c(m - k\sigma_X^*) + \frac{(\bar{\mu}' + c\tau k)\sigma_X^*}{1 - \tau} = 0. \quad (48)$$

Second-order condition for τ^* and derivation of (24).

We differentiate the first-order condition (48) in order to obtain

$$B_\tau = \frac{(1 - \tau)ck\sigma_X^* + (\bar{\mu}' + c\tau k)\sigma_X^*}{(1 - \tau)^2} > 0 \quad (49)$$

$$B_{\sigma_X} = ck + \frac{\sigma_X^* \cdot \bar{\mu}'' + \bar{\mu}' + c\tau k}{1 - \tau} \quad (50)$$

$$B_a = 0. \quad (51)$$

The second-order condition for τ^* is now given by

$$B_\tau + B_{\sigma_X} \frac{\partial \sigma_X^*}{\partial \tau} < 0, \quad (52)$$

which is satisfied for $i_\mu = 0$, $\bar{\mu}''(\sigma_X^*) < -\frac{\bar{\mu}'(\sigma_X^*)}{\sigma_X^*}$ and small welfare costs c (in this case $B_{\sigma_X} < 0$ as well as $\frac{\partial \sigma_X^*}{\partial \tau} = -\frac{A_\tau}{A_{\sigma_X}} > 0$ hold). Then, (24) follows directly from $\frac{\partial \tau^*}{\partial a} = -\frac{B_a + B_{\sigma_X} \cdot \frac{\partial \sigma_X^*}{\partial a}}{B_\tau + B_{\sigma_X} \cdot \frac{\partial \sigma_X^*}{\partial \tau}} = -\frac{B_{\sigma_X} \cdot \frac{\partial \sigma_X^*}{\partial a}}{B_\tau + B_{\sigma_X} \cdot \frac{\partial \sigma_X^*}{\partial \tau}}$.

Second-order condition for $\tilde{\sigma}_X$ and derivation of (27).

The first-order condition is given by

$$C := -(1 - \tau)i - \frac{aV'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} + \bar{\mu}' + \tau k = 0. \quad (53)$$

Substituting $\mu_Y = \bar{\mu} - c\tau(m - k\sigma_X)$ and differentiating (53) we obtain:

$$\begin{aligned}
C_{\sigma_X} &= \bar{\mu}'' - (1 - \tau)^2 \frac{d^2 \mu_Y}{d\sigma_Y^2} - \frac{aV'}{U_\mu} \cdot \left[(1 - \tau) i_\mu \cdot \frac{\partial s_Y}{\partial \sigma_X} + \frac{\partial^2 s_Y}{\partial \sigma_X^2} \right] \\
&\quad - \frac{aV''}{U_\mu} \cdot \left[\frac{\partial s_Y}{\partial \sigma_X} \right]^2 + \left[\frac{aV'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} \right]^2 \cdot \frac{U_{\mu\mu}}{U_\mu} - \frac{i_\mu \cdot (1 - \tau) aV'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} \\
&\quad - (1 - c)\tau k [(1 - \tau) i_\mu + U_{\mu\mu}]
\end{aligned} \tag{54}$$

$$\begin{aligned}
C_\tau &= k + i + c(1 - \tau) i_\mu (m - k\sigma_X) + (1 - \tau) i_\sigma \sigma_X - \frac{aV''}{U_\mu^2} \cdot \frac{\partial s_Y}{\partial \tau} \\
&\quad - \frac{aV'}{U_\mu^2} \cdot \left[\frac{\partial s_Y}{\partial \sigma_X} (c[m - k\sigma_X] + U_{\mu\sigma} \sigma_X) + U_\mu \cdot \frac{\partial^2 s_Y}{\partial \sigma_X \partial \tau} \right]
\end{aligned} \tag{55}$$

$$C_a = -\frac{V'}{U_\mu} \cdot \frac{\partial s_Y}{\partial \sigma_X} < 0. \tag{56}$$

C_{σ_X} follows by making use of the first-order condition (26), $i_\mu \cdot i + i_\sigma = \frac{d^2 \mu_Y}{d\sigma_Y^2}$ and $-\frac{U_{\mu\mu} U_\sigma}{U_\mu} + \frac{U_{\mu\sigma} U_\mu}{U_\mu} = -U_\mu i_\mu$. Since all terms of C_{σ_X} except the second term are negative we can constitute that $i_\mu = 0$ is a sufficient (but not necessary) condition to satisfy the second-order condition, i.e. $C_{\sigma_X} < 0$. Then, expression (27) follows immediately from $\frac{\partial \tilde{\sigma}_X}{\partial a} = -\frac{C_a}{C_{\sigma_X}}$.