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Fair tax evasion and majority voting over redistributive taxation

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Abstract

We shed some light on fairness preferences regarding tax evasion. Individuals perceive income inequality which they are responsible for as fair (e.g. work effort) while inequality resulting from factors outside their reach is regarded as unfair (e.g. productivity or wage rate). This affects the incentives to hide income from tax authorities and supply labor. We set up a model where individuals simultaneously choose unreported income and work effort given a linear taxation scheme. We show the conditions for which individuals respond with lower or higher unreported income and work effort when fair tax evasion is introduced. Beyond, it can be shown that unreported income increases while work effort decreases when the tax rate is raised. Finally, we consider a majority voting over redistributive taxation. Thereby, it is shown that the median voter prefers lower (higher) taxation if she evades less (more) taxes than would be fair since raising the tax rate would enlarge (reduce) the deviation from fair tax evasion. This affects the moral cost as perceived by the individuals.

JEL classification: D31, D78, H26, H30

Key words: redistributive taxation, majority voting, fairness, tax evasion, labor supply
1 Introduction

The European sovereign-debt crisis in the aftermath of the 2007-2008 global financial crisis emphasizes the role of tax compliance behavior for balanced national budgets which has, not least, recently been illustrated by the case of Greece. The subject of tax evasion has found its way into modern economic theory in the 1970s. In their seminal work Allingham and Sandmo (1972) analyze the individual behavior regarding the declaration of income to the tax authorities. As individuals face a probability of getting caught and penalized the income reporting decision takes place under uncertainty. They analyze the interrelations between individual tax evasion, risk attitudes and the detection probability. Based on this, further theoretical works (see e.g. Christiansen, 1980; Sandmo, 1981; Yitzhaki, 1987; Falkinger, 1991; Andreoni et al., 1998 or Bernasconi, 1998) on individual tax compliance behavior emerged.

Beyond that, the case of tax evasion has been considered from an empirical (e.g. Clotfelter, 1983; Slemrod, 1985; Feinstein, 1991; Pommerehne and Weck-Hannemann, 1996) or experimental perspective (e.g. Becker et al., 1987; Alm et al., 1992; Mittone, 2006; Kleven et al., 2011). As has already been pointed out by Gordon (1989), Sandmo (2005) or Traxler (2010) the conventional portfolio choice approach to tax evasion is not able to explain a crucial part of the empirical results whereupon people seem to evade less taxes than would be expected from a pure risk perspective. According to Alm et al. (1992) individuals comply with the tax law even when the probability of being detected is zero. A common feature of the standard tax evasion models is that non-pecuniary motives and fairness perceptions are neglected. These social motives could play a vital role for individual tax compliance behavior (see e.g. Bosco and Mittone, 1997).

Consequently, social incentives has received growing attention in the theory of tax evasion. The works of Gordon (1989), Myles and Naylor (1996) as well as Traxler (2010) extends the standard tax evasion literature by social norms and group conformity. In these settings the expected degree of tax evasion in society and therefore stigma influences the individual evasion decision. In a similar vein, Kim (2003) considers the relation between the distribution of income and tax evasion stigma. Beyond, the approach of Bordignon (1993) sheds some light on fairness perceptions regarding the fiscal treatment with the provision of public goods as well as the behavior of other tax payers. Empirical evidence from Spicer and Becker (1980) shows that individuals feeling unfairly treated by the tax scheme tend to evade more taxes. From a theoretical perspective Falkinger (1995) analyzes the role of income equity for the tax payer’s risk preference and the corresponding tax evasion decision. However, it is worth noting that the relevance of distributive fairness perceptions for individual work effort and the tax evasion decision is scarcely considered in the beforementioned literature.

From a liberal egalitarian perspective of distributive justice individuals seem to consider the responsibility of factors underlying income inequality as being important (e.g. Dworkin, 1981; Fleurbaey, 1995; Kolm, 1996; Roemer 1998 or Cappelen and Tungodden, 2009). If influencable factors drive income inequality individuals perceive this inequity as rather fair. However, income

\[ \text{See Cowell (1992) for a basic survey on inequity concepts and tax evasion.} \]
inequality resulting from factors which could not be influenced by individuals is considered as unfair. According to Barth et al. (2013) work effort could be interpreted as individual responsibility factor while wage rates reflecting talent or productivity are assumed to be not in an individual’s reach. As governments can only tax the entire individual income this perceived distinction between justifiable and unjustifiable income inequality plays a vital role for tax compliance or morale, respectively. Barth et al. (2013) set up an individual decision framework and confirms the empirical relevance of fair tax evasion. It deserves to be mentioned that this concept of distributive fairness is also closely correlated to the fairness approach of Alesina et al. (2002). In their seminal work redistributive tax policies are analyzed where income variance is divided into a fair (based on effort and investments) and an unfair part (resulting from luck or illegal activities) where governments are not able to distinguish between the sources of income inequality. Yet the relevance of tax evasion is neglected in this framework.

Hitherto, the question of how fair tax evasion influences tax rates resulting from a majority voting has received less attention. Some works consider different aspects of majority voting and tax evasion (e.g. Alm et al., 1999; Borck, 2004, 2009; Fuest and Huber 2001; Traxler, 2009, 2012). A common feature of these approaches reveals in neglecting social motives for individual behavior. Thus, our approach aims to contribute to the politico-economic strand of the tax evasion literature by extending the analysis with individual perceptions of distributive fairness.

We employ the framework of Barth et al. (2013) in order to analyze how exogenous changes in taxation affects the degree of tax evasion and work effort chosen by individuals. The fair income depends on work effort as individuals expect to be rewarded for their effort. It is also perceived as fair to receive the average wage rate. These fairness perceptions affect the fair tax liability and subsequently the degree of tax evasion which is regarded as fair. Then, deviations from the fair level of tax evasion give rise to a moral cost. This moral cost could be influenced by varying the unreported income as well as the individual work effort. We show how individuals changes the income which is not declared to the tax authorities as well as the labor supply when the fairness preference concerning tax evasion is introduced. The decision depends on whether the individual receives an unfairly high or low net wage as well as on the difference between actual and fair tax evasion. Further, it can be shown that individuals declare less income to the tax authorities and reduce work effort when the tax rate is raised. Based on this, we consider the political equilibrium regarding redistributive taxation constituted by majority voting. Thereby, we show that the median voter prefers higher (lower) taxation if she evades more (less) taxes than would be fair since raising the tax rate would reduce (enlarge) the deviation from fairness and hence affects the moral cost.

The remainder of this paper proceeds as follows. Section 2 presents and defines the underlying tax evasion fairness concept. The individual decision on tax evasion and work effort for a given tax policy is considered in section 3. Based on this, we analyze how changes in the fairness preference as well as in the tax policy affects the individual decisions to work and to declare income to the tax authorities. In section 4, we turn to the political equilibrium concerning redistributive taxation which is constituted by means of majority voting. The analysis is then concluded in section 5.
2 The model

We consider an economy inhabited by \( n \) individuals. Each individual \( i \) receives a wage rate \( w_i \) which reflects talent or productivity and decides on her work effort \( \ell_i \). The individual pre-tax income is then given by \( x_i = w_i \ell_i \). Further, let us denote \( \bar{w} = 1/n \sum_i w_i \) and \( \bar{\ell} = 1/n \sum_i \ell_i \) as the average wage rate and work effort, respectively. Bear in mind, that each individual is presumed to neglect the effect of changes in her own wage rate or work effort on the average values. This seems plausible for a large number \( n \) of individuals. The government levies a linear income tax rate \( \tau \in [0, 1] \) which serves to finance a lumpsum transfer \( B = \tau \bar{w} \bar{\ell} \). This indirectly progressive taxation scheme leads to an individual net-tax liability \( T_i = \tau w_i \ell_i - B \). Each individual takes the public benefit as exogenously given. Hence, individuals are presumed to not anticipate the balanced governmental budget (fiscal illusion) which seems to be reasonable for a large population. Further note that income redistribution is presumed to be the only purpose of the tax system. Hence, we abstract from the possibility that governments provide public goods.

**Tax evasion and post-tax income.** Each individual chooses to hide the amount \( u_i \in [0, x_i] \) of income from the tax authorities. This leads to the amount \( e = \tau u_i \) of evaded taxes. The probability of being detected is given by \( p(u_i) \). By following Yitzhaki (1987) we assume the detection probability as being increasing (and convex) in unreported income, i.e. \( p' > 0 \ (p'' > 0) \).

If tax evasion is uncovered the individual has to pay a penalty tax \( t(u_i) > e \) which also increases (at an increasing rate) with the unreported income: \( t' > 0 \ (t'' > 0) \). Thus, the expected penalty is given by \( \phi(u_i) = p(u_i)t(u_i) \) with \( \phi' > 0 \ (\phi'' > 0) \). Taking this into account

\[
y(w_i, \ell_i, u_i, \tau_i) = (1 - \tau)w_i\ell_i + B + \tau u_i - \phi(u_i)
\]

represents the expected net income each individual \( i \) receives. In the following this actual net income has to be distinguished from the income individuals perceive as fair.

**Fair income and justifiable tax evasion.** According to Barth et al. (2013) we define \( x^*_i = \ell_i \bar{w} \) as the fair income perceived by each individual. Hence, individuals consider the fair income as being proportional to the work effort while it would be fair if any individual receive the average wage rate. In other words, differences in the wage rate would result in unfair income inequality. This is in line with the abovementioned liberal egalitarian theories of distributive justice whereupon individuals consider income inequality resulting from differences in impressionable factors, i.e. work effort, as fair while inequalities caused by suggestable factors, i.e. wage rate or talent, as unfair. Thus, individuals disclaim tax-induced reductions in income inequality resulting from differences in work effort. As governments can only tax the entire income and are therefore not able to differ between the sources of inequality individuals could consider tax evasion as fair or unfair, respectively. Based on this we obtain the fair tax liability

\[
T^* = x_i - x^*_i = \ell_i (w_i - \bar{w}),
\]
which would lead to the fair income. For low-skilled workers ($w_i < \bar{w}$) the fair tax payment is negative while it turns out to be positive for high-skilled workers ($w_i > \bar{w}$). This reflects the individual desire for income redistribution in order to reduce wage rate or productivity differences.\textsuperscript{2} The fair amount of tax evasion is then given by
\[ e^*_i = T_i - T^*_i = \ell_i [\bar{w} - (1 - \tau)w_i] - B \] (3)

which could be positive (for $T_i > T^*_i$) or negative (for $T_i < T^*_i$). In other words, individuals with a low productivity perceive tax evasion as fair while high-productivity individuals do not. We can directly see that the fair tax evasion is strictly decreasing with the wage rate ($\partial e^*_i / \partial w_i < 0$) while the effect of work effort depends on the wage differential ($\partial e^*_i / \partial \ell_i = \bar{w} - (1 - \tau)w_i \geq 0$).

If the net wage is unfairly low ($(1 - \tau)w_i < \bar{w}$) the effect of a higher work effort on actual net income would be larger than on the fair income. Consequently, the fair tax payment grows faster than the actual tax liability which enhances the fair amount of tax evasion. The opposite holds for an unfairly high wage rate ($(1 - \tau)w_i > \bar{w}$).

Eventually, we define $d_i = e_i - e^*_i$ as the difference between actual and fair tax evasion. Partially differentiating $d_i$ with respect to unreported income $u_i$, work effort $\ell_i$ and the tax rate $\tau$ yields:
\[
\begin{align*}
\frac{\partial d_i}{\partial u_i} &= \tau \geq 0, \\
\frac{\partial d_i}{\partial \ell_i} &= -\frac{\partial e^*}{\partial \ell_i} = -[\bar{w} - (1 - \tau)w_i], \\
\frac{\partial d_i}{\partial \tau} &= u_i - w_i \ell_i \leq 0.
\end{align*}
\] (4-6)

Obviously, the difference $d_i$ between actual and fair tax evasion increases with unreported income $u_i$ for positive tax rates $\tau > 0$ and decreases with the tax rate $\tau$ if not the entire pre-tax income is hided from the tax authorities, i.e. $u_i \in [0, w_i \ell_i)$. However, analogously to the fair tax evasion the effect of individual work effort on $d_i$ depends on the difference between the net wage rate and the fair wage. In case of unfairly high net wages ($(1 - \tau)w_i > \bar{w}$) the difference $d_i$ is increasing with individual work effort while for an unfairly low net wage ($(1 - \tau)w_i < \bar{w}$) raising work effort reduces $d_i$. It is worth mentioning that we obtain the inverse relation of $\frac{\partial e^*}{\partial \ell_i}$ since changing work effort affects the fair tax evasion while the actual tax evasion remains unchanged.

**Individual preferences.** In the following, we assume individuals to behave risk neutral concerning changes in expected post-tax income.\textsuperscript{3} This allows us to separately focus on the effects of tax fairness on tax evasion and work effort behavior. Individual preferences are represented by the additively-separable utility function
\[ V_i = V(u_i, \ell_i) = y(w_i, \ell_i, u_i) - \alpha \omega(\ell_i) - \beta \psi(d_i), \] (7)

\textsuperscript{2}For $w_i = \bar{w}$ the fair tax amount would be zero and hence the individual prefers zero taxation.\textsuperscript{3}In this case, individuals maximize expected income rather than expected utility.
where \( y > 0 \) denotes the expected income and \( \omega > 0 \) captures the disutility from labor which is increasing (and convex) in work effort, i.e. \( \omega' > 0 \ (\omega'' > 0) \). The parameter \( \alpha \) represents work morale or laziness. The third term of (7) with \( \psi(d_i) > 0 \) reflects the moral cost of deviating from fair tax evasion. If the actual tax evasion exceeds (deededs) the fair level, the psychic cost \( \psi \) is assumed to increase (decrease) with the difference \( d_i \): \( \psi' < 0 \) for \( d_i < 0 \) and \( \psi' > 0 \) for \( d_i \geq 0 \). Furthermore the moral cost function is convex in its argument, i.e. \( \psi'' > 0 \). The parameter \( \beta \) represents the strength of the tax fairness considerations.

### 3 Individual choice of tax evasion and work effort

We turn to the individual decision on unreported income \( u_i \) and labor supply \( \ell_i \). Taken the tax policy as exogenously given each individual \( i = 1, ..., n \) maximizes utility (7). Differentiating with respect to \( u_i \) and \( \ell_i \) yields the first-order conditions:

\[
\frac{\partial V_i}{\partial u_i} = \tau - \omega'(u_i) - \beta \psi'(d_i) = 0, \quad (8)
\]

\[
\frac{\partial V_i}{\partial \ell_i} = (1 - \tau)w_i - \alpha \omega'(\ell_i) - \beta \psi'(d_i) \frac{\partial d_i}{\partial \ell_i} = 0. \quad (9)
\]

**Unreported income.** According to (8) increasing the individual unreported income exhibits three marginal effects on utility. (A) First, ceteris paribus increasing \( u_i \) would enhance expected income by \( \tau \) which captures the additional tax payment avoided by the individual \( i \). (B) In contrast, the expected penalty raises by \( \phi' \) when more income remains unreported as the detection probability as well as the penalty tax in case of being caught increases with \( u_i \). While effect (A) constitutes a marginal benefit of tax evasion effect (B) represents the marginal cost. (C) The third term of (8) captures how an increase in tax evasion affects the moral cost from evading more or less taxes as would be fair. The sign of effect (C) depends on the sign of \( \psi' \). For individuals evading less than it would be fair \( d_i < 0 \) raising \( d_i \) reduces the moral cost \( \psi' < 0 \). As increasing \( u_i \) enhances the difference \( d_i \) by \( \tau \) this would be utility enhancing. If, however, an individual hides more than the fair income \( d_i > 0 \) from tax authorities a further increase in \( d_i \) would result in a larger moral cost \( \psi' > 0 \). Again, increasing the unreported income \( u_i \) leads to a higher difference which then raises the moral cost and hence reduces utility. Thus, effect (C) turns out to be a marginal benefit \( d_i < 0 \) or cost \( d_i > 0 \) depending on whether the individual evades more or less taxes than it is perceived as fair. According to (8) the marginal cost and benefits has to be equal in the optimum.

**Work effort.** The first-order condition (9) describes the marginal effects of higher individual work effort on utility. (a) First, a ceteris paribus raise in work effort enhances the individual net income by \( (1 - \tau)w_i \) which increases utility. (b) On the contrary, as captured by \( \alpha \omega' \) higher work effort results in a higher disutility from labor. The marginal benefit of work effort is

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4 For \( d_i = 0 \) we get \( \psi' = 0 \). In this case, the individual evades the fair amount of taxes and therefore the moral cost effect vanishes.
represented by effect (a) while effect (b) constitutes the marginal cost. (c) Analogously to the decision on unreported income, increasing the work effort ceteris paribus affects the moral cost through a change in the difference $d_i$. Whether the moral cost effect of work effort (c) is a marginal benefit or marginal cost depends on the deviation of actual tax evasion from the fair level and as well as on the difference between the net wage and the fair wage. Thereby, we have to consider four cases.

(i) Suppose the individual evades less than the fair level ($d_i < 0 \Rightarrow \psi' < 0$) and receives an unfairly low net wage ($(1 - \tau)w_i < \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell_i} < 0$). Then, increasing work effort would magnify the deviation from fair tax evasion and hence constitutes a marginal cost.

(ii) If, however, the individual receives an unfairly high net wage ($(1 - \tau)w_i > \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell_i} > 0$) while evading less taxes than would be fair ($d_i < 0 \Rightarrow \psi' < 0$) raising work effort would reduce the deviation from the fair tax evasion. Hence, a raise in $\ell_i$ goes along with a marginal benefit.

(iii) Now let us turn to the case where the individual evades more than the fair level ($d_i > 0 \Rightarrow \psi' > 0$). In case of receiving an unfairly low net wage ($(1 - \tau)w_i < \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell_i} < 0$) the individual could reduce the deviation from fairness or the moral cost by working more. Thus, raising $\ell_i$ is associated with a marginal benefit.

(iv) Finally, we consider the case where the individual evades more than the fair level ($d_i > 0 \Rightarrow \psi' > 0$) while receiving an unfairly high net wage ($(1 - \tau)w_i > \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell_i} > 0$). Then, raising work effort goes along with a higher deviation from fairness and therefore enlarges the moral cost. Subsequently, increasing $\ell_i$ gives rise to a marginal cost.

In a nutshell, each individual is able to reduce the psychic or moral cost of deviating from fair tax evasion by changing individual work effort or shifting unreported income. While varying individual work effort affects the fair level of tax evasion changes in unreported income influences the actual individual tax evasion.

3.1 Changes in the fairness preference

This paragraph explores how changes in the individual desire for fair tax evasion affects the choice of unreported income and work effort. Therefore, we implicitly differentiate the first-order conditions (8) and (9) with respect to the fairness preference parameter $\beta$ in order to get

$$\frac{\partial u_i}{\partial \beta} = -\frac{1}{V_{uu}} \left[ -\tau \psi' - \beta \tau \psi' \frac{\partial d_i}{\partial \ell_i} \frac{\partial \ell_i}{\partial \beta} \right],$$

(10)

$$\frac{\partial \ell_i}{\partial \beta} = -\frac{1}{V_{\ell \ell}} \left[ -\psi' \frac{\partial d_i}{\partial \ell_i} - \beta \psi' \frac{\partial d_i}{\partial \ell_i} \frac{\partial u_i}{\partial \beta} \right],$$

(11)
with $V_{uu} < 0$ and $V_{\ell\ell} < 0$ as utility is concave in the unreported income $u_i$ and the individual work effort $\ell_i$. Thus, it follows $-\frac{1}{V_{uu}} > 0$ and $-\frac{1}{V_{\ell\ell}} > 0$.

**Unreported income and tax evasion fairness.** An increase in the fairness preference exhibits the following marginal utility effects. According to the first term in (10) increasing $\beta$ affects the moral cost effect of unreported income $u_i$ directly ($\frac{\partial}{\partial \beta} \frac{\partial V}{\partial u}$). Beyond, a raise in $\beta$ affects the moral cost effect of unreported income indirectly via a change in work effort ($\frac{\partial}{\partial \beta} \frac{\partial V}{\partial \ell}$), cf. the second term of (10). Whether an increase in $\ell_i$ would be positive or negative depends on how work effort changes the difference $d_i$ between actual and fair tax evasion. As shown before, for individuals with an unfairly high net wage rate $([1 - \tau] w_i > \bar{w})$ increasing work effort enlarges the difference ($\frac{\partial d_i}{\partial \ell_i} > 0$). Consequently, a higher individual work effort reduces the marginal benefit (when $d_i < 0$) or enlarges the marginal cost (for $d_i > 0$) of declaring less income such that hiding income gets less desirable at the margin ($-\frac{\partial}{\partial \ell} \frac{\partial V}{\partial u} < 0$). However, if the individual net wage is unfairly low $([1 - \tau] w_i < \bar{w})$ a higher work effort goes along with a smaller deviation from the fair level of tax evasion ($\frac{\partial d_i}{\partial \ell_i} < 0$). At the margin, this renders declaring less income to the tax authorities more desirable ($-\frac{\partial}{\partial \ell} \frac{\partial V}{\partial u} > 0$).

**Work effort and tax evasion fairness.** With respect to the individual work effort we can also observe a direct as well as an indirect effect of a higher preference strength $\beta$. First, raising $\beta$ directly changes the moral cost effect of work effort ($\frac{\partial}{\partial \beta} \frac{\partial V}{\partial \ell}$). If the moral cost effect is positive ($\psi' \frac{\partial d_i}{\partial \ell_i} > 0$) increasing work effort becomes more desirable at the margin. Vice versa, raising work effort gets less desirable if the moral cost effect is negative ($\psi' \frac{\partial d_i}{\partial \ell_i} < 0$). Finally, the indirect effect of the preference strength $\beta$ on the marginal cost and benefit of work effort is represented by the second term in brackets of (11). This effect is similar to the second term of (10) except that it relates to a change in unreported income ($\frac{\partial}{\partial \beta} \frac{\partial V}{\partial \ell} = \frac{\partial}{\partial \beta} \frac{\partial V}{\partial u} \frac{\partial d_i}{\partial \ell_i}$). Increasing unreported income would ceteris paribus increase $d_i$ by $\tau$. Further increasing work effort when $\frac{\partial d_i}{\partial \ell_i} > 0$ would then enlarge the marginal cost (if $d_i > 0$) or reduce the marginal benefit (for $d_i < 0$) such that working more gets less desirable at the margin ($-\frac{\partial}{\partial \ell} \frac{\partial V}{\partial \ell} < 0$). Contrary, if a higher work effort goes along with a smaller difference, i.e. $\frac{\partial d_i}{\partial \ell_i} < 0$, an increase in $\ell_i$ gets more desirable at the margin ($-\frac{\partial}{\partial \ell} \frac{\partial V}{\partial \ell} > 0$) since the marginal cost shrinks (for $d_i < 0$) or the marginal benefit rises (if $d_i > 0$).

Solving the equation system (10) and (11) leads to the following comparative static results:  \[ \text{(12)} \]

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$\frac{\partial u_i}{\partial \beta}$</td>
<td>$\text{sign} \left[-\psi'\right]$</td>
</tr>
<tr>
<td>$\frac{\partial \ell_i}{\partial \beta}$</td>
<td>$\text{sign} \left[-\psi' \frac{\partial d_i}{\partial \ell_i}\right]$</td>
</tr>
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We can directly see that the indirect cross effects of tax evasion and work effort play no role since both effects cancel each other out. According to (12) the sign of $\frac{\partial u_i}{\partial \beta}$ depends on the sign of $\psi'$. If the individual evades less taxes than it would be fair ($d_i < 0 \Rightarrow \psi' < 0$) a raise in unreported income would reduce the moral cost as the deviation from the fair level gets smaller. Hence raising $u_i$ constitutes a marginal benefit. Introducing or strengthening the preference would

\[5\text{A detailed derivation is delegated to the appendix.}\]
enlarge this marginal benefit such that unreported income $u_i$ increases with $\beta$ ($\frac{\partial u_i}{\partial \beta} > 0$). If, however, the individual evades more than the fair level ($d_i > 0 \Rightarrow \psi' > 0$) a further raise in the deviation from fairness would decrease utility. By reducing unreported income the individual is able to decrease the deviation from the fair tax evasion which enhances utility. Increasing $\beta$ would enlarge the positive effect of a reduction in unreported income. Thus, $u_i$ is decreasing in the preference strength ($\frac{\partial u_i}{\partial \beta} < 0$).

**Proposition 1** Introducing or strengthening the fairness preference induces individuals to hide more income from tax authorities if their actual tax evasion is unfairly high. If, however, individuals evade less taxes than it would be fair, strengthening the desire for tax evasion fairness leads to a higher unreported income.

Further, the sign of $\frac{\partial \ell^*}{\partial \beta}$ is determined by the sign of $\psi' \frac{\partial d_i}{\partial \ell^*}$, cf. expression (13). At this, we can distinguish between four cases depending on whether the moral cost effect of work effort turns out to be positive or negative. As shown before, the moral cost effect of work effort is negative if

(i) the actual tax evasion is smaller than it would be fair ($d_i < 0 \Rightarrow \psi' < 0$) and the individual net wage is unfairly low ($(1 - \tau)w_i < \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell^*} < 0$), or

(ii) the actual tax evasion is larger than it would be fair ($d_i > 0 \Rightarrow \psi' > 0$) and the individual net wage is unfairly high ($(1 - \tau)w_i > \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell^*} > 0$).

Increasing the preference strength would then enlarge this marginal cost of working. Thus, individual work effort is decreasing with the preference parameter ($\frac{\partial \ell^*}{\partial \beta} < 0$). However, the moral cost effect constitutes a marginal benefit if

(iii) the actual tax evasion is smaller than it would be fair ($d_i < 0 \Rightarrow \psi' < 0$) and the individual net wage is unfairly high ($(1 - \tau)w_i > \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell^*} > 0$), or

(iv) the actual tax evasion is larger than it would be fair ($d_i > 0 \Rightarrow \psi' > 0$) and the individual net wage is unfairly low ($(1 - \tau)w_i < \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell^*} < 0$).

Then, raising the preference strength enhances the marginal benefit of supplying labor. Subsequently, the individual responds with a higher work effort if the preference parameter increases ($\frac{\partial \ell^*}{\partial \beta} > 0$). These results are then summarized in the following proposition.

**Proposition 2** Individual work effort is increasing in the fairness preference if an individual with an unfairly high (low) net wage evades less (more) taxes than it would be fair. If, however, individuals with an unfairly high (low) net wage evades more (less) taxes than the fair level, individuals respond with lower work effort when the fairness preference is introduced or strengthened.

The intuition behind these results is based on the moral cost effect of work effort as described in the first-order condition (9). After having determined the effect of the fairness preference, we turn to changes in taxation in the next section.
3.2 Tax policy changes

Now, we strive to focus on tax-induced changes in unreported income $u_i$ and work effort $\ell_i$ chosen by each individual. Implicit differentiation of the first-order conditions (8) and (9) with respect to the choice variables and the tax rate $\tau$ leads to

$$\frac{\partial u_i}{\partial \tau} = -\frac{1}{V_{uu}} \left[ 1 - \beta \psi' \frac{\partial d_i}{\partial \tau} - \beta \tau \psi'' \frac{\partial d_i}{\partial \tau} - \beta \psi' \frac{\partial \ell_i}{\partial \tau} \right],$$

(14)

$$\frac{\partial \ell_i}{\partial \tau} = -\frac{1}{V_{\ell\ell}} \left[ w_i + w_i \beta \psi' \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \tau} - \beta \psi' \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \tau} \right],$$

(15)

with $V_{uu} < 0$ and $V_{\ell\ell} < 0$ since individual utility is concave in its arguments. Based on this, the partial effects of taxation on the marginal cost and benefits of hiding income and working are described.

**Unreported income and taxation.** According to (14) a tax increase affects the marginal benefit and cost of hiding income directly ($\frac{\partial}{\partial \tau} \frac{\partial V}{\partial u}$). First, the avoided tax amount increases with the tax rate by 1. Further, increasing taxation changes the fairness effect by $-\beta \psi'$ and depends on the sign of $\psi'$. For $\psi' > 0$ the fairness effect is negative and therefore a marginal cost. If, however, $\psi' < 0$ the opposite holds and increasing unreported income proves to be a marginal benefit. As increasing the tax rate ceteris paribus reinforces the positive or negative fairness effect the marginal benefit or cost of hiding income rise. Third, a higher tax rate goes along with a smaller difference $d_i$ between actual and fair evasion ($\frac{\partial d_i}{\partial \tau} < 0$). As the moral cost $\psi$ is concave in the deviation from fairness ($\psi'' > 0$) this effect is positive. Ceteris paribus the individual willingness to evade taxes increases as hiding more income would boost the actual tax evasion.

The indirect effect of the tax rate on the moral cost effect of unreported income is captured by the fourth term of (14). This effect is based on how changes in work effort would affect the marginal benefit and cost of hiding income which is equivalent to $\frac{\partial}{\partial \tau} \frac{\partial V}{\partial u}$ from expression (10). To keep the exposition brief, we refer to the interpretation of the indirect effect in expression (10). In contrast, the individual work effort response is now induced by changes in redistributive taxation ($\frac{\partial \ell}{\partial \tau}$). Raising work effort would render hiding income more desirable for individuals receiving an unfairly low net wage ($((1 - \tau)w_i < \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell_i} < 0 \Rightarrow -\frac{\partial}{\partial \tau} \frac{\partial V}{\partial u_i} > 0$). The opposite holds for an unfairly high net wage rate ($((1 - \tau)w_i > \bar{w} \Rightarrow \frac{\partial d_i}{\partial \ell_i} > 0 \Rightarrow -\frac{\partial}{\partial \tau} \frac{\partial V}{\partial u_i} < 0$)

**Work effort and taxation.** Turning to expression (15) the effects of a change in taxation on the marginal benefit and cost of work effort can be identified. Increasing taxation again results in a direct effect as well as an indirect effect via a tax-induced change in unreported income. At first, a higher tax rate would directly reduce the income effect and therefore the marginal benefit of work effort by $-w_i$. As a higher share of income is claimed by the government enlarging work effort gets less desirable. The second term captures the effect of taxation on the influence of work effort on fairness deviation ($\frac{\partial d_i}{\partial \ell, \tau}$). It can be shown that increasing the tax
rate reduces $\frac{\partial d_i}{\partial \tau}$ by $-w_i$. Therefore, the growth (decline) of $d_i$ via a work effort increase gets weaker (stronger) with the tax rate for an unfairly high net wage (unfairly low wage). This enhances the marginal benefit if an individual evades more taxes than would be fair ($d_i > 0$ as a smaller difference is preferred ($\psi' > 0$). In case of an unfairly low level of tax evasion ($d_i < 0$) the opposite holds. Since the individual prefers a larger difference ($\psi' < 0$) the tempering effect of the tax rate on $\frac{\partial d_i}{\partial \tau}$ reduces the marginal benefit of work effort. Beyond, the tax rate cuts the difference between actual and fair tax evasion ($\frac{\partial d_i}{\partial \tau} < 0$) as captured by the third term in the brackets of expression (15). This effect is negative (positive) when the difference $d_i$ is decreasing (increasing) with the work effort.

Again, increasing taxation changes the moral cost effect of work effort via a change in unreported income, cf. the fourth term in (15). This effect is equivalent to $\frac{\partial}{\partial u} \frac{\partial V}{\partial \ell_i}$ as explained above. Thus we refer to the interpretation of the indirect effect in (10). However, the change in unreported income would now be induced by a variation of the tax rate. A raise in unreported income would render working more desirable for individuals with an unfairly low net wage $((1 - \tau)w_i < \tilde{w} \Rightarrow \frac{\partial d_i}{\partial \tau} < 0 \Rightarrow -\frac{\partial}{\partial u} \frac{\partial V}{\partial \ell_i} > 0)$. If, however, an individual receives an unfairly high net wage rate, the opposite holds $((1 - \tau)w_i > \bar{w} \Rightarrow \frac{\partial d_i}{\partial \tau} > 0 \Rightarrow -\frac{\partial}{\partial u} \frac{\partial V}{\partial \ell_i} < 0)$.

In order to determine the comparative static effects the equation system (14) and (15) has to be solved. We can show that unreported income is increasing with the tax rate while individual work effort shrinks with taxation, i.e.

$$\frac{\partial u^*_i}{\partial \tau} > 0,$$

$$\frac{\partial \ell^*_i}{\partial \tau} < 0,$$

if either

(i) the actual wage is unfairly low ($w_i < \tilde{w}$) and the disutility effect from labor is sufficiently as well as the change in the detection probability is sufficiently small. From $w_i < \tilde{w}$ follows an unfairly low net wage $((1 - \tau)w_i < \tilde{w})$. In this case, the reduction in labor supply would reduce $d_i$ ($\frac{\partial d_i}{\partial \tau} < 0$) which renders hiding income less desirable ($-\frac{\partial}{\partial u} \frac{\partial V}{\partial \ell_i} > 0$), cf. (14). Thus, the indirect effect is negative. In contrast, it can be shown that the direct effect of taxation is unambiguously positive. For a sufficiently small labor disutility effect the positive direct effect dominates the negative indirect effect such that unreported income $u^*_i$ increases in the tax rate $\tau$. Turning to the case of work effort, we can show that for an unfairly low net wage the indirect effect is negative $((1 - \tau)w_i < \tilde{w} \Rightarrow \frac{\partial d_i}{\partial \tau} < 0)$ since the increase in unreported income renders working more desirable at the margin ($-\frac{\partial}{\partial u} \frac{\partial V}{\partial \ell_i} > 0$), cf. expression (15). By contrast, the direct effect of taxation is negative. For a sufficiently small change in the detection probability the negative direct effect outweighs the positive indirect effect such that individual work effort $\ell^*_i$ decreases in taxation $\tau$.

6We have shown in the appendix that $\alpha \omega' < \tilde{w}$ and $\phi' < \phi' \frac{\partial d_i}{\partial \tau}(1 + \tau w_i)$ must hold.
(ii) or the actual wage is unfairly high \((w_i > \bar{w})\) and the disutility effect from labor as well as the change in the detection probability due to a higher \(u_i\) is sufficiently large.\(^7\) In this case, the individual net wage could be unfairly high or low and hence the indirect effects regarding \(u_i\) and \(\ell_i\) are positive or negative. Nevertheless, it can be shown that for a sufficiently large \(\phi'\) the direct effect dominates the indirect effects if both differs in sign.

These results are summarized in the following proposition.

**Proposition 3** Suppose an unfairly low wage rate and the disutility effect from labor as well as the change in detection probability to be sufficiently small. Then, unreported income increases with the tax rate while individuals reduce the work effort. This result also holds for an unfairly high wage if the disutility effect from labor as well as the change in detection probability are presumed to be sufficiently large.

Beyond, we can show that if the changes in the moral cost function are supposed to be zero \((\psi'' = 0)\) unreported income is unambiguously raising in taxation while work effort is decreasing with the tax rate. In this case, the indirect, cross effects in (14) and (15) vanishes such that only the positive direct effect regarding \(u_i\) and the negative direct effect with respect to \(\ell_i\) remain.

Thus, changes in the tax policy alters the marginal benefits and costs of using tax evasion and work effort to alleviate the moral cost which is induced by the deviation from the fair level of tax evasion. After determining the conditions under which unreported income and work effort increase or decrease with taxation we turn to the majority voting on redistributive taxation.

## 4 Majority voting on redistributive taxation

This section considers the political equilibrium regarding the redistributive tax policy constituted by means of majority voting. Therefore the median voter plays a decisive role for the majority tax rate (see e.g. Roberts, 1977; Meltzer and Richard, 1981).

It deserves to be mentioned that we presume the moral cost function to be linear in the difference \(d_i\) between actual and fair tax evasion, i.e. \(\psi'' = 0\). A linear moral cost functions induce the indirect cross effects between reporting income and working to be zero. Thus, the direct effects remain and we obtain \(\frac{\partial u_i^*}{\partial \tau} > 0\) and \(\frac{\partial \ell_i^*}{\partial \tau} < 0\) irrespective of the functional form of \(\phi\). According to this, unreported income is unambiguously increasing with taxation while individual work effort decreases with the tax rate.

Beyond, we assume the wage distribution to be right skewed such that the majority of individuals exhibit a productivity below the average. This seems to reflect empirical observations adequately. At this, the wage of the median voter is unfairly low, i.e. \(w_m < \bar{w}\). For positive tax rates \(\tau > 0\) the net wage is unfairly low, too. Hence, \((1 - \tau)w_m < \bar{w}\) holds.

\(^7\)From the appendix follows that \(\alpha \omega' > \bar{w}\) and \(\phi' > \phi'' \frac{\partial d_i}{\partial \tau}(1 + \tau w_i)\) must hold.
The median utility under risk neutrality is given by:

\[ V^m = (1 - \tau)w_m\ell_m + B + \tau u_m - \phi(u_m) - \alpha\omega(\ell_m) - \beta\psi(d_m), \] (18)

where \( m \) denotes the median voter. Beyond that, the government has to take the fiscal budget constraint into account, i.e.

\[ B = \tau \frac{1}{n} \sum_i w_i\ell_i. \] (19)

According to (19) the average tax yield corresponds to the lumpsum transfer in order to balance the fiscal budget. Further, the government anticipates the optimal individual decisions regarding unreported income and work effort. Therefore, the indirect utility

\[ V^* := (1 - \tau)w_m\ell_m^* + \tau \frac{1}{n} \sum_i w_i\ell_i^* + \tau u_m^* - \phi(u_m^*) - \alpha\omega(\ell_m^*) - \beta\psi(d_m^*) \] (20)

is maximized with respect to the tax rate \( \tau \). This leads to the following first-order condition for the tax rate:\(^8\)

\[ -w_m\ell_m^* + \bar{w}\ell + \tau \bar{w} - \alpha\omega(\ell_m) - \beta\psi(d_m) - \beta\psi'(d_m) = 0. \] (21)

Increasing taxation exhibits five marginal effects on median voter utility. To begin with, the term (I) represents an income effect of redistributive taxation. Increasing the tax rate induces post-tax income to shrink. Hence, effect (I) is negative and constitutes a marginal cost. Second, the effect (II) captures a positive income effect. Raising the tax rate ceteris paribus allows to finance a higher lumpsum transfer. This leads to a higher post-tax income which is utility enhancing. Therefore, term (II) is a marginal benefit of taxation. The public transfer is also affected by a change in work effort, cf. term (III). A higher (lower) work effort ceteris paribus increases (reduces) the tax revenue which enhances (cuts) the post-tax income through a higher (lower) transfer. Beyond, enlarging the tax rate ceteris paribus goes along with a larger amount of evaded taxes as captured by effect (IV). This it utility enhancing since post-tax income rises. Hence, effect (IV) constitutes a marginal benefit of redistributive taxation. Finally, a higher tax rate induces the deviation from the fair level of tax evasion to shrink (\( \frac{\partial d_i}{\partial \tau} < 0 \)). This is represented by the moral cost effect of taxation, cf. term (V). The sign of (V) depends on the actual deviation from fair tax evasion. If the median voter evades less taxes than it would be fair (\( d_i < 0 \Rightarrow \psi' < 0 \)) the tax-induced reduction in \( d_i \) enlarges the deviation from fairness and therefore the moral cost. Then, (V) is negative and represents a marginal cost of taxation. If, however, the median voter evades more than the fair amount of taxes (\( d_i > 0 \Rightarrow \psi' > 0 \)) raising the tax rate reduces the deviation from fairness as \( d_i \) decreases. This leads to a lower moral cost and hence gives rise to a marginal benefit.

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\(^8\)For a derivation of the first-order condition (21) as well as the second-order condition we refer to the appendix.
Fairness preference and the majority tax rate. Next, we strive to analyze the comparative static effect with respect to the preference strength $\beta$ and the tax rate $\tau^m$ resulting from majority voting. Then, we are able to derive the following comparative static result:

$$\text{sign} \frac{\partial \tau^*}{\partial \beta} = \text{sign} [d_m].$$

(22)

if the change in detection probability $\phi'$ induced by a higher unreported income is not too large.\(^9\) Whether the tax rate increases with the preference strength regarding fairness depends on whether the median voter evades more or less taxes than would be fair. This result is summarized in the following proposition.

**Proposition 4** Suppose the change in detection probability due to the larger unreported income to be not too large. Then, we can show that the tax rate preferred by the majority increases (decreases) with the fairness preference if the median voter evades more (less) than the fair level of taxes.

To provide a further intuition behind this result, we consider the effect of the tax rate on the deviation from fairness. Raising the tax rate would ceteris paribus lead to a smaller difference between actual and fair tax evasion. If the median voter evades less taxes than would be fair higher taxation would then enlarge the deviation from fairness and therefore the moral cost. In this case, the median voter prefers lower taxation. If, however, the median voter’s tax evasion exceeds the fair level a higher tax rate would lead to a smaller deviation from fairness. This reduces the moral cost and thus the median voter prefers higher taxation.

5 Conclusion

This paper sheds some light on how taxation interrelates with the individual decisions on work effort and the degree of tax evasion if individuals exhibit a desire for fairness concerning the sources of income inequality and subsequently regarding the level of tax evasion which is perceived as fair. Thereby we employ the fair tax evasion concept of Barth et al. (2013) which is in the vein of liberal egalitarian theories on distributive justice. According to this approach, work effort should be rewarded and is therefore expected to increase the fair income. By contrast, differences outside an individual’s reach, i.e. the productivity or the wage rate respectively, are regarded as unfair and should therefore not be taxed. A similar concept has also been applied by Alesina et al. (2002) to study redistributive tax policies from a median voter perspective.

We find that introducing or strengthening the fairness preference regarding tax evasion induces individuals evading less taxes than would be fair to reduce the income which is reported to the tax authorities. If, however, an individual evades an unfairly high amount of taxes, she responds with a lower unreported income when the fairness preference is introduced. Concerning individual work effort it can be shown that the behaviour depends on whether the individual

\(^9\)See the appendix for a detailed derivation of the comparative static effect. We can show that this result holds if $\phi' < \min\{\frac{\tau^m}{\bar{n}}, \frac{\tau^m}{n_0} \bar{\omega}'\}'$. 

13
is able to reduce the deviation from fair tax evasion and thus the moral cost by working more. This depends on how the individual net wage as well as the actual tax evasion differs from the fair levels. Individuals receiving an unfairly high (low) net wage while evading less (more) taxes than would be fair can reduce the moral cost by increasing labor supply. Hence, introducing or strengthening the desire for fairness leads to a higher work effort. The opposite holds for individuals with an unfairly high (low) net wage rate evading more (less) than the fair tax evasion. In this case, reducing labor supply is capable of decreasing the deviation from fairness and therefore the moral cost. In a nutshell, individuals are able to reduce the moral cost by varying unreported income and work effort.

Turning to changes in the tax policy, we obtain the following result. It can be shown that individuals declare less income to the tax authorities while working less if the tax rate is raised. We present conditions for which this result holds irrespective of the wage deviation from fairness. Increasing redistributive taxation affects the marginal benefit and cost of evading taxes and supplying labor. Our finding shows that the direct effects of taxation dominate, i.e. the higher amount of taxes which can ceteris paribus avoided as well as the reduction of work incentives.

Beyond, we consider the political equilibrium regarding redistributive taxation. The tax policy is constituted by means of majority voting. In this case, the median voter decides on the tax rate. Supposing a right-skewed wage distribution where the median voter receives an unfairly low net wage, we show how introducing fair tax evasion affects the preferred tax rate. If the median voter evades more taxes than would be fair the tax rate is increasing with the fairness preference since taxation reduces the deviation from fairness and therefore the moral cost. However, if the median voter evades less taxes than would be fair raising the tax rate enlarges deviation from fairness. This enhances the moral cost such that she prefers lower taxation.

Finally, there are some remarks which are left open for future research. First, we presumed individuals to behave risk neutral. This allows us to analyze the effect of fair tax evasion on individual behavior and the tax policy in separation from responses based on income risk. Nevertheless, it would be interesting to consider how risk averse individuals behave when they exhibit fairness preferences regarding tax evasion. Beyond, we omit to analyze the efficiency properties of the political equilibrium. Fair tax evasion would affect socially optimal taxation and the deviation of the majority tax rate from the efficient allocation. Eventually, we introduced a linear taxation scheme. It could be questioned which effect fair tax evasion would have under a non-linear tax policy.
References


Appendix (only for the referees)

(Note: In case of publication the following detailed appendix would be condensed.)

For the following derivations we define the first-order conditions as:

\[ F^1 := \tau - \phi' - \tau \beta \psi' = 0, \quad (A1) \]
\[ F^2 := (1 - \tau) w_i - \alpha \omega' - \beta \psi' \cdot \frac{\partial d_i}{\partial \ell_i} = 0. \quad (A2) \]

Both conditions together determine the optimal individual amounts of unreported income and work effort.

Second-order condition with respect to \( u_i \) and \( \ell_i \)

The Hessian matrix is given by

\[ H := \begin{pmatrix} F^1_u & F^1_{\ell} \\ F^2_u & F^2_{\ell} \end{pmatrix}. \quad (A3) \]

with

\[ F^1_u = -\phi'' - \beta \tau^2 \psi'', \quad (A4) \]
\[ F^2_u = -\beta \tau \psi'' \frac{\partial d_i}{\partial \ell_i}, \quad (A5) \]
\[ F^1_{\ell} = -\beta \tau \psi'' \frac{\partial d_i}{\partial \ell_i}, \quad (A6) \]
\[ F^2_{\ell} = -\alpha \omega'' - \beta \psi'' \left( \frac{\partial d_i}{\partial \ell_i} \right)^2. \quad (A7) \]

We can directly see that due to \( \phi'' > 0 \) and \( \psi'' > 0 \) the first principal minor is negative, i.e. \( F^1_u = -\phi'' - \beta \tau^2 \psi'' < 0 \). Beyond, the determinant of the Hessian matrix is given by:

\[ |H| = F^1_u F^2_{\ell} - F^2_u F^1_{\ell} = (\phi'' - \beta \tau^2 \psi'') \cdot \left( -\alpha \omega'' - \beta \psi'' \left( \frac{\partial d_i}{\partial \ell_i} \right)^2 \right) - \left( -\beta \tau \psi'' \frac{\partial d_i}{\partial \ell_i} \right) \cdot \left( -\beta \tau \psi'' \frac{\partial d_i}{\partial \ell_i} \right). \quad (A8) \]

Expanding and rearranging (A8) then yields:

\[ |H| = \alpha \omega'' \left( \phi'' + \beta \tau^2 \psi'' \right) + \beta \phi'' \psi'' \left( \frac{\partial d_i}{\partial \ell_i} \right)^2 > 0, \quad (A9) \]

and therefore the Hessian matrix is negative definit. Thus, the second-order condition for a utility maximum is satisfied.
Comparative statics of $u_i$ and $\ell_i$ with respect to $\beta$

Totally differentiating (A1) and (A2) leads to the equation system

$$\mathbf{H} \cdot \begin{pmatrix} \frac{\partial u_i}{\partial \beta} \\ \frac{\partial \ell_i}{\partial \beta} \end{pmatrix} = \begin{pmatrix} -F_1^\beta \\ -F_2^\beta \end{pmatrix}$$

(A10)

displayed in matrix form for $d\alpha = 0$, $d\tau = 0$ and $dB = 0$ with

$$F_1^\beta = -\tau \psi',$$

(A11)

$$F_2^\beta = -\psi' \frac{\partial d_i}{\partial \ell_i},$$

(A12)

where $\mathbf{H}$ represents the Hessian matrix as introduced above. Applying Cramer’s Rule then solves the equation system (A25) to

$$\frac{\partial u_i}{\partial \beta} = \frac{-F_1^\beta F_2^\ell + F_2^\beta F_1^\ell}{|\mathbf{H}|},$$

(A13)

$$\frac{\partial \ell_i}{\partial \beta} = \frac{-F_1^\beta F_2^\ell + F_2^\beta F_1^\ell}{|\mathbf{H}|}.$$  

(A14)

(i) Unreported income. Since $|\mathbf{H}| > 0$ from (A13) follows

$$\text{sign} \frac{\partial u_i}{\partial \beta} = \text{sign} \left[ -F_1^\beta F_2^\ell + F_2^\beta F_1^\ell \right] = \text{sign} [A],$$

(A15)

where $A := -F_1^\beta F_2^\ell + F_2^\beta F_1^\ell$. Substituting (A6), (A7), (A11) and (A12) in $A$ gives:

$$A = -(-\tau \psi') \cdot \left(-\alpha \omega'' - \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 \right) + \left(-\psi' \frac{\partial d_i}{\partial \ell_i} \right) \cdot \left(-\tau \psi'' \frac{\partial d_i}{\partial \ell_i} \right).$$

(A16)

Expanding expression (A16) leads to:

$$A = -\alpha \omega'' \tau \psi' - \tau \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 + \tau \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2.$$

(A17)

As $-\tau \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 + \tau \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 = 0$ we obtain:

$$A = -\alpha \omega'' \tau \psi'.$$

(A18)

From (A18) we can derive:

$$\text{sign} \frac{\partial u_i}{\partial \beta} = \text{sign} \left[-\psi' \right].$$

(A19)

since $\alpha \omega'' \tau > 0$.
(ii) Work effort. Again, as $|\mathbf{H}| > 0$ we obtain

$$\text{sign} \left( \frac{\partial \ell_i}{\partial \beta} \right) = \text{sign} \left[ -F^1_u F^2_\beta + F^2_u F^1_\beta \right] = \text{sign} [B],$$  \hspace{1cm} (A20)

from (A14) with $B := -F^1_u F^2_\beta + F^2_u F^1_\beta$. Inserting (A4), (A5), (A11) and (A12) in $B$ we can write:

$$B = -(-\phi'' - \beta \tau^2 \psi'') \cdot \left( -\psi' \frac{\partial d_i}{\partial \ell_i} \right) + \left( -\beta \tau \psi'' \frac{\partial d_i}{\partial \ell_i} \right) \cdot (-\tau \psi')$$  \hspace{1cm} (A21)

Multiplying (A21) out yields:

$$B = -\phi'' \psi' \frac{\partial d_i}{\partial \ell_i} - \tau^2 \beta \psi \psi'' \frac{\partial d_i}{\partial \ell_i} + \tau^2 \beta \psi \psi'' \frac{\partial d_i}{\partial \ell_i}.$$  \hspace{1cm} (A22)

Since $-\tau^2 \beta \psi \psi'' \frac{\partial d_i}{\partial \ell_i} + \tau^2 \beta \psi \psi'' \frac{\partial d_i}{\partial \ell_i} = 0$ we get:

$$B = -\phi'' \psi' \frac{\partial d_i}{\partial \ell_i}.$$  \hspace{1cm} (A23)

Then, we can conclude

$$\text{sign} \left( \frac{\partial \ell_i}{\partial \beta} \right) = \text{sign} \left[ -\psi' \frac{\partial d_i}{\partial \ell_i} \right].$$  \hspace{1cm} (A24)

from (A23) as $\phi'' > 0$

**Comparative statics of $u_i$ and $\ell_i$ with respect to $\tau$**

Totally differentiating (A1) and (A2) leads to the equation system

$$\mathbf{H} \cdot \begin{pmatrix} \frac{\partial u_i}{\partial \tau} \\ \frac{\partial \ell_i}{\partial \tau} \end{pmatrix} = \begin{pmatrix} -F^1_\tau \\ -F^2_\tau \end{pmatrix}$$  \hspace{1cm} (A25)

displayed in matrix form for $d\alpha = 0$, $d\beta = 0$ and $dB = 0$ with

$$F^1_\tau = 1 - \beta \psi' - \tau \beta \psi'' \frac{\partial d_i}{\partial \tau},$$  \hspace{1cm} (A26)

$$F^2_\tau = -w_i - \beta \psi'' \frac{\partial d_i}{\partial \tau} + \beta \psi' w_i,$$  \hspace{1cm} (A27)

where $\mathbf{H}$ represents the Hessian matrix as defined above. Applying Cramer’s Rule then solves the equation system (A25) to

$$\frac{\partial u_i}{\partial \tau} = \frac{-F^1_u F^2_\tau + F^2_u F^1_\tau}{|\mathbf{H}|},$$  \hspace{1cm} (A28)

$$\frac{\partial \ell_i}{\partial \tau} = \frac{-F^1_u F^2_\ell + F^2_u F^1_\ell}{|\mathbf{H}|}. $$  \hspace{1cm} (A29)

(i) **Unreported income.** To start with, as $|\mathbf{H}| > 0$ from (A28) follows:

$$\text{sign} \left( \frac{\partial u_i}{\partial \tau} \right) = \text{sign} \left[ -F^1_u F^2_\tau + F^2_u F^1_\tau \right],$$  \hspace{1cm} (A30)
where we denote \( C := -F_\tau^1 F_{\ell}^2 + F_\tau^2 F_{\ell}^1 \). Plugging in (A6), (A7), (A26) and (A27) yields:

\[
C = - \left( 1 - \beta \psi' - \tau \beta \psi' \frac{\partial d_i}{\partial \tau} \right) \cdot \left( -\alpha'' - \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 \right) \\
+ \left( -w_i - \beta \psi'' \frac{\partial d_i}{\partial \tau} \right) \cdot \left( \beta \psi' w_i \right) + \left( -\beta \tau \psi'' \frac{\partial d_i}{\partial \ell_i} \right). \tag{A31}
\]

Expanding (A31) we can write:

\[
C = \alpha'' + \alpha'' \beta \psi' - \alpha'' \beta \psi' \frac{\partial d_i}{\partial \tau} \\
+ \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 - \beta^2 \psi' \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 + w_i \beta \tau \frac{\partial d_i}{\partial \ell_i} - \beta^2 w_i \tau \psi' \psi'' \frac{\partial d_i}{\partial \ell_i} \\
- \tau \beta^2 \psi'' \frac{\partial d_i}{\partial \tau} \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 + \tau \beta^2 \psi'' \frac{\partial d_i}{\partial \tau} \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2. \tag{A32}
\]

Taking into account that \(-\beta \tau^2 \psi'' \frac{\partial d_i}{\partial \tau} \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 + \tau \beta^2 \psi'' \frac{\partial d_i}{\partial \tau} \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 = 0\) and factoring out \(\alpha''\) as well as \(\beta \psi''\), (A32) changes to:

\[
C = \alpha'' \left( 1 - \beta \psi' - \beta \tau \psi' \frac{\partial d_i}{\partial \tau} \right) \\
+ \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 - \beta \psi' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 - w_i \beta \psi' \frac{\partial d_i}{\partial \ell_i} + w_i \tau \frac{\partial d_i}{\partial \ell_i}. \tag{A33}
\]

From (A2) follows \(\beta \psi' \frac{\partial d_i}{\partial \ell_i} = (1 - \tau) w_i - \alpha'\). Substituting this into (A33) leads to:

\[
C = \alpha'' \left( 1 - \beta \psi' - \beta \tau \psi' \frac{\partial d_i}{\partial \tau} \right) \\
+ \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 - [(1 - \tau) w_i - \alpha'] \frac{\partial d_i}{\partial \ell_i} - w_i \tau \left[ (1 - \tau) w_i - \alpha' + w_i \tau \frac{\partial d_i}{\partial \ell_i} \right]. \tag{A34}
\]

Rearranging the terms in the brackets of the second term of (A34), we can write:

\[
C = \alpha'' \left( 1 - \beta \psi' - \beta \tau \psi' \frac{\partial d_i}{\partial \tau} \right) \\
+ \beta \psi'' \left[ \frac{\partial d_i}{\partial \ell_i} \right]^2 - (1 - \tau) \frac{\partial d_i}{\partial \ell_i} + \alpha' \frac{\partial d_i}{\partial \ell_i} - \frac{w_i^2}{\tau} (1 - \tau) + w_i \tau \alpha' + w_i \tau \frac{\partial d_i}{\partial \ell_i}. \tag{A35}
\]

Using \(\frac{\partial d_i}{\partial \ell_i} = -\bar{w} + w_i - \tau w_i\) in (A35) gives:

\[
C = \alpha'' \left( 1 - \beta \psi' - \beta \tau \psi' \frac{\partial d_i}{\partial \tau} \right) + \beta \psi'' \left[ -\bar{w} + w_i - \tau w_i \right]^2 \\
- (1 - \tau) \frac{\partial d_i}{\partial \ell_i} - \bar{w} + w_i - \tau w_i + \alpha' \left[ -\bar{w} + w_i - \tau w_i \right] \\
- \frac{w_i^2}{\tau} + \frac{w_i^2}{\tau} + w_i \tau \alpha' + w_i \tau \left[ -\bar{w} + w_i - \tau w_i \right]. \tag{A36}
\]
Using the rule for trinomials \((a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\) and expanding the brackets in the second term leads to:

\[
C = \alpha'' \left(1 - \beta \psi' - \beta \tau \psi'' \frac{\partial d_i}{\partial \tau}\right) + \beta \psi'' \left(\bar{w}^2 + w_i^2 + \tau^2 w_i^2 - 2\bar{w}w_i\right) + 2\tau \bar{w}w_i - 2\tau w_i^2 + w_i \bar{w} - w_i^2 + \tau w_i^2 - \tau w_i \bar{w} + \tau w_i^2 - \tau^2 w_i^2 - \alpha' \bar{w}
+ \alpha' w_i - \alpha' \tau w_i - \bar{w}^2 \tau + w_i \tau \alpha' - \tau w_i \bar{w} + \tau w_i^2 - \tau^2 w_i^2. \tag{A37}
\]

Then, taking into account that \(\tau^2 w_i^2 - \tau^2 w_i^2 = 0, w_i \tau \alpha' - \alpha' \tau w_i = 0, w_i^2 - w_i^2 = 0, \tau w_i^2 - w_i^2 \tau = 0, w_i^2 \tau^2 - w_i^2 \tau = 0, 2\tau \bar{w}w_i - \tau w_i \bar{w} - \tau w_i \bar{w} = 0\) as well as \(\tau w_i^2 + \tau w_i^2 - 2\tau w_i^2 = 0\) and rearranging (A36) results in:

\[
C = \alpha'' \left(1 - \beta \psi' - \beta \tau \psi'' \frac{\partial d_i}{\partial \tau}\right) + \beta \psi'' \left(\bar{w}^2 - \bar{w}w_i + \alpha' [w_i - \bar{w}]\right). \tag{A38}
\]

From (A1) follows \(\beta \psi' = 1 - \frac{\varphi'}{\tau}\). Inserting this into (A38) and rearranging the second bracket term yields:

\[
C = \alpha'' \left(\frac{\varphi'}{\tau} - \beta \tau \psi'' \frac{\partial d_i}{\partial \tau}\right) + \beta \psi'' (\bar{w} (w_i - \bar{w}) + \alpha' [w_i - \bar{w}]). \tag{A39}
\]

Finally, (A39) can be rearranged to:

\[
C = \alpha'' \left(\frac{\varphi'}{\tau} - \beta \tau \psi'' \frac{\partial d_i}{\partial \tau}\right) + \beta \psi'' (w_i - \bar{w}) (\alpha' - \bar{w}). \tag{A40}
\]

We can see that the first term of (A40) (direct effect) is unambiguously positive due to \(\alpha'' > 0, \frac{\varphi'}{\tau} > 0\) and \(\beta \tau \psi'' \frac{\partial d_i}{\partial \tau} < 0\). In contrast, the second term of (A40) (indirect effect) is indeterminate in sign. It becomes positive if sign \((w_i - \bar{w}) = \text{sign} (\alpha' - \bar{w})\). Thus, for \(w_i > \bar{w}\) and \(\alpha' > \bar{w}\) or \(w_i < \bar{w}\) and \(\alpha' < \bar{w}\) the term gets positive such that \(C\) is unambiguously positive. In this case we get \(\frac{\partial w_i}{\partial \tau} > 0\). If, however, sign \((w_i - \bar{w}) = -\text{sign} (\alpha' - \bar{w})\) the second term is negative. This holds for \(w_i < \bar{w}\) and \(\alpha' > \bar{w}\) or \(w_i > \bar{w}\) and \(\alpha' < \bar{w}\). Then, \(C\) remains positive if \(\psi'' = 0\) and hence \(\frac{\partial w_i}{\partial \tau} > 0\) follows.

(ii) Work effort. Furthermore, from (A29) and \(|H| > 0\) we can derive

\[
\text{sign} \frac{\partial \ell_i}{\partial \tau} = \text{sign} \left[-F^1_u F^2_r + F^2_u F^1_r\right]. \tag{A41}
\]

with \(D := -F^1_u F^2_r + F^2_u F^1_r\). Again, by using (A4), (A5), (A26) and (A27) we obtain:

\[
D = - (\phi'' - \beta \tau \psi'') \cdot \left(-w_i - \beta \psi'' \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \ell_i} + \beta \psi' w_i\right) + \left(-\beta \tau \psi'' \frac{\partial d_i}{\partial \ell_i}\right) \cdot \left(1 - \beta \psi' - \tau \beta \psi'' \frac{\partial d_i}{\partial \tau}\right). \tag{A42}
\]
Multiplying out the brackets yields:

\[
D = -w_i \phi'' - \beta \psi'' \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \ell_i} + \phi'' \psi' w_i - w_i \tau \beta \psi'' - \beta^2 \tau^2 [\psi']^2 \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \ell_i} + \beta^2 \tau^2 \psi'' w_i - \tau \beta \psi'' \frac{\partial d_i}{\partial \ell_i} + \beta^2 \tau^2 [\psi']^2 \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \ell_i}. \tag{A43}
\]

Taking into account that \(\tau^2 \beta^2 [\psi']^2 \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \ell_i} = 0\) and factoring out \(\phi'' w_i\) as well as \(\beta \psi''\), (A43) is rewritten to:

\[
D = \phi'' w_i (\beta \psi' - 1) + \beta \psi'' \left( -\phi'' \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \ell_i} - w_i \tau^2 + \beta \tau^2 \psi' w_i - \tau \frac{\partial d_i}{\partial \ell_i} + \beta \tau \psi' \frac{\partial d_i}{\partial \ell_i} \right). \tag{A44}
\]

Making use of \(\frac{\partial d_i}{\partial \ell_i} = -\bar{w} + w_i - \tau w_i\) gives:

\[
D = \phi'' w_i (\beta \psi' - 1) + \beta \psi'' \left( -\phi'' \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \ell_i} - w_i \tau^2 + \beta \tau^2 \psi' w_i - \tau \frac{\partial d_i}{\partial \ell_i} - \tau \bar{w} + w_i - \tau w_i \right) + \beta \tau \psi' \left( -\bar{w} + w_i - \tau w_i \right). \tag{A45}
\]

Rearranging (A45) leads to:

\[
D = \phi'' w_i (\beta \psi' - 1) + \beta \psi'' \left( -\phi'' \frac{\partial d_i}{\partial \tau} \frac{\partial d_i}{\partial \ell_i} - w_i \tau^2 + \beta \tau^2 \psi' w_i - \tau \bar{w} + w_i - \tau w_i + \tau w_i - \beta \tau \psi' \bar{w} + \beta \tau \psi' w_i - \beta \tau^2 \psi' w_i \right). \tag{A46}
\]

Consider that \(\tau^2 w_i - w_i \tau^2 = 0\) as well as \(\beta \tau^2 \psi' w_i - \beta \tau^2 \psi' w_i = 0\) and rearranging (A46) we obtain:

\[
D = \phi'' w_i (\beta \psi' - 1) + \beta \psi'' (\bar{w} - w_i) \left( \phi'' \frac{\partial d_i}{\partial \tau} [1 + \tau w_i] + \tau \left[ 1 - \beta \psi' \right] \right). \tag{A47}
\]

From (A1) follows \(\beta \psi' = 1 - \frac{\phi'}{\tau}\). Substituting this into (A47) and rearranging gives:

\[
D = -\frac{\phi' \phi'' w_i}{\tau} + \beta \psi'' (\bar{w} - w_i) \left( \phi'' \frac{\partial d_i}{\partial \tau} [1 + \tau w_i] + \phi' \right). \tag{A48}
\]

We can directly see that the first term of (A48) is unambiguously negative due to \(\phi' > 0\) and \(\phi'' > 0\). However, the second term is per se indeterminate in sign. It becomes negative for i) \(w_i > \bar{w}\) and \(\phi' > \phi'' \frac{\partial d_i}{\partial \tau} (1 + \tau w_i)\) or if ii) \(w_i < \bar{w}\) and \(\phi' < \phi'' \frac{\partial d_i}{\partial \tau} (1 + \tau w_i)\). In these cases, both terms are negative and thus \(D\) is unambiguously negative. This leads to \(\frac{\partial d_i}{\partial \tau} < 0\). If i) or ii) do not hold the second term is positive and hence the sign of \(D\) is not clear. In case of \(\psi'' = 0\), the second term vanishes and then \(D\) turns negative such that \(\frac{\partial d_i}{\partial \tau} < 0\).
First-order condition for $\tau$

In the following we presume the moral cost function to be linear, i.e. $\psi'' = 0$. For $\psi'' = 0$ the comparative static effects of $u_m$ and $\ell_m$ regarding the tax rate $\tau$ as well as the fairness preference $\beta$ change to:

\[
\frac{\partial u^*_m}{\partial \beta} = -\frac{\tau \psi'}{\phi''}, \tag{A49}
\]

\[
\frac{\partial \ell^*_m}{\partial \beta} = -\frac{\psi'}{\alpha \omega'} \frac{\partial d_m}{\partial \ell_m}, \tag{A50}
\]

\[
\frac{\partial u^*_m}{\partial \tau} = \phi' \frac{\partial \ell^*_m}{\partial \tau}, \tag{A51}
\]

\[
\frac{\partial \ell^*_m}{\partial \tau} = -\frac{w_i \phi'}{\tau \alpha \omega''}. \tag{A52}
\]

We differentiate indirect utility (18) with respect to the tax rate $\tau$:

\[
\frac{\partial V^*}{\partial \tau} = -w_m \ell_m + (1 - \tau)w_m \frac{\partial \ell^*_m}{\partial \tau} + \bar{w} \bar{\ell} + \tau w \frac{1}{n} \frac{\partial \ell^*_m}{\partial \tau} + u^*_m
\]

\[
+ \tau \frac{\partial u^*_m}{\partial \tau} - \phi' \frac{\partial u^*_m}{\partial \tau} - \alpha \omega' \frac{\partial \ell^*_m}{\partial \tau} - \beta \psi' \left[ \frac{\partial d_m}{\partial \tau} + \frac{\partial d_m}{\partial u_m} \frac{\partial u^*_m}{\partial \tau} + \frac{\partial d_m}{\partial \ell_m} \frac{\partial \ell^*_m}{\partial \tau} \right] = 0. \tag{A53}
\]

Then, (A53) can be rearranged to:

\[
\frac{\partial V^*}{\partial \tau} = -w_m \ell_m + \bar{w} \bar{\ell} + \tau \bar{w} \frac{1}{n} \frac{\partial \ell^*_m}{\partial \tau} + u^*_m - \beta \psi' \frac{\partial d_m}{\partial \tau}
\]

\[
+ \frac{\partial \ell^*_m}{\partial \tau} \left[ (1 - \tau)w_m - \alpha \omega' - \beta \psi' \frac{\partial d_m}{\partial \ell_m} \right]
\]

\[
+ \frac{\partial u^*_m}{\partial \tau} \left[ \tau - \phi' - \beta \psi' \frac{\partial d_m}{\partial u_m} \right] = 0. \tag{A54}
\]

Substituting $\frac{\partial d_m}{\partial \tau} = \tau$ and making use of the first-order conditions (A1) and (A2) we can directly see that the second and third line of (A54) vanishes. Thus, the first-order condition for the tax rate is given by:

\[
\frac{\partial V^*}{\partial \tau} = -w_m \ell_m + \bar{w} \bar{\ell} + \tau \bar{w} \frac{1}{n} \frac{\partial \ell^*_m}{\partial \tau} + u^*_m - \beta \psi' \frac{\partial d_m}{\partial \tau} = 0. \tag{A55}
\]

Second-order condition for $\tau$

Differentiating (A55) with respect to the tax rate $\tau$ yields:

\[
\frac{\partial^2 V^*}{\partial \tau^2} = -w_m \frac{\partial \ell^*_m}{\partial \tau} + w \frac{1}{n} \frac{\partial \ell^*_m}{\partial \tau} + \tau w \frac{1}{n} \frac{\partial^2 \ell^*_m}{\partial \tau^2} + \frac{\partial u^*_m}{\partial \tau} - \beta \psi' \frac{\partial^2 d_m}{\partial \tau^2}. \tag{A56}
\]

As $\frac{\partial^2 d_m}{\partial \tau^2} = \frac{\partial u^*_m}{\partial \tau} - w_m \frac{\partial \ell^*_m}{\partial \tau}$ we can write:

\[
\frac{\partial^2 V^*}{\partial \tau^2} = -w_m \frac{\partial \ell^*_m}{\partial \tau} + w \frac{1}{n} \frac{\partial \ell^*_m}{\partial \tau} + \tau w \frac{1}{n} \frac{\partial^2 \ell^*_m}{\partial \tau^2} + \frac{\partial u^*_m}{\partial \tau} - \beta \psi' \left[ \frac{\partial u^*_m}{\partial \tau} - w_m \frac{\partial \ell^*_m}{\partial \tau} \right]. \tag{A57}
\]
By rearranging (A57) we obtain:

\[
\frac{\partial^2 V^s}{\partial \tau^2} = \frac{\partial F^s}{\partial \tau} \left[ \frac{\bar{w}}{n} - w_m + \beta \psi' w_m \right] + \frac{\bar{w}}{n} \frac{\partial^2 F^s}{\partial \tau^2} + \frac{\partial u^s}{\partial \tau} \left[ 1 - \beta \psi' \right]. \tag{A58}
\]

Then, we substitute (A51), (A52) as well as \( \beta \psi' = 1 - \frac{\phi'}{\tau} \) from (A1) into (A58) in order to write:

\[
\frac{\partial^2 V^s}{\partial \tau^2} = \left( - \frac{w_m \phi'}{\tau \alpha \omega''} \right) \left[ \frac{\bar{w}}{n} - w_m + w_m \left( 1 - \frac{\phi'}{\tau} \right) \right] + \frac{\bar{w}}{n} \frac{\partial^2 F^s}{\partial \tau^2} + \frac{\phi'}{\tau \phi''} \left[ - \frac{\phi'}{\tau} \right]. \tag{A59}
\]

From (A52) follows \( \frac{\partial^2 V^s}{\partial \tau^2} = \left[ - \frac{w_m \phi'}{\tau \alpha \omega''} \left( - \frac{\phi'}{\tau^2} + \frac{\phi'}{\tau} \right) \right] \) as third partial derivatives are supposed to be zero, i.e. \( \omega'' = 0 \). Using this in (A59) we get:

\[
\frac{\partial^2 V^s}{\partial \tau^2} = \left( - \frac{w_m \phi'}{\tau \alpha \omega''} \right) \left[ \frac{\bar{w}}{n} - w_m + w_m \left( 1 - \frac{\phi'}{\tau} \right) \right] + \frac{\bar{w}}{n} \frac{\partial^2 F^s}{\partial \tau^2} + \frac{\phi'}{\tau \phi''} \left[ - \frac{\phi'}{\tau} \right]. \tag{A60}
\]

We rearrange (A60) to obtain:

\[
\frac{\partial^2 V^s}{\partial \tau^2} = - \frac{w_m \phi'}{\tau \alpha \omega''} \left[ \frac{\bar{w}}{n} - \frac{w_m \phi'}{\tau} \right] + \frac{\bar{w}}{n} \left[ \frac{w_m \phi'}{\alpha \omega'' \tau^2} - \frac{w_m \phi'}{n \alpha \omega''} \right] - \frac{[\phi']^2}{\tau^2 \phi''}. \tag{A61}
\]

Multiypling out the brackets in (A61) yields:

\[
\frac{\partial^2 V^s}{\partial \tau^2} = - \frac{w_m \bar{w} \phi'}{n \tau \alpha \omega''} + \frac{w_m^2 \phi'^2}{n \alpha \omega'' \tau^2} + \frac{w_m \bar{w} \phi'}{n \alpha \omega''} - \frac{w_m \bar{w} \phi'}{n \alpha \omega''} \frac{[\phi']^2}{\tau^2 \phi''}. \tag{A62}
\]

We take into account that \( \frac{w_m \bar{w} \phi'}{n \tau \alpha \omega''} - \frac{w_m \bar{w} \phi'}{n \alpha \omega''} = 0 \) in order to obtain:

\[
\frac{\partial^2 V^s}{\partial \tau^2} = - \frac{w_m^2 \phi'^2}{n \alpha \omega'' \tau^2} + \frac{w_m \bar{w} \phi'}{n \alpha \omega''} - \frac{[\phi']^2}{\tau^2 \phi''}. \tag{A63}
\]

Finally, by rearranging (A63) changes to:

\[
\frac{\partial^2 V^s}{\partial \tau^2} = \left( \frac{\phi'}{\tau} \right)^2 \left[ \frac{w_m^2}{\alpha \omega''} - \frac{1}{\phi''} \right] - \frac{w_m \bar{w} \phi'}{n \alpha \omega''}. \tag{A64}
\]

The second term of (A64) is unambiguously negative. Turning to the first term, we can see that \( \left( \frac{\phi'}{\tau} \right)^2 > 0 \) while the bracket term is indeterminate in sign. The first term \( \frac{w_m^2}{\alpha \omega''} \) in the brackets is positive and the second term \( - \frac{1}{\phi''} \) is negative. The term in brackets becomes negative if \( \frac{w_m^2}{\alpha \omega''} - \frac{1}{\phi''} < 0 \). This holds for \( \alpha \omega'' > \frac{w_m^2}{\phi''} \) which constitutes a sufficient condition for the second-order condition to be satisfied, i.e. \( \frac{\partial^2 V^s}{\partial \tau^2} < 0 \).
Comparative static effect of $\tau$ with respect to $\beta$

Implicitly differentiating the first-order condition for the tax rate with respect to the preference parameter $\beta$ yields:

$$\frac{\partial \tau^*}{\partial \beta} = \frac{\partial V^*}{\partial \tau \partial \beta}.$$  \hfill (A65)

As the second-order condition regarding taxation is satisfied, $\frac{\partial^2 V^*}{\partial \tau^2} < 0$, we obtain

$$\text{sign} \frac{\partial \tau^*}{\partial \beta} = \text{sign} \left[ \frac{\partial V^*}{\partial \tau \partial \beta} \right],$$  \hfill (A66)

with

$$\frac{\partial V^*}{\partial \tau \partial \beta} = -w_m \frac{\partial \ell_m^*}{\partial \beta} + \frac{\bar{w}}{n} \frac{\partial \ell_m^*}{\partial \beta} + \tau \frac{\bar{w}}{n} \frac{\partial^2 \ell_m^*}{\partial \beta^2} + \frac{\partial u_m^*}{\partial \beta} \frac{\partial d_m}{\partial \tau} - \psi \frac{\partial^2 d_m}{\partial \tau \partial \beta}.$$  \hfill (A67)

From $\frac{\partial d_m}{\partial \tau} = u_m - w_m \ell_m$ follows

$$\frac{\partial^2 d_m}{\partial \tau \partial \beta} = \frac{\partial u_m}{\partial \beta} - w_m \frac{\partial \ell_m}{\partial \beta}.$$  \hfill (A68)

Substituting (A68) as well as $\frac{\partial^2 \ell_m^*}{\partial \tau \partial \beta} = \frac{\partial u_m^*}{\tau \omega''} \frac{\partial \ell_m^*}{\partial \beta}$ derived from (A52) into (A67), we can write:

$$\frac{\partial V^*}{\partial \tau \partial \beta} = -w_m \frac{\partial \ell_m^*}{\partial \beta} + \frac{\bar{w}}{n} \frac{\partial \ell_m^*}{\partial \beta} + \tau \frac{\bar{w}}{n} \frac{\partial^2 \ell_m^*}{\partial \beta^2} + \frac{\partial u_m^*}{\partial \beta} \frac{\partial d_m}{\partial \tau} - \psi \frac{\partial^2 d_m}{\partial \tau \partial \beta}.$$  \hfill (A70)

Rearranging (A70) then yields:

$$\frac{\partial V^*}{\partial \tau \partial \beta} = \frac{\partial \ell_m^*}{\partial \beta} \left[ \frac{\bar{w}}{n} - w_m + w_m \beta \psi' \right] + \frac{\partial u_m^*}{\partial \beta} \left[ 1 - \frac{w_m \bar{w} \phi''}{\tau \omega''} - \beta \psi' \right] - \psi \frac{\partial d_m}{\partial \tau}.$$  \hfill (A71)

We substitute (A49), (A50) and $\beta \psi' = 1 - \frac{\phi'}{\tau}$ from (A1) in order to get:

$$\frac{\partial V^*}{\partial \tau \partial \beta} = \frac{\psi'}{\phi''} \frac{\partial \ell_m^*}{\partial \beta} \left[ \frac{\bar{w}}{n} - w_m + w_m \left( 1 - \frac{\phi'}{\tau} \right) \right]$$

$$+ \left( -\frac{\tau \psi \phi''}{\phi''} \left[ \frac{\phi'}{\tau} - \frac{w_m \bar{w} \phi''}{\tau \omega''} \right] - \psi \frac{\partial d_m}{\partial \tau}, \right.$$  \hfill (A72)

Finally, we rearrange (A72) and obtain:

$$\frac{\partial V^*}{\partial \tau \partial \beta} = -\frac{\psi'}{\phi''} \frac{\partial \ell_m^*}{\partial \beta} \left[ \frac{\bar{w}}{n} - \frac{\phi'}{\tau} \right] - \psi \frac{\phi'}{\phi''} - \frac{\tau w_m \bar{w}}{\tau \omega''} - \psi \frac{\partial d_m}{\partial \tau}.$$  \hfill (A73)
We assumed \( w_m < \bar{w} \) and therefore \((1 - \tau)w_m < \bar{w} \Rightarrow \frac{\partial d_m}{\partial \tau_m} < 0 \) follows. The bracket in the first term is positive if \( \frac{\bar{w}}{n} - \frac{\phi'}{\tau} > 0 \). This holds for \( \phi' < \frac{\bar{w}}{n} \). In contrast, the bracket in the second term is negative if \( \frac{\phi'}{\phi''} - \frac{\tau w_m \bar{w}}{\alpha \omega \phi''} < 0 \). Thus, the negative sign of the bracket term holds for \( \phi' < \frac{\tau w_m \bar{w}}{\alpha \omega \phi''} \). Using the relation \( \phi' < \min\{ \frac{\bar{w}}{n}, \frac{\tau w_m \bar{w}}{\alpha \omega \phi''} \} \) we can state that the overall expression (A73) is positive and therefore \( \frac{\partial V^*}{\partial d_m} > 0 \) when \( d_m < 0 \Rightarrow \psi' < 0 \). In this case, we obtain \( \frac{\partial V^*}{\partial \beta} > 0 \) according to (A66). Vice versa, if \( d_m > 0 \Rightarrow \psi' > 0 \) expression (A73) becomes negative and therefore \( \frac{\partial V^*}{\partial \beta} < 0 \) holds. Then, from (A66) we can derive the result \( \frac{\partial V^*}{\partial \beta} \). If, however, \( \phi' > \min\{ \frac{\bar{w}}{n}, \frac{\tau w_m \bar{w}}{\alpha \omega \phi''} \} \) the sign of expression (A73) remains unclear.