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Abstract

Comparing labor markets of the United States and Germany over the period $1980 - 2004$ uncovers three stylized differences: (1) transition rates from unemployment to employment (UE) were lower by a factor of 5 and inflow rates from employment to unemployment (EU) were lower by a factor of 4 in Germany. (2) The volatility of the UE rate was equal but the EU rate was 2.3 times more volatile in Germany. (3) In Germany EU flows contributed $60 - 70\%$ to the unemployment volatility while in the U.S. they contributed only $30 - 40\%$. We show that these differences can be largely explained by a single factor, namely a lower efficiency in matching unemployed workers to open positions in Germany. Alternative explanations like employment protection, the benefit system, union power, or rigid earnings are likely not the main driving force for the cross-country difference. The lower matching efficiency leads to a substantial propagation of shocks. After an adverse shock peak unemployment is reached after 3 quarters in the United States but only after 9 quarters in Germany.

JEL: E02, E24, E32

Keywords: Business Cycle Fluctuations, Labor Market Institutions, Unemployment, Endogenous Separation

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1 Introduction

Compared to the United States the European labor market over the period from $1980-2004$ was characterized by high unemployment rates and a sluggish response to shocks. For example, Germany displayed a prolonged period of high unemployment rates in the aftermath of the large oil price shocks, while the U.S. at that time recovered fairly quickly. We document in this paper three important cross-country differences comparing U.S. and German labor market flows: the transition rate from unemployment to employment (UE rate) is lower by a factor of 5 and inflow rates from employment to unemployment (EU rate) are lower by a factor of 4. Second, while (log) UE rates are as volatile, the volatility of the (log) EU rate is 2.3 times larger in Germany compared to the United States. Third, if we decompose the unemployment rate volatility into contributions of EU and UE flows, we find that in Germany the EU flows dominate and account for $60-70\%$ of the unemployment volatility, while in the U.S. they account for only $30-40\%$.[1]

In this paper we propose an explanation for all three differences that is based on a common source, a lower efficiency in matching unemployed workers to open positions in Germany. We show that the empirical cross country comparison offers identification restrictions that can be used to disentangle our explanation from prominent alternatives that have been proposed in the literature to rationalize either the lower average transition rates across country or the differences in the volatilities. To our knowledge our paper is the first to simultaneously look at salient labor market features across the two countries both in the mean rates and the business cycle dynamics and to link them to structural differences in a common framework.

For this purpose, we develop a simple labor market search and matching model with endogenous separations. We adapt the model to study business cycles in a similar fashion as den Haan, Ramey, and Watson (2000) and Ramey (2008). We derive simple closed form solutions for the second moments, so that we can analytically characterize the implications of institutional changes on the reaction to business cycle shocks. To study the different explanations in a unified framework we allow for worker and firm specific human capital accumulation, persistent idiosyncratic shocks as in Costain, Jimeno, and Thomas (2010) and tenure-dependent firing taxes.

A lower efficiency in the matching process in Germany relative to the U.S. leads to a decline in the frequency of UE transitions due to an effective increase in the cost of creating an open position. Simultaneously the average match surplus increases due to a deterioration of the outside opportunities of employed workers induced by the longer search duration. The increase in the average match surplus makes is less likely that negative idiosyncratic shocks destroy a match, so the frequency of transitions from employment into unemployment declines. However, differences in matching efficiency not only influence average transition rates. The increase in the average surplus makes German workers more sensitive to business cycle shocks.

[1] For the U.S. Hall (2005) and Shimer (2007) emphasizes the importance of the UE flows in understanding labor market dynamics while Fujita and Ramey (2009) and Elsby, Michaels, and Solon (2009) focus more on the EU flows.

Consider a German worker at the beginning of a boom. In case she separates, she has to search longer to find a new match compared to a U.S. worker due to the lower average UE rate. She would miss a larger fraction of the most profitable time of being employed. This will make her more reluctant to separate. Similarly, at the onset of a recession the German worker is more willing to separate because she will only miss the least profitable time of being employed while searching for a job. As a result, the German EU rate decreases more strongly in booms and increases more strongly in recessions, i.e. it is more volatile. The EU rate volatility is driven by the absolute change in the surplus of a match while the UE rate volatility is driven by the relative change of the surplus to its long run average. This remains largely unaffected due to the simultaneous increase in the average surplus and the increase in the sensitivity of the surplus to shocks. Hence, the contribution of the EU rate in the unemployment volatility increases. A lower matching efficiency can therefore explain both the difference in average transition rates as well as the differences in the second moments between the two countries.

We consider four alternative explanations and show that these cannot explain all three cross-country differences at the same time. First, explaining the lower UE rates in Germany by a more benevolent unemployment insurance system, larger firing taxes and/or an increase in micro-economic turbulence Ljungqvist and Sargent (2008); Wasmer (2006) will lower the average match surplus and increase the outflow volatility by more than the inflow volatility, inconsistent with our empirical facts. The second alternative, we consider, is a stronger bargaining position of the worker in Germany possibly induced by the employment protection legislation Blanchard and Portugal (2001). This can explain the lower UE rate but not the larger EU rate volatility. We show that at the Hosios condition Hosios (1990) both the average surplus and, as a consequence, the EU rate volatility are minimized. A deviation from the Hosios condition is quantitatively too small to jointly account for the lower UE rates and the large differences in the volatilities we observe in the data. Third, we consider differences in firing taxes between low and high tenured worker Bentolila, Cahuc, Dolado, and Barbanchon (2010); Costain, Jimeno, and Thomas (2010). Yet, these lead to an increase in the UE rate volatility and to inconsistencies in the tenure pattern of the transition rates which we document empirically. Fourth, explanations based on rigidities in the wage setting process Shimer (2010); Elsby and Michaels (2010) affect, in models with endogenous destruction, the EU rate and the UE rate volatility symmetrically, leaving the contribution of inflows to the unemployment volatility unaffected, again inconsistent with the empirical facts.

We then show that a different efficiency in the matching process matters for the transmission of business cycle shocks. In our quantitative model, estimated to reproduce the empirical differences across both labor markets, an adverse shock hitting the U.S. economy leads to a peak in the unemployment rate after 3 quarters and levels off fairly quickly afterwards. In contrast, the German unemployment rate peaks 9 quarters after the initial shock and even five years later the deviation of the unemployment rate from its long-run trend is still twice as large in Germany relative to the

United States.

The paper is related to the growing body of literature studying the European ins and outs of unemployment Petrongolo and Pissarides (2008); Pissarides (2009) based on micro-data and Elsby, Hobijn, and Sahin (2010) using aggregate OECD data. We provide a detailed account on the "ins and outs" for Germany using a large micro data set on employment histories.[2] We extend the unemployment volatility decomposition developed in Fujita and Ramey (2005) and Petrongolo and Pissarides (2008) to a three state, six transition rate decomposition to particularly control for flows in and out of inactivity. We provide new evidence on the transition rates by tenure to shed light on the impact of differential firing taxes and the skill accumulation process. Finally we give a complete account on the earning dynamics in Germany, controlling for selection effects using various methods proposed in Bils (1985), Solon, Barsky, and Parker (1994) and Haefke, Sonntag, and van Rens (2007) to empirically assess the possible importance of wage rigidities across countries.

The remainder of the paper is organized as follows: Section 2 documents labor market facts for Germany, section 3 develops the model, section 4 characterizes the results, extensions are in section 5, and section 6 concludes.

2 Data

2.1 Data description

Our dataset is the IAB[3] employment panel that comprises a 2% representative sample taken from the German social security and unemployment records for the period $1980 - 2004$. The sample contains employees that are covered by the compulsory German social security system, and excludes self-employed and civil servants ('Beamte'). It covers about 80% of Germany's labor force. Since the East German labor market was subject to additional regulations and restructuring after the reunification, we exclude all persons with employment spells in East Germany from our sample.[4] For each worker in the sample we observe the entire employment history (social security status) on a daily basis. We choose as our basic period length one month and construct monthly employment

[2]Burda and Wyplosz (1994) summarize evidence on average transition rates for Europe. There are two other studies on worker flows using the IAB in Germany that show a limited amount of overlap with our results. Bachmann (2005) uses a slightly different concept to measure worker flows. He measures worker flows on a monthly frequency but focuses for the dynamics at an annual frequency. However, his results regarding average transition rates are consistent with our findings. Very recently Gartner, Merkl, and Rothe (2009) also report some basic facts but use different definitions for labor market states, for example they do not control for inactivity, and work with a quarterly aggregation so that their results are not comparable to our findings.

[3]This study uses the factually anonymous BA-Employment Panel (Years $1975 - 2004$). Data access was provided via a Scientific Use File supplied by the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB).

[4]We do a first step sample selection where we remove very few individuals with missing observations. Details on this step and further information on the data set can be found in the appendix.

histories from the daily data.[5] We account explicitly for periods of inactivity and transitions out of the labor force, e.g. (early) retirement or maternity leave.

Aggregate data for Germany are from the German statistical office ('Statistische Bundesamt'). The official unemployment rate is from the German Employment Agency ('Bundesagentur für Arbeit').[6] The data for the U.S. is from the BLS for the aggregate time series and from Shimer (2007) for the labor market transition rates. The numbers on employer-to-employer transitions are from Fallick and Fleischman (2004). In the decomposition analysis of the unemployment volatility we use in addition data from Fujita and Ramey (2009).

2.2 Labor market flows

Following the work of Shimer (2005) for the United States and Petrongolo and Pissarides (2008) for several European countries this section provides a comprehensive analysis of the 'ins and outs' of unemployment for Germany and compares the results to existing evidence for the United States.[7] Table 1 summarizes our findings and presents a cross-country comparison along three dimensions: aggregate business cycle fluctuations, mean labor market transition rates, and volatilities of the transition rates.[8] Two facts are striking, while the aggregate business cycle fluctuations look very much alike (see left part of the table), the transition rates in the right part of the table uncover a labor market that is substantially different both in mean rates and in volatilities (see right part of the table).

More specifically, measures of aggregate economic activity GDP, labor productivity, and earnings have similar volatilities in both countries. The aggregate measures of the labor market are slightly more volatile in Germany compared to the U.S.. The unemployment rate is 1.2 times as volatile and vacancies[9] are 1.6 times as volatile. Correlations with GDP have the same sign and similar magnitudes across the two countries. Additionally the Beveridge curve, the correlation between unemployment rates and vacancies, is strongly negative in Germany (correlation -0.85) and the U.S. (correlation -0.91). Altogether, the picture that emerges on an aggregate level is fairly similar. This changes once we look at labor market transition rates and volatilities in the right part of the table. We find average rates that are substantially lower in Germany. The EU rate is lower by a factor of 4 and the EE and EN rates differ by a factor of approximately 3. The UE rate is also

[5]In the appendix we describe in detail how we construct the employment histories and labor market states. Bachmann (2005) studies an earlier version of our dataset covering the period $1975 - 2001$ and applies a different approach to measure labor market transition rates. His results account for all transitions within a month but are virtually unchanged compared to our findings.

[6]Further details especially on the adjustment for the German reunification can be found in the appendix.

[7]The data discussed here refers to all workers and an online appendix to this paper provides an extended analysis where we separate worker flows based on sex and education.

[8]We do not report NU and NE transition rates because we do not observe the universe of all non-employed so that transition rates can not be computed. The online appendix reports the correlation and volatilities for these flows.

[9]We only have notified open positions at the job centers that do not constitute the whole universe of open positions. Indeed, comparison of recent firm survey data with the data on registered vacancies suggest that about 1/3 of all open positions are announced to job centers. We take it therefore only as an indicator.

Table 1: GDP, unemployment rates, and transition rates over the business cycle

	Statistic	mean	std	corr	Transition rate	mean	std	corr
Germany	GDP		2.4	1	EU	0.5	15.1	-0.81
U.S.			2.6	1		2.0	6.5	-0.72
Germany	Productivity		1.6	0.77	UE	6.2	10.4	0.40
U.S.			1.4	0.44		30.7	11.2	0.82
Germany	Earnings		1.7	0.84	EE	0.9	15.6	0.65
U.S.			1.8	0.42		2.6	6.3	0.65
Germany	Vacancies		33.4	0.82	EN	1.0	6.2	0.53
U.S.			20.4	0.85		2.7	4.6	0.44
Germany	Urate	8.4	18.1	-0.76	UN	4.9	10.3	0.45
U.S.		6.3	15.0	-0.89		26.6	9.1	0.73

Notes: Standard deviations (STD) are given as percentage deviations from an HP-filtered trend ($\lambda = 100000$) of the rates (in logs). Correlations (CORR) give the correlation coefficient with GDP. Our productivity measure is GDP per employed. Source: Authors' own calculations based on IAB data.

substantially lower and differs by a factor of 5.[10] A reverse picture arises for the volatilities. While the UE rates in both countries are still equally volatile, the German EU rate turns out to be 2.3 times more volatile than the U.S. rate.[11] Figure 1(a) visualizes the close connection of the cyclical component of the EU rate and the unemployment rate in Germany while the link is present but not as close in the U.S. (Figure 1(b)).

(a) Germany (b) United States

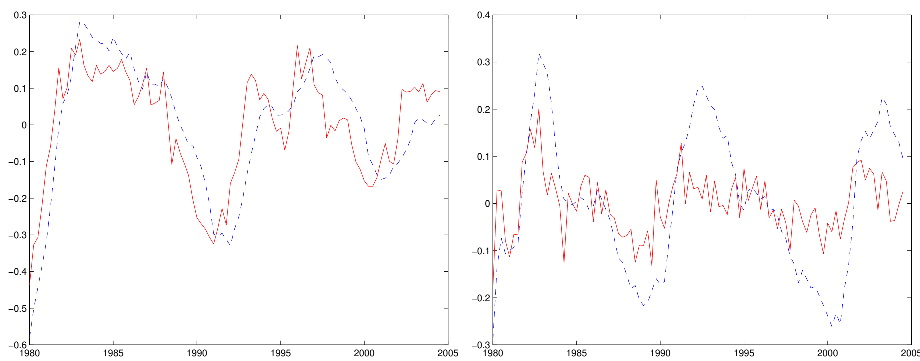

Figure 1: Cyclical component of EU rate and unemployment rate

Notes: The figure shows the cyclical component of the EU rate and the official unemployment rate based on an HP-filter ($\lambda = 100000$). The red solid line is the EU rate and the blue dashed line is the unemployment rate.

The *exit rate* (EU + EN) tends towards acyclicality both in Germany (correlation -0.47) and the

[10]These lower rates can be observed throughout the sample period and are not an artifact of the developments in the nineties. In 1980, the average UE rate in Germany is 10.9% declining over time to 4.7% in the mid-nineties (1995). During the same time period the EU rate increased from 0.4% to 0.5%.

[11]Given that we study the interaction of long-run means and cyclical volatilities and the long lasting consequences of business cycle shocks we prefer a high smoothing parameter to the HP Filter. We follow Shimer (2005) in this choice who finds that a lower smoothing parameter like the traditionally chosen $\lambda = 1,600$ for business cycle analysis removes a lot of the cyclical variation of interest.

U.S. (correlation -0.24). This is due to countercyclical EU rates and procyclical EN rates. Only for Germany, we have EE flows for the whole sample period. If we add these to the total separation rate (EU + EN + EE), the correlation turns positive (correlation 0.46) as a consequence of the procyclical EE flows. These results suggest that EN flows are rather different to EU flows and seem to have more in common with EE flows than with EU flows. Support for this view comes from Nagypal (2005). She shows that for the U.S. many EN flows are reverted one month later suggesting possibly the move to a new employer with an intervening month of inactivity.

2.3 Unemployment decomposition

To address the importance of in- and outflows in explaining unemployment volatility, we use the methodology proposed in Fujita and Ramey (2009) but develop also an extended decomposition with three states and six transition rates to control for flows into inactivity. Details on the volatility decomposition of Fujita and Ramey (2009) and our extension can be found in Appendix A.1.[12] Table 2 summarizes our finding based on the two state and three state decomposition where the numbers present the share in unemployment volatility attributed to the respective rates.

Table 2: Unemployment Volatility Decomposition

Country	Data	EU	UE	NE	EN	NU	UN	ε
Germany	IAB	61.1	38.6					0.3
	IAB	42.5	24.6	20.0	-4.5	6.6	11.0	-0.3
U.S.	Shimer	32.6	67.6					-0.2
	Fujita/Ramey	38.4	61.9					-0.2
	Shimer	20.1	48.6	8.8	-3.8	10.4	15.2	0.7

Notes: Data is HP-filtered ($\lambda = 100,000$) for the period $1980q1 - 2004q4$. For Germany the transition rates are for all workers. The U.S. data is obtained from Shimer (2007) and Fujita and Ramey (2009). Contribution shares are given as percentage numbers. Source: Authors' own calculations based on IAB data.

Based on a two state decomposition the contribution of EU rates account for more than 60% of the volatility in unemployment while in the U.S. it accounts for 30−40%. The three state decomposition indicates that German EU rates contribute about twice as much to the unemployment volatility as the UE rates, while in the U.S. the opposite is the case. EU and UE rates taken together account in both countries for around 2/3 of the unemployment volatility possibly justifying the focus on a

[12] Petrongolo and Pissarides (2008) analyze the contribution of job in- and outflow rates to the fluctuations in unemployment for the UK, France, and Spain. Fujita and Ramey (2009) present an analysis for the U.S. Elsby, Hobijn, and Sahin (2010) estimate in- and outflow rates from aggregate data for OECD countries to study the decomposition of unemployment dynamics. In their analysis the estimated transition rates for most European countries yield an insufficient approximation of the steady state unemployment rate. Once they account in their decomposition for deviations from steady state their results are again consistent with our findings. The analysis in all papers is based on a first-order approximation around trend unemployment but the detrending methods and the considered labor market flows differ. The analysis in Petrongolo and Pissarides is based on a first difference filter allowing for four aggregate transition rates whereas Fujita and Ramey use the HP-Filter and a two state decomposition. Fujita and Ramey show that the first difference filter is typically very sensitive to high-frequency fluctuations. The online appendix provides sensitivity with respect to these methods.

two-state decomposition.

Our empirical findings show that the German labor market is characterized by substantially lower average transitions rates compared to the U.S. Despite a similar UE rate volatility the German EU rate volatility though is substantially larger. In reaction to shocks the German labor market relies more heavily on adjusting the inflow rate while the U.S. labor market relies more on the outflow rate.

3 Model

To understand the differential labor markets in the two countries we develop an extended version of a Mortensen-Pissarides-style search and matching model. In the general version of the model we allow for a generic idiosyncratic state process which we will later attach specific forms in order to model tenure on the job, individual or match specific skills.

There is a continuum of workers with measure one. Workers and firms are risk neutral. Workers can be either employed or unemployed denoted by $\tilde{e} \in \{e, u\}$. The aggregate technology state A is random and follows a Markov process. Additionally, there is an idiosyncratic state attached to each worker denoted by $x \in \mathbb{X}$. This state also follows a Markov process. We allow this process to depend on the labor market transition from the current labor market state $\tilde{e}$ to next period's state $\tilde{e}'$, for example to model the loss of firm specific human capital after an EU transition (turbulence). This means the model has different conditional distributions over tomorrow's idiosyncratic state depending on current and future employment status. We denote these distributions by $p_{ee}(x'|x)$, $p_{eu}(x'|x)$, $p_{ue}(x'|x)$, and $p_{uu}(x'|x)$ depending on wether the agent stays employed, moves into unemployment, out of unemployment or stays unemployed, respectively.

The measure of unemployed workers in the different idiosyncratic states is denoted by $u(x)$ and for employed workers by $l(x)$. The joint distribution over employment states $\tilde{e}$ and idiosyncratic states x is $\lambda : \{e, u\} \times \mathbb{X} \rightarrow [0, 1]$ where Λ denotes the set of possible joint distributions.

Time is discrete. Workers who are currently in a match bargain jointly and efficiently over the wage and the separation decision for the next period. If the bargaining is successful, they produce output according to the production technology $Ag(x)$ where the aggregate technology A evolves exogenously and common to all matches, and $g(x)$ summarizes the individual productivity for a worker of type x. At the end of the period, the firm receives an idiosyncratic cost shock ε. We assume that ε is i.i.d. across firms and over time and logistically distributed with mean zero and variance $\frac{\pi^2}{3}\psi_\varepsilon^2$. The assumption of a logistic distribution allows us to obtain closed form solutions and is done for convenience. The firm has to pay the costs ϵ only if it wishes to continue the production process. The costs are sunk after the period and will not affect any future decision. At the bargaining stage the firm and the worker agree upon a threshold value $\bar{\epsilon}$ for the continuation costs ε.

If the realized continuation costs ϵ are larger than the threshold value $\bar{\epsilon}$, the match dissolves and the firm has to pay a state dependent firing tax $\tau(x)$ to the government and the worker becomes unemployed. The transition probability for the individual state in this case is $p_{eu}(x'|x)$.[13] If the costs ϵ are smaller than the cut-off value $\bar{\epsilon}$, then they the firm pays the continuation costs and continues the match. In this case, the worker transits to a new idiosyncratic state with probability $p_{ee}(x'|x)$. This structure of the optimal decision allows us to cast the separation decision solely in terms of cut-off values.[14]

An unemployed worker searches for a job and is matched in a matching market governed by a standard Cobb-Douglas matching function. Search is random so unemployed workers receive job offers from firms with probability π_{ue}. Together with the offer comes a realized idiosyncratic productivity component. The probability distribution for the idiosyncratic state is $p_{ue}(x'|x)$. In case the worker does not receive an offer, a new idiosyncratic state is drawn according to $p_{uu}(x'|x)$. While unemployed, a worker has a utility flow $\tilde{b}(A, x)$ which might depend on the current idiosyncratic state but also on the aggregate state. We include the dependence on the aggregate and idiosyncratic states to capture in a simple way the effects of wage rigidity. Specifically, we use $\tilde{b}(A, x) = \exp(\varphi(x)\log(A))b$. This functional form includes the different cases studied in the literature. Using $\varphi = 1$, we mimic very flexible wages, using $\varphi = 0$, we have our benchmark case used in Shimer (2005) and Hagedorn and Manovskii (2008), and using $\varphi < 0$, we can mimic a stronger form of wage rigidity inducing a countercyclical element to the surplus that will amplify shocks.[15]

Consider a worker-firm pair at the beginning of the period. The firm discounts the future, as does the worker, with a constant discount factor β. For given wages $w : \mathbb{R} \times \mathbb{X} \times \Lambda \to \mathbb{R}_+$ and cut-off

[13] Note that $\tau(x)$ is expressed as a firing tax, or a reorganization cost and does not include severance payments. In our framework, severance payments are efficiently bargained away and would have no effect on the equilibrium outcomes. The government transfers all income lump sum back to the worker, so under risk-neutrality, there is no need to formally specify governmental behavior.

[14] Technically, the cut-off value represents a quantile of the cost shock distribution and the cost shock ε itself will not appear explicitly in any further expression.

[15] We allow for a possible type dependence on the idiosyncratic state to include the case were newly employed workers have a different degree of wage rigidity as continuously employed workers, see section 5.3.

strategies $\bar{\epsilon} : \mathbb{R} \times \mathbb{X} \times \Lambda \to \mathbb{R}_+$ the firm's surplus and the probability π_{eu} are given by[16]

$$\begin{aligned}
J(A,x,\lambda) &= Ag(x) - w(A,x,\lambda) \\
&\quad + \int_{-\infty}^{\bar{\epsilon}} \left(\beta \mathbb{E}\left[\sum_{x'} p_{ee}(x'|x) J(A',x',\lambda') \right] - \varepsilon \right) df(\varepsilon) - \int_{\bar{\epsilon}}^{\infty} \tau(x) df(\varepsilon) \qquad (1) \\
\pi_{eu}(A,x,\lambda) &= 1 - Prob(\varepsilon < \bar{\epsilon}) = \left(1 + \exp\left(\frac{\bar{\epsilon}(A,x,\lambda)}{\psi_\varepsilon} \right) \right)^{-1}.
\end{aligned}$$

The value functions for employed workers $V_e : \mathbb{R}_+ \times \mathbb{X} \times \Lambda \to \mathbb{R}$ and unemployed workers $V_u : \mathbb{R}_+ \times \mathbb{X} \times \Lambda \to \mathbb{R}$ are given by

$$\begin{aligned}
V_e(A,x,\lambda) &= w(A,x,\lambda) + (1 - \pi_{eu}(A,x,\lambda)) \beta \mathbb{E}\left[\sum_{x'} p_{ee}(x'|x) V_e(A',x',\lambda') \right] \\
&\quad + \pi_{eu}(A,x,\lambda) \beta \mathbb{E}\left[\sum_{x'} p_{eu}(x'|x) V_u(A',x',\lambda') \right] \qquad (2) \\
V_u(A,x,\lambda) &= b \exp(\varphi(x) \log(A)) + (1 - \pi_{ue}(A,\lambda)) \beta \mathbb{E}\left[\sum_{x''} p_{uu}(x''|x) V_u(A',x'',\lambda') \right] \\
&\quad + \pi_{ue}(A,\lambda) \beta \mathbb{E}\left[\sum_{x'} p_{ue}(x'|x) V_e(A',x',\lambda') \right]. \qquad (3)
\end{aligned}$$

We denote the worker's surplus by $\Delta(A,x,\lambda) = V_e(A,x,\lambda) - V_u(A,x,\lambda)$ and the match surplus as $S(A,x,\lambda) = J(A,x,\lambda) + V_e(A,x,\lambda) - V_u(A,x,\lambda)$.

New matches are formed by a standard Cobb-Douglas matching technology that links searching workers to vacancies. The measure of unemployed workers is denoted by u, the vacancies posted are denoted by v, and the resulting matches by m.

$$\begin{aligned}
m &= \varkappa v^{1-\varrho} u^{\varrho} \\
u &= \sum_{x \in \mathbb{X}} u(x).
\end{aligned}$$

[16]Solving the conditional expectation for $\pi_{eu}(A,x)$ the firm's profit is

$$J(A,x,\lambda) = Ag(x) - w(A,x,\lambda) + (1 - \pi_{eu}(A,x,\lambda)) \beta \mathbb{E}\left[\sum_{x'} p_{ee}(x'|x) J(A',x',\lambda') \right] - \pi_{eu}(A,x,\lambda)\tau(x) + \Psi(A,x,\lambda).$$

Evaluating the integrals under the distributional assumptions yields the given functional form. The option value Ψ follow directly from the assumption of a logistically distributed cost shock and captures the value of having a choice to continue the match and is always positive

$$\Psi(A,x) = -\psi_\varepsilon \Big((1 - \pi_{eu}(A,x,\lambda)) \log(1 - \pi_{eu}(A,x,\lambda)) + \pi_{eu}(A,x,\lambda) \log(\pi_{eu}(A,x,\lambda)) \Big).$$

The matching efficiency $\varkappa$ measures how effectively unemployed workers are matched to open positions. Examples for factors that can influence the matching efficiency are the willingness of workers to move for a new job, skill mismatch, or bureaucracy in employment agencies. Labor market tightness is defined as usual as the ratio of vacancies to searching workers $\theta := \frac{v}{u}$. The probability that a searching worker will meet a firm is

$$\pi_{ue}(A, \lambda) = \frac{m}{u} = \varkappa\theta^{1-\varrho}$$

and the probability that a firm posting a vacancy will meet some worker is given by

$$\pi_{ve} = \frac{m}{v} = \varkappa\theta^{-\varrho}.$$

To determine the number of vacancies posted, we impose a standard free entry condition

$$\kappa = \pi_{ve} \sum_{x \in \mathbb{X}} \frac{u(x)}{u} \beta \mathbb{E} \left[\sum_{x'} J(A', x', \lambda') p_{ue}(x'|x) \right].$$

The probability of meeting a specific worker with characteristics x is $\frac{u(x)}{u}$. We assume Nash bargaining jointly over wages and cut-off values. The outcome of the bargaining process is characterized by

$$\{w, \bar{\omega}\} = \arg\max_{w, \bar{\omega}} \quad \mu \log\left(\Delta(A, x, \lambda)\right) + (1-\mu)\log\left(J(A, x, \lambda)\right)$$

where μ denotes the bargaining power of the worker. First-order conditions deliver

$$\begin{aligned} \bar{\omega}(A, x, \lambda) &= \beta \mathbb{E} \left[\sum_{x'} p_{ee}(x'|x)(J(A', x', \lambda') \right. \\ &\quad \left. + V_e(A', x', \lambda')) - \sum_{x'} p_{eu}(x'|x) V_u(A', x', \lambda') \right] + \tau(x) \\ \frac{\mu}{1-\mu} &= \frac{\Delta(A, x, \lambda)}{J(A, x, \lambda)}. \end{aligned}$$

Technology evolves exogenously according to

$$A = \exp(a) \qquad a' = \rho a + \eta'$$

where ρ denotes the auto-correlation coefficient and innovations η are normally distributed. Additionally to aggregate productivity we have in general to keep track of employment states by skill status whose laws of motion are given by:

$$\begin{aligned} l'(x') &= \sum_x (1-\pi_{eu}(A,x,\lambda))p_{ee}(x'|x)l(x) + \sum_x \pi_{ue}(A,\lambda)p_{ue}(x'|x)u(x) \\ u'(x') &= \sum_x \pi_{eu}(A,x,\lambda)p_{eu}(x'|x)l(x) + \sum_x (1-\pi_{ue}(A,\lambda))p_{uu}(x'|x)u(x) \\ 1 &= \sum_x u(x) + \sum_x l(x). \end{aligned}$$

4 Baseline Specification and Results

This section explains why the cross-country differences we document empirically can be explained by differences in the matching efficiency. In our baseline specification presented in this section we specialize to the homogeneous worker case abstracting from idiosyncratic shocks, i.e. we set $x = 1$, so all policy rules are functions of the aggregate state only.[17] We first characterize the working of the model and explain why it accounts for the cross-country differences in the first and the second moments jointly. To do so, we provide an analytic link between the structural parameters of the model capturing institutional differences across countries and these moments. Finally, we offer a quantitative exploration.

4.1 Basic Mechanism

Table 3 shows the steady state of the model and gives an analytical characterization of the volatilities based on a first-order approximation, where σ_y captures the deviation of a variable y from its steady state value $\bar{y}$ when the productivity state is a, i.e. $y - \bar{y} = \sigma_y a$ and $\tilde{\sigma}_y = \frac{\sigma_y}{\bar{y}}$ denotes the percentage deviation. The absolute value of $\tilde{\sigma}_y$ coincides with the log standard deviations relative to the standard deviation of productivity. We also report some simple approximations to the resulting expressions to gain intuition.

As Table 3 shows the *EU rate volatility* ($|\tilde{\sigma}_{eu}|$) is linear in the surplus reaction σ_S scaled by the standard deviation ψ of the continuation-cost. Intuitively, less dispersed cost shocks lead to a larger fraction of firms living around the cut-off value $\bar{\epsilon}$. As a consequence, a change in the surplus after a business cycle shock will lead to more firms that draw cost shocks below the cut-off value and decide to dissolve. At the aggregate level this implies an increase in the EU rate after a negative business cycle shock making EU rates countercyclical.[18]

Unlike the EU rate volatility the *UE rate volatility* ($|\tilde{\sigma}_{ue}|$) is linear in the relative surplus volatility $\frac{\sigma_S}{S}$. This makes the UE rate volatility a direct function of the outside option b (see approximation), a

[17] In the absence of idiosyncratic shocks the model is block-recursive in the sense of Menzio and Shi (2009) so the employment measure does not enter the policy functions.

[18] This is the standard logic of generating countercyclical EU rates and applies as well to models using log-normal multiplicative shocks.

Table 3: Analytic Expressions for the First and Second Moments

	Exact	Approximation
$\bar{S}$	$\frac{\bar{A}-b-\psi\log(1-\bar{\pi}_{eu})}{1-\beta(1-\bar{\pi}_{ue}\mu)}$	$\frac{\bar{A}-b}{\bar{\pi}_{ue}\mu}$
$\bar{\pi}_{ue}$	$\varkappa\left(\frac{(1-\mu)\varkappa\beta\bar{S}}{\kappa}\right)^{\frac{1-\varrho}{\varrho}}$	$\varkappa\left(\frac{1-\mu}{\mu}\frac{\bar{A}-b}{\kappa}\right)^{1-\varrho}$
$\bar{\pi}_{eu}$	$\left(1+\exp\left(\frac{\beta\bar{S}+\tau}{\psi}\right)\right)^{-1}$	$\left(1+\exp\left(\frac{\bar{A}-b}{\bar{\pi}_{ue}\mu\psi}+\frac{\tau}{\psi}\right)\right)^{-1}$
σ_S	$\left(1-\beta\rho\left(1-\bar{\pi}_{eu}+\bar{\pi}_{ue}\frac{\mu}{\varrho}\right)\right)^{-1}$	$\frac{\varrho}{\bar{A}-b}\bar{S}$
$\tilde{\sigma}_{ue}$	$(1-\varrho)\frac{\rho}{\varrho}\frac{\sigma_S}{\bar{S}}$	$\frac{1-\varrho}{\bar{A}-b}$
$\tilde{\sigma}_{eu}$	$-(1-\bar{\pi}_{eu})\frac{\rho\beta}{\psi}\sigma_S$	$-\frac{\varrho}{\bar{A}-b}\frac{\bar{S}}{\psi}$

Notes: Analytic exact expressions for steady states are in the first column. Approximations using $\beta\rho \approx 1$ and $\pi_{EU} \approx 0$ are given in the second column.

fact that has been discussed in the recent literature Shimer (2005); Hagedorn and Manovskii (2008). As a consequence, our calibration will require an outside option b that is close to productivity to match the UE rate volatility, but this is not decisive for our argument as we show in section 5. The presence of endogenous destruction has no first order effect on the UE rate volatility. Separations are efficiently bargained and reflect a choice of the match, so their impact is of second order on the dynamics of the relative change in the surplus. However, endogenous destruction affects the model's unemployment rate volatility.

$$\begin{aligned} |\tilde{\sigma}_u| &= \frac{|\sigma_{eu}(1-\bar{u})-\sigma_{ue}\bar{u}|}{\bar{u}\sqrt{1-(1-\bar{\pi}_{ue}-\bar{\pi}_{eu})^2}}\sqrt{\frac{1+\rho(1-\bar{\pi}_{ue}-\bar{\pi}_{eu})}{1-\rho(1-\bar{\pi}_{ue}-\bar{\pi}_{eu})}} \\ &\approx (|\tilde{\sigma}_{eu}|+|\tilde{\sigma}_{ue}|)(1-\bar{u}) \end{aligned}$$

The contribution of the EU rate to the unemployment volatility is essentially driven by the ratio $|\tilde{\sigma}_{eu}|$ to $|\tilde{\sigma}_{ue}|$.[19] Using the approximation from table 3, we see that the contribution of the EU rate to the unemployment volatility is proportional to the average surplus

$$\frac{|\tilde{\sigma}_{eu}|}{|\tilde{\sigma}_{ue}|} = \frac{\varrho}{1-\varrho}\frac{\bar{S}}{\psi}$$

and to explain a higher contribution of EU transitions to the volatility of the unemployment in

[19] The formula shows that a cross-country comparison should be based on the volatility of the percentage deviation not the rates in levels. Two countries with similar unemployment rates but different UE and EU rates would have an identical reaction to shocks if the log volatilities are identical, while they would differ substantially if the rate volatilities would be identical.

Germany we directly see that the average surplus has to be larger.

The EU rate volatility $|\tilde{\sigma}_{eu}|$ is an increasing function of the average match surplus and is inversely related to the average UE rate (cp. approximation in table 3). The intuition for the inverse relationship has its seeds in the reemployment prospects of workers after separation. This can be seen by looking at the recursive formula of the surplus obtained from equations (1), (2) and (3), where we set $\psi_\epsilon log(1-\pi_{eu}) \approx 0$ for simplicity

$$S \approx A - b + \beta \mathbb{E}\left[S'\right] - \pi_{UE}\mu\mathbb{E}\left[S'\right].$$

We see that the current surplus is the discounted surplus of the current match $A - b + \beta\mathbb{E}[S']$ net of the *outside opportunity* $\pi_{UE}\mu\mathbb{E}[S']$ in an alternative match of the worker.[20]

Consider now how a positive business cycle shock affects these two values. The surplus of the current match increases making it less likely that an idiosyncratic shocks hitting the match will lead to a separation. As a result, the EU rate falls making it countercyclical. At the same time, the increase in the surplus of the current match is dampened by the reaction of the outside opportunity of the worker which enters negatively into the total surplus and will therefore lower the reaction σ_S. In a boom the outside opportunity will increase because the prospects of finding a job quickly increase and the expected surplus of an alternative match also rises.

The cross-country differences in the outside opportunity explains the differences in the reaction to shocks. Take Germany that has a lower average UE rate. At the onset of a boom, the opportunity costs of separating for the German worker are higher because she misses particularly productive times. On average the German worker searches for a new job for roughly a year and the worker has missed the most profitable times of being employed. This makes her more reluctant to separate in reaction to the shock. The U.S. worker instead needs to search on average only three months and is still able to benefit from the booming conditions by quickly accepting a new job offer. As a consequence the increase in the outside opportunity in the U.S. is stronger at the beginning of a boom compared to Germany. This dampens the reaction of the surplus and ultimately lowers the EU rate volatility.[21]

[20] We use the term *outside opportunity* because it captures the expected value of an alternative match in case of separation.

[21] A corresponding argument shows that at the beginning of a recession the German worker will be more willing to separate from the current job compared to the U.S. worker in reaction to the shock. Now, the current match surplus falls. The expected outside opportunity of the worker falls, too, dampening the decline in the total surplus. However, the dampening effect will be smaller in Germany given that the outside opportunity receives less weight due to the lower UE rate. As a result, the surplus reaction in Germany will be stronger and the German worker will effectively spend a longer time being unemployed, i.e. the relative value of unemployment has increased by more than in the U.S.. As discussed above, the decrease in the surplus makes it more likely that an idiosyncratic shock destroys the match, and therefore, EU rates react more strongly in Germany.

4.2 Institutional Factors

What institutional factors can explain the observed differences in labor market outcomes between the U.S. and Germany? Our intuition developed so far has focused on the transmission from mean transition rates to volatilities, which are both endogenous objects. We now provide the link to the underlying structural parameters. Table 4 reports the analytic elasticities of the average rates and the volatilities with respect to a change in the underlying parameter. They can be used to sign the impact of each of the structural parameters on the four endogenous dimensions considered in this paper.[22]

Table 4: Analytic approximations of steady state elasticities

p	Parameter	$\frac{p}{\pi_{ue}}\frac{d\pi_{ue}}{dp}$	$\frac{p}{\pi_{eu}}\frac{d\pi_{eu}}{dp}$	$\frac{p}{\lvert\tilde{\sigma}_{eu}\rvert}\frac{d\lvert\tilde{\sigma}_{eu}\rvert}{dp}$	$\frac{p}{\tilde{\sigma}_{ue}}\frac{d\tilde{\sigma}_{ue}}{dp}$
$\varkappa$	Matching Efficiency	$\frac{\varrho}{1-\varrho}$	$\frac{\varrho}{1-\varrho}\frac{\bar{S}}{\psi}$	$-\frac{\varrho}{1-\varrho}$	0
μ	Bargaining Power	$-\frac{1-\varrho}{1-\mu}$	$-\frac{\mu-\varrho}{1-\mu}\frac{\bar{S}}{\psi}$	$\frac{\mu-\varrho}{1-\mu}$	0
b	Outside Option	$-\frac{(1-\varrho)b}{\bar{A}-b}$	$\frac{\varrho b}{\mu\psi\bar{\pi}_{ue}}$	$(1-\varrho)\frac{b}{\bar{A}-b}$	$\frac{b}{\bar{A}-b}$
τ	Firing Tax	$-\frac{(1-\varrho)\tau\bar{\pi}_{eu}}{\bar{A}-b}$	$-(1-\frac{\varrho\bar{\pi}_{eu}}{\mu\bar{\pi}_{ue}})\frac{\tau}{\psi}$	$(1-\varrho)\frac{\tau\bar{\pi}_{eu}}{\bar{A}-b}$	$\frac{\tau\bar{\pi}_{eu}}{\bar{A}-b}$
ψ	Shock Variance	$-\frac{\bar{\Psi}(1-\varrho)}{\bar{A}-b}$	$\frac{\tau+\bar{S}}{\psi}+\frac{\bar{\Psi}\varrho}{\psi\mu\bar{\pi}_{ue}}$	$-\left(1-\frac{\bar{\Psi}(1-\varrho)}{\bar{A}-b}\right)$	$\frac{\bar{\Psi}}{\bar{A}-b}$

Notes: Approximation to the steady state elasticities. $\bar{\Psi}$ is the steady state value of the option value from the separation decision. The approximation is based on $\beta\rho \simeq 1$.

The upper part of table 3 shows that there are essentially three options to generate lower average UE rates in Germany: First, a lower *efficiency of the matching function* $\varkappa$ in Germany and an associated increase in the effective cost per unit of vacancies posted. The parameter captures in a reduced form sense frictions in the entry process like for example skill, occupational, or regional mismatch. A decline lowers the average UE rate, increases the surplus of the match, and lowers the average EU rate. The UE rate volatility remains unchanged because the increase in the surplus is accompanied by an increase in the effective cost to post a vacancy, keeping the percentage change in the surplus largely unaffected. The EU rate volatility increases by the same factor as the average UE rate declines (cp. table 4, first row) matching therefore all of our stylized facts qualitatively.
Second, *higher benefits b* as argued for in Ljungqvist and Sargent (2008) lowers the surplus of the match, lowers profits, and the average UE rate. The lower surplus would lead to a counterfactual

[22] To obtain the elasticities we make use of the fact that both the average rates and the volatilities are simple functions of the average match surplus (see table 3). Implicitly differentiating the steady state surplus equation yields the results. To ease readability we use again some simple approximations. The online appendix reports the exact elasticities without the approximation and explains the derivation in detail. The approximation captures the effects quantitatively reasonably well as we show in the next section.

increase in the average EU rate, so this option has to rely on additional firing taxes τ to jointly explain the mean rate differences across countries. Still, the mechanism will be inconsistent with the second moments of the data. The third row in table 4 shows that the reaction of the EU rate volatility ($|\tilde{\sigma}_{eu}|$) is always lower by a factor $1-\varrho$ compared to the reaction of the UE rate volatility ($|\tilde{\sigma}_{ue}|$). Therefore a decline in the surplus will unambiguously decrease the contribution of inflows relative to outflows in the unemployment volatility and will be inconsistent with the empirical evidence. A similar argument can be made for higher firing taxes (cp. table 4, fourth row).

Third, a higher *bargaining power* of the worker μ in Germany lowers the share of the surplus accruing to the firm. This lowers the incentives to create jobs, and thereby lowers the average UE rate. This mechanism is used for example in Blanchard and Portugal (2001) who argue that the employment protection legislation implicitly increases the threat point of the worker, and therefore effectively raises the bargaining power. The effect of a higher bargaining power on the average surplus is ambiguous and depends on the distance to the Hosios point of efficiency (cp. table 4, second row). Two counteracting forces are at work: a higher bargaining power lowers the UE rate which tends to increase the average surplus as explained above. But at the same time the outside opportunity of the worker raises relative to the current match which tends to lower the average surplus. Exactly at the Hosios condition the surplus is minimized[23] and the bargaining power of the worker μ is equal to the matching elasticity ϱ. To see this, we implicitly differentiate the steady state surplus with respect to the bargaining power

$$\frac{\partial S}{\partial \mu} = \frac{\mu - \varrho}{1-\mu} \frac{\beta \bar{S} \bar{\pi}_{ue}}{\varrho \left(1 - \beta + \beta \left(\bar{\pi}_{eu} + \frac{\mu}{\varrho} \bar{\pi}_{eu}\right)\right)}$$

It can be immediately verified that the surplus has its minimum at the Hosios condition.[24] Intuitively, the benchmark scenario of a perfectly competitive market without search and matching friction would compete the surplus to zero, making all workers employed, and force wages to be equal to productivity. The matching frictions impose a deviation from this benchmark leading to a positive surplus. The social planner minimizes this deviation by putting the economy at the Hosios condition given all other parameters. As a result, the EU rate volatility is also minimized. Due to the sign switch in the elasticity of $|\tilde{\sigma}_{eu}|$ at the Hosios condition (cp. table 4, second row) a cross-country change in the bargaining power can therefore increase or decrease the EU rate volatility depending on the initial conditions. To the extend that the change in the bargaining power is large enough the channel works similarly to a change in the match efficiency. It lowers the gains from posting a vacancy and simultaneously increase the surplus of the match. However, the outside opportunity of the worker is directly affected which tends to dampen the EU rate volatility

[23] Despite our endogenous destruction mechanism, it is straightforward to show that the Hosios condition still holds in our framework, conditional on interpreting the outside option as home-production or the value of leisure, not as a choice of the government.

[24] The second term is always positive, so that the extremum must be a minimum.

quantitatively.
Table 4 shows that larger firing taxes τ or differences in the continuation-cost variance ψ also affect the average UE rate but this is only through their effect on the average EU rate, so their impact turns out to be quantitatively small.

4.3 Quantitative Results

A lower matching efficiency moves the economy qualitatively in the right direction. We now show that it can explain the cross-country differences quantitatively.
In the calibration we harmonize 4 parameters to be equal across country and allow 5 parameters to vary. Data moments and estimated parameters are given in Table 5. We set the autocorrelation of the aggregate shock to $\rho = 0.975$ implying a standard estimate of 0.95 on a quarterly base, and normalize the productivity volatility to 1.4% for both countries in line with our empirical findings for the U.S. We set the discount factor $\beta = 0.996$ implying an annual interest rate of 4% and the matching elasticity $\varrho = 0.5$ in line with estimates reported in Petrongolo and Pissarides (2001). We normalize vacancy posting cost $\kappa = 0.38$ to obtain a probability of filling a vacancy of 90% per month for the U.S.[25] We assume these four parameters to be equal across countries. The remaining parameters $b, \psi, \tau, \varkappa$ are chosen to exactly match the average rates and the volatilities. Additionally we follow Hagedorn and Manovskii (2008) and choose the bargaining power μ to match the wage elasticity $|\sigma_w|$. We target $\sigma_w = 0.8$ in both countries in line with our empirical estimates reported below.[26] We see in Table 5 that the benefit level b, the firing tax τ and the idiosyncratic shock variance ψ appear to be similar across country. The main difference that arises is a substantially lower matching efficiency that declined by 65% and an increase in the bargaining power.
Next we investigate more carefully which of these differences are most important, i.e. explain

[25] The value is in between the estimates used in Shimer (2005) ($\kappa = 0.21$) and Hall (2008) $\kappa = 0.43$. The model depends essentially on the ratio $\frac{\varkappa}{\kappa}$ so our findings would also hold for an increase in vacancy posting cost. However, an increase in vacancy posting cost turns out to increase the probability of finding a worker, while evidence on open positions suggest that firms search considerably longer in Germany, in line with a decline in the average match efficiency. For the benchmark calibration we find $\pi_{ve} = 0.64$. Davis, Faberman, and Haltiwanger (2009) documents that the average job filling rate for the U.S. ranges from 16 to 25 working days during the period 2001 - 2006. Adding in weekends and holidays this period increases to 19 to 29 days. For West Germany the search duration, i.e. from the begin of search to signing the work contract, averages to 48 days for the period 1989 - 2001 with a low at 38 days in 1997 and a high in 1991 and 1992 of 57 days. The average time for which open positions are registered at the Employment Agencies shows similar pattern over time and the same level of 48 days for the corresponding period. The time of registration for open positions is available back until 1980 and averages to 43 days if the whole period is considered. If we consider the period from the begin of search to starting work instead of signing the contract, then the search time for the period 1989 - 2001 increases significantly to 76 days. The data on duration of open positions has been kindly provided by the IAB.

[26] The first-order approximation for the wage elasticity is

$$\sigma_w = \mu\sigma_S\left(1 - \beta\rho(1 - \bar{\pi}_{eu} - \bar{\pi}_{ue}) + \beta\rho\bar{\pi}_{ue}\frac{1-\varrho}{\varrho} - \bar{\pi}_{eu}(1 - \bar{\pi}_{eu})\beta\frac{\bar{S}}{\psi}\right)$$

For Germany and the U.S. $\sigma_w = 0.8$ is at the upper range of the estimates as we will discuss below and delivers fairly flexible wages, see Haefke, Sonntag, and van Rens (2007) for a U.S. estimate.

Table 5: Calibration

Parameter	$\varkappa$	μ	ψ	b/w	τ
U.S.	0.52	0.27	0.98	0.95	3.23
Germany	0.18	0.52	0.9	0.95	3.38
Data target	$\bar{\pi}_{ue}$	$\bar{\pi}_{eu}$	$\lvert\tilde{\sigma}_{eu}\rvert$	$\tilde{\sigma}_{ue}$	$\tilde{\sigma}_{w}$
U.S.	30.6	2.0	6.5	11.2	0.8
Germany	6.8	0.53	15.1	10.4	0.8

Notes: Data targets and calibrated parameters.

the bulk of the different labor market targets. We start from the calibrated U.S. economy and change one parameter at a time to match one target for the German economy (bold number). Table 6 reports in the first column the parameter that has been changed relative to the calibrated U.S. economy and the corresponding value. The cases $\mu = 0.5$ (Hosios condition) and $\mu = 0.73$ (volatilities identical to the U.S. benchmark) are included to highlight the changing effect of the bargaining power on $\tilde{\sigma}_{eu}$. Some points are worth noticing: (1) A decline in the efficiency of the

Table 6: Parameter experiments

		$\bar{\pi}_{ue}$	$\bar{\pi}_{eu}$	$\lvert\tilde{\sigma}_{eu}\rvert$	$\lvert\tilde{\sigma}_{ue}\rvert$	$\lvert\tilde{\sigma}_{w}\rvert$	$\frac{\lvert\tilde{\sigma}_{eu}\rvert}{\lvert\tilde{\sigma}_{eu}\rvert+\lvert\tilde{\sigma}_{ue}\rvert}$
	Germany (data)	6.8	0.53	15.1	10.4	0.8	61.1
(1)	$\varkappa = 0.14$	**6.8**	0.67	19.4	11.5	0.6	62.8
(2)	$\mu = 0.5$	19.3	2.12	5.9	11.3	0.87	34.3
(3)	$\mu = 0.73$	11.4	2.0	6.5	11.2	0.87	36.7
(4)	$\mu = 0.88$	**6.8**	1.75	8.4	11.3	0.87	42.6
(5)	$b/w = 0.99$	**6.8**	3.15	14.3	112	0.5	11.3
(6)	$\tau = 4.6$	26	**0.53**	8.1	16.8	0.85	32.5
(7)	$\psi = 0.7$	25	**0.53**	11.6	17.1	0.85	40.4

Notes: The first column gives the parameter and the corresponding value that has been changed relative to the calibrated U.S. economy. The bold number shows the targeted data point. The two cases where no data point is targeted examine the non-monotonic effect of μ on $\tilde{\sigma}_{ue}$.

matching process ($\varkappa$) can qualitatively and largely quantitatively account for the bulk of the cross country differences in the means and the volatilities. The EU rate volatility is a bit too high while the wage elasticity is too low. An increase in the bargaining power dampens both effects and allows us to align model and data (see Tables 4 and 5). As discussed above starting below the Hosios condition for the U.S. we observe a decline in the EU rate volatility for values below $\mu = 0.73$ [27], so our final parameter choice $\mu^{GER} = 0.52$ dampens the EU rate volatility. (2) An increase in the bargaining power alone, beyond the point of $\mu = 0.73$, starts to increase the average surplus in Germany and would qualitatively move the economy in the right direction, but leaves us

[27] We consider $\mu = 0.73$ in particular because there it holds that $\mu = \varrho + 0.23$ and we had $\mu^{US} = \varrho - 0.23$.

quantitatively substantially away from the observed differences. Both the changes in the average EU rate as well as in the EU rate volatility are too small. (3) As expected an increase in benefits (b) will increase the UE rate volatility substantially whereas the EU rate volatility increases only slightly. As we will show below this effect is not an artifact of the small surplus calibration but will also hold more broadly in a *'large surplus'* calibration with rigid wages. (4) An increase in firing taxes mechanically lowers the average EU rate, but has only a very modest impact on the average UE rate and almost no impact on the EU volatility while increasing the UE volatility. (5) Finally, the variance of the idiosyncratic shock process ψ lowers the average EU rate, but increases both the EU and UE volatility, leaving the contribution of the ins and the outs in the decomposition of the unemployment volatility unaffected.

To align model and data our findings suggest that a large fraction of the cross-country differences are due to a substantially lower matching efficiency in Germany.

4.4 Transmission of Shocks

In this section we ask whether the highlighted differences matter for the transmission of shocks. We first present some evidence that the simple shock structure still captures important aspects of the data. We then report impulse response function to highlight differences in the transmission of shocks.

We evaluate the performance of the model by studying its predictive power. We estimate for both countries the underlying shock processes using a Kalman filter on GDP growth. We feed the estimated processes into the model using the estimated parameters of Table 5 and predict all endogenous variables applying an HP-filter ($\lambda = 100,000$) to the resulting time-series. Figure 2 graphically illustrates the successes and failures of the simple model. The time series patterns of the unemployment rate are predicted well and the model captures the EU rate and the UE rate dynamics in both countries. The model reproduces the time series pattern of earnings in Germany very well, while it fails to predict the earnings in the nineties for the U.S.. Still, the success for both countries lends some credit to the underlying mechanism explored in this paper.

Figure 3 shows impulse-response functions for the calibrated economies after a large negative shock of 4% that roughly matches the increase in the unemployment rate at the beginning of the big recession in Germany in the 80s after the second oil crisis. The impulse-responses uncover the key cross-country difference in the reaction to a shock. In the U.S. the unemployment rate peaks three quarters after the initial shock, while in Germany it peaks after nine quarters uncovering a substantial propagation to shocks in Germany. Hence, the German recovery is very sluggish. In fact, five years after the shock has hit the economy the German unemployment rate is still 23% away from its long-run average while the U.S. is only 12% above its steady state value. Although peak unemployment is similar across the two countries, the unconditional standard deviation of the unemployment rate in the model for Germany is still 29% larger than for the U.S., consistent with

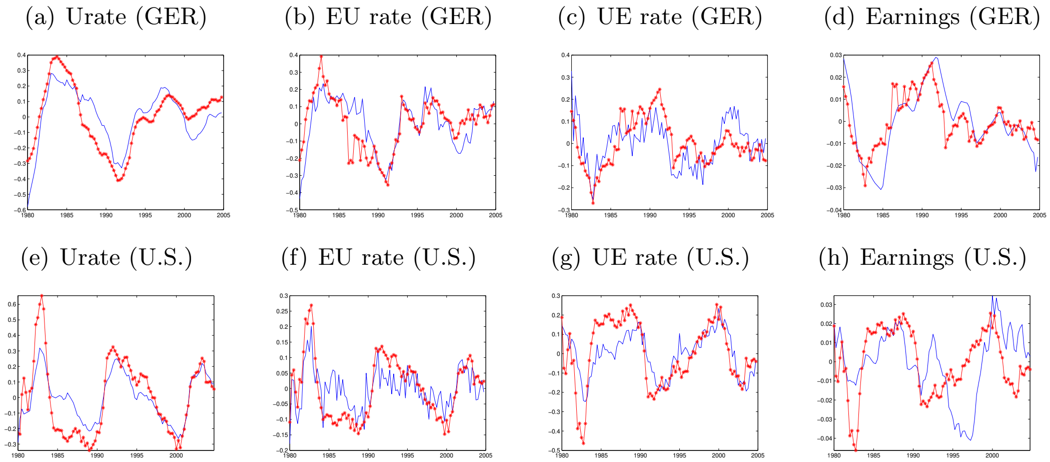

Figure 2: Data and predicted series

Notes: The figure plots the model predictions (red dotted lines) and the data (blue solid line). The prediction is based on a technology process obtained from a Kalman filter on GDP growth. Model and data are in logs and are HP-filtered with $\lambda = 100,000$. Earnings for Germany refer to median earnings obtained from the microdata.

our empirical findings.

The difference in the reaction of the unemployment rate to shocks are not generated by differences in the reaction of wages. Despite the lower bargaining power in the U.S. the wage reaction was targeted to be the same across the two countries, and is confirmed in figure 3(e). The difference is also not due to differences in the UE reaction given that the reaction is almost identical in both countries (figure 3(d)). What causes the sluggish response in Germany is an interplay of the strong reaction in the EU rate causing a strong rise in unemployment (figure 3(c)) and the low reemployment probabilities due to the lower average UE rate caused by the lower matching efficiency in Germany. The low reemployment probability in Germany leads to a situation where we observe output growth after 6 quarters in combination with increasing unemployment rates for additional 3-4 quarters. For large shocks such a recovery might therefore well look like a period of a *jobless recovery* Shimer (2010).

5 Additional Explanations and Robustness

The analysis of the benchmark model has shown that a simple version of firing tax is likely not the main driving force for the observed cross-country differences. A large literature though has argued that the employment protection legislation in general might be an important source for the cross-country differences at least in the average transition rates. Ljungqvist and Sargent (2008) argue that a combination of higher benefits, larger firing taxes and micro-economic turbulence, that is skill losses after a separation, can explain the U.S.-Europe differences in the mean rates. Moreover the employment protection legislation might shield high tenured and low tenured workers

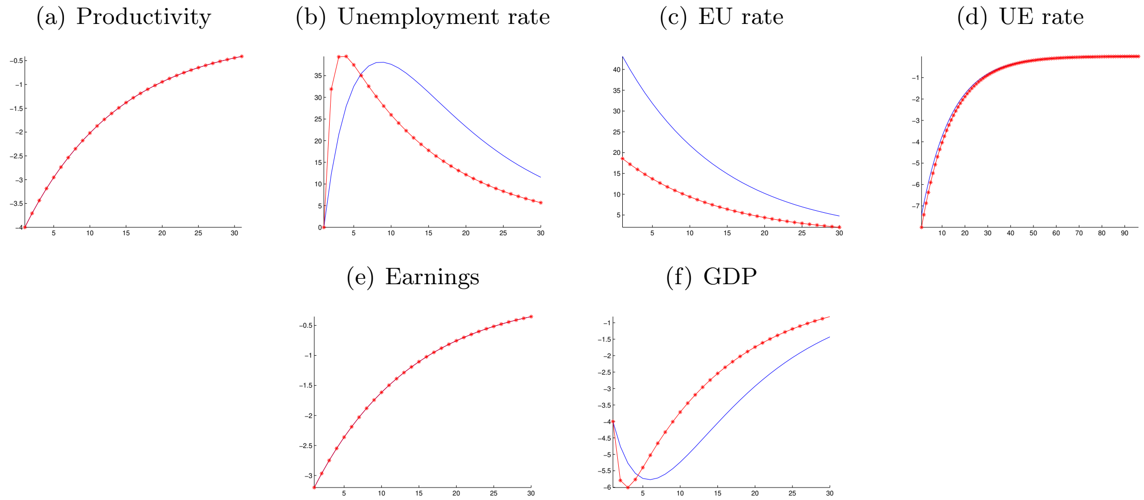

Figure 3: Impulse response functions

Notes: The figure plots the impulse response functions for the U.S. (red dotted lines) and Germany (blue solid line) on a quarterly scale.

differentially. This effect might give firms incentives to circumvent firing taxes for low tenured workers using for example short-term employment contracts Costain, Jimeno, and Thomas (2010); Bentolila, Cahuc, Dolado, and Barbanchon (2010).

In this section we examine whether similar mechanisms could also explain the stylized facts for the cross-country differences between Germany and the U.S. For this purpose we first present empirical results for the labor market dynamics in Germany controlling for tenure. We then offer a theoretical exploration based on the augmented model with idiosyncratic shocks that uses our empirical findings to discriminate between these alternative explanations.

5.1 Tenure - Data

To examine the role of skill accumulation and employment protection empirically we construct transition rates conditioning on tenure for four tenure classes. For Germany this data can be constructed from the employment histories. For the U.S. we rely on irregular supplements to the CPS that report information on tenure with the current employer.[28] For both countries we report in Table 7 time averages of monthly rates across all available observations.

We find that both countries show a strongly declining pattern of transition rates with tenure. This holds for all separation rates either to a new firm, to unemployment or to inactivity. In Germany

[28] We use the *Occupational Mobility and Job Tenure* supplements for the years 1983, 1987, 1991, 1996, 1998, 2000, 2002, 2004, and 2006 where information on tenure is available. We link the supplement information to the basic monthly data files as described in Shimer (2007). Using the linked monthly files, we construct gross flow rates by tenure for the nine year/month pairs where tenure information is available. For the EE flows we can only use data after 1994 because EE flows can not be identified before (For details see Fallick and Fleischman (2004)). For details, see the technical appendix.

the average rates are substantially below the U.S. rates in all tenure classes. However, the decline across tenure groups is very similar in the two countries. In both countries the share of low tenured worker[29] in all EU transitions is larger than 50% but is a bit smaller in the U.S. (60%) compared to Germany (72%). For Germany we can also look at the volatilities of transition rates across tenure

Table 7: Transition rates by tenure classes

	TENURE IN YEARS	< 1	$1-2$	$2-5$	> 5
EU	Germany	1.8	0.7	0.4	0.2
	U.S.	4.7	2.4	1.6	0.8
EE	Germany	1.8	1.1	0.8	0.4
	U.S.	4.7	2.9	2.5	1.6
EN	Germany	1.9	0.6	0.4	0.2
	U.S.	5.0	2.8	2.2	1.7

Notes: Tenure categories are given in years. All transition rates are given as percentages of the workers in the respective tenure group and are averages over time. For Germany only workers in full-time employment over the period $1980-2004$ are considered. The U.S. rates are derived using the January and February supplements to the Current Population survey (CPS) using available supplements in the period $1983-2006$. Due to the rotation of the panel and the point in time information on tenure in the CPS, we report only transition rates in the month were tenure is available. U.S. transition rates are adjusted for seasonal effects and time aggregation to match their unconditional averages. The transition rates for Germany are constructed from employment histories and seasonally adjusted.

classes.[30] Interestingly, we find that the EU rate volatility is very large for all tenure classes and is, if anything, increasing over tenure.[31] We conclude from these observations that our findings of substantially larger EU rate volatilities is not driven by low tenured workers moving in and out of employment alone but that this facts holds more broadly over tenure classes.

5.2 Augmented Model

To investigate whether differences in the human capital accumulation process or differential firing taxes are major drivers of the labor market differences pointed out in section 2 we augment our benchmark model to allow for worker and match-specific human capital accumulation. To economize on the state space, we assume that employed workers can be in three tenure states, low, medium and high (L, M, H). We assume that workers stochastically gain match-specific skills by staying at their firm, i.e. accumulating tenure. We normalize the initial state and set match specific productivity in the lowest tenure state to $s_L = 1$. The worker needs on average 2 years to transit

[29]We define low tenured as tenure below 2 years.

[30]Since we only have information at a limited set of points in time for the U.S., we can not calculate reasonable volatilities for the different tenure classes there.

[31]The exact numbers for the standard deviations are 19.6, 17.4, 23.0, 23.4 where the first number refers to the lowest tenure class and all numbers are given as percentages. The correlation is strongly negative also across tenure classes. More details are provided in the online appendix.

to tenure level M, and another three years to transit to the final tenure state H. Workers with $2-5$ years of tenure (state M) have a skill level $s_M = (1+g_M)$ and workers with 5 years or more of tenure (state L) have skill level $s_H = (1+g_H)$. Upon separation the worker loses tenure. We target g_M and g_H to reproduce the declining EU transition rates in the United States. We find $g_M = 0.034$ and $g_H = 0.067$ so the yearly increase of skills in tenure is roughly 1.3%.[32].

To study skill losses we additionally assume that the worker can be in one of three worker specific skill states, namely bad, normal, or good with productivity denoted by z_B,z_N, and z_G respectively, so that the total number of idiosyncratic productivity states is nine. We assume that the skill process attached to the worker z_i is given by a discrete approximation to an AR(1) process with three states.[33] We set the autocorrelation coefficient at 0.98 on a monthly basis to generate a persistent process as in Costain, Jimeno, and Thomas (2010) and set the standard deviation to imply a shock size of 10% in our discrete approximation, normalizing $z_N = 1$.[34] Upon unemployment the workers also switch states according to this AR(1) process, so we have to keep track of the distribution of employed worker by skill and tenure level and unemployed worker by skill level.

Worker and match-specific transitions follow independent stochastic processes, so we calculate the appropriate transitions functions p_{ee}, p_{eu},p_{ue} and p_{uu} on the stacked vector of idiosyncratic states as the convolution of the two processes and assume that a particular individual state is the multiplication of the two processes.[35] We aggregate over the worker specific states and report the average for each tenure class.

We re-calibrate the remaining parameters to match the same aggregate statistics as in the benchmark case.[36] The upper part of Table 8 reports the calibrated U.S. economy together with the empirical targets. The last line in the upper part reports the data targets for Germany. In the lower part of the table we perform foure experiments similar to the ones in table 6. Again, we change parameters (first column) starting from the calibrated U.S. economy to match a German data target (bold number).[37]

[32] Altonji and Williams (2005) reports gain to tenure of 11% for ten years for the U.S., roughly in line with these numbers. Dustmann and Meghir (2005) report returns to tenure for skilled German worker between $1.7-2.4\%$.

[33] We use the method of Kopecky and Suen (2010) to obtain the conditional Markov transition kernel numerically.

[34] In contrast to standard models with endogenous destruction the variance of the worker specific shock process is less important for the business cycle dynamics given that separation rates are still governed by idiosyncratic match specific shocks with variance proportional to ψ which we again calibrate to reproduce the aggregate EU rate volatility of the U.S.. We varied the standard deviation between $5-20\%$ and re-calibrated ψ without affecting the results.

[35] That is the first state is $x_1 = s_L z_B, x_2 = s_L z_N, \ldots, x_9 = s_H z_G$. The resulting transition matrix $p_{ue}(x, x\prime)$ for example takes care of the fact that unemployed workers can only switch to low tenured jobs.

[36] We additionally introduce a stochastic probability of retiring to generate the tenure distribution. We set the work-life to 40 years as in as in Costain, Jimeno, and Thomas (2010) and assume that newly born workers are born with skill levels according to the invariant distribution of the Markov transition. We adjust the model equation accordingly. We see that heterogeneity lowers the average net replacement rate, but only very modestly. All other parameters are very similar to the benchmark case. This results for the U.S. economy in the following parameters $\kappa = 0.26$, $\varkappa = 0.52$, $\mu = 0.35$, $\psi = 1.08$, $\frac{b}{w} = 0.926$, and $\tau = 3.05$.

[37] We rely numerically throughout on a first order approximation, given that the state space has to include all employment states by skill, implying 18 state variables in the model.

Table 8: Experiments

		$\pi_{EU,L}$	$\pi_{eu,M}$	$\pi_{eu,H}$	π_{ue}	$\tilde{\sigma}_{ue}$	$\tilde{\sigma}_{eu,L}$	$\tilde{\sigma}_{eu,M}$	$\tilde{\sigma}_{eu,H}$	σ_w
	U.S. (Data)	3.55	1.68	0.8	30.6	11.2		*6.5*		0.8
	U.S. (Model)	3.55	1.68	0.8	30.6	11.2	4.6	5.8	6.7	0.8
	GER (Data)	1.3	0.4	0.2	6.2	10.5	18.4	23.5	23.4	0.8
(1)	$\varkappa = 0.12$	1.4	0.5	0.2	**6.2**	10.1	14.6	16.8	19.1	0.67
(2)	$\tau_M, \tau_H = 4.9$	3.7	**0.4**	**0.2**	28.9	14.3	5.2	9.0	10.0	0.82
(3)	$\tau_M, \tau_H = 4.5$ $\mu = 0.92$	2.9	**0.4**	**0.2**	**6.2**	12	8.0	12.6	14.5	0.94
(4)	*Turbulence*	2.6	0.9	0.4	21.5	19	6.4	9.1	12.3	0.85

Notes: The upper part reports the data. The lower part reports the experiments. $\pi_{eu,L}$, $\pi_{eu,M}$ and $\pi_{eu,H}$ denotes the EU rate for low (medium, high) tenured worker averaged over all idiosyncratic skill levels. The same applies for $\tilde{\sigma}_{eu}$. The value on the EU rate volatility for the U.S. marked by * is the average over all tenure classes due to data limitations.

5.2.1 Matching Efficiency Revisited

The first experiment decreases the matching efficiency ($\varkappa$) to show that the identified mechanism from the previous section still works in the extended model. The average UE rate falls. The surplus in each tenure class increases in Germany due to the lower average UE rates, so accumulated skills get more valuable. Upon separation high tenured workers will lose their tenure. Due to the long search duration it takes longer to accumulate human capital in a new match which makes German workers more reluctant to separate. The average EU rates fall in a way consistent with the observed tenure patter. Moreover, due to the larger surplus in each tenure class the EU rate volatilities increase.

5.2.2 Differential Firing Taxes

If differential firing taxes were an important driving force of the cross-country difference, one might suspect that short-term employment contracts would be one possibility to circumvent this friction.[38] To shed light on this alternative explanation we use in the second and third experiment differential firing taxes to explain the decline in Germany for higher tenured worker. We keep τ_L at its U.S. value and increase τ_M and τ_H to target the observed EU rates in Germany. We see in experiment 2 that the presence of tenure dependent firing taxes lead to a decline in the EU rates for protected workers and to an increase for unprotected workers. The EU rate volatility modestly increases for higher tenured workers due to a larger surplus, and remains largely unchanged for low tenured worker. The unemployment volatility is amplified because both the UE rate as well as the EU rate

[38]There is no direct evidence that short-term employment contracts increased substantially in Germany during the sample period, in contrast to southern European countries like Spain, which have witnessed large increases along this margin.

volatility increase. The contribution of the EU rate volatility though falls because the increase in the UE rate volatility dominates, in line with our findings for the benchmark model.

A firing tax by itself has only a very small impact on the average UE rate. If firing taxes affect in addition the threat point of the bargaining, the implicit bargaining power increases. The third experiment varies therefore jointly the firing taxes as well as workers bargaining power. As analyzed before, a substantial increase in the bargaining power will raise the surplus, if the deviation from the Hosios condition is large enough (we need a $\mu = 0.92$). Again, we would see a larger decline in the EU rates for high tenured workers by construction, a counterfactually high average EU rate for low tenured workers and a counterfactually low EU rate volatility. Moreover, the surplus of new hires tends to decline, increasing the UE rate volatility.

5.2.3 Human Capital Accumulation and Turbulence

The final experiment considers a version of *turbulence* along the lines of Ljungqvist and Sargent (2008) to study the role of worker and firm specific human capital. We assume that skills are more firm specific in Germany and might be lost after a separation. Concretely, we assume that workers with a good skill level lose their skills and become a normal type upon separation, while workers with normal skill level become bad types. That is a large fraction of the work force lose 10% of their skill level upon separation. This assumption transforms skills that are attached to the worker in the U.S. to skills that are more specific to the match in Germany. [39]

As a consequence of the higher risk of losing skills the surplus for medium and high skilled workers increases due to the deterioration of the outside opportunity. As a result the average EU rates decline for these groups. For low tenured workers the decline is not as pronounced as observed in the data. Two effects are at work: The increase in the average surplus tends to *increase* the average UE rate making it more attractive for firms to post vacancies because there is more to split. However, the composition of the unemployment pool changes. There are more bad types in the search pool, making it less attractive to post vacancies. In our calibration there are 44% bad types in the unemployment pool for the U.S. while in Germany, due to the skill losses, the number increases to 75%. If differences in the skill processes were the main driving force in explaining the empirical labor market differences, the deterioration of skill effect has necessarily to dominate to explain the low average UE rates observed in Germany as it does on our calibration. However, the resulting decline in the expected surplus from creating an open position implies that the UE rate volatility will increase and we find a counterfactual decline in the contribution of the EU flows relative to the UE flows in the unemployment volatility.

Our experiments show that the behavior of the transitions rates by tenure are potentially infor-

[39] We choose this calibration that is at the upper end of empirically plausible values (Fujita (2008), Burda and Mertens (2001)) to get the largest effects from our experiment. The flexible specification of the transition matrices would also allow for specifications where only a share of workers looses its skills, however, effects then would be even smaller.

mative to discriminate between different explanations studied in the literature. Differential firing taxes do not explain the low average transition rates of low tenured workers in Germany which should be less affected by firing restrictions. Differences in the idiosyncratic skill processes either increase the surplus, if they lead to more match specific skills in Germany, or decrease the expected surplus, if the cost to re-training low tenured workers is large or the search pool has very bad skills. In the former case the average UE rate should increase, because firms can exploit the worker better, in the latter case the contribution of the UE rate volatility would increase. Both implications are counter-factual. To explain the data one needs a mechanism that jointly increases the surplus and lowers the average UE rate.

Our quantitative results so far have relied on the small surplus calibration of Hagedorn and Manovskii (2008). We now show that our results still hold under an alternative set of assumptions that allow for a larger surplus calibration and study the impact of wage rigidities on the volatilities.

5.3 Rigid Earnings

The recent literature has stressed versions of wage rigidities as a potentially important source of an amplification mechanism that can explain the large hiring rate volatilities without relying on an outside option close to productivity (Shimer (2005), Hall (2005) and more recently Elsby and Michaels (2010)). Maybe surprisingly, we find empirically that German earnings[40] are not any more rigid in Germany compared to the U.S., though confidence bands are large. We show on theoretical grounds that strong versions of wage rigidity will affect the EU rate and the UE rate volatility symmetrically, leaving the contribution rate to the unemployment volatility unaffected.

5.3.1 Empirical Estimates

As an empirical measure of earning rigidity the literature typically uses an elasticity estimate on the reaction of wages or earnings with respect to a measure of the business cycle. As this measure of business cycle either productivity or the unemployment rate have been used.

These studies also differ in the way how they control for selection effects. Several approaches have been proposed to control for this composition bias. Following Solon, Barsky, and Parker (1994) we use a fixed group of individuals of continuously employed workers who stayed at the same firm over the whole sample period. This selection rules out work force composition effects because the composition of the group is fixed in terms of all observable characteristics.[41] The drawback of this

[40] We focus on earnings because our dataset does not have an hours worked measure. The online appendix documents that our earnings measure and aggregate measures of wages move almost one to one, and that the behavior of hours worked is likely not an important source of the cycle variation of earnings in Germany.

[41] This group is informative about the cyclicality of earnings because if repeated annual collective bargaining about earnings is very prevalent in Germany, earnings of continuously employed will follow the same cyclical pattern as earnings in overall labor market. It also addresses concerns regarding job composition over the cycle raised by Gertler and Trigari (2009) given that no transitions occur.

approach is the fact that that no transition occurs might be endogenous so the selected group might not be very representative. In a second approach we follow Bils (1985) by estimating individual earnings growth equations to difference out fixed effects. This rules out that observable composition effects affect average earnings growth rates over time. The last earnings reported for unemployed workers though might be a long period of time ago, possibly biasing our estimates. In a final approach we construct an earnings index following Haefke, Sonntag, and van Rens (2007). They propose to run a first stage regression of earnings on individual controls and to use the residuals of the regression for the index construction separately for all labor market transitions averaging over residuals in the cross-section.[42]

Table 9 shows the results. Haefke, Sonntag, and van Rens (2007) report earning elasticities for newly employed workers around 0.8 and for stayer of around 0.4 for the U.S. In Germany we obtain an elasticity estimate roughly between $0.5 - 0.8$ across all methods and subgroups used. The point estimates of the different methods controlling for selection give conflicting evidence on the question whether earnings for newly employed workers are more or less rigid than earnings for continuously employed. Overall the different subgroups seem to behave similarly over the cycle in line with union wage arrangements that apply to all workers in a particular industry. However, given the large confidence bands, we can not rule out the possibility that German earnings are more or less rigid compared to the U.S. We therefore turn briefly to a theoretical exploration.

Table 9: earnings elasticities

	EE	UE	stay	cont. empl.
index	0.69	0.53	0.68	0.80
s.e.	0.21	0.25	0.21	0.25
growth	0.33	0.86	0.66	0.62
s.e.	0.12	0.24	0.13	0.15
correlation	0.60	0.42	0.49	0.52

Notes: Annual earnings elasticities for full-time employed workers. *index* refers to the earnings index (mean) using a first difference filter. *growth* refers to the estimation using individual growth rates. All elasticities are for annual changes and with respect to productivity (GDP p. empl.).

[42] We refer for details of the estimation procedure and a variety of robustness checks to the online appendix accompanying this paper.

5.3.2 Theoretical Exploration of Wage Rigidities

While the micro-foundations on the form of wage rigidities differ substantially across papers[43], the basic mechanism is similar: making wages rigid over the cycle increases firm profits in a boom more than proportional, so the percentage change in firm profits is amplified and in turn the UE rate volatility increases. We capture this effects using a countercyclical outside payment $b\exp(\varphi(x)a)$ with $\varphi(x) < 0$.[44] Conditioning on the tenure status we can make wages for different subgroups, i.e. newly employed worker or continuously employed workers, rigid to a different degree.

We use the same calibration strategy as before with the exception that we now target a benefit to output ratio of 80%, which we call a 'large surplus' calibration.[45] We then use $\varphi = -2.9$ to generate the UE volatility observed in the data.[46] We still target a wage elasticity of 0.8 initially, so the bargaining power for the U.S. has to increase substantially. Compared to the benchmark calibration the total surplus of a match increases but average profits accruing to firms remain small due to the large bargaining power of workers. Together with the countercyclical reaction of benefits the model is able to generate large UE rate volatilities despite a substantial decline in the level of the outside option.

Table 10 shows again the results for the calibrated U.S. economy in the upper part together with the data targets for the U.S. and Germany. The lower part again comprises the parameter experiments. The first experiment reproduces the outcomes for a change in the matching efficiency. We see the same picture emerging as in the the last section suggesting that our results are not driven by the small surplus calibration. The second experiment shows a change in the outside option consistent with a decline in the average UE rate. If larger benefits were the main driver in explaining the *average* UE rate differences we would need a large increase in the outside option given that the underlying elasticities changed Costain and Reiter (2008). At the same time, the impact on the volatilities is comparable to the baseline model. A similar argument applies to all other parameters discussed in section 4.

Making wages more (less) rigid in the third (fourth) experiment has the effect of increasing (decreasing) both the UE rate as well as the EU rate volatility in our model leaving the ratio, and therefore,

[43] Hall and Milgrom (2008), Shimer (2005), Rudanko (2009) and Elsby and Michaels (2010) among others propose different rationales and/or micro-foundations for these wage rigidities. The interesting mechanism in Elsby and Michaels (2010) for example differentiates between the marginal wage and the average wage. Firms with decreasing returns to scale take the impact on the average wage of an additional marginal hire into account. An extra worker in a boom not only adds to output, but reduces average wages, making profits more pro-cyclical. This mechanism though seems less appealing for Germany, where industry wide wage arrangements imply that a firm accepts wage changes for all of their workers and their hiring decision has no impact on the wages paid to other workers at the same firm.

[44] One way to think about this are extensions in the duration of benefit payments in a recession as has been recently observed in the United States.

[45] The number is taken from Elsby and Michaels (2010) who generate this outside option endogenously in a model with decreasing returns to scale, endogenous separation and a wage bargaining mechanism that differentiated the marginal wage and the average wage paid. Hall and Milgrom (2008) argue for a similar number.

[46] This results for the U.S. economy in the following parameters $\kappa = 0.06$, $\varkappa = 0.52$, $\mu = 0.91$, $\psi = 1.8$, $\frac{b}{w} = 0.8$, and $\tau = 5.15$.

Table 10: Experiments

		$\pi_{eu,L}$	$\pi_{eu,M}$	$\pi_{eu,H}$	π_{ue}	$\tilde{\sigma}_{ue}$	$\tilde{\sigma}_{eu,L}$	$\tilde{\sigma}_{eu,M}$	$\tilde{\sigma}_{eu,H}$	σ_w
	U.S. (Data)	3.55	1.68	0.8	30.6	11.2	-	-	-	0.8
	U.S. (Model)	3.55	1.68	0.8	30.6	11.2	4.4	6.3	7.7	0.8
	GER (Data)	1.3	0.4	0.2	6.2	10.5	18.4	23.0	23.4	0.8
(1)	$\varkappa = 0.11$	1.0	0.35	0.15	**6.2**	11.5	19.3	22.3	24.1	0.76
(2)	$b/w = 0.97$	4.8	2.5	1.3	**6.2**	146	12.0	13.4	17.2	0.82
(3)	$\varphi = -14.5$	3.6	1.7	0.8	30.6	38.6	14.9	19.4	**22.5**	0.47
(4)	$\varphi = -1.45$	3.6	1.7	0.8	30.6	7.3	2.9	4.3	5.6	0.85
(5)	$\varphi_L = -13.6$ $(\varphi_M, \varphi_H) = -2.9$	3.6	1.7	0.8	30.6	46.2	**18.1**	3.6	4.7	0.77
(6)	$\varphi_L = -2.9$ $(\varphi_M, \varphi_H) = -13.6$	3.6	1.7	0.8	30.6	9.2	3.4	20.5	**23.7**	0.5

Notes: The upper part reports the data. The lower part reports the experiments. $\pi_{EU,L}$, $\pi_{EU,M}$ and $\pi_{EU,H}$ denotes the EU-rate for low (medium, high) tenured worker averaged over all idiosyncratic skill levels. The same applies for $\tilde{\sigma}_{EU,\cdot}$.

the decomposition of the unemployment volatility almost unaffected. If wage rigidities affect the surplus they will, in a model with endogenous separation, also affect the EU rate volatility.

The fifth experiment looks at wage rigidities that only affect newly employed workers, i.e. we make wages for low tenured jobs more rigid. We see that this channel will increase the EU rate volatility for low tenured workers, but also the UE rate volatility. However, the EU rate volatility is reduced for high tenured workers, which is counterfactual. The last experiment reverts the argument and makes wages for medium and high tenured worker's more rigid, while leaving the wage rigidity for low tenured workers at their U.S. value. We see that although the EU rate volatility increases for high tenured workers the price to pay would be an EU rate volatility that is substantially too low for low tenured workers.

Our findings imply that there is a tight connection between a version of rigid wages and the EU rate volatility. Subgroups of workers, i.e. newly employed workers, which might have a different wage elasticity would in our model also experience a different behavior of the separation decision compared to workers who were continuously employed. Our data on the uniform increase in the EU rate volatility by tenure and our empirical finding of a similar wage elasticity across subgroups though suggests that differences in wage rigidities are likely not the prime driving force for the U.S.-German cross country differences in the second moments.

6 Conclusions

We document in this paper large differences in the average transition rates and the behavior of the EU rate volatility in Germany in comparison to the U.S. The second moments of the data offer identification restrictions that help to disentangle the importance of institutional factors in explaining the large cross-country differences in the first moments. We show that some of the usual 'suspects' for the transatlantic division, employment protection, union bargaining or the benefit system are likely not the main driving force of the observed differences. We traced the differences between Germany and the U.S. back to one factor, namely inefficiencies in the matching process. Matching inefficiencies in our model capture, in a reduced form sense, frictions in the entry process of creating new positions. Our findings suggest that understanding the details of this labor market friction in a more micro-founded way will be a quantitatively important factor in understanding the cross-country differences in labor markets and in the transmission of shocks.

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A Appendix

A.1 Unemployment decomposition

We describe here briefly the decomposition proposed in Fujita and Ramey (2009) and our extension. The decomposition of Fujita and Ramey is a two state, two transition rates decomposition. The idea of the decomposition of the unemployment volatility into contribution rates from EU and UE flows is to take an approximation around trend unemployment

$$\begin{aligned} u_t &\approx \frac{\pi_{eu,t}}{\pi_{eu,t}+\pi_{ue,t}} \\ \log\left(\frac{u_t}{\bar{u}_t}\right) &= (1-\bar{u}_t)\log\left(\frac{\pi_{ue,t}}{\bar{\pi}_{ue,t}}\right) - (1-\bar{u}_t)\log\left(\frac{\pi_{eu,t}}{\bar{\pi}_{eu,t}}\right) + \epsilon_t \\ du_t &= dUE_t + dEU_t + \epsilon_t \end{aligned}$$

where $\pi_{eu,t}$ denotes the EU rate and $\pi_{ue,t}$ is the UE rate. A bar denotes the trend component of the respective variable. $\log(u_t/\bar{u}_t)$ measures the relative deviation of the unemployment rate from its trend.

Fujita and Ramey (2009) show that the variance of $\ln(u_t/\bar{u}_t)$ can then be decomposed such that $1 = \beta_{\pi_{ue}} + \beta_{\pi_{eu}} + \beta_\epsilon$ where $\beta_x = \frac{cov(du_t, d\pi_x)}{var(du_t)}$. Their decomposition allows us to obtain two separate components (and an error term) for the importance of the respective series in explaining the cyclical variation of the unemployment rate. Using an equivalent steady state approximation for the three state case and defining weights $\alpha := \frac{\bar{\pi}_{nu}}{\bar{\pi}_{ne}+\bar{\pi}_{nu}}$ and $\lambda_{ij} := (1-\bar{u})\frac{\bar{\pi}_{ij}}{\bar{\pi}_u}$, as well as the (weighted) average of separation and hiring rates $\bar{\pi}_u := \bar{\pi}_{eu} + \frac{\bar{\pi}_{nu}}{\bar{\pi}_{ne}+\bar{\pi}_{nu}}\bar{\pi}_{en}$ and $\bar{\pi}_e := \bar{\pi}_{ue} + \frac{\bar{\pi}_{un}}{\bar{\pi}_{ne}+\bar{\pi}_{nu}}\bar{\pi}_{ne}$, we obtain an extended decomposition

$$\begin{aligned}
\log\left(\frac{u_t}{\bar{u}}\right) &= \log\left(\frac{\pi_{eu,t}}{\bar{\pi}_{eu}}\right)\lambda_{eu} - \log\left(\frac{\pi_{ue,t}}{\bar{\pi}_{ue}}\right)\lambda_{ue} \\
&\quad + \log\left(\frac{\pi_{en,t}}{\bar{\pi}_{en}}\right)\alpha\lambda_{en} - \log\left(\frac{\pi_{ne,t}}{\bar{\pi}_{ne}}\right)(1-\alpha)(\lambda_{ue}+\lambda_{un}-\lambda_{eu}) \\
&\quad + \log\left(\frac{\pi_{nu,t}}{\bar{\pi}_{nu}}\right)\alpha(\lambda_{eu}+\lambda_{en}-\lambda_{ue}) - \log\left(\frac{\pi_{un,t}}{\bar{\pi}_{un}}\right)(1-\alpha)\lambda_{un} + \varepsilon_t \\
du_t &= dEU_t + dUE_t + dEN_t + dNE_t + dNU_t + dUN_t + \varepsilon_t
\end{aligned}$$

Using again $\beta_x = \frac{cov(du_t, d\pi_x)}{var(du_t)}$ a similar covariance decomposition as in Fujita and Ramey (2009) of the form $1 = \sum_{i=1}^{n}\beta_i + \varepsilon_t$ applies.[47]

A.2 Data

A.2.1 Data description

The data is taken from the IAB regional files that cover the period from January 1975 to December 2004. The data consists of daily employment records of workers that have been employed for at least one day in a job under mandatory social security. The dataset comprises a 2% representative subsample of workers drawn from these records. Once an individual has been put into the sample, the full employment history of this individual during the sampling period is observed. The employment history consists of employment spells that are subject to mandatory social security and unemployment spells where social security benefits have been paid. The sample therefore does not contain spells in public service (*'Beamte'*), self-employment, and periods of inactivity. We describe below in detail how we control for these periods by constructing artifical spells. Still, the data covers about 80% of the German workforce.

A.2.2 Sampling period and sample selection

Due to measurement problems in unemployment during the years 1977 and 1978 we use the first five years $(1975-1979)$ only as a pre-sample and start our main analysis in 1980.

In a first step sample selection, we drop all individuals where the East-West information is missing (2,787 individuals dropped) or information regarding the current job[48] (14,490 individuals dropped). Furthermore, we drop homeworkers ('Heimarbeiter') from the sample (7,315 individuals

[47]The formula is similar to the first difference filter obtained in Petrongolo and Pissarides (2008), though they essentially lump together the rates $dEN_t + dUN_t$ and the corresponding inflow rate into $dNE_t + dNU_t$. In fact the inactivity flows are hard to interpret in their decomposition. It is important to note that the decomposition does not rely on knowing the state of non-employed workers, which is not available for Germany but that only the (gross) flows are needed. A detailed derivation is available upon request.

[48]stib information missing.

dropped). This results in a dropping rate of 1.81% for the whole sample, and leaves us with a sample of employment histories for $1,336,357$ individuals. After the German reunification the data contains employment histories with spells that are located in East Germany. Since the East German labor market was subject to additional regulations and restructuring after the reunification, we exclude in a second step all persons with employment spells in the East from our sample. This leaves us with a final sample of $1,087,555$ individuals. From these records we drop all marginal employment spells to avoid mismeasurement because marginal employment spells are only reported for the last five years of the sample period.

A.2.3 Construction of monthly employment histories

The employment history is given as a collection of employment spells on a daily basis. A new spell can either occur due to administrative reasons of the social security system or changes within a given firm. Importantly for our analysis, every change of employers or the beginning of an unemployment or a inactivity spell is recorded in the data. Regularly, individuals have periods of parallel employment in the sample. This is reported as multiple spells. For every spell, we observe whether it is full-time, part-time, or starting in 1999 marginal employment We apply a hierarchical ordering to classify these spells.[49]

Our basic time-period is one month. We adopt the ILO timing convention to measure the employment status of a person in a given month. For each month we determine the Monday of the second week in the month and take the week starting from this Monday as our reference week. We look at all spells that overlap with this week. If only one spell overlaps, then this spell determines the labor market status in the current month. If several spells overlap, we again use a hierarchical ordering of spells[50] From this classification of monthly employment states, we construct time-series at monthly frequency. To check whether a person stays with the same employer, we use the establishment number of the employment spells. This implies that a transition of a person between establishments but within the same firm is counted as a job-to-job transition. The definition of who is counted as unemployed follows from the content of the dataset. A person is unemployed if she receives unemployment benefits or other benefits on the basis of the Social Security Code III ('Sozialgestzbuch III'). We can not follow the ILO definition that is based on interview questions

[49]If persons have parallel spells in their employment history, we consider only what we call *primary spells*. The idea is to consider the employment spell that generates the most income and occupies the most working time of an individual. To identify the *primary spell*, we apply a hierarchical selection procedure. If a person is simultaneously employed full-time and part-time, we label him or her as full-time employed and drop the part-time spells, if a person has two part-time employments, we follow the ordering in the dataset that applies a hierarchical ordering based on income and part-time status over parallel spells, finally, if a person has simultaneously employment and unemployment spells, we label the employment spells as primary to be consistent with the procedure in the next step of determining the employment status.This problem only arises with marginal employment and can therefore be disregarded for the analysis in this paper.

[50]A full-time employment spell dominates part-time spells and any employment spell trumps unemployment or inactivity spells.

on job search and willingness to take up employment because this is unobservable in our sample. We label inactive employment that is reported in the dataset as currently not working. These spells are periods of sustained employment relationships which are currently inactive, i.e. the worker does not work and no income is paid. Examples for these periods are maternity leave, long periods of illness, or sabbaticals. We construct additional inactivity spells as residual spells in the dataset. The additional spells are included if a person is not observed in the sample for some time period between two spells. To deal with persons entering the sample or dropping out of the sample, we introduce additional labor market states that we label *labor market entry* and *retirement*.[51]

A.2.4 Earnings

The earnings reported at one spell is the average daily earnings of an individual during the employment spell[52]. We do not observe hours worked but observe whether the person is full-time, part-time, or from 1999 on in marginal employment. We use earnings of the *primary spell* for the analysis in this paper. We deflate earnings using the annual German CPI obtained from the Bundesbank. We adjust observed earnings in the sample along two dimensions, we impute earnings below and above the social security thresholds following Gartner (2005) and we adjust for the change in earnings reporting in 1983 following Fitzenberger (1999). An extensive discussion about the methods can be found in the online appendix.

A.2.5 Aggregate data

Our GDP measure for Germany is GDP per capita. We use GDP per capita because of the large inflows to West Germany after the fall of the Berlin Wall but before the official reunification. To obtain productivity, we divide by the number of employed persons. The time series for West Germany at quarterly frequency are only available until $1992Q4$ afterwards only GDP series for Germany are available at a quarterly frequency. We merge the two series in $1992Q4$ and run an ARIMA X-12 outlier correction on the combined series. The outlier correction controls for additive outlier, temporary, and permanent shifts in the data. The earnings series for Germany are median earnings of full-time employed workers from our dataset. We deflate all series using the CPI. The unemployment rates for Germany are available at monthly frequency for West Germany and we aggregate to quarterly frequency by taking quarterly averages of monthly rates. GDP, GDP

[51]The labor market entry state is an artificial state that we add before the first employment state. The retirement state is an artificial state at the end of the labor market history. We assign it to persons that are 55 or older when they have their last observed spell. The retirement state is by construction an absorbing state and persons that enter will be dropped from the analysis one month later. Persons that are below 55 and have no future spells in the sample are labeled as *other employment* and are no longer considered after the transition into this inactivity state, i.e. they do not generate transitions out of inactivity. Persons that are below 55 but have future spells are labeled as *out of the labor force*. The *labor market entry* state, the reported spells of inactivity, and the *out of the labor force* spells constitute the pool from which all inactivity transitions originate.

[52]The working period is not adjusted for weekends or holidays.

per employed (productivity), earnings, and unemployment rates for the U.S. have been obtained from the bureau of labor statistics (BLS). Except for the EE rates that have been obtained from Fallick and Fleischman (2004) all data on transition rates has been obtained from Shimer (2007). The transition rates by tenure groups for the U.S. are authors' calculations. We rely on irregular supplements to the CPS that report information on tenure with the current employer. These are the *Occupational Mobility and Job Tenure* supplements for the years 1983, 1987, 1991, 1996, 1998, 2000, 2002, 2004, and 2006. We link the supplement information to the basic monthly data files as described in Shimer (2007). Using the linked monthly files, we construct gross flow rates by tenure for the nine months where tenure information is available. The reported transition rates are time averages. Due to the rotation of the panel and the point in time information on tenure in the CPS, we report only transition rates in the month where tenure is available. U.S. transition rates are adjusted for seasonal effects and time aggregation to match their unconditional averages. The transition rates for Germany are constructed using the employment histories that have been constructed from the IAB panel. All data that is generated based on our own calculations is seasonally adjusted at a monthly frequency using the X-12 ARIMA method.

A.3 Data details

A.3.1 Imputation method for censored earnings

Earnings in the sample are top-coded at the upper contribution limit ('Beitragsbemessungsgrenze') of the German social security system, and bottom-coded at the marginal employment contribution level ('Geringfuegigkeitsgrenze'). For some of the steps in the analysis we need an uncensored earnings distribution. For these steps we impute earnings above and below the two censoring points using the method proposed in Gartner (2005). The same approach is used in Dustmann, Ludsteck, and Schönberg (2009) who use the previous vintage of the same dataset. The method runs a Tobit regression on log earnings. As controls, we use a fourth order polynomial in potential experience, a sex dummy, a foreigner dummy, and education dummies for the three groups of low, medium, and high education. We run the regression separately for each year and each employment state (full-time, two part-time states, and apprenticeship). We impute earnings under the assumption of a normally distributed error term. This assumption is also used in Dustmann, Ludsteck, and Schönberg (2009) who do an extensive sensitivity analysis to this specification.

A.3.2 Correction for structural breaks

Starting in 1984 the earnings data also includes overtime and bonus payments. We correct for this structural break using the method proposed in Fitzenberger (1999). His procedure leaves the median and all observations below the median unchanged and corrects earnings observations only above the median. The approach is based on measuring the excess growth of the upper earnings quantiles between 1983 and 1984. It corrects all earnings before 1984 if observed earnings in 1984 are in the upper half of the earnings distrinution using an adjustment factor that is a combination of the excess growth rate from 1983 to 1984 and the quantile position at that point in time. For further details see Fitzenberger (1999).

A.3.3 Measurement error

For variables regarding the job status, earnings, or the duration of the job the data contains virtually no measurement error because it is taken from the social security records that are used to determine social security contributions and benefits. The personal characteristics that we observe with every spell, such as year of birth, education, industry, and location of the employer may, however, contain measurement error. Fitzenberger, Osikominu, and R.Voelter (2006) point out that the education variable may be subject to higher measurement error and provide imputation and correction rules for this variable. We adopt their imputation and correction procedure and determine the highest attained education level of an individual over the employment history to group persons into education groups. The low education group comprises all individuals with no vocational training, the medium education group all individuals with vocational training or high-school ('Abitur') but no vocational training, and high-school and vocational training, and the high education group all individuals with technical college degree or university degree. The variable year of birth is censored for all observations in the employment history, if a person is at at least one spell below 16 or older than 62.[53] In the first case, we set year of birth as if the person is 15 at the first spell and in the second case as if the person is 63 at the last spell. We recover age at all spells consistently.

A.4 Labor market transition rates

Table 11 gives the transition rates for males and females and table 12 gives transition rates by education levels. For a definition of the different education groups see A.3.3.
Table 13 gives the cyclical properties of employment outflow rates for different tenure classes in Germany. Since we observe the U.S. data only at a limited set of points in time, we can not provide reasonable statistics for the U.S.. Most importantly and discussed in the main text the volatility of EU rates is always larger than the unconditional EU rate for the U.S. and is increasing across tenure classes. Table 14 reports the cyclical properties of the outflow rates from inactivity. For Germany the mean rates are given in brackets because in our dataset we do not observe the whole universe of persons in inactivity. The cyclical properties align with the U.S. counterparts especially regarding the correlation with GDP.

[53]Due to the sample length both cases can not occur simultaneously.

Table 11: Labor market transition rates by sex

	sex	mean	std	corr.
EU	Males	0.6	18.5	−0.81
	Females	0.5	10.5	−0.76
EN	Males	0.9	7.3	0.52
	Females	1.1	5.9	0.48
UE	Males	6.8	11.7	0.36
	Females	5.4	10.4	0.59
UN	Males	4.4	11.4	0.48
	Females	5.6	9.5	0.28
EE	Males	0.9	15.6	0.61
	Females	0.9	16.1	0.68

Notes: Transition rates and standard deviations are given as percentages. Correlations give the correlation coefficient with GDP.

A.5 Unemployment decomposition

Table 15 provides the decomposition of the unemployment volatility based on the decomposition proposed by Petrongolo and Pissarides (2008). This decomposition includes transitions from and to inactivity (not in the labor force) but does not allow to separate the contributions of EN and NU flows from the contribution of UN and NE flows.
Table 16 provides the decomposition for different subgroups using the decomposition proposed by Fujita and Ramey (2009) and our extension to the three state case.

A.6 Earnings cyclicality

A.6.1 Estimation

If transitions from employment to unemployment and vice versa over the business cycle do not uniformly occur over all groups of workers the composition of the characteristics of employed workers changes over the business cycle. For example, it might be the case that during a recession especially low-skilled, i.e. with lower education, or younger workers account for the increasing flow from employment to unemployment. In this case we should expect that average earnings resp. wages of the group that remains employed rises mechanically because average worker quality rises. This composition bias would bias earnings resp. wages upward in recessions. Similarly, during a boom the average worker quality might decrease because now the lower skilled and/or younger workers make up for a large share of the unemployment to employment flows so that average earnings resp. wages are now biased downward for all employed. In total, this composition bias might lead to less comovement of aggregate series over the business cycle than it is the case at the

Table 12: Labor market transition rates by education

	education	mean	std	corr.
	low	0.5	13.8	-0.60
EU	medium	0.5	16.2	-0.83
	high	0.4	12.5	-0.53
	low	1.4	9.0	0.37
EN	medium	0.9	6.1	0.59
	high	1.1	12.6	0.17
	low	3.4	14.5	0.42
UE	medium	6.8	10.2	0.41
	high	6.6	11.7	0.52
	low	5.2	12.0	0.23
UN	medium	4.8	10.5	0.50
	high	5.5	9.2	0.23
	low	0.5	19.4	0.56
EE	medium	0.9	15.5	0.67
	high	1.1	14.1	0.50

Notes: Transition rates for all workers. Transition rates and standard deviations are given as percentages. Correlations give the correlation coefficient with GDP.

individual level.[54] Next, we describe several approaches to control for the composition bias that have been proposed in the literature and that we apply to obtain the estimates reported in the main paper.

Solon, Barsky, and Parker (1994) use a group selection procedure to fix the group of individuals in order to avoid changes in the composition over time. We follow their approach and identify ongoing job relations that exist not just on a year-to-year basis but over the whole sample period. This constitutes a particularly homogeneous subpanel of workers, namely those who had a job in 1975 and were continuously employed full-time at the same firm until 2004. In other words, for this group, we ensure that no EE transition and no EU transition happened during their entire work experience.[55] For this group we only have earnings information at annual frequency. Although the group of continuously employed workers is highly selective, it allows us to examine the earnings dynamics of very stable jobs. The selection procedure addresses, therefore, concerns regarding job quality over the cycle raised by Gertler and Trigari (2009). We consider this group to be especially informative because if collective union bargaining is important for individual earnings in Germany,

[54] Since the composition bias results from changes in the workforce composition over the business cycle, the bias should be increasing in the transition rates between labor market states. Given that transition rates for Germany are lower by a factor of $4-5$, we expect the composition effect to be lower for Germany. This is also suggested by a comparison of the correlation of earnings with GDP for the U.S. and Germany in table 1 in the main text.

[55] The group still consists of approximately $5,969$ workers and is therefore large enough to provide reasonable estimates.

Table 13: Transition rates by tenure over the business cycle over the period $1980-2004$

TENURE IN YEARS	<1	$1-2$	$2-5$	>5
		EU		
MEAN	1.8	0.7	0.4	0.2
STD	19.6	17.4	23.0	23.4
REL. SHARE	58.5	13.5	14.6	13.5
CORR	-0.77	-0.74	-0.73	-0.57
		EE		
MEAN	1.8	1.1	0.8	0.4
STD	12.0	15.9	17.4	14.6
REL. SHARE	41.7	15.1	21.8	21.5
CORR	0.58	0.58	0.61	0.51
		EN		
MEAN	1.9	0.6	0.4	0.2
STD	11.4	13.5	15.6	17.2
REL. SHARE	59.4	10.7	15.3	14.7
CORR	0.20	0.20	0.29	0.27

Notes: Tenure categories are given in years. All transition rates are given as percentages of the workers in the respective tenure group and are averages over time. Only workers in full-time employment are considered. Standard deviations are given as percentage deviations from trend of the rates (in logs). Correlations give the correlation coefficient with GDP.

then this group should obey the same cyclical pattern as other groups yet it will not be affected by the described composition bias. We report estimates of earnings elasticities for this group for each of the two methods proposed below. Furthermore, we run a regression directly on median earnings. Since this group is not affected by composition bias, the estimates should be the same as for the methods where we control for the composition bias.

As a second correctional approach, we follow Bils (1985) and estimate individual earnings growth equations using first differences to control for individual specific fixed effects. This approach might be restrictive if only a short panel dimension is available. In particular, we do not observe last earnings of unemployed workers. We overcome this problem by exploiting the panel dimension. We keep track of last earnings of unemployed workers and use them as a proxy for unobserved earnings in the regressions. We construct a sample comprising all spells with certain labor market transitions, e.g. UE transitions. For this sample we regress individual earnings growth for the particular labor market event on several individual control variables and productivity growth. We include a constant in the earnings growth regression. This constant captures the initial level effect after reemployment Jacobson, LaLonde, and Sullivan (1993); Burda and Mertens (2001). We perform a sensitivity check with respect to the length of the unemployment spell and the previous employment spell for EE transitions below. The labor market events are grouped by years, and

Table 14: Inactivity transition rates over the business cycle

Statistic	Transition rate	mean	std	corr
Germany	NE	(6.5)	9.4	0.49
U.S.		4.2	5.9	0.64
Germany	NU	(2.3)	8.5	-0.47
U.S.		3.6	7.1	-0.58

Notes: Mean transition rates for Germany are given in brackets because the pool of inactive workers is not observed in the dataset. Standard deviations are given as percentage deviations from trend of the rates (in logs). Correlations give the correlation coefficient with GDP.

Table 15: Unemployment decomposition sensitivity to filter choice

Country	Data	EU	UE	NU + EN	UN + NE	ε
Germany	IAB (Δ)	49.0	51.0			0
	IAB (Δ)	34.6	28.4	20.6	16.4	0.0
	IAB (HP)	42.5	24.6	-2.7	31.0	-0.3
U.S.	Shimer (Δ)	64.3	35.7			0.0
	Fujita/Ramey (Δ)	51.7	48.3			0.0
	Shimer (Δ)	40.1	30.5	12.1	17.3	0.0
	Shimer (HP)	20.1	48.6	6.6	24.0	0.7

Notes: Data period $1980q1 - 2004q4$. For Germany the transition rates are for all workers. The U.S. data is obtained from Shimer (2007) and Fujita and Ramey (2009). Contribution shares are given as percentage numbers. (Δ) denotes the cases where a 1st difference filter has been used. HP refers to the cases where a HP-Filter ($\lambda = 100,000$) has been used.

individual controls are a fourth order polynomial in potential labor market experience, dummies for sex, three education groups, and for foreigners. We also include a time-trend.

Although, the panel dimension of our dataset allows us to overcome missing pre-employment earnings for UE transitions, there might still be concerns regarding this approach. To overcome potential concerns, we follow Haefke, Sonntag, and van Rens (2007) who propose a wage index construction. They propose to control for observable characteristics like age, sex, education, and experience and to focus on the behavior of the residual. We follow their procedure and construct earnings indices for UE, EE, persons who stayed at the same firm throughout the year (stayer), and for the group of continuously employed workers described above. We use the same controls as in the individual earnings growth regression.

A.6.2 Results

Results are given in 9 in the main text. The elasticity estimates for the group of continuously employed workers is 0.80 if we consider the earnings index and 0.62 in the regression using individual growth rates. If we regress the growth rate of median (mean) earnings of this group directly on

Table 16: Unemployment decomposition for different groups

Sample	EU	UE	NE	EN	NU	UN	ε
males	64.3	35.4					0.3
	45.7	24.9	16.9	-4.1	8.3	8.4	-0.1
females	49.4	50.3					0.3
	33.5	28.0	25.4	-4.6	3.7	14.5	-0.3
low skilled	48.0	51.9					0.1
	29.2	26.8	31.6	-3.2	7.9	7.9	-0.2
medium skilled	63.7	35.9					0.3
	45.0	23.8	18.1	-4.5	7.1	10.8	-0.2
high skilled	51.7	48.2					0.1
	36.5	26.2	20.9	-1.0	6.7	10.3	0.4

Notes: Contribution of labor market transitions to unemployment fluctuations. Data is detrended using the HP-filter ($\lambda = 100,000$) for the period $1980q1 - 2004q3$.

productivity without correction for composition bias, we find an elasticity of 0.81(0.75) in line with hypothesis that there is no composition bias for this group. Table 17 provides a sensitivity check for estimates for the earnings elasticity of EU and EE transitions from the earnings growth regression. For EE transitions we consider only transitions out of jobs that lasted for a certain minimum number of days (column 1). For UE transitions we only include transitions that take place before a maximum number of days in unemployment (column 3). The exclusion of particularly short (long) employment (unemployment) spells, allows us to focus on homogeneous transitions that are not affected by composition effects in the spell duration before the transition. Different length of spell duration before the transition could be correlated with unobserved worker quality or in the case of a transition out of employment with the quality of the match or in case of a transition out of unemployment with the amount of depreciated human capital. We see that for the EE transitions the estimate is unaffected while it increases slightly if the considered transitions include also longer unemployment spells.

In table 18, we run the regression of individual growth rates using a least absolute deviations (LAD) estimation to reduce the influence of outliers on the earnings elasticity estimate. The estimates for workers staying on the job remain unaffected while for UE transitions the focus on the median decreases the estimate slightly while it increases it for the EE transitions.

In table 19, we use instead of first differencing the data the HP filter to obtain the cyclical component of the earnings index. The estimated elasticities increase slightly.

Table 17: Sensitivity of earnings elasticity to tenure and unemployment duration

$min(E)$	EE	$max(U)$	UE
30	33.6 (11.6)	30	63.2 (19.3)
90	37.8 (12.0)	60	70.3 (17.6)
180	38.1 (13.1)	90	69.2 (16.3)
360	39.0 (12.9)	120	67.8 (16.6)
720	36.0 (13.1)	180	72.4 (17.5)
1080	30.4 (12.3)	240	75.8 (18.7)
1440	33.5 (12.3)	300	75.2 (20.1)
1800	36.0 (13.3)	360	76.0 (21.1)

Notes: Annual earnings elasticity from individual earnings growth equation for job-to-job movers (EE) and job finder (UE). $min(E)$ gives the minimum days of tenure before the job-to-job transition for the transition to be considered. $max(U)$ gives the maximum unemployment duration in days before the transition to employment for the transition to be considered. Standard errors are clustered by years.

Table 18: Earnings elasticities using LAD

	EE	UE	stay	cont. stay
LAD	0.48	0.68	0.64	0.62
s.e.	0.12	0.23	0.13	0.19

Notes: Annual earnings elasticities for full-time employed workers. Earnings elasticities are estimated using a least absolute deviations (LAD). Standard errors are bootstrapped and clustered by years.

Table 19: Earnings elasticities using the HP-Filter

	EE	UE	stay	cont. stay
	91.3	63.8	68.1	88.7
s.e.	18.7	22.2	23.3	26.3
corr	0.71	0.51	0.52	0.58

Notes: Earnings elasticities using the earnings index and the HP filter ($\lambda = 100,000$).

A.7 Analytic elasticities

In the paper, we report analytic approximations for the elasticities of $\bar{\pi}_{eu}$, $\bar{\pi}_{ue}$, $\tilde{\sigma}_{eu}$, and $\tilde{\sigma}_{ue}$. Here we give the exact analytic expressions. We use $\eta(x, p)$ to denote the elasticity of expression x with respect to parameter p. To ease readability, we use the following shorthand expressions

$$\begin{aligned}
\Sigma &= 1-\beta+\beta\left(\pi_{eu}+\frac{\mu}{\varrho}\pi_{ue}\right) \\
\bar{\Psi} &= \psi\left((1-\pi_{eu})\log(1-\pi_{eu})+\pi_{eu}\log(\pi_{eu})\right)
\end{aligned}$$

$$\begin{aligned}
\eta\left(\pi_{\mathrm{eu}},\kappa\right) &= -\frac{\beta^2\mu S\left(\pi_{\mathrm{eu}}-1\right)\left(\varrho-1\right)\pi_{\mathrm{ue}}}{\Sigma\psi\varrho} \\
\eta\left(\pi_{\mathrm{ue}},\kappa\right) &= \frac{\varrho-1}{\varrho}+\frac{\beta\mu(\varrho-1)^2\pi_{\mathrm{ue}}}{\Sigma\,\varrho^2} \\
\eta\left(\pi_{\mathrm{eu}},\mu\right) &= -\frac{\beta^2\mu S\left(\mu-\varrho\right)\left(\pi_{\mathrm{eu}}-1\right)\pi_{\mathrm{ue}}}{\Sigma\psi\varrho\left(\mu-1\right)} \\
\eta\left(\pi_{\mathrm{ue}},\mu\right) &= \frac{\beta\mu\left(\mu-\varrho\right)\left(\varrho-1\right)\pi_{\mathrm{ue}}}{\Sigma\varrho^2\ \left(\mu-1\right)}-\frac{\mu\left(\varrho-1\right)}{\varrho\left(\mu-1\right)} \\
\eta\left(\pi_{\mathrm{eu}},\psi\right) &= -\frac{\left(\pi_{\mathrm{eu}}-1\right)\left(\tau+\beta\,S\right)}{\psi}-\frac{\bar{\Psi}\beta\left(\pi_{\mathrm{eu}}-1\right)}{\psi\Sigma} \\
\eta\left(\pi_{\mathrm{ue}},\psi\right) &= \frac{\bar{\Psi}\left(\varrho-1\right)}{\Sigma\varrho S} \\
\eta\left(\pi_{\mathrm{eu}},b\right) &= -\frac{\beta b\left(\pi_{\mathrm{eu}}-1\right)}{\Sigma\psi} \\
\eta\left(\pi_{\mathrm{ue}},b\right) &= \frac{b\left(\varrho-1\right)}{\Sigma\varrho S} \\
\eta\left(\pi_{\mathrm{eu}},\tau\right) &= \frac{\tau\left(\pi_{\mathrm{eu}}-1\right)\left(\Sigma-\beta\pi_{\mathrm{eu}}\right)}{\Sigma\psi} \\
\eta\left(\pi_{\mathrm{ue}},\tau\right) &= \frac{\tau\left(\varrho-1\right)\pi_{\mathrm{eu}}}{\Sigma\varrho S} \\
\eta\left(\pi_{\mathrm{eu}},\varkappa\right) &= \frac{\beta^2\mu S\left(\pi_{\mathrm{eu}}-1\right)\pi_{\mathrm{ue}}}{\Sigma\psi\left(\varrho-1\right)} \\
\eta\left(\pi_{\mathrm{ue}},\varkappa\right) &= -\frac{1}{\varrho-1}-\frac{\beta\mu\pi_{\mathrm{ue}}}{\Sigma\varrho}
\end{aligned}$$

$$
\begin{aligned}
\eta\left(\tilde{\sigma}_{eu},\kappa\right) &= -\frac{\beta\kappa\mu\rho\left(\frac{(\varrho-1)\pi_{\mathrm{ue}}}{\kappa\varrho}+\frac{\beta\mu(\varrho-1)^2{\pi_{\mathrm{ue}}}^2}{\Sigma\kappa\varrho^2}\right)\sigma_S}{\varrho}-\frac{\beta^2\mu S\left(\frac{\beta\rho\sigma_S}{\psi}-\beta\rho\sigma_S\tilde{\sigma}_{eu}\right)\left(\pi_{\mathrm{eu}}-1\right)\left(\varrho-1\right)\pi_{\mathrm{eu}}\pi_{\mathrm{ue}}}{\Sigma\psi\varrho\tilde{\sigma}_{eu}}\\
\eta\left(\tilde{\sigma}_{ue},\kappa\right) &= \frac{\beta\mu\left(\varrho-1\right)\pi_{\mathrm{ue}}\left(\varrho^2-\Sigma\rho\varrho\sigma_S+\beta\mu\rho\sigma_S\pi_{\mathrm{ue}}-\beta\mu\rho\varrho\sigma_S\pi_{\mathrm{ue}}\right)}{\Sigma\varrho^3}\\
&\quad -\frac{\beta\mu\left(\varrho-1\right)\left(\beta^2\rho\varrho^2S\sigma_S\pi_{\mathrm{eu}}-\beta^2\rho\varrho^2S\sigma_S{\pi_{\mathrm{eu}}}^2\right)\pi_{\mathrm{ue}}}{\Sigma\psi\varrho^3}\\
\eta\left(\tilde{\sigma}_{eu},\mu\right) &= \frac{\beta\mu^2\rho\left(\frac{(\varrho-1)\pi_{\mathrm{ue}}}{\varrho(\mu-1)}-\frac{\beta(\mu-\varrho)(\varrho-1){\pi_{\mathrm{ue}}}^2}{\Sigma\varrho^2(\mu-1)}\right)\sigma_S}{\varrho}-\frac{\beta\mu\rho\sigma_S\pi_{\mathrm{ue}}}{\varrho}\\
&\quad -\frac{\beta^2\mu S\left(\mu-\varrho\right)\left(\frac{\beta\rho\sigma_S}{\psi}-\beta\rho\sigma_S\tilde{\sigma}_{eu}\right)\left(\pi_{\mathrm{eu}}-1\right)\pi_{\mathrm{eu}}\pi_{\mathrm{ue}}}{\Sigma\psi\varrho\left(\mu-1\right)\tilde{\sigma}_{eu}}\\
\eta\left(\tilde{\sigma}_{ue},\mu\right) &= \frac{\beta\mu\left(\mu-\varrho\right)\pi_{\mathrm{ue}}\left(\varrho^2-\Sigma\rho\varrho\sigma_S+\beta\mu\rho\sigma_S\pi_{\mathrm{ue}}-\beta\mu\rho\varrho\sigma_S\pi_{\mathrm{ue}}\right)}{\Sigma\varrho^3\left(\mu-1\right)}\\
&\quad -\frac{\beta\mu\left(\mu-\varrho\right)\left(\beta^2\rho\varrho^2S\sigma_S\pi_{\mathrm{eu}}-\beta^2\rho\varrho^2S\sigma_S{\pi_{\mathrm{eu}}}^2\right)\pi_{\mathrm{ue}}}{\Sigma\psi\varrho^3\left(\mu-1\right)}\\
\eta\left(\tilde{\sigma}_{eu},\psi\right) &= -\frac{\left(\frac{\beta\rho\sigma_S}{\psi}-\beta\rho\sigma_S\tilde{\sigma}_{eu}\right)\left(\frac{(\pi_{\mathrm{eu}}-1)(\tau+\beta S)\pi_{\mathrm{eu}}}{\psi}+\frac{\bar{\Psi}\beta(\pi_{\mathrm{eu}}-1)\pi_{\mathrm{eu}}}{\Sigma\psi}\right)}{\tilde{\sigma}_{eu}}-\frac{\bar{\Psi}\beta\mu\rho\left(\varrho-1\right)\sigma_S\pi_{\mathrm{ue}}}{\Sigma\varrho^2S}-1\\
\eta\left(\tilde{\sigma}_{ue},\psi\right) &= \frac{\bar{\Psi}}{\Sigma S}+\beta\rho\left(\frac{\left(\pi_{\mathrm{eu}}-1\right)\left(\tau+\beta S\right)\pi_{\mathrm{eu}}}{\psi}+\frac{\bar{\Psi}\beta\left(\pi_{\mathrm{eu}}-1\right)\pi_{\mathrm{eu}}}{\Sigma\psi}\right)\sigma_S-\frac{\bar{\Psi}\beta\mu\rho\left(\varrho-1\right)\sigma_S\pi_{\mathrm{ue}}}{\Sigma\varrho^2S}\\
\eta\left(\tilde{\sigma}_{eu},b\right) &= -\frac{\beta b\left(\frac{\beta\rho\sigma_S}{\psi}-\beta\rho\sigma_S\tilde{\sigma}_{eu}\right)\left(\pi_{\mathrm{eu}}-1\right)\pi_{\mathrm{eu}}}{\Sigma\psi\tilde{\sigma}_{eu}}-\frac{\beta\mu\rho b\left(\varrho-1\right)\sigma_S\pi_{\mathrm{ue}}}{\Sigma\varrho^2S}\\
\eta\left(\tilde{\sigma}_{ue},b\right) &= \frac{b}{\Sigma S}+\frac{\beta^2\rho b\left(\pi_{\mathrm{eu}}-1\right)\sigma_S\pi_{\mathrm{eu}}}{\Sigma\psi}-\frac{\beta\mu\rho b\left(\varrho-1\right)\sigma_S\pi_{\mathrm{ue}}}{\Sigma\varrho^2S}\\
\eta\left(\tilde{\sigma}_{eu},\tau\right) &= \frac{\tau\left(\frac{\beta\rho\sigma_S}{\psi}-\beta\rho\sigma_S\tilde{\sigma}_{eu}\right)\left(\frac{(\pi_{\mathrm{eu}}-1)\pi_{\mathrm{eu}}}{\psi}-\frac{\beta(\pi_{\mathrm{eu}}-1){\pi_{\mathrm{eu}}}^2}{\Sigma\psi}\right)}{\tilde{\sigma}_{eu}}-\frac{\beta\mu\rho\tau\left(\varrho-1\right)\sigma_S\pi_{\mathrm{eu}}\pi_{\mathrm{ue}}}{\Sigma\varrho^2S}\\
\eta\left(\tilde{\sigma}_{ue},\tau\right) &= \frac{\tau\pi_{\mathrm{eu}}}{\Sigma S}-\beta\rho\tau\left(\frac{\left(\pi_{\mathrm{eu}}-1\right)\pi_{\mathrm{eu}}}{\psi}-\frac{\beta\left(\pi_{\mathrm{eu}}-1\right){\pi_{\mathrm{eu}}}^2}{\Sigma\psi}\right)\sigma_S-\frac{\beta\mu\rho\tau\left(\varrho-1\right)\sigma_S\pi_{\mathrm{eu}}\pi_{\mathrm{ue}}}{\Sigma\varrho^2S}\\
\eta\left(\tilde{\sigma}_{eu},\varkappa\right) &= \frac{\beta\mu\rho\varkappa\left(\frac{\pi_{\mathrm{ue}}}{\varkappa(\varrho-1)}+\frac{\beta\mu{\pi_{\mathrm{ue}}}^2}{\Sigma\varrho\varkappa}\right)\sigma_S}{\varrho}+\frac{\beta^2\mu S\left(\frac{\beta\rho\sigma_S}{\psi}-\beta\rho\sigma_S\tilde{\sigma}_{eu}\right)\left(\pi_{\mathrm{eu}}-1\right)\pi_{\mathrm{eu}}\pi_{\mathrm{ue}}}{\Sigma\psi\left(\varrho-1\right)\tilde{\sigma}_{eu}}\\
\eta\left(\tilde{\sigma}_{ue},\varkappa\right) &= \frac{\beta\mu\rho\varkappa\left(\frac{\pi_{\mathrm{ue}}}{\varkappa(\varrho-1)}+\frac{\beta\mu{\pi_{\mathrm{ue}}}^2}{\Sigma\varrho\varkappa}\right)\sigma_S}{\varrho}-\frac{\beta\mu\pi_{\mathrm{ue}}}{\Sigma\left(\varrho-1\right)}-\frac{\beta^3\mu\rho S\left(\pi_{\mathrm{eu}}-1\right)\sigma_S\pi_{\mathrm{eu}}\pi_{\mathrm{ue}}}{\Sigma\psi\left(\varrho-1\right)}
\end{aligned}
$$