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Abstract

We perform the Multifractal Detrended Fluctuation Analysis (MF-DFA) method to investigate the multifractal properties of the Moroccan All Shared Index (MASI) and the Moroccan Most Active Shares Index (MADEX) from the Casablanca Stock Exchange (CSE). By applying the MF-DFA method we first calculate the generalized Hurst exponents and we then deduce the Rényi exponents as well as the singularity spectrum of the MASI and MADEX indices. Furthermore, we perform the shuffling and the phase-randomization techniques to detect the sources of the multifractality. We show that there are two major sources of multifractality, the long-range temporal correlations and the fat-tail distribution. We show notably that the first source contributes mainly to the multifractality of MASI index while the two sources contribute almost equally to the multifractality of the MADEX index. By comparing the multifractal behavior of the MASI and MADEX indices we find finally that the first one exhibits a richer multifractal feature than the second one. This permits us to conclude that the greater is the stock market the more complex is the dynamics of the stock market index representing all of the market, which is traduced by richer multifractal behavior of this index. This study leads to the principal conclusion that the Casablanca Stock Exchange is characterized by a multifractal behavior.

Keywords: Multifractality, Generalized Hurst exponent, Rényi exponent, Singularity spectrum

1. Introduction

In financial markets, the variation of prices over time is governed by a highly complex law. This complexity is mainly caused by nonlinear interactions among heterogeneous agents and by complex events occurring in external environment. Recently, many researchers prove that financial markets exhibit properties resembling to those of fluid turbulence. In the two fields, it has been proved that fluctuations are characterized by turbulent feature, multiscaling properties and multifractal behavior. The multifractality is one of the most important concepts introduced in dynamical complex system of many fields as fluid, earthquakes, biology as well as finance.

Recently, dynamics of financial markets has aroused much interest of researchers from physical field as well as economical field leading to the emergence of a new field well known by Econophysics [1, 2]. The multifractality of financial markets has been handled by various econophysists using two powerful methods, the wavelet transform modulus maxima WTMM method [3,4,5,6,7] and the multifractal detrended fluctuation analysis (MF-DFA) method [5,6,7,8,9,10,11,12,13,14,15,16]. In this paper we deal with the last method as we consider it has the advantage of involving easy implementation and robust estimations [34].

The MF-DFA method is a robust and powerful technique which is proposed by Kantelhardt *et* al [17]. It generalizes the Detrended Fluctuation Analysis (DFA) method [18,19,20,21,22,28,29,30,31] which is a widely-used technique for the determination of mono-fractal scaling properties and the detection of long-range correlations in stationary time series. The advantages of MF-DFA over many techniques are the fact that it permits the detection of the multifractalilty behavior in stationary as well as non-stationary time series.

Many researchers have successfully applied the MF-DFA method to prove the multifractal behavior of financial time series of developed markets as well as of emerging markets. In this work, we applied the MF-DFA method to investigate the multifractal properties of the Moroccan All Shared Index (MASI) and the Moroccan Most Active Shares Index (MADEX) from the Casablanca Stock Exchange (CSE).

This paper is organized as follows. In Section 2, we give a brief description of the data investigated in the current study. Section 3 introduces in detail the MF-DFA method. In section 4, the MF-DFA method is used to analyze the multifractality properties of the Moroccan All Shared Index (MASI) and the Moroccan Most Active Shares Index (MADEX) from the Casablanca Stock Exchange (CSE). In this section, we show first how we obtain the generalized Hurst exponents and how we deduce the Rényi exponents as well as the singularity spectrum, and then we perform the shuffling and the phase-randomization techniques to detect the sources of the multifractality. In section 5, we finally conclude by a summarization of the main results of this paper.

2. Data

The Casablanca Stock Exchange (CSE) is the Africa's second largest Stock Exchange after Johannesburg Stock Exchange in South Africa. It achieves one of the best performances in the region of the Middle East and North Africa (MENA).

In this paper, we aim to apply the MF-DFA method to two daily closing prices data of the Casablanca Stock Exchange (CSE) from January 2nd, 1992 to June 4th, 2010: Moroccan All Shared Index (MASI) and Moroccan Most Active Shares Index (MADEX) (data source: http://www.casablanca-bourse.com/). The MASI index is a stock index that tracks the performance of all companies listed in the Casablanca Stock Exchange. It is one of two main indices at the Stock Exchange, the other being the MADEX which includes only the most active shares.

If we designate by P_t the price of the index on day t, we have then 4486 prices for each index in the studied period. In the present paper, the method is applied to the logarithmic returns of the index defined by:

$$r_t = \log\left(\frac{p_{t+1}}{p_t}\right) \tag{1}$$

We obtained then 4485 logarithmic returns for each index. MASI and MADEX logarithmic returns are respectively illustrated in figure 1 and figure 2.

Figure 1: Logarithmic returns of MASI series covering the time period January 2nd, 1992 to June 4th, 2010

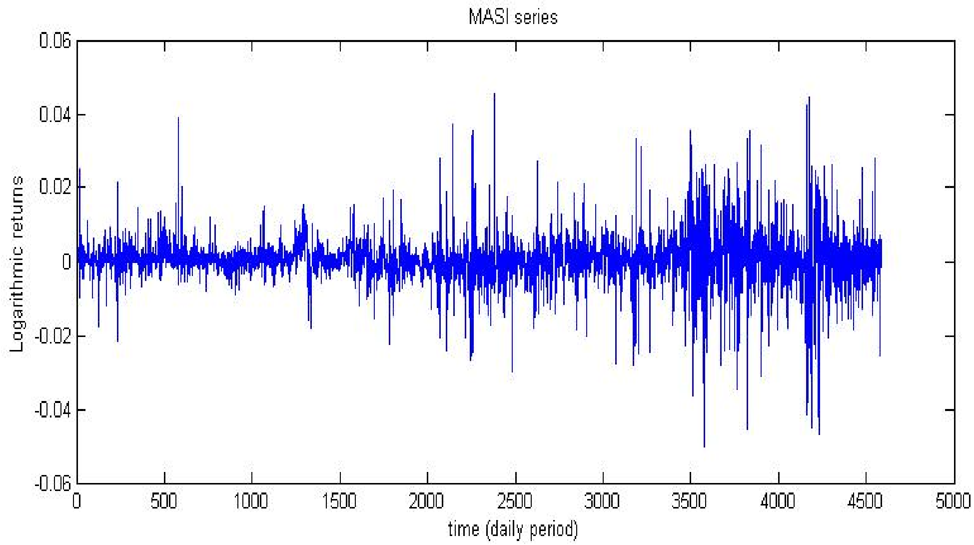
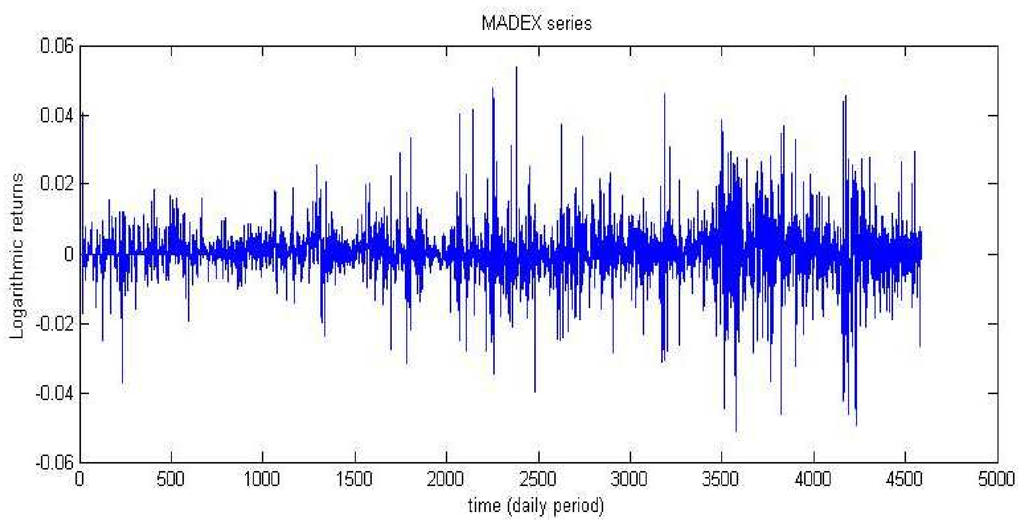


Figure 2: Logarithmic returns of MADEX series covering the time period January 2nd, 1992 to June 4th, 2010



3. Theoretical Background

As we have already seen, Multifractal Detrended Fluctuation Analysis (MF-DFA) method is a generalization of Detrended Fluctuation Analysis (DFA) method.

The MF-DFA algorithm consists of five steps. Let $x(k)$ be a time series of length N representing a logarithmic returns. We suppose that this time series is of compact support, i.e. $x(k)=0$ for an insignificant fraction of values only.

Step 1 : we determine the accumulated profile $y(i)$ of the time series $x(k)$ for $i=1,\dots,N$

$$y(i)=\sum_{k=1}^{i}\left[x(k)-\bar{x}\right] \tag{2}$$

where $\bar{x}$ denotes the mean of the time series $x(k)$ We can easily verify that $y(N)=0$

Step 2 : For a given time scale s, we divide the profile $y(i)$ into $N_s = int(N/s)$ non-overlapping segments of equal length s, where $int(\ )$ denotes the function which gives the integer part of a real number. Since in general N is not often a multiple of s, a short part of the end of the profile may be disregarded. In order to incorporate this ignored part of the series, we repeat the same procedure starting from the end of the profile. We obtain thus $2N_s$ segments.

Step 3: The aim of this step is first to estimate for each of the $2N_s$ segments a local trend by fitting a polynomial to the data. We then calculate the variances by two formulas depending on the segment v

for each segment $v = 1, \dots, N_s$

$$F^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left[y\left((v-1)s+i\right) - p_v^n(i)\right]^2 \tag{3}$$

- for each segment $v = N_s + 1, \dots, 2N_s$

$$F^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left[y\left((N-v-N_s)s+i\right) - p_v^n(i)\right]^2 \tag{4}$$

where $p_v^n(i)$ is the *n*-th order fitting polynomial in the segment v We can use linear DFA1, quadratic DFA2, cubic DFA3 or higher order polynomials DFAn for n>3.

Step 4 : By averaging the variances over all segments we obtain the qth order fluctuation function:

for $q \neq 0$

$$F_q(s) = \left[\frac{1}{2N_s}\sum_{v=1}^{2N_s}\left[F^2(v,s)\right]^{q/2}\right]^{1/q} \tag{5}$$

for $q = 0$

$$F_0(s) = \exp\left[\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln\left[F^2(v,s)\right]\right] \tag{6}$$

The aim of the MF-DFA procedure is principally to determine the behavior of the q dependent fluctuation functions $F_q(s)$ with regard to the time scale s, for various values of q Hence, steps 2 till 4 must be repeated for different values of time scales s leading us to a final step.

Step 5 : we analyze the multi-scaling behavior of the fluctuation functions $F_q(s)$ by estimating the slope of log-log plots of $F_q(s)$ vs. s or different values of q If the analyzed time series presents a long-range power law correlation as fractal proprieties, the fluctuation function $F_q(s)$ will behave, for large sufficiently values of s, as the following power-law scaling

$$F_q(s) \sim s^{h(q)} \tag{7}$$

In order to estimate the values of $h(q)$ for various values of q we regress the time series $h(q)$ on the time series $F_q(s)$

Generally the exponent $h(q)$ is a function depending on the variable q If we are in presence of stationary time series we obtain only the exponent $h(2)$ which is identically equal to the standard Hurst exponent H. Therefore, the exponent $h(q)$ generalize the Hurst exponent H and is commonly called the generalized Hurst exponent. To distinguish between monofractal and multifractal time series, we can say that if $h(q) = H$ (constant) for all values of q then the time series under study is monofractal, otherwise $h(q)$ is a monotonously decreasing function of q and the corresponding time series is multifractal. From equations (4) and (5) we can infer that for positive values of q the averaging fluctuation function $F_q(s)$ is dominated by the segments v holding large variances $F^2(v,s)$ Thus, for positive values of q, the generalized Hurst exponents $h(q)$ describe the scaling properties of large fluctuations. On the contrary, for negative values of q exponents $h(q)$ describe the scaling properties of small fluctuations.

It is well known that the generalized Hurst exponent $h(q)$ defined by the MF-DFA method is directly related to the multifractal scaling exponent τ(q) commonly known as the Rényi exponent :

$$\tau(q) = qh(q) - 1 \tag{8}$$

In the standard multifractal formalism, the multifractal scaling exponent τ(q) is usually defined via the partition function. Using the standard box counting formalism, Stanley et al [17] give a proof of equation (8).

It is clear that the monofractal time series are characterized by a linear form for the Rényi exponent:

$$\tau(q) = q \times H - 1 \tag{9}$$

where H is the Hurst exponent.

Another interesting way to characterize the multifractality of time series is to use the so called Hölder spectrum or singularity spectrum $f(\alpha)$ of the Hölder exponent α It is well-known that the singularity spectrum $f(\alpha)$ is related to the Rényi exponent $\tau(q)$ by the Legendre transform:

$$\alpha = \tau^{'}(q) \text{ and } f(\alpha) = q\alpha - \tau(q) \tag{10}$$

The Hölder exponent α characterizes the strength of the singularity and the singularity spectrum $f(\alpha)$ represents the Hausdorff dimension of the fractal subset with the exponent α The richness of multifractality can be determined by the spectrum width $\Delta\alpha = \alpha_{max-}\alpha_{min}$ .Thus, the wider is the spectrum, the richer is the multifractality behavior of the analyzed time series.

We can easily deduce from equation (8) the relation between the generalized Hurst exponent $h(q)$ and the singularity spectrum $f(\alpha)$

$$\alpha = h(q) + qh^{'}(q) \text{ and } f(\alpha) = q\left[\alpha - h(q)\right] + 1 \tag{11}$$

4. Empirical Results

In this paper, we perform the MF-DFA method over the MASI and MADEX indices of the Casablanca Stock Exchange (CSE), covering the time period January 2nd, 1992 to June 4th, 2010.

4.1. Multifractality Behavior of the MASI Index

As was previously seen, the MASI index is the main stock index representing all the companies listed in the Casablanca Stock Exchange.

The multi-scaling behavior of the fluctuation functions $F_q(s)$ versus the time scales s corresponding to the MASI index logarithmic return is investigated. The figures Fig.3 and Fig. 4 show respectively the log-log plot of $F_q(s)$ versus s for 54 values and 9 values of chosen values of q We can see also on fig. 4 that the plotted points for each of the 9 values of q are well fitted by least-square lines.

Figure 3: The plotting of $\log F_q(s)$ vs. $\log s$ of the MASI series corresponding to 54 values of q

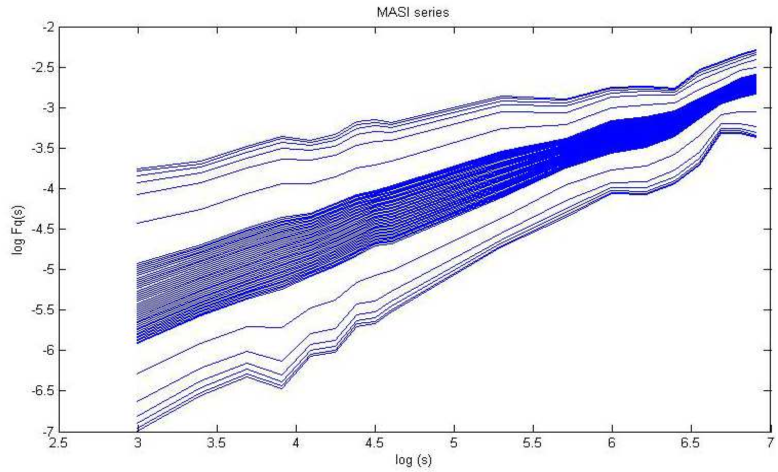
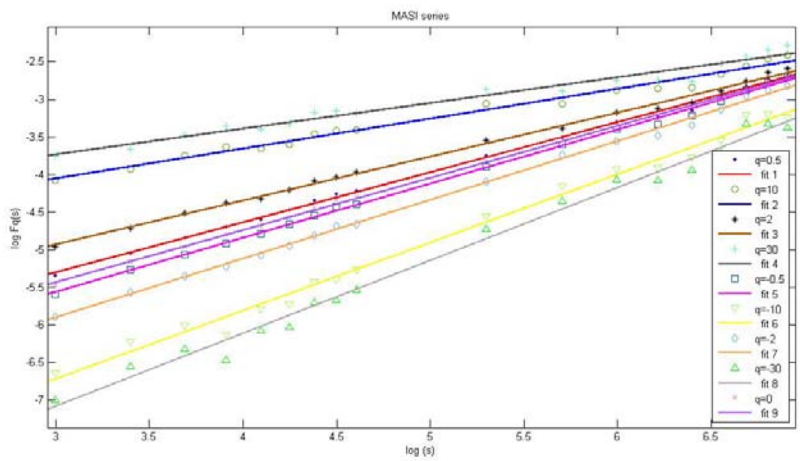
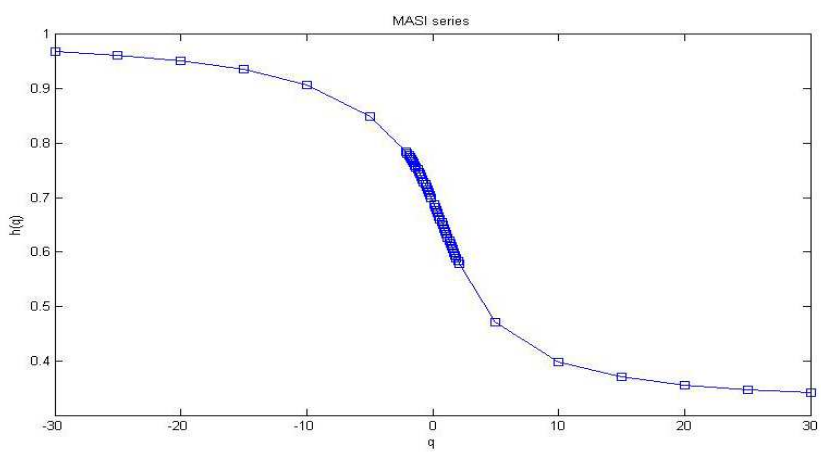


Figure 4: The plotting of $\log F_q(s)$ vs. $\log s$ of the MASI series corresponding to 9 values of q



The generalized Hurst exponents $h(q)$ are given by the slopes of the straight lines obtained by a least-square fit as shown in fig. 4. The figure Fig. 5 displays the generalized Hurst function $h(q)$ versus the variable q

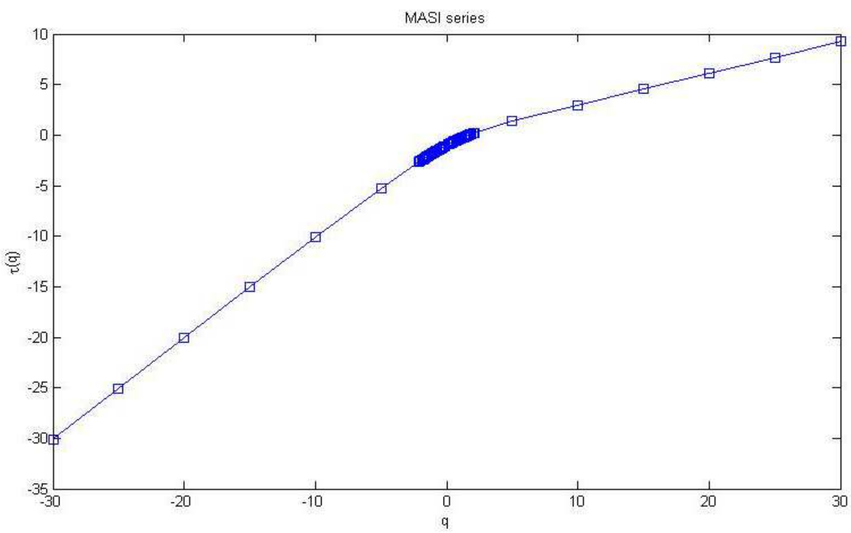
Figure 5: The generalized Hurst exponent $h(q)$ vs. q for the MASI series



We can remark that the function $h(q)$ presents a nonlinear decreasing form for increasing values of q which reveals the multifractality feature of MASI time series. When the slope is null it is obvious that the time series is monofractal.

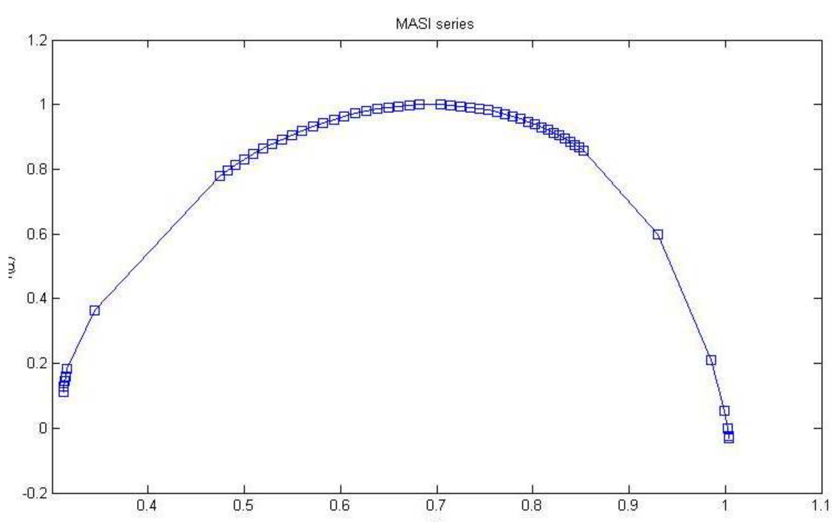
Another interesting way to detect the multifractality behavior of the time series under study is to calculate the Rényi exponent $\tau(q)$ related to the generalized Hurst exponents $h(q)$ by eq. 8. In fig. 6 is displayed the curve of the function $\tau(q)$ with respect the variable q thus, the nonlinearity shape of this curve reveals a multifractal behavior of the MASI data. The nonlinearity is confined in the vicinity of $q = 0$ corresponding to sufficiently small values of $|q|$

Figure 6: The Rényi exponent $\tau(q)$ vs. q for the MASI time series



Another nice way characterizing the multifractality behavior is the singularity spectrum $f(\alpha)$ The illustration of the calculus of $f(\alpha)$ with regard α is given in fig. 7.

Figure 7: The singularity spectrum $f(\alpha)$ vs. α of the MASI time series



We can notice in this figure that the singularity spectrum curve of MASI data has an inverted parabola shape. It is well known in case of monofractal time series that the curve reduces theoretically to a single point $\alpha = H$ with $f(\alpha) = 1$.

As we have already seen, we can measure the degree of multifractality by the calculus of the spectrum width $\Delta\alpha = \alpha_{max-}\alpha_{min}$ In our case we obtain $\Delta\alpha \simeq 0.6919$ The wider $\Delta\alpha$ is with regard to $\Delta\alpha = 0$ the richer and stronger the multifractality of data is.

Finally, we investigate the nature of the multifractality of MASI time series. It is generally admitted that there are two principal sources of multifractality for time series. The first one is the long-range temporal correlations and the second one is the fat-tail distribution [32]. To detect the contribution of each of the sources to the multifractality of MASI time series we performed two techniques: the shuffling and the phase-randomization.

The shuffling technique consists to destroy from the original time series any temporal correlations by performing the following steps:

(i) Generate pairs (p, q) of random integer numbers $(with\ p, q \leq N)$ where N is the total length of the time series under study.
(ii) Interchange entries p and q.
(iii) Repeat two above steps for 20N times.

The phase-randomization [33] consists on the randomization of the Fourier phases of original time series in order to destroy the nonlinearities stored in the phases. The time series generated by this procedure is generally called surrogate data. In the literature, there are various techniques of phase-randomization:

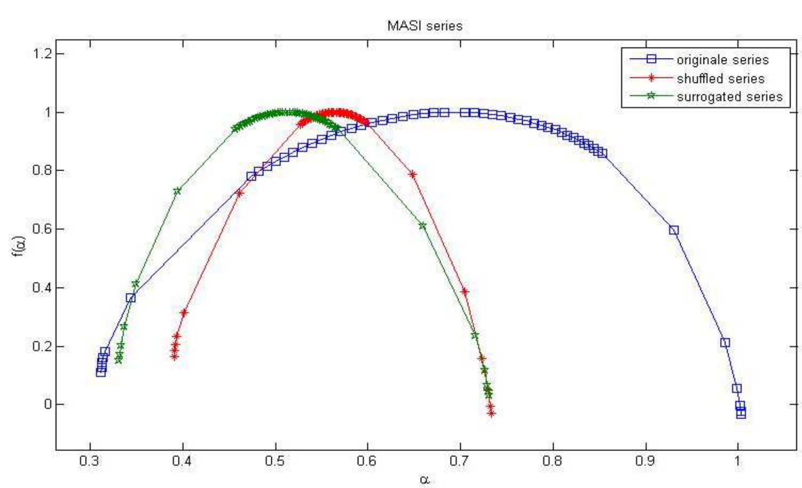
- Fourier transform (FT) algorithm [23],
- Amplitude Adjusted Fourier Transform (AAFT) [23],
- Iterated AAFT (iAAFT) algorithm [24],
- Statically Transformed Autoregressive Process (STAP) [25].

We can find a comparison of the three algorithms AAFT, iAAFT and STAP in [26, 27]. In our paper we choose the STAP method as it seems to be more appropriate and doesn't suffer from the bias in autocorrelation with regard to AAFT and iAAFT methods.

By means of the MF-DFA algorithm, we calculate for the shuffled and surrogate time series the generalized Hurst exponent and deduce the Rényi exponent as well as the singularity spectrum. We then compare with the results obtained for the original time series.

Figure 8: Shows the singularity spectrum for the original, shuffled and surrogate of MASI time series.

Figure 8: The singularity spectrum $f(\alpha)$ vs. α for original, shuffled and surrogated MASI time series

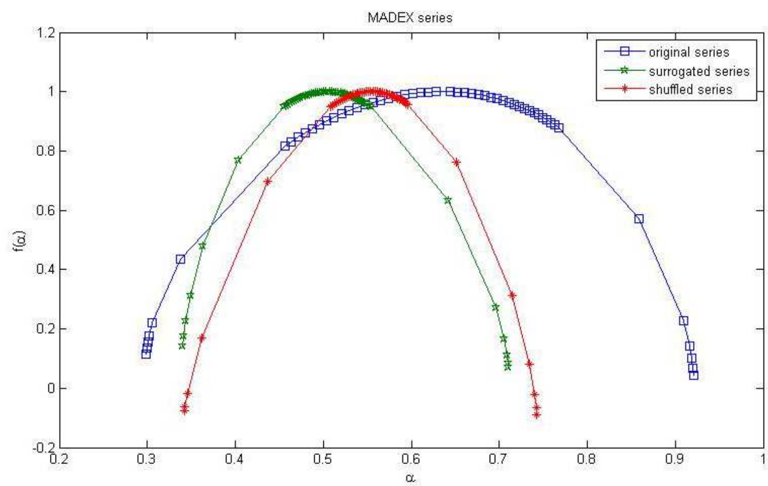


We can deduce from this figure and also from the calculus that the multifractality of the original time series has the greatest spectrum width $\Delta\alpha = 0.6919$ while the shuffled and the surrogate time series have respectively $\Delta\alpha = 0.3424$ and $\Delta\alpha = 0.399$.This proves that the shuffling and the phase-randomization techniques reduce all together the multifractality strength of the original time series and demonstrates that the long-range correlation contributes more to multifractality than the fat-tail distribution.

4.2. Multifractality Behavior of the MADEX Index

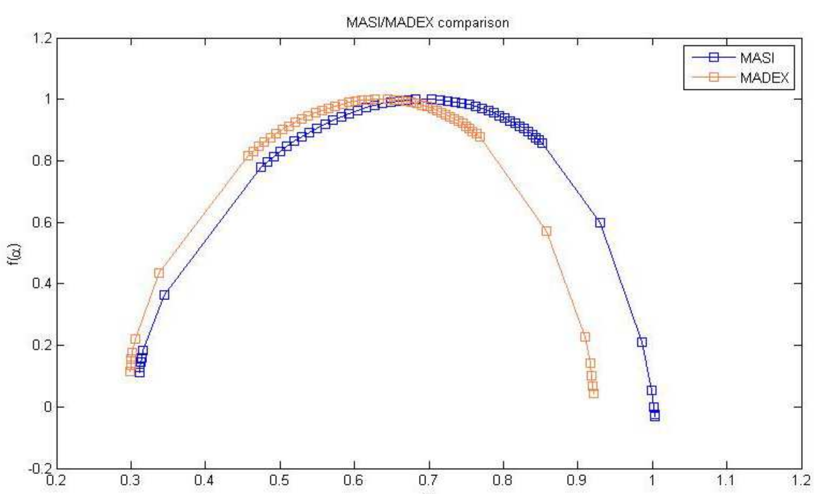
The MADEX index includes only the most active shares in the Casablanca Stock Exchange. We can notice from Fig. 9 clearly that the original, shuffled and surrogate MADEX times series exhibit all together a multifractal behavior. The original time series presents the strongest nonlinearity while the shuffled and surrogate ones have practically the similar nonlinear curve, with a slight dominance for the surrogate data. Thus the multifractality has been almost equally reduced by shuffling and phase-randomizing the MADEX original time series. This proves that the long-range correlation contributes slightly more to the multifractality of the MADEX time series than the fat-tail distribution.

Figure 9: The singularity spectrum $f(\alpha)$ vs. α for original, shuffled and surrogate MADEX time series



In Fig. 10 we compared the multifractal behavior of the MASI and MADEX indices.

Figure 10: The singularity spectrum $f(\alpha)$ vs. α of the original MASI and MADEX data



It is obvious to deduce from this last figure that the MASI time series shows a stronger multifractal behavior than the MADEX time series. This can be perhaps explained by the fact that the latter time series contains only the main active shares in the Casablanca Stock Exchange and consequently the dynamics of the financial market is less complex if it is represented by MADEX index.

5. Conclusion

In this paper, we investigate the multifractal properties of MASI and MADEX logarithmic returns using the multifractal detrended fluctuation analysis MF-DFA. We first calculate the fluctuation functions from which we estimate the generalized Hurst exponents. Then we deduce the Rényi exponents and the singularity spectrum of the two indices. Moreover, in order to detect the sources contributing to the multifractality, we perform the shuffling and phase-randomization (surrogate) techniques on the originate time series. Our results suggest that there are two principal sources of multifractality for MASI and MADEX indices: the long-range temporal correlations and the fat-tail distribution. By observing the curves representing the generalized Hurst exponents, the Rényi exponents as well as the singularity spectrum, we conclude that the long-range temporal correlations contributes mainly to the multifractality of MASI time series than do the fat-tail distribution. However, the two sources contribute almost equally to the multifractality of the MADEX time series with a slight dominance for the first source. Finally, the comparison of the multifractal behavior of MASI and MADEX data leads us to conclude that the first data reveals a richer multifractality feature than the second one. This result can be perhaps explained by the fact that the dynamics of the financial market is more complex by considering the MASI index which includes all the shares of the Casablanca Stock Exchange. This study leads to the principal conclusion that the Casablanca Stock Exchange is characterized by a multifractal behavior.

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