The Optimal Order Execution Problem within the Framework of a High-Frequency Trading - Sample Model

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THE OPTIMAL ORDER EXECUTION PROBLEM WITHIN THE FRAMEWORK OF A HIGH-FREQUENCY TRADING SAMPLE MODEL

Preliminaries

U.S. May 6, 2010 flash crash and the tremendous increase in trading volumes of High-Frequency Trading (HFT) has recently drawn massive public attention. The way HFT is perceived in public, media and regulatory discussions shows, that new academic research in this field is in demand to examine the real impact of HFT on market quality and liquidity.

Before we proceed to the main model, some terminology like High-Frequency trading and the optimal execution problem have to be clarified.

High-Frequency Trading

HFT enables sophisticated market participant to achieve higher rewards on their investments in technology and compensation for their operational risk exposure. But HFT is not a trading strategy. It is a technical means to implement established trading strategies[1]. HFT applies the latest technological advances in market and market data access like collocation, low latency and order routing to maximize returns on established trading strategies.

From this point of view HFT is nothing more than technological evolution[2], a form of automating real time tactical decisions based on previous experience and new back tested strategies. Despite many regulatory concerns this process is irreversible in terms of the technical means applied and HFT market share is growing very rapidly, as many industry and academic studies show. This tendency will continue in the next years due to fact that many exchanges have announced the implementation of infrastructure that enables HFT execution.

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[1] P. Gomber, B. Arndt, M. Lutat, T. Uhle: *High-Frequency Trading*, Deutsche Börse AG, Frankfurt 2011, p. 1.

[2] Compare: M. Durbin, *All About High-Frequency Trading*, McGraw-Hill, 2010, p. 193–194.

Table 1

High-Frequency Trading Market Sizing

Origin	Date of publication	US	Europe	Australia
TABB Group	Sep-09	61%		
Celent	Dec-09	42% of US trade volume	Rapidly growing	
Rosenblatt Securities	Sep-09	66%	~35% and growing fast	
Broogard	Nov-10	68% of Nasdaq trade volume		
Jarnecic and Snape	Jun-10		20% and 32% of LSE total trades and 19% and 28% of total volume	
Tradeworx	Apr-10	40%		
ASX	Feb-10			10% of ASX trade volume
Swinburne	Nov-10	70%	40%	
TABB Group	Jan-11		35% of overall UK market and 77% of turnover in continuous markets	

Source: P. Gomber et al.: *High-Frequency Trading*. Deutsche Börse AG, Frankfurt 2011, p. 73.

Optimal execution problem

In classical market theories it is assumed that assets are perfectly liquid. But in the real market we have to face many liquidity risks, like transaction costs and the uncertainty of trading.

An optimal execution problem exists, if a trader has a certain amount of securities $X_0 > 0$ to be traded in a limited time horizon T, with the aim of minimizing the expected cost due to the relative illiquidity of such asset. In this case he is challenged by the problem of choosing an optimal trading strategy to be executed within a trading period $[0, T]$ short enough to reduce the risk of uncertainty of future prices S.

The Model

In our problem the investor having to liquidate $X_0 > 0$ units of assets during the trading period $[0, T]$ is looking for a strategy that minimizes execution costs, but is also trying to minimize the risk of price decrease as a function of increased supply.

Therefore he chooses quantities $\xi_k \geq 0$ to sell at discrete times $t_k = k_\tau$ for $k = 1, ..., N$ such that $\Sigma_{k=1}^{N} \xi_k = X_0$, where N is the number of equal intervals of length $\tau = T/N$. For the moment, the investor applies this very naive strategy. This strategy needs to be improved in the first steps within the HFT framework.

In the Bertsimas and Lo approach to this problem[3], the price process of the asset could be considered as

$$\tilde{S}_k = \tilde{S}_{k-1} - \gamma\xi_\kappa + \epsilon_k, \quad \gamma > 0,$$

where $\{\epsilon_k\}_{k=1}^{N}$ is the zero mean and σ_ϵ^2 the variance sequence of independent and identically distributed random variables. The total cost associated to this strategy is

$$C(X) = X_0 S_0 - \Sigma_{k=1}^{N} \xi_\kappa \tilde{S}_k,$$

whereas the aim is to minimize under the constraint that $\Sigma_{k=1}^{N} \xi_\kappa = X_0$, the execution costs

$$\min_{\{\epsilon_k\}_{k=1}^{N}} E[C(X)].$$

There is a price dependence within this model, namely the price ξ_κ at time t_k depends on the price at time t_{k-1} which is again defined by the trade $\xi_{\kappa-1}$ at time t_{k-1}. This dependency is an additional element of risk which has to be minimized.

HFT knows such strategy: it is *time slicing*[4]. The idea is to display only a portion ξ_k of an order size X_0 that to the market will appear in "chunks" of seemingly unrelated orders. In addition, the next order is displayed after the previous one trades and after some additional "waiting" time interval has been reached. To make this strategy $\mathfrak{G}$ even more difficult to discover, the amount of shares ξ_k should be chosen randomly

$$\xi_1 = K_1 X_0, \quad \xi_2 = K_2 (X_0 - \xi_1), \ldots, \xi_N = K_N (X_0 - \textstyle\sum_{\mathrm{k}=1}^{N-1} \xi_k),$$

where $K: \Omega \to (0,1]_{\mathbb{Q}}$ is a rational number[5] defined in the probability space $(\Omega, \mathcal{F}, P)$ that

$$\{\omega: K(\omega) \le r\} \in \mathcal{F} \quad \forall r \in (0,1]_{\mathbb{Q}}$$

under the condition $\Sigma_{k=1}^{N} K_k = 1$.

Given the X_0 and the market situation $\mathfrak{M}\mathfrak{G}$ we are choosing the strategy dynamically $\mathfrak{G} = f(X_0, \mathfrak{M})$, which determines the number of trading intervals $N = f(\mathfrak{G})$.

Again the trading time t_k will be decomposed into semi-constant and only from the trading infrastructure latency[6] $\mathfrak{L}$ dependent execution time $t_e(\mathfrak{L})$ and randomly chosen "waiting" times:

$$t_w = (T - Nt_e)/K_k, \quad k = 1, 2, ..., N-1,$$

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[3] D. Bertsimas, A. Lo: *Optimal control of execution costs*, „Journal of Financial Markets" 1998, 1, p. 1–50.

[4] M. Durbin: *All About High Frequency Trading*, McGraw-Hill, 2010, p. 55–56.

[5] The rational numbers better fit to the general case where fractional orders are allowed. With no impact on the model structure we can limit this condition to integers.

[6] The round trip delay time is not constant; it varies within a wide range. There seems to be a non-deterministic factor that impacts latency variability and for a very short time period can be considered as constant. More about see: R. Kay: *Pragmatic Network Latency Engineering: Fundamental Facts and Analysis*, cPacket Networks 2009, Working Paper.

that

$$t_k = t_e + t_w, \qquad \Sigma_{e=1}^{N} t_e + \sum_{w=1}^{N-1} t_w = T.$$

In practice, this strategy improves our model in a way that minimizes the risk related to price decrease caused by the execution of a large selling order. A growing number of market participants benefit from direct access to this trading infrastructure. This again minimizes order execution costs, even in the case that one order will be divided into thousands of suborders.

In order to illustrate the line-up of this order execution strategy, the following drawing is presented:

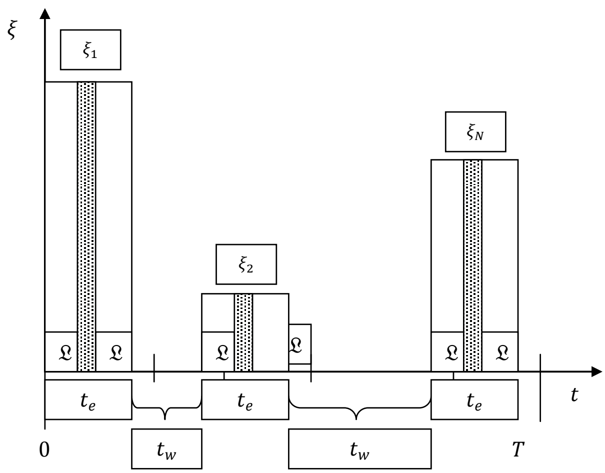

Figure 1. X_0 order execution line-up

Source: author.

This strategy can be applies to an opposite (short) position as well. In this case $X_0 < 0$ and the other conditions remain unchanged.

Conclusions and directions for future research

HFT improvements in optimal execution brings with it the ability to sell large risky assets under conditions that minimize costs. Dividing large orders into small random portions and selling them within randomly chosen time intervals, we are able to minimize the impact of supply-side factors on price formation. The trading period T can be shortened to the network infrastructure limits, up to its latency barrier.

On the other hand, by executing orders within very short time intervals, we are providing better liquidity to the market.[7] In this sense the presence of high-frequency strategies in the market provides a benefit, when they act as liquidity providers.

Such strategies are hard to discover and almost invisible for traditional low frequency market participants. This information asymmetry brings with it a new question. The question of: is there a broader market benefit in using HFT? Albert Menkveld in his UK Government's Foresight Study argues, that: *The presence of high-frequency traders in electronic markets improves welfare when they act as liquidity suppliers and thereby reduce the informational friction that exists between nonsynchronously arriving investors. It, however, reduces welfare when HFTs picks off investors' quotes at superhuman speed on information that would have been revealed to investors at a somewhat lower frequency*[8].

This question is not easy to be answered. For sure the growing HFT market brings many sophisticated solutions to solve not only the optimal execution problem, but also solutions to many others, which in their daily work thousands of traders have to face. But before the final economic balance sheet will be drawn, more academic research must be performed to explore the real impact of HFT on the benefit of there use.

Literature

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[7] Cf. P. Gomber: *op.cit.*, p. 59.

[8] A. Menkveld: *Foresight Driver Review: Electronic Trading and Market Structure*, UK Government's Foresight Project, DR16, 2011, p. 12.

[9] Email: boguslaw.blawat@yahoo.com, Financial support of this research and publication by the *"Doctoral Research Fellowships Chance for Development of the Region"* EU Project is gratefully acknowledged. To Mike O'Hara and his *HFT Review* many thanks for generous knowledge sharing.

Summary

Optimal execution of large orders is examined within the technical framework of High-Frequency Trading (HFT). A sample model is proposed, which extends an existing strategy through HFT means like time slicing with random splitting of the order volume and time shifting. As this strategy brings some information asymmetry to the trading parties, a general question about its impact on market benefit is raised and proposed for further academic research.

PROBLEM OPTYMALNEGO WYKONANIA ZLECENIA W OBSZARZE TRADINGU WYSOKIEJ CZĘSTOTLIWOŚCI – PRZYKŁADOWY MODEL

Streszczenie

Prezentowany artykuł analizuje problem optymalnego wykonania dużych zleceń metodami tradingu wysokiej częstotliwości (HFT). Zaproponowany został przykładowy model, który rozszerza istniejące strategie za pomocą takich środków HFT jak time slicing – podział zlecenia w czasie z losowo generowanymi fragmentami wykonywanymi w przypadkowych interwałach. Ponieważ tego typu strategia przynosi ze sobą asymetrię informacji między uczestnikami transakcji, postawione zostało pytanie o jej wpływ na ekonomię, na które odpowiedzi dostarczyć mogą dalsze badania naukowe.