Measuring the Economic Stock of Money

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Abstract

Aggregation theoretic measures of the capital stock of money have in the past been criticized for their dependence on future expectations. I attempt to answer some of those objections by using several forecasting methods to generate expectations needed for calculating the economic stock of money. I show that targeted factor model forecasting improves the accuracy of monetary capital stock measurements slightly. However, I also find, as has previous research, that monetary capital stock calculations are robust to assumptions about future expectation. I believe these findings tend to support the conclusion that concerns about the dependency of theoretical monetary stock aggregates on forecasted future expectations have been overstated.

Key words: Monetary Aggregation, Money Stock, Economic Stock of Money, Targeted Factor Models

JEL classification codes: E49

1 Introduction

Aggregation theoretic measures of the capital stock of money have in the past been criticized for their dependence on forecasted future expectations.

I attempt to answer some of those objections by using several forecasting methods to generate expectations needed for calculating the economic stock of money. Factor model forecasts have shown much promise in recent years, tending to outperform other models, but Barnett et al. (Forthcoming) found that the calculation of the economic stock of money is robust to assumptions about future expectation. Thus, the purpose of this study is to: one, improve the current measurement of the economic stock of money; and two, confirm the robustness result of Barnett et al..

Stock and Watson (1999) showed that approximate factor models tend to outperform other forecasting methods because they allow the use of a large panel of data. This study will utilize a data set with 112 variables. Bai and Ng (2002) derive information criteria for determining the number of factors that underlie a large panel of time series data, and in a 2006 working paper Bai and Ng (2006a) examine the problem of which variables should be included in the panel. They use a sequential model selection algorithm, known as least angle regression (LARS), developed by Efron et al. (2004) to target the panel of explanatory variables to the variable being forecasted. They found that these "targeted factor models" outperformed an AR(4) model in forecasting annual inflation.

I found that the use of targeted factor model forecasting methods improves the accuracy of the calculation of the economic stock of money slightly, but because distant future service flows are heavily discounted, I confirm the robustness result of Barnett et al. (Forthcoming); that is, I confirmed that calculating the economic stock of money is robust to assumptions about expectations formation. This robustness result tends to support the conclusion of Barnett et al. that concerns about the dependency of theoretical monetary stock aggregates on forecasted future expectations have been overstated.

The remainder of this paper is organized as follows. Sections two reviews the relevant monetary aggregation theory. Section three presents the forecasting methodology used, and evaluates that methodology. Section four explains the calculation of the economic stock of money. Section five presents the results of calculating the economic stock of money using targeted factor model forecasts and concludes.

2 The Economic Capital Stock of Money

2.1 Definition Under Perfect Foresight

Following Barnett (1978), let the representative consumer's current period inter temporal utility function, u_t, be weakly separable in each period's consumption of goods and monetary assets.[1] Let period t be the current time period, and let T be the length of the planning horizon, possibly infinity, such that the representative consumer plans for all periods, $s = t, t+1, \ldots t+T$. Now define the following variable for period s:

$C_s =$ $(I \times 1)$ vector of planned per capital consumption of goods and services during period s,

$p_s =$ $(I \times 1)$ vector of goods and services expected prices and of durable goods expected rental prices,

$p^*_s =$ the true cost of living index,

$m_s =$ $(N \times 1)$ vector of planned real balances of monetary assets,

$M_s =$ $(N \times 1)$ vector of planned nominal balances of monetary assets,

[1] It is worth noting that u_t is not an elementary utility function, since it contains monetary assets. However, Arrow and Hahn (1971) proved that if money has positive value, then there exists a derived utility function containing money.

$r_s =$ $(N \times 1)$ vector of expected nominal holding period yields on monetary assets,

$R_s =$ the expected nominal one-period holding yield on the benchmark asset.

Further assume that there exists $p_s^* = p_s^*(p_s)$, which is the true cost of living index that can be used to deflate nominal values in period s. Barnett (1991) recursively combines the $T+1$-period budget constraints from the representative agents problem into the single discounted Fisherine wealth constraint. From this discounted Fisherine wealth constraint, Barnett defines the ESM, V_t, as

$$V_t \equiv \sum_{s=t}^{T} \sum_{n=1}^{N} \left[\frac{p_s^*}{\rho_s} - \frac{p_s^*(1 + r_{n,s})}{\rho_{s+1}} \right] m_{ns}, \tag{1}$$

where the discount rate for period s is

$$\rho_s = \begin{cases} 1 & s = t \\ \prod_{u=t}^{s-1}(1 + R_u) & s \neq t \end{cases}. \tag{2}$$

Following Barnett et al. (2005), (1) can be rewritten as

$$V_t = \sum_{s=t}^{\infty} \sum_{n=1}^{N} \left[m_{ns} \left(p_s^* \cdot \frac{R_s + r_{n,s}}{1 + R_s} \right) \frac{1}{\rho_s} \right] = \sum_{s=t}^{\infty} \frac{TE_s}{\rho_s}, \tag{3}$$

where TE_s is the total nominal expenditure on monetary services in period s, and T is allowed to approach infinity.

2.2 Extension to Uncertainty

Barnett (1995) and Barnett et al. (1997) showed that, assuming inter-temporally strong separability, all the results on user cost and Divisia aggregation can be extended to the case of risk neutrality by replacing all random variables with their expectations. Thus, applying the consumption-based capital asset pricing model theory,[2] the formulas for the economics capital stock of money under inter-temporal strong separability becomes

$$V_t = E_t\left(\sum_{s=t}^{\infty}\Gamma_s TE_s\right), \tag{4}$$

where

$$\Gamma_s = \beta^{s-t}\frac{\partial u}{\partial C_s}/\frac{\partial u}{\partial C_t} \tag{5}$$

is the subjectively-discounted marginal rate of inter-temporal substitution between consumption in the current period t and the future period s.

3 Forecasting Methodology and Evaluation

I will look at monthly data from 1960:03 - 2006:02 that was collected from Economic Data - FRED® database maintained by the Saint Louis Federal Reserve,[3] the Bridge Commodity Research Bureau[4] and the United States Bureau of Labor Statistics[5]. I limit the study to the 1960:03 - 2004:03 time period to remain consistent with Barnett et al. (2005) for comparison purposes.

3.1 Variables to be Forecasted

The variables to be forecasted are total expenditures on monetary services provided by monetary assets included in M1, M2 and M3 monetary aggregates.

[2] See Blanchard and Fischer (1989) section 6.3 and Cochrane (2005).
[3] Saint Louis Federal Reserve: 2006, *Economic Data - FRED®*. http://research.stlouisfed.org/fred2/
[4] Bridge Commodity Research Bureau: 2006. http://www.crbtrader.com/
[5] United States Bureau of Labor Statistics: 2006. http://www.bls.gov

Figures 1 plots each variable. Each variable was tested for stationarity using the Augmented Dickey-Fuller test and the Dickey-Fuller GLS (Elliott et al., 1996). Table 1 summarizes the results of these tests. Each variable is found to be I(1) non-stationary, and so each is differenced once.

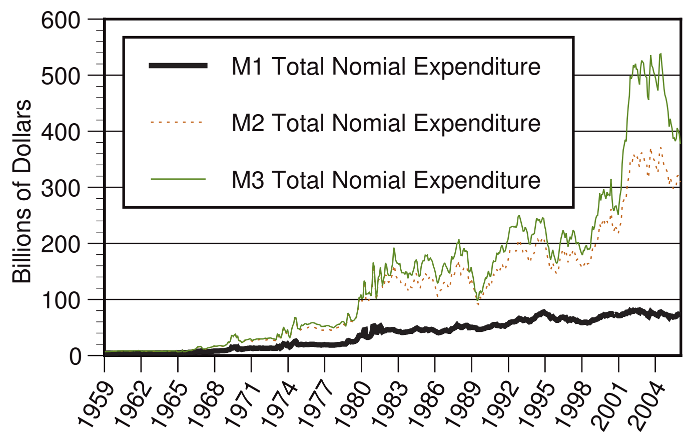

Figure 1. Total Expenditure (M1): 1959:01 - 2004:03

Table 1
Summary of Unit Root Tests on Total Expenditure Variables

		ADF Test Statistic	DF-GLS Test Statistic
Critical Values:	1%	-3.975	-3.480
	5%	-3.418	-2.890
	10%	-3.131	-2.570
TE (M1)	Level	-2.975	-2.135
	1st Diff.	-18.586 *	-18.555 *
TE (M2)	Level	-2.103	-1.529
	1st Diff.	-19.581 *	-6.462 *
TE (M3)	Level	-0.651	-0.742
	1st Diff.	-6.154 *	-5.877 *

* Reject that the series has a unit root at the 5% confidence level.

3.2 Explanatory Data

The panel of explanatory variables includes 118 series including selected long-term and short-term interest rates, unemployment data, aggregate price data, monetary aggregate data and other macroeconomic time series data, see table A. Beveridge and Nelson (1981) and Nelson and Plosser (1982) argue that many of these series are I(1) non-stationary or contain an I(1) component, and therefore each variable was tested for stationarity using the Augmented Dickey-Fuller test and the Dickey-Fuller GLS (Elliott et al., 1996). Each variable is transformed by taking logs, first or second differences as needed.

3.3 Forecasting Models

The forecasting methodology used in this section is based on approximate factor models which I will discuss in section 3.3.1. Approximate factor models tend to outperform other forecasting techniques because they eliminate the need to throw-away useful data. However, Boivin and Ng (2006) found that adding predictors that bear little information about factor components does not necessarily improve forecasts. Thus, the forecasting performance of approximate factor models could be significantly improved by targeting the panel of explanatory variables to the series being forecasted. Bai and Ng (2006b) propose a method for targeting the panel of explanatory variables that I will discuss in section 3.3.3 and 3.3.4.

3.3.1 Approximate Factor Model

Let $X_{(T \times N)}$ be a matrix of N observed variables over T periods. Then consider the model suggested by Bai and Ng (2002),

$$X = F\Lambda' + e, \tag{6}$$

where $\Lambda = (\lambda_1 \ldots \lambda_N)'$ is a $(N \times r)$ matrix of loading factors, F is a $(T \times r)$ matrix of common factors, and e is a $(T \times N)$ matrix of idiosyncratic errors. See Bai and Ng (2002) for the necessary assumptions for consistent estimation of the r common factors. The factors are estimated by the method of asymptotic principle components.

In order to estimate the number of common factors, r, I minimize, by choosing k, the following information criterion:

$$IC_{p1}(k) = \ln\left[V\left(k, \hat{F}^k\right)\right] + k\left(\frac{N+T}{NT}\right)\ln\left(\frac{NT}{N+T}\right), \tag{7}$$

where $\hat{F}^k$ equals $\sqrt{T}$ times the eigenvectors corresponding to the k largest eigenvalues of the $(T \times T)$ matrix XX' and

$$V\left(k, \hat{F}^k\right) = \min_{\Lambda}\left[(NT)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{it} - \lambda_i^{k\prime} F_t^k\right)\right].$$

I use the forecasting equation suggested by Bai and Ng (2006b):

$$\hat{y}^h_{T+h|T} = \hat{\alpha}' W_T + \hat{\beta}' \hat{F_T}^k, \tag{8}$$

where $\hat{y}^h_{T+h}$ is the h-period forecast of the variable y_t given the information available as of time period T, W_T is a vector of predetermined variables that could include a constant and/or lags of y_{t+h}, $\hat{F_T}^k$ is a vector of k common factors of X, and the parameters α and β are obtained from the ordinary least squares estimation of

$$y^h_{t+h} = \alpha' W_T + \beta' \hat{F_T}^k + \varepsilon_{t+h}.$$

3.3.2 Squared Principle Components

One limitation of the method of principle components is that it presupposes a linear linking function between the data and the latent factors. Bai and Ng (2006b) propose a more flexible approach that allows for rudimentary non-linearity in the factor linking function. Define X^* to be X augmented by a subset of the unique cross-products of X. Specifically, consider $X^* = \{X_{tn}, X^2_{tn}\}$, which Bai and Ng referred to as squared principle components[6] (SPC). Estimation of the common factors of X^* proceeds by the usual principle components method. In the case of SPC, there are $2N$ predictors, and the dimension X^* could be much larger if other cross-products are included. Thus I am left with the problem of determining which predictors are really necessary.

3.3.3 Targeted Factor Model

Boivin and Ng (2006) found that adding additional predictors that bear little information about factor components does not necessarily improve forecasts. They found that when the data panel is too noisy, it is better to eliminate some of the data. The optimal panel of predictors could be determined by the use

[6] Bai and Ng (2006b) also experimented with the incorporation of cross-products, $X_{ti}X_{tj}$ where $i \neq j$, but they found that this was computationally demanding and did not significantly improve forecasting performance.

of an information criteria, such as BIC. However, with N possible predictors, there are 2^Npossible sets to consider. Hence, this method is impractical. Bai and Ng (2006b) examine the use of several methods by which the panel of predictors can be targeted to the variable being forecasted. Bai and Ng found that the method of least angle regression (LARS) developed by Efron et al. (2004) was the most successful at forecasting inflation, thus I use LARS to target our panel.

3.3.4 Least Angle Regression

I shall briefly discuss the LARS algorithm. Let $\hat{\mu}_k$be the current estimate of y with k predictors and define

$$\hat{c} = X'(y - \hat{\mu}_k)$$

to be the "current correlation." (Note: it is assumed that each column of X has been standardized.) Choose j to maximize $|\hat{c}_j|$ and consider the updating rule

$$\hat{\mu}_{k+1} = \hat{\mu}_k + \hat{\gamma}\text{sign}(\hat{c}_j) X_j. \tag{9}$$

At each step, the $\hat{\gamma}$ is chosen endogenously so that the algorithm proceeds equiangularly between the variables in the most correlated set until the next variable is found. After k steps, k variables will have been selected; thus, determining the optimal set of predictors becomes a problem of determining a stopping rule for k.

The LARS algorithm begins with $\hat{\mu}_0 = 0$. Let $\hat{\mu}$ be the current estimate of y, let $\hat{c} = X'(y - \hat{\mu})$, and define K to be the set of indices corresponding to the variables in X with the largest absolute "current correlation," i.e. the "in set,"

$$\hat{C} = \max_j |\hat{c}_j| \qquad K = \left\{ j : |\hat{c}_j| = \hat{C} \right\}.$$

Let $s_j = \text{sign}(\hat{c}_j)$ and thus the active matrix corresponding to K is

$$X_K = (\dots\, s_j x_j\, \dots)_{j \in K}.$$

Let

$$G_K = X_K' X_K \quad \text{and} \quad A_K = (1_K' G_K 1_K)^{-\frac{1}{2}},$$

where 1_K is a vector of ones of length equal to the size of K. The unit equiangular-vector with the columns of the active matrix is

$$u_K = X_K w_K, \quad w_K = A_K G_K^{-1} 1_K,$$

so that

$$X_K u_K = A_K 1_K \quad \text{and} \quad \|u_K\|^2 = 1.$$

LARS then updates $\hat{\mu}$ using the LARS variant of (9),

$$\hat{\mu}^{new} = \hat{\mu} + \hat{\gamma} u_K,$$

where

$$\hat{\gamma} = \overset{+}{\min_{j \in A_K^c}} \left\{ \frac{\hat{C} - \hat{c}_j}{A_K - a_j}, \frac{\hat{C} + \hat{c}_j}{A_K + a_j} \right\},$$

where $a_K = X' u_K$, $\overset{+}{\min}$ indicates that the minimum is taken over only positive components within each choice of j, and A_K^c is the set of indices corresponding to the variables not yet in the "in set." If LARS is repeated N times, it returns an ordering of the N predictors from best to worst.

3.4 Comparison Forecasting Models

I evaluate the performance of targeted factor model forecasts as compared to the following models. For our first comparison model, I estimate a simple auto-regressive process of p lags using the Bayesian information criterion to select p. I selected an AR(p) model as a model for comparison because of its long standing usefulness in forecasting of all types. In many instances, the AR(p) model has been shown to outperform much more complicated models. Thus, the AR(p) is a natural benchmark for comparing the performance of any new forecasting methodology. I will refer to this model as AR in all following tables and figures. I will also estimate an auto regressive model where I use the LARS algorithm to determine the lags that are included. I will refer to this model as AR_LARS.

Our second comparison model is a Martingale forecast. The Martingale forecast model is chosen as a model for comparison because of the long tradition of modeling interest rates as Martingale processes. Arguments supporting Martingale expectations date back to Sargent (1976) and Pesando (1979). Elliott and Baier (1979) found empirical evidence for the use of Martingale forecasts of interest rates. The Martingale forecast model is also chosen as a comparison model because it is a common assumption in the calculation of theoretical monetary stock aggregates, such as the currency equivalent index (Barnett, 1991). I will refer to this model as Martingale in all following tables and figures.

3.5 Forecasting Results

3.5.1 Criterion for Evaluation of Forecasting Performance

To evaluate the forecasting performance of each model, we calculate root mean squared error, Theil's U statistic and combined forecast regression. Let TE_{t+h} be the observed value of TE in period $t+h$, and let $\hat{TE}_{t+h|t}$ be the h-period ahead forecast of TE conditional on information available in period t. Then

$$RMSE_H\left(model\right) = \sqrt{\frac{1}{H}\sum_{t=1}^{H}\left(TE_{t+h} - \hat{TE}_{t+h|t}\right)^2} \tag{10}$$

and

$$U_H(model) = \frac{\sqrt{\frac{1}{H}\sum_{t=1}^{H}\left(TE_{t+h} - \hat{TE}_{t+h|t}\right)^2}}{\sqrt{\frac{1}{H}\sum_{t=1}^{H}\left(TE_{t+h} - TE_t\right)^2}} \tag{11}$$

are calculated, where h is the forecasting horizon and H is the total number of forecasts. Theil's U statistic compares a model's forecasting performance to that of the no change model. When U is less than one, the model forecast performs better than the no change forecast. When U is greater than one, the model performs more poorly than the no change forecast.

3.5.2 Forecasting Results

In order to evaluate each of the forecasting models in question, we compare each to actual observed values.[7] Figures 2, 3 and 4 plot the best fitting forecasts, based on RMSE, and the actual total expenditure realized at the M1, M2 and M3 levels of aggregation and at the six month, 12 month, 24 month and 36 month forecasting horizon. Tables 2, 3, 4 and 5 report the performance

[7] The results are generated using Ox version 4.00 (Doornik, 2006). See http://www.doornik.com for further information.

of each forecast by measuring the root mean squared error (RMSE) and Theil's U statistic.

We find that the targeted factor model index outperforms all other models, based on Theil's U statistic, at the M2 and the M3 levels of aggregation and a forecasting horizons of six and 12 months and targeted factor models are also highly competitive at the M2 and M3 levels of aggregation at longer time horizons. Finally, we found the Martingale forecasts outperform all other models at all forecasting horizons at the M1 level of aggregation.

Table 2
Forecasting Results (6 Month Time Horizon)

Variable	Forecasting Method	Root Mean Squared Error	Theil's U Statistic
Total Expenditure (M1)	Martingale	19.58065	1.00000
	AR	23.13588	1.18157
	AR_LARS	20.78049	1.06128
	LARS	27.04081	1.38100
	TFM	21.46643	1.09631
Total Expenditure (M2)	Martingale	528.68123	1.00000
	AR	532.18815	1.00663
	AR_LARS	528.28452	0.99925
	LARS	523.38807	0.98999
	TFM	513.46344	0.97122
Total Expenditure (M3)	Martingale	1450.73554	1.00000
	AR	1159.16294	0.79902
	AR_LARS	1187.70426	0.81869
	LARS	1107.41616	0.76335
	TFM	1068.20332	0.73632

Table 3
Forecasting Results (12 Month Time Horizon)

Variable	Forecasting Method	Root Mean Squared Error	Theil's U Statistic
Total Expenditure (M1)	Martingale	36.77069	1.00000
	AR	38.79432	1.05503
	AR_LARS	38.69613	1.05236
	LARS	47.02705	1.27893
	TFM	40.06596	1.08962
Total Expenditure (M2)	Martingale	1217.54731	1.00000
	AR	1163.32466	0.95547
	AR_LARS	1186.59930	0.97458
	LARS	1195.15246	0.98161
	TFM	1158.32468	0.95136
Total Expenditure (M3)	Martingale	3342.65402	1.00000
	AR	2908.17955	0.87002
	AR_LARS	2922.34333	0.87426
	LARS	2863.17481	0.85656
	TFM	2766.15834	0.82753

Table 4
Forecasting Results (24 Month Time Horizon)

Variable	Forecasting Method	Root Mean Squared Error	Theil's U Statistic
Total Expenditure (M1)	Martingale	54.38643	1.00000
	AR	60.17125	1.10637
	AR_LARS	60.69944	1.11608
	LARS	70.02686	1.28758
	TFM	61.03253	1.12220
Total Expenditure (M2)	Martingale	2142.22520	1.00000
	AR	1928.17834	0.90008
	AR_LARS	1896.27860	0.88519
	LARS	2013.62204	0.93997
	TFM	1937.90244	0.90462
Total Expenditure (M3)	Martingale	6150.45427	1.00000
	AR	5355.93840	0.87082
	AR_LARS	5385.85992	0.87568
	LARS	5636.30650	0.91640
	TFM	5440.54167	0.88458

Table 5
Forecasting Results (36 Month Time Horizon)

Variable	Forecasting Method	Root Mean Squared Error	Theil's U Statistic
Total Expenditure (M1)	Martingale	80.35107	1.00000
	AR	86.14545	1.07211
	AR_LARS	90.81359	1.13021
	LARS	102.41892	1.27464
	TFM	93.98284	1.16965
Total Expenditure (M2)	Martingale	3325.83595	1.00000
	AR	2940.10811	0.88402
	AR_LARS	2920.11263	0.87801
	LARS	3015.66041	0.90674
	TFM	2960.97874	0.89030
Total Expenditure (M3)	Martingale	9813.96862	1.00000
	AR	8457.48247	0.86178
	AR_LARS	8664.67006	0.88289
	LARS	9343.70693	0.95208
	TFM	8815.59455	0.89827

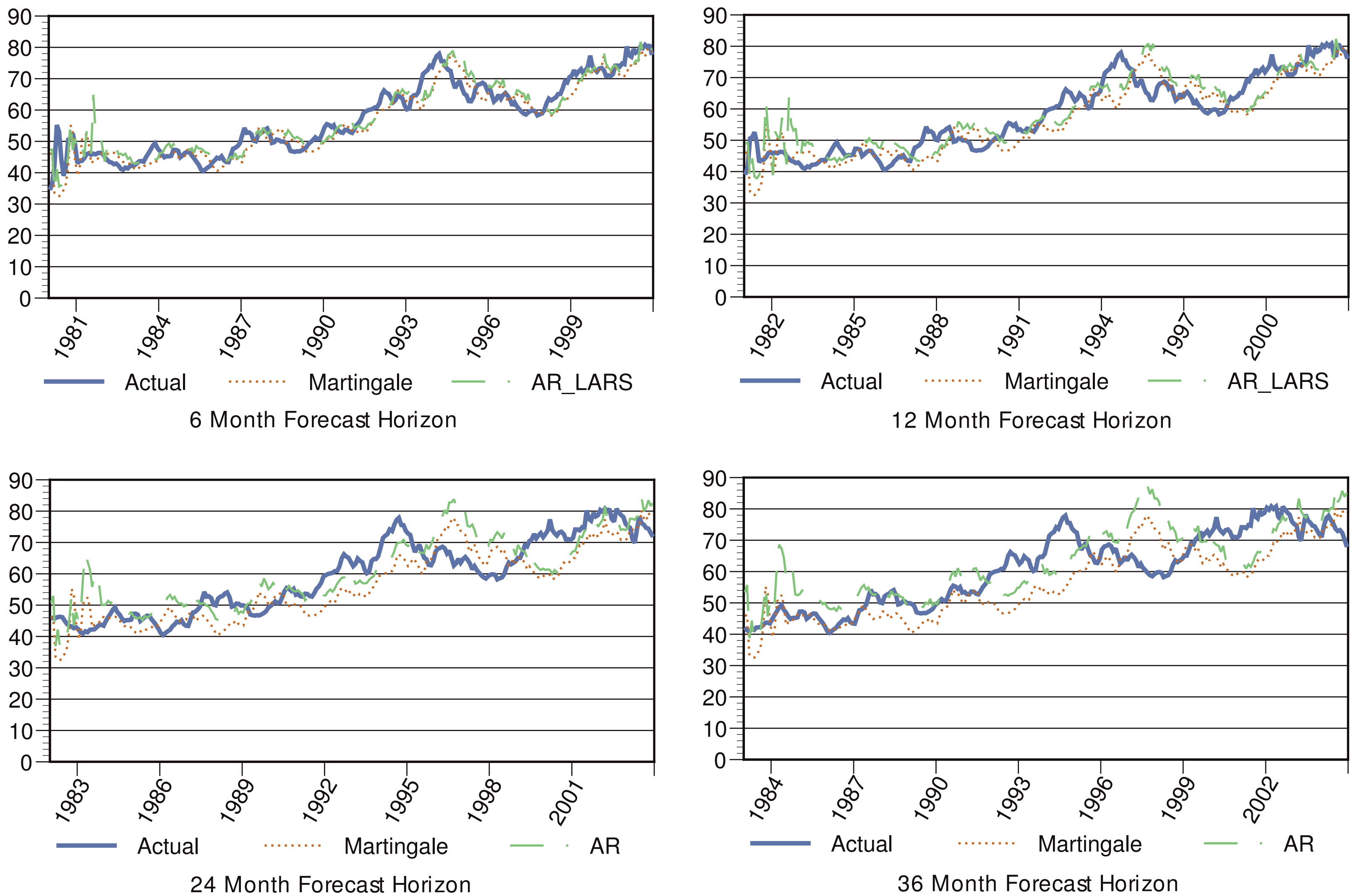

Figure 2. Forecasts of the Total Expenditure on Monetary Service (M1)

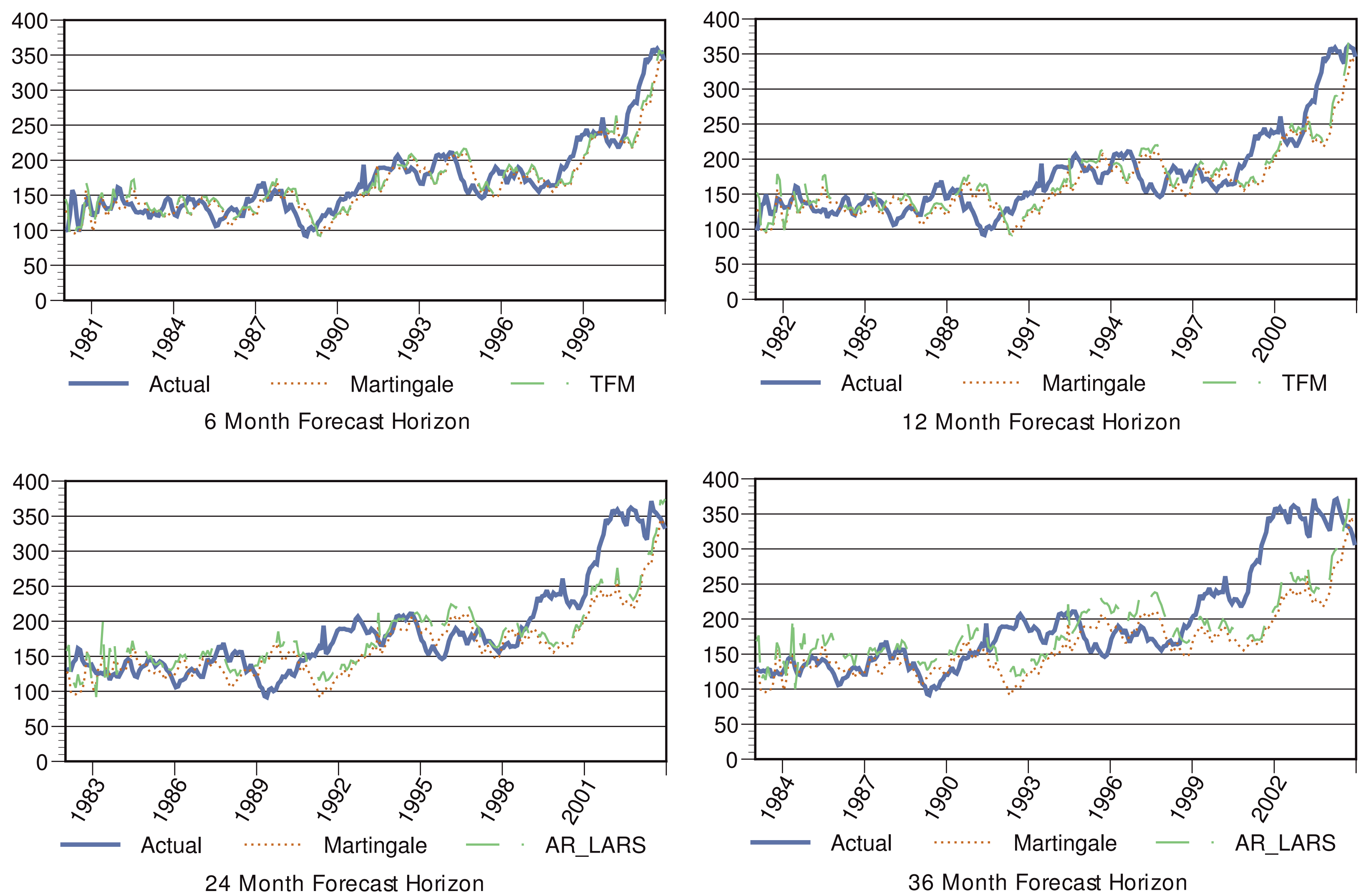

Figure 3. Forecasts of the Total Expenditure on Monetary Service (M2)

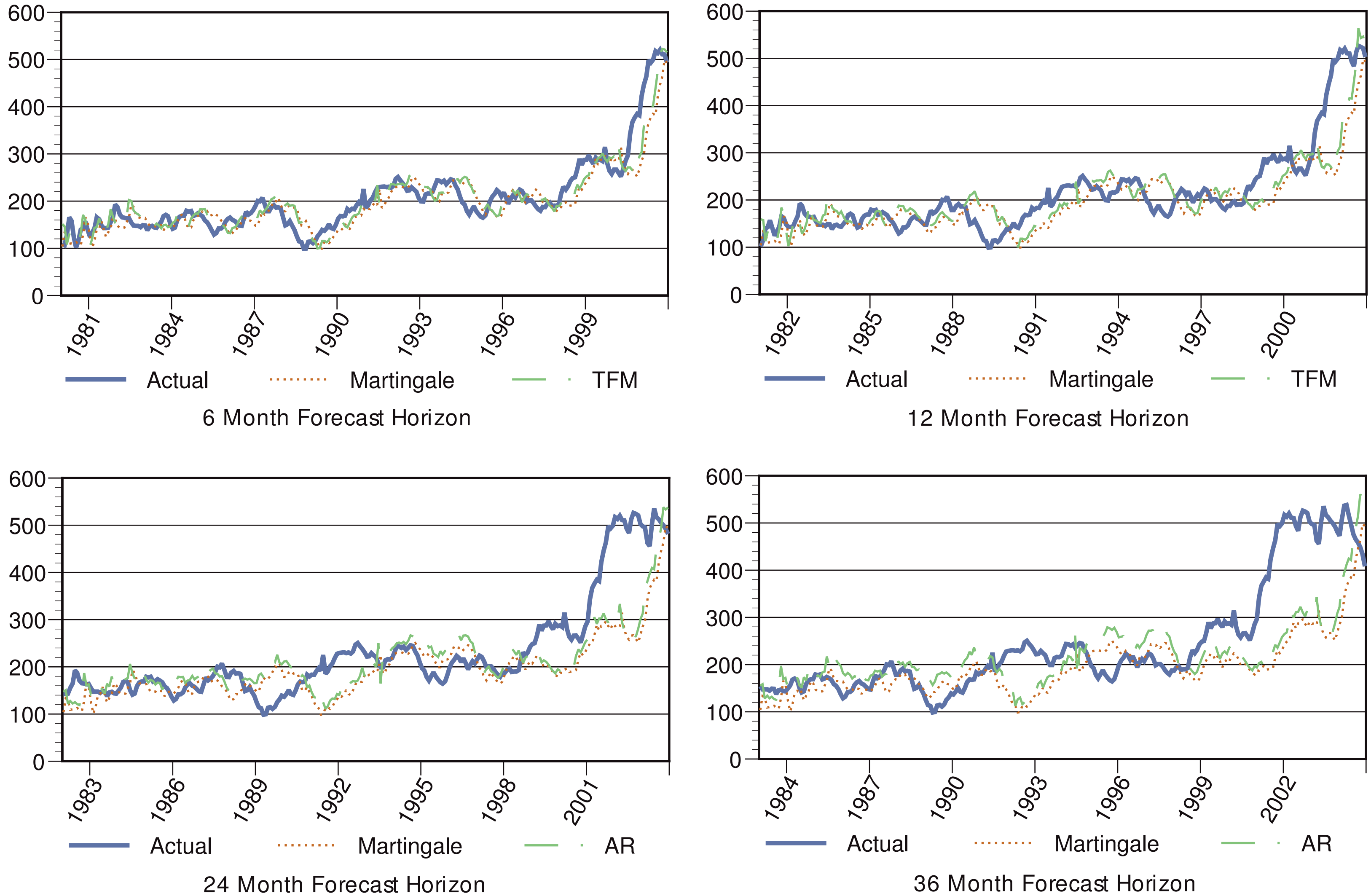

Figure 4. Forecasts of the Total Expenditure on Monetary Service (M3)

4 Calculating The Economic Stock of Money

Following Barnett et al. (2005), the ESM, (3),

$$V_t = \sum_{s=t}^{\infty} \frac{TE_s}{\rho_s},$$

is calculated by assuming perfect foresight and thus use actual future data to compute (3). The perfect foresight ESM (ESM_PF) is not a feasible index number since future data cannot be known ex ante, but as in Barnett et al. (2005), I use ESM_PF to evaluate the performance of our measures of the ESM that are based on forecasted data.

I calculate a feasible measure of the ESM by assuming risk neutrality and using forecasted data in (**??**),

$$V_t = E_t \left(\sum_{s=t}^{\infty} \beta^{s-t} TE_s \right).$$

The expected value of a nonlinear function is equal to the function evaluated at the expected value of each variable plus covariance terms. I follow method 3 of Barnett et al. (Forthcoming) and set each of these covariance terms to zero. It is well known from asset pricing theory that

$$i = \frac{1-\beta}{\beta},$$

where, in our case, the interest rate i is the benchmark rate. Substituting the benchmark rate and solving for β yields

$$\beta = \frac{1}{1+R_t}.$$

Thus, by setting the covariance terms to zero, I am assuming that the covariance between total expenditure on monetary assets and the benchmark rate is zero.

I also assume the benchmark rate follows a Martingale process. This assumption is consistent with **?** who found that Martingale forecasts of the benchmark rate tend to outperform more sophisticated forecasting models. Assuming Martingale expectations corresponds to method 2 of Barnett et al. (Forthcoming). Total expenditure is forecasted using the targeted factor model described in section 3.

In practice, (3) must be evaluated for a finite number of periods, H, so that (3) becomes

$$V_t = \sum_{s=t}^{H} \frac{TE_s}{\rho_s}.$$

To determine the number of iterations, H, needed to calculate the ESM index number, I chose the smallest H that satisfies the stopping criterion

$$\left| \frac{\sum_{s=t}^{H} \frac{TE_s}{\rho_s} - \sum_{s=t}^{H-1} \frac{TE_s}{\rho_s}}{\sum_{s=t}^{H-1} \frac{TE_s}{\rho_s}} \right| < 10^{-4}. \tag{12}$$

5 Results and Conclusion

In order to evaluate how well each of the models in question is able to measure the economic stock of money, I compare each of the indices calculated to the perfect foresight ESM described above. [8] Figures 5, 6 and 7 plot the best fitting index calculated in this paper, the best fitting index calculated by Barnett et al. (2005) and the perfect foresight ESM at the M1, M2 and M3 levels of

[8] The results are generated using Ox version 4.00 (Doornik, 2006). See http://www.doornik.com for further information.

aggregation. Table 6 reports the performance of each index calculated in this paper and the indexes calculated by Barnett et al. (2005)[9] by measuring the mean percent error (MPE), mean absolute percent error (MAPE) and root mean squared error (RMSE) of each index relative to perfect foresight ESM.

I find that the targeted factor model index outperforms all other models, based on RMSE, at the M2 and M3 levels of aggregation, but I fail to improve upon Barnett's calculations of ESM at the M1 level of aggregation. Finally, I find that while measurements of the economic stock of money can be improved with better forecasting, such improvement is relatively small, and thus, I concur with Barnett et al. (2005) that aggregation theoretic measures of the economic stock of money are robust to assumptions about future expectations.

[9] I compare our results to method 1, 2 , and 3 using Bayesian VAR forecasting calculated by Barnett et al. (2005).

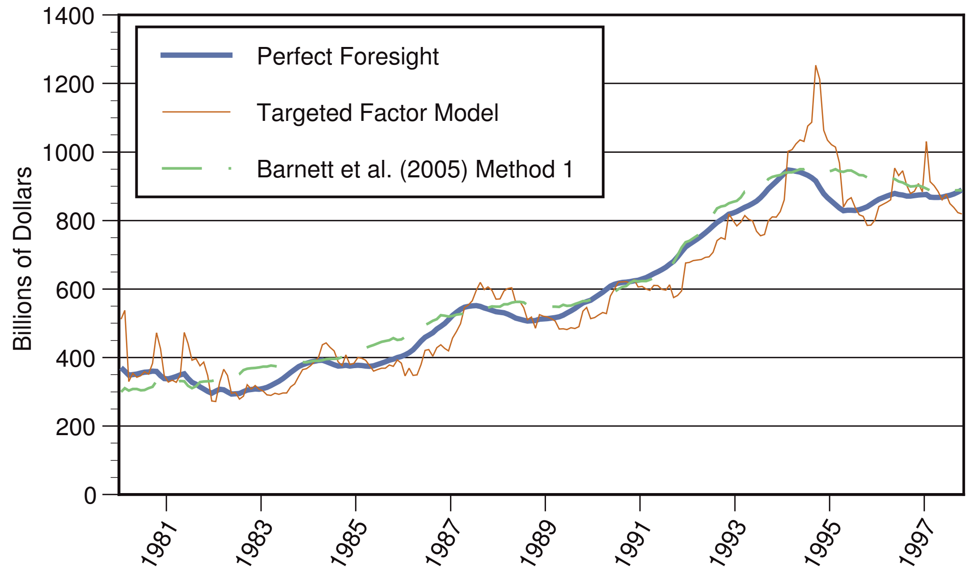

Figure 5. The Economic Stock of Money (M1)

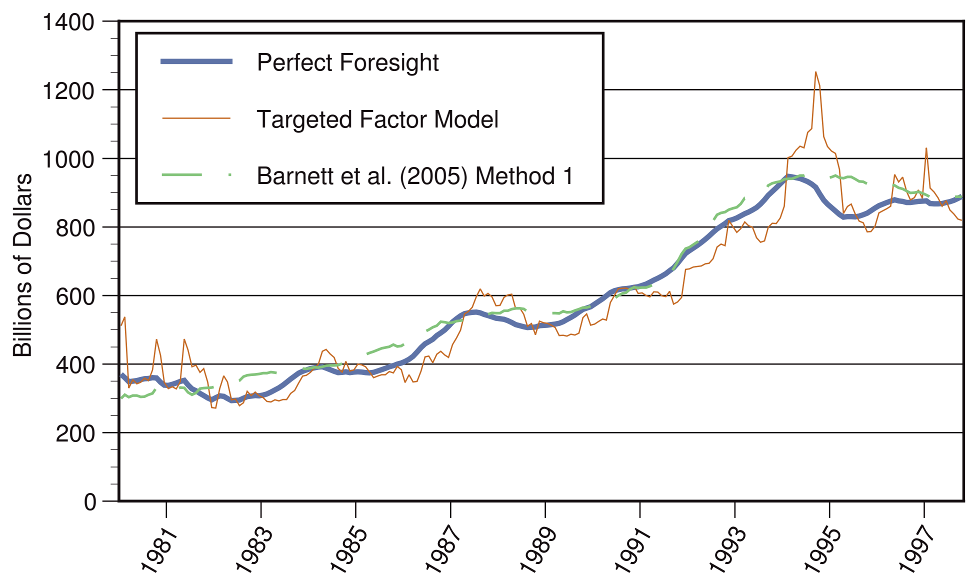

Figure 6. The Economic Stock of Money (M2)

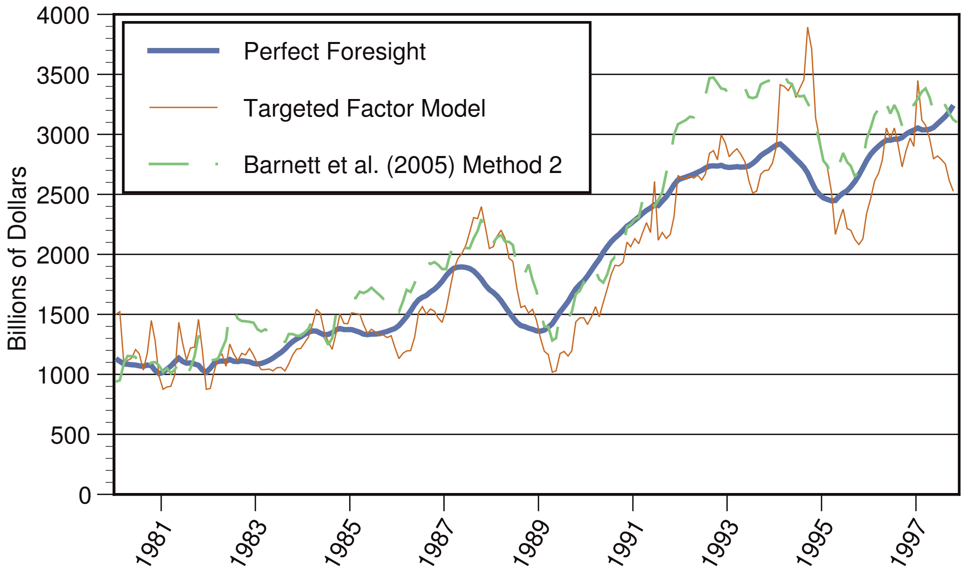

Figure 7. The Economic Stock of Money (M3)

Table 6. Comparison of Methods of Calculating the Economic Stock of Money

Variable	Forecasting Method	Root Mean Squared Error	Mean Absolute Percent Error	Mean Percent Error
ESM (M1)	Auto Regression Model	68.757	8.58%	0.46%
	Auto Regression/LARS Model	69.562	8.80%	0.94%
	Targeted Factor Model	67.393	8.39%	0.18%
	Barnett et al. (2005) Method 1	40.656	6.47%	4.23%
	Barnett et al. (2005) Method 2	42.380	5.98%	2.13%
	Barnett et al. (2005) Method 3	41.616	6.76%	4.75%
ESM (M2)	Auto Regression Model	235.414	10.85%	0.00%
	Auto Regression/LARS Model	233.885	11.02%	0.35%
	Targeted Factor Model	232.033	10.96%	-0.01%
	Barnett et al. (2005) Method 1	287.450	15.82%	15.24%
	Barnett et al. (2005) Method 2	279.541	15.48%	14.52%
	Barnett et al. (2005) Method 3	322.190	18.77%	18.32%
ESM (M3)	Auto Regression Model	295.926	11.59%	-0.23%
	Auto Regression/LARS Model	293.788	11.76%	-0.24%
	Targeted Factor Model	289.632	11.74%	-0.25%
	Barnett et al. (2005) Method 1	341.800	15.22%	13.88%
	Barnett et al. (2005) Method 2	324.316	13.50%	11.46%
	Barnett et al. (2005) Method 3	344.253	15.39%	14.20%

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A Data Descriptions and Transformations

Mnemonic	*Tran.*	*Description*
HOUST	ln	Housing Starts: Total: New Privately Owned Housing Units Started, Thous. of Units, SAAR
HOUST1F	ln	Privately Owned Housing Starts: 1-Unit Structures, Thous. of Units, SAAR
INDPRO	ln	Industrial Production Index, Index 2002=100, SA
NAPM	lv	ISM Manufacturing: PMI Composite Index, Index, SA
PERMIT	ln	New Private Housing Units Authorized by Building Permit, Thous. of Units, SAAR
CCOCB	Δlv	Major Holders of Consumer Credit Outstanding: Commercial Banks, Millions of dollars, NSA
CCOCU	Δlv	Major Holders of Consumer Credit Outstanding: Credit Unions, Millions of dollars, NSA
CCOFC	ln	Major Holders of Consumer Credit Outstanding: Finance Companies, Millions of dollars, NSA
CCONFB	Δ^2 ln	Major Holders of Consumer Credit Outstanding: Nonfinancial Business, Millions of dollars, NSA
CCOPSA	Δlv	Major Holders of Consumer Credit Outstanding: Pools of Securitized Assets, Millions of dollars, NSA
CCOT	Δ ln	Major Holders of Consumer Credit Outstanding: Total, Millions of dollars, NSA
CE16OV	Δ ln	Civilian Employment: Sixteen Years & Over, Thous., SA
CLF16OV	Δ ln	Civilian Labor Force, Thous., SA
CNP16OV	Δ ln	Civilian Noninstitutional Population, Thous., NSA
EMRATIO	lv	Civilian Employment-Population Ratio, %, SA
MANEMP	Δ ln	Employees on Nonfarm Payrolls: Manufacturing, Thous., SA
PAYEMS	Δ ln	Total Nonfarm Payrolls: All Employees, Thous., SA
POP	Δ ln	Total Population: All Ages including Armed Forces Overseas, Thous., NA
SRVPRD	Δ ln	All Employees: Service-Providing Industries, Thous., SA

Mnemonic	*Tran.*	*Description*
UEMP15OV	ln	Civilians Unemployed - 15 Weeks & Over, Thous., SA
UEMPLT5	$\Delta \ln$	Civilians Unemployed - Less Than 5 Weeks, Thous., SA
USCONS	ln	All Employees: Construction, Thous., SA
USEHS	$\Delta \ln$	All Employees: Education & Health Services, Thous., SA
USFIRE	$\Delta \ln$	All Employees: Financial Activities, Thous., SA
USGOOD	$\Delta \ln$	All Employees: Goods-Producing Industries, Thous., SA
USGOVT	$\Delta \ln$	All Employees: Government, Thous., SA
USINFO	$\Delta \ln$	All Employees: Information Services, Thous., SA
USLAH	$\Delta \ln$	All Employees: Leisure & Hospitality, Thous., SA
USMINE	$\Delta \ln$	All Employees: Natural Resources & Mining, Thous., SA
USPBS	ln	All Employees: Professional & Business Services, Thous., SA
USSERV	$\Delta \ln$	All Employees: Other Services, Thous., SA
USTPU	$\Delta \ln$	All Employees: Trade, Transportation & Utilities, Thous., SA
USTRADE	$\Delta \ln$	All Employees: Retail Trade, Thous., SA
USWTRADE	$\Delta \ln$	All Employees: Wholesale Trade, Thous., SA
DSPI	$\Delta \ln$	Disposable Personal Income, Bil. of $, SAAR
DSPIC96	$\Delta \ln$	Real Disposable Personal Income, Bil. of Chained 2000 $, SAAR
PCE	$\Delta \ln$	Personal Consumption Expenditures, Bil. of $, SAAR
PCEC96	ln	Real Personal Consumption Expenditures, Bil. of Chained 2000 $, SAAR
PCEDG	$\Delta \ln$	Personal Consumption Expenditures: Durable Goods, Bil. of $, SAAR
PCEDGC96	$\Delta \ln$	Real Personal Consumption Expenditures: Durable Goods, Bil. of Chained 2000 $, SAAR
PCEND	$\Delta \ln$	Personal Consumption Expenditures: Nondurable Goods, Bil. of $, SAAR

Mnemonic	*Tran.*	*Description*
PCENDC96	$\Delta \ln$	Real Personal Consumption Expenditures: Nondurable Goods, Bil. of Chained 2000 \$, SAAR
PCEPI	$\Delta^2 \ln$	Personal Consumption Expenditures: Chain-type Price Index, Index 2000=100, SA
PCEPILFE	$\Delta^2 \ln$	Personal Consumption Expenditures: Chain-Type Price Index Less Food and Energy, Index 2000=100, SA
PCES	$\Delta \ln$	Personal Consumption Expenditures: Services, Bil. of \$, SAAR
PCESC96	$\Delta \ln$	Real Personal Consumption Expenditures: Services, Bil. of Chained 2000 \$, SAAR
PI	$\Delta \ln$	Personal Income, Bil. of \$, SAAR
10TCM	Δlv	10-year Treasury Constant Maturity, %, N/A
1YTCM	Δlv	1-year Treasury Constant Maturity, %, N/A
20MBI	Δlv	Bond Buyer GO 20-Year Bond Municipal Bond Index, %, N/A
20TCM	Δlv	20-year Treasury Constant Maturity, %, N/A
3MTB2M	Δlv	3-month Treasury Bills - Secondary Market, %, N/A
3YTCM	Δlv	3-year Treasury Constant Maturity, %, N/A
5YTCM	Δlv	5-year Treasury Constant Maturity, %, N/A
6MTB2M	Δlv	6-month Treasury Bills - Secondary Market , %, N/A
AAA	Δlv	Moody's Seasoned Aaa, %, N/A
BAA	Δlv	Moody's Seasoned Baa, %, N/A
FFR	lv	Federal funds (effective), %, N/A
IPIAP	$\Delta \ln$	Industrial Production Index: Automotive products, Index 2002=100, SA
IPIBE	ln	Industrial Production Index: Business equipment, Index 2002=100, SA
IPIBSUP	$\Delta \ln$	Industrial Production Index: Business supplies, Index 2002=100, SA
IPICEP	$\Delta \ln$	Industrial Production Index: Consumer energy products, Index 2002=100, SA
IPICG	ln	Industrial Production Index: Consumer goods, Index 2002=100, SA

Mnemonic	*Tran.*	*Description*
IPICLO	$\Delta \ln$	Industrial Production Index: Clothing, Index 2002=100, SA
IPICSUP	ln	Industrial Production Index: Construction supplies, Index 2002=100, SA
IPIDG	$\Delta \ln$	Industrial Production Index: Durable consumer goods, Index 2002=100, SA
IPIDGMAT	$\Delta \ln$	Industrial Production Index: Durable goods materials, Index 2002=100, SA
IPIDSE	$\Delta \ln$	Industrial Production Index: Defense and space equipment, Index 2002=100, SA
IPIEMAT	$\Delta \ln$	Industrial Production Index: Energy materials, Index 2002=100, SA
IPIFP	ln	Industrial Production Index: Final products and non-industrial supplies, Index 2002=100, SA
IPIFT	$\Delta \ln$	Industrial Production Index: Foods and tobacco, Index 2002=100, SA
IPIMAN	ln	Industrial Production Index: Manufacturing (SIC), Index 2002=100, SA
IPIMAT	$\Delta \ln$	Industrial Production Index: Materials, Index 2002=100, SA
IPINDCG	$\Delta \ln$	Industrial Production Index: Nondurable consumer goods, Index 2002=100, SA
IPINDGMAT	$\Delta \ln$	Industrial Production Index: Nondurable goods materials, Index 2002=100, SA
IPITE	$\Delta \ln$	Industrial Production Index: Transit equipment, Index 2002=100, SA
IDMNODI	ln	ISM Manufacturing New Orders Diffusion Index, Index, SA
ISMIDI	ln	ISM Manufacturing Inventories Diffusion Index, Index, SA
ISMPMI	ln	ISM PMI (Purchasing Managers' Index), Index, SA
PSMSDDI	ln	ISM Manufacturing Supplier Deliveries Diffusion Index, Index, SA
BENCH	Δlv	Benchmark Rate, N/A
MSIM1	$\Delta^2 \ln$	Nominal (Tornqvist-Theil) Monetary Services Index For M1, SA

Mnemonic	*Tran.*	*Description*
MSIM2	$\Delta \ln$	Nominal (Tornqvist-Theil) Monetary Services Index For M2, SA
MSIM3	$\Delta^2 \ln$	Nominal (Tornqvist-Theil) Monetary Services Index For M3, SA
TOTEXPM1	$\Delta \ln$	Total Nominal Expenditures For M1, SA
TOTEXPM2	$\Delta \ln$	Total Nominal Expenditures For M2, SA
TOTEXPM3	$\Delta \ln$	Total Nominal Expenditures For M3, SA
ADJRESSL	$\Delta \ln$	St. Louis Adjusted Reserves, Bil. of $, SA
AMBSL	$\Delta \ln$	St. Louis Adjusted Monetary Base, Bil. of $, SA
BOGAMBSL	ln	Board of Governors Monetary Base, Adjusted for Changes in Reserve Requirements, Bil. of $, SA
BOGNONBR	$\Delta \ln$	Non-Borrowed Reserves of Depository Institutions, Bil. of $, SA
BOGUMBNS	ln	Board of Governors Monetary Base, Not Adjusted for Changes in Reserve Requirements, Bil. of $, NSA
BORROW	ln	Total Borrowings of Depository Institutions from the Federal Reserve, Bil. of $, NSA
CURRCIR	$\Delta \ln$	Currency in Circulation, Bil. of $, NSA
EXCRESNS	$\Delta \ln$	Excess Reserves of Depository Institutions, Bil. of $, NSA
NFORBRES	lv	Net Free or Borrowed Reserves of Depository Institutions, Bil. of $, NSA
REQRESNS	$\Delta \ln$	Required Reserves, Not Adjusted for Changes in Reserve Requirements, Bil. of $, NSA
RESBALNS	$\Delta \ln$	Reserve Balances with Federal Reserve Banks, Not Adjusted for Changes in Reserve Requirements, Bil. of $, NSA
SBASENS	ln	St. Louis Source Base, Bil. of $, NSA
TRARR	$\Delta \ln$	Board of Governors Total Reserves, Adjusted for Changes in Reserve Requirements, Bil. of $, SA
VAULT	$\Delta \ln$	Vault Cash Used to Satisfy Required Reserves, Not Adjusted for Changes in Reserve Requirements, Bil. of $, NSA
PPICMAT	$\Delta \ln$	Producer Price Index: Crude Materials for Further Processing, Index 1982=100, SA

Mnemonic	*Tran.*	*Description*
DJIPRATIO	$\Delta \ln$	DJI/Industrial Production (2000 basis)//period=monthly, NSA
DJOPEN	$\Delta \ln$	DJ Industrial Average Open, NSA
DJTOPEN	$\Delta \ln$	DJ Transportation Average Open, NSA
DJUOPEN	$\Delta \ln$	DJ Utility Average Open, NSA
SP5IPRATIO	$\Delta \ln$	SP500/Industrial Production (2000 basis)//period=monthly, NSA
SP5OPEN	$\Delta \ln$	S&P 500 Open, NSA
SP5VOL	$\Delta \ln$	S&P 500 Volume, NSA