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Abstract

This study demonstrates that a Phillips curve relationship can be derived from a model with efficiency wages and imperfect information about aggregate wages or prices. The model's first-order conditions produce equations for the dynamic labor demand (DLD) curve and the dynamic efficiency wage-setting (DEWS) condition, and the paths of inflation and unemployment depend on the interaction between these curves. If one of these expressions is substituted into the other, a third relationship is derived, and this relationship has the characteristics of a Phillips curve, in which unemployment and inflation are both endogenously determined. The Phillips curve is a much more convenient and parsimonious specification than the DEWS condition. Depending on how often wages are adjusted, the Phillips curve may be either purely backward looking or have both a forward-looking and backward-looking component. The model has an equilibrium unemployment rate, and the Phillips curve and DLD curve can be used to show the dynamics of inflation and unemployment as they adjust from their initial equilibrium to their new equilibrium in response to demand shocks. The predicted coefficient on the unemployment rate in the Phillips curve is reasonably close to values that have been estimated with U.S. data, and the model does a very good job of explaining the typical unemployment dynamics in post-WWII recessions.

Keywords: Phillips curve; Efficiency wages
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I. Introduction

The Phillips curve was originally developed as a relationship between unemployment and the rate of either wage or price inflation.[1] Subsequent work by Friedman (1968) and Phelps (1968) argued that expected inflation should be included as an independent variable in a Phillips curve, with a predicted coefficient of 1. While researchers have found empirical evidence for the expectations-augmented Phillips curve,[2] it has been much more difficult to provide theoretical justification for it. For example, Mankiw (2001, p. C46) states, "The [Phillips curve] tradeoff remains mysterious, however, for the economics profession has yet to produce a satisfactory theory to explain it."[3] In addition, the Phillips curve does not have a counterpart curve in unemployment – inflation space, which means that it shows the combinations of unemployment and inflation that are possible but does not predict the actual values of these variables.

This study demonstrates that a downward-sloping Phillips curve results from the profit-maximizing behavior of firms, under the assumptions that firms pay efficiency wages and that workers and/or firms have imperfect information about aggregate wages or prices. Because information is imperfect, workers and firms predict aggregate wages or prices either through a sticky information process, as in Mankiw and Reis (2002), or through mixed rational and adaptive expectations, as in Campbell (2010) and Levine et al. (2012). The Phillips curve is purely backward looking when wages can be adjusted once each period, but has both a backward-looking and forward-looking component when wages are adjusted less often.

A wage-wage Phillips curve is obtained if workers' efficiency depends on their wages relative to the average wage, and a price-price Phillips curve is obtained if their efficiency depends on real wages. In addition, this study derives the counterpart to the Phillips curve (referred to as the dynamic labor demand curve) in inflation – unemployment space from the same framework used to derive the Phillips curve. This curve is upward sloping when the dependent variable is wage inflation and is downward sloping when the dependent variable is price inflation. Shifts in the Phillips curve and the dynamic labor demand curve trace out the paths of inflation and unemployment in response to demand or technology shocks.

Previous Phillips curve research is discussed in Section II. Section III develops a model in which efficiency depends on relative wages and in which workers have imperfect information about average wages. It is first assumed that firms can adjust wages each period. While this assumption is not realistic for modeling quarterly dynamics, it is useful for showing how the Phillips curve is derived from the interaction of the dynamic labor demand (DLD) curve and the dynamic efficiency wage-setting (DEWS) condition. These equations are obtained by substituting the production function and the unemployment equation into the model's first-order conditions and taking first differences. In both the DLD curve and the DEWS condition, wage inflation is a function of the unemployment rate, as well as other variables.

In response to shocks to the growth rate of nominal demand, shifts in the DLD curve and DEWS condition determine the paths of wage inflation and unemployment. A rise (fall) in demand growth initially reduces (raises) unemployment and raises (reduces) inflation by less than demand growth changes. Over time, unemployment returns to its initial level, and wage inflation equals the new growth rate of demand. Thus, the model is characterized by a natural rate of unemployment.

There is another way to derive the transition paths of inflation and unemployment in response to exogenous shocks. If labor demand is substituted into the efficiency wage-setting condition, a third relationship is obtained, and the transition can be illustrated by the intersections between the dynamic labor demand curve and this new relationship. This third relationship has the characteristics of a Phillips curve in which wage inflation is related to unemployment and expected wage inflation, with the coefficient on expected inflation equaling 1. In this relationship, unemployment and inflation are both endogenously determined.

While the economy's transition path can be illustrated either by the DLD curve and the DEWS condition or by the DLD curve and the Phillips curve, the latter is a more parsimonious and convenient specification. The Phillips curve depends on just expected inflation and unemployment, and the coefficient on expected inflation equals one. In contrast, the DEWS condition depends on more variables, including the change in nominal demand (which is nearly impossible to observe), and the coefficient on expected inflation will generally not equal one.

The model in Section III is then modified by allowing only a fraction of firms to adjust wages each period, making it more suitable to model quarterly dynamics. In addition to workers having imperfect knowledge about current average wages, in this case it is also assumed that firms' expectations of future wages are partly adaptive. It is demonstrated that current wage inflation depends on unemployment and on both lagged and expected future wage inflation. With this extension, the Phillips curve's predicted slope is reasonably close to empirically estimated values, and the model makes realistic predictions about the time it takes unemployment to reach its maximum value and for it to return to the natural rate, following a recessionary shock.

Section IV develops a model in which efficiency depends on workers' real wages. With this specification, the interaction between the DLD curve and the DEWS condition results in a Phillips curve in which price inflation depends on expected price inflation (with a coefficient of 1), the unemployment rate, and technology shocks. The DLD – Phillips curve framework is used to show the effects of both demand and technology shocks.

In Section V, the issue of whether workers' efficiency is more likely to depend on real or relative wages is discussed. Section VI considers an extension of the model in which effort is a function of the ratio between workers' wages and their reference wages, and it is argued that this modification enables the model to explain a wider set of phenomena. Section VII concludes.

This study expands upon the work of Campbell (2010), which develops a barebones version of the model in this study. This previous study derives equations for the wage-wage and price-price Phillips curves, but does not derive the dynamic labor demand curve or the dynamic efficiency wage-setting condition, nor does it consider staggered wage adjustments. The present study shows how a Phillips curve results from shifts of the dynamic labor demand curve and the dynamic efficiency wage-setting condition, it derives the transition paths the economy follows in response to demand and technology shocks, it provides a more complete specification of workers' behavior, and it allows wages to be adjusted sporadically.

II. Relation to Other Phillips Curve Models

Two models of the Phillips curve that have been developed in recent years are the New Keynesian Phillips curve and the sticky information Phillips curve. Roberts (1995) shows that the New Keynesian Phillips curve can be derived from the staggered contract models of Taylor (1979, 1980) and Calvo (1983) and from the quadratic adjustment cost model of Rotemberg (1982). Roberts demonstrates that these sticky price models all yield the prediction that inflation depends on expectations of future inflation and on the output gap.

While the sticky price model is widely used in policy analysis,[4] it has been criticized on several grounds. Fuhrer and Moore (1995) find that it predicts much less inflation persistence than is observed in actual data, and Ball (1994) shows that announced, credible disinflations may cause booms in this model. The New Keynesian Phillips curve predicts that inflation depends on output and expected future inflation, yet several studies find evidence against models in which expectations are purely forward looking. For example, Fuhrer (1997) regresses current inflation on lagged inflation, expected future inflation, and the output gap. He finds that the sum of coefficients is much higher on lagged inflation than on future inflation, and he cannot reject the hypothesis that expectations are purely backward looking. Rudd and Whelan (2005) rewrite the New Keynesian Phillips curve, by repeated substitution, as a relationship between current inflation and the discounted sum of expected future output gaps. They then demonstrate that there is little evidence for forward-looking expectations and that forward-looking behavior is strongly dominated by backward-looking behavior.[5] In fact, some studies that incorporate the New Keynesian Phillips curve, such as Christiano, Eichenbaum, and Evans (2005), assume that firms index their prices to lagged inflation if they cannot reset them, to enable their models to more accurately describe macroeconomic dynamics.

Galí and Gertler (1999) develop another variant of the sticky price Phillips curve in which inflation depends on marginal cost, which is measured by labor's share of national income, and they demonstrate that their model outperforms a model in which inflation depends on the output gap. While they show that price inflation depends on the behavior of wages, their study does not analyze the factors that determine wages.

Models with overlapping wage contracts are developed in Erceg, Henderson, and Levin (2000), Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005), and Galí (2011), under the assumption that workers set their own wages. However, this assumption does not characterize wage setting for the vast majority of workers, and it means that unemployment cannot legitimately be considered as involuntary.

In the present study inflation displays persistence, since the Phillips curve depends, at least partly, on lagged inflation. In contrast to the sticky-price derivation of the Phillips curve, a downward-sloping Phillips curve can be derived if firms are free to set wages and prices in each period, although including wage contracts results in a flatter Phillips curve. The Phillips curve derived in this study also differs from the New Keynesian Phillips curve in that it is a relationship between inflation and unemployment, as the Phillips curve was initially specified, rather than between inflation and output. To econometrically estimate the Phillips curve, the former is a more useful specification. The measured output gap depends on the calculated value of potential GDP, which is estimated imprecisely because of uncertainties about the natural rate and about the Okun's law relationship between unemployment deviations and output deviations.[6] On the other hand, only the first uncertainty is relevant when the Phillips curve is estimated with the difference between actual unemployment and the natural rate on the right-hand side.

In the sticky information model of Mankiw and Reis (2002, 2006), a firm's optimal price depends on aggregate prices and output. In each period, a fraction of firms receives information that enables them to compute optimal prices, while the remaining firms operate with out-of-date information. In Mankiw and Reis (2006), it is also assumed that workers set their own wages, but have sticky information concerning the determinants of the optimal wage.

The present model is similar to Mankiw and Reis in that economic fluctuations result from imperfect information, and one type of information imperfection considered is sporadic updating by workers about the values of aggregate wages or prices. However, the present study differs from Mankiw and Reis by assuming that it is firms, rather than workers, who set wages, an assumption that more realistically describes actual practice. Also, as previously discussed, unemployment in models in which workers set wages cannot be viewed as truly involuntary. In addition, the Phillips curve derived here is a relationship between inflation and unemployment, whereas the sticky information Phillips curve relates inflation to real output.

The present study also differs from both the New Keynesian and the sticky information Phillips curves by deriving the counterpart curve to the Phillips curve and by developing a model in which the economy is characterized by equilibrium unemployment.

III. The Wage-Wage Phillips Curve

The economy is assumed to be populated by a continuum of *ex ante* identical individuals and firms. It is also assumed that firms make random errors in setting wages, but that the profit-maximizing wage is set on average. Because wages vary across firms, workers do not know the mean of the aggregate wage distribution with certainty. The rest of this section considers the assumptions about individuals' behavior, firms' behavior, and the derivations of the dynamic labor demand curve, the dynamic efficiency wage-setting condition, and the Phillips curve.

Assumptions about individuals' behavior

This subsection develops the theoretical framework to explain individuals' decisions regarding labor supply, effort, product demand, and information acquisition. It is assumed that individuals' utility depends positively on their consumption and their leisure, with ρ representing the relative weight placed on leisure. As in Dixit and Stiglitz (1977), total consumption (c) is the composite of the output purchased from individual firms. Assuming a continuity of firms, indexed from 0 to 1, total consumption can be expressed as

$$c_{t+i} = \left[\int_0^1 c_{t+i}(f)^{\frac{\gamma-1}{\gamma}} df \right]^{\frac{\gamma}{\gamma-1}}.$$

Utility is also assumed to depend on workers' effort (*e*), with the marginal utility being negative in equilibrium. While increased effort lowers workers' current utility, it also reduces the probability of him or her being dismissed, which increases expected future utility. A rational worker balances the costs and benefits of effort in deciding how hard to work. Since a dismissed worker needs to find work elsewhere, the optimal level of effort depends on the average wage paid by other firms, which workers may not know with certainty since wages vary across firms. Workers who form incorrect expectations of the average wage will exert a non-optimal level of effort and incur a utility loss as a result.[7] However, information about current average wages is assumed to be costly, so a rational worker may not acquire all available information.

A worker seeks to maximize

$$
\begin{aligned}
E[U] = {} & E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+\delta} \right)^i \left\{ \log \left[\int_0^1 c_{t+i}(f)^{\frac{\gamma-1}{\gamma}} df \right]^{\frac{\gamma}{\gamma-1}} + \rho \Pr[Emp_{t+i}] \log(X_{t+i}) \right. \\
& + \rho (1 - \Pr[Emp_{t+i}]) \log(T) \\
& \left. + (T - X_{t+i} - zI_{t+i}) \Pr[Emp_{t+i}][\alpha e_{t+i} - \eta e_{t+i}^2 + \zeta \frac{W_{t+i}}{\overline{W}_{t+i}^{e}} e_{t+i} - \Gamma(I_{t+i})] \right\}
\end{aligned} \tag{1}
$$

$$
\text{s.t.} \quad \sum_{i=0}^{\infty} \frac{\int_0^1 P_{t+i}(f) c_{t+i}(f) df}{\prod_{j=1}^{i} (1 + r_{t+j-1})} = \sum_{i=0}^{\infty} \frac{1}{\prod_{j=1}^{i} (1 + r_{t+j-1})} W_{t+i} (T - X_{t+i} - zI_{t+i}) \Pr[Emp_{t+i}] \ ,
$$

where δ is the discount rate, $\Pr[Emp_{t+i}]$ is the probability that the worker is employed in period *t+i*, *X* is leisure in the periods in which the individual is employed, *T* is time allotment (allocated between labor, leisure, and acquiring information), α, η, and ζ are parameters representing the utility or disutility of effort, *W* is the wage, $\overline{W}^{e}$ represents workers' expectations of the average wage, *r* is the interest rate, and *P*(*f*) is the price of the *f*th firm's output. In addition, *I* represents the amount of information acquired about average wages, $\Gamma(I)$ is the expected utility loss resulting from imperfect information about average wages (with $\Gamma'(I)<0$), and z is the time spent obtaining information. The first term in the utility function is the utility from consumption, the second is the utility from leisure when the employee is working, the third is the utility from leisure when the individual is unemployed, and the fourth is the utility or disutility of effort when the employee is working.

If the first-order conditions are approximated around their steady-state equilibria, it can be demonstrated that labor supply can be expressed as

$$N\left(\frac{W_{t+i} / \overline{P}^{e}_{t+i}}{W^{SS} / \overline{P}^{SS}}\right), \text{ with } N' \geq 0, \tag{2}$$

where W^{SS} and $\overline{P}^{SS}$ represent the steady-state values of wages and prices.[8] The short-run labor-supply elasticity (i.e., N'/N) will be denoted by ψ.

This constrained maximization problem can also be used to derive an expression for workers' effort, although this derivation is quite complex. Campbell (2006) develops a model of workers' effort with a similar utility function and budget constraint and makes assumptions about the probability of dismissal (as a function of a worker's effort) and the probability of an unemployed worker being hired.[9] It is demonstrated that workers' efficiency depends on the ratio of their current wage to the average wage at other firms and on the unemployment rate, such that

$$e = e[W_t / \overline{W}_t^{e}, u_t] \qquad \text{with } e_W > 0, \quad e_u > 0, \quad e_{WW} < 0, \quad e_{Wu} < 0, \tag{3}$$

where W_t is the wage at a worker's current firm, $\overline{W}_t^{e}$ denotes workers' expectations of the average wage rate, and u_t is the unemployment rate.[10]

Given the assumption about total consumption being the composite of individual goods, Dixit and Stiglitz (1977) demonstrate that the demand curve facing each firm can be expressed as

$$Q_t^D = Y_t \left(\frac{P_t}{\overline{P}_t} \right)^{-\gamma}, \tag{4}$$

where P is the firm's price, $\overline{P}$ is the aggregate price level, γ is the price elasticity of demand, and Y is real aggregate demand per firm.

Since it is costly to obtain information about average wages, workers may choose not to acquire all available information that enables them to predict the average wage (which they need to estimate to optimize their effort), particularly because the relevant comparison is wages for workers in the same occupational group, and current data on occupational wages are not easily obtainable.[11] As a result, their expectations may not necessarily satisfy the criteria for rational expectations and may be based partly on old information. This study considers two ways that workers may form expectations when information is costly. One is that only a fraction of workers gets updated information in each period, as in the sticky information model, and the other is that each worker's expectations are a mixture of rational and adaptive expectations.

Assumptions about firms' behavior

1. Firms produce output (Q) with the Cobb-Douglas production function,

$$Q_t = A_t^{\phi} L_t^{\phi} K_0^{1-\phi} e\left[W_t / \overline{W}_t^{\,e}, u_t \right]^{\phi}, \tag{5}$$

where A represents technology (assumed to be exogenous and labor augmenting), L is labor, and K is capital (assumed to be fixed). It is assumed that firms have unbiased expectations about workers' expectations of the average wage ($\overline{W}_t^{\,e}$).

2. Real aggregate demand per firm is determined from the constant velocity specification,

$$Y_t = M_t / \overline{P}_t, \tag{6}$$

where M is nominal demand per firm.

3. Parameters are such that firms pay efficiency wages, yielding excess supply of labor.[12] The unemployment rate can be expressed as

$$u_t = \frac{N\left(\dfrac{W_t / \overline{P}_t^e}{W^{SS} / \overline{P}^{SS}}\right) - L_t}{N\left(\dfrac{W_t / \overline{P}_t^e}{W^{SS} / \overline{P}^{SS}}\right)}. \tag{7}$$

4. All firms share a common efficiency function, but individual firms do not know the effect of wages on efficiency with certainty. As a result, each firm views its value of e_W as

$$e_{W,i}\left[W_t/\overline{W}_t^e, u_t\right] = e_W\left[W_t/\overline{W}_t^e, u_t\right] + \varepsilon_i ,$$

where ε_i is a firm-specific white noise error. As a result of these random errors in estimating e_W, wages vary across firms, but the profit-maximizing wage is set on average.

5. It is costly for firms both to acquire information about next period's average wage and to set its current wage at a non-optimal level as a result of incorrect expectations about next period's average wage. (This assumption is only relevant when only a fraction of firms can adjust wages each period.) Because firms may not acquire all relevant information, their expectations may not be fully rational. These costs could be incorporated into the profit function, although the solution for information acquisition would be trivial.

Derivations of the DLD, DEWS, and Phillips curves when wages can be adjusted each period

By solving (4) for P_t and multiplying by Q_t, total revenue is given by

$$P_t Q_t = Y_t^{\frac{1}{\gamma}} Q_t^{\frac{\gamma-1}{\gamma}} \overline{P}_t .$$

Thus, profits in period t are

$$\Pi_t = Y_t^{\frac{1}{\gamma}} \left[A_t^{\phi} L_t^{\phi} K_0^{1-\phi} e[W_t / \overline{W}_t^{\,e}, u_t]^{\phi} \right]^{\frac{\gamma-1}{\gamma}} \overline{P}_t - W_t L_t - rK_0 . \tag{8}$$

It is first assumed that firms are free to adjust wages and prices each period. With this assumption, it is demonstrated that the interaction between the DLD curve and the DEWS condition produces a Phillips curve and that the Phillips curve is downward sloping, even if all firms can adjust wages and prices. By differentiating the profit function with respect to wages and employment, the following first-order conditions for the average firm are obtained:

$$\frac{d\Pi_t}{dL_t} = 0 = \frac{\phi(\gamma-1)}{\gamma} Y_t^{\frac{1}{\gamma}} A_t^{\frac{\phi(\gamma-1)}{\gamma}} L_t^{\frac{\phi(\gamma-1)}{\gamma}-1} K_0^{\frac{(1-\phi)(\gamma-1)}{\gamma}} e[\bullet]^{\frac{\phi(\gamma-1)}{\gamma}} \overline{P}_t - W_t , \tag{9a}$$

and

$$\frac{d\Pi_t}{dW_t} = 0 = \frac{\phi(\gamma-1)}{\gamma} Y_t^{\frac{1}{\gamma}} A_t^{\frac{\phi(\gamma-1)}{\gamma}} L_t^{\frac{\phi(\gamma-1)}{\gamma}} K_0^{\frac{(1-\phi)(\gamma-1)}{\gamma}} e[\bullet]^{\frac{\phi(\gamma-1)}{\gamma}-1} e_W[\bullet] \frac{1}{\overline{W}_t^{\,e}} \overline{P}_t - L_t . \tag{9b}$$

Combining (9a) and (9b) and taking steady-state values (i.e., $W = \overline{W}$) results in the equilibrium condition,

$$e_W[1,u] e^{-1}[1,u] = 1. \tag{10}$$

The steady-state condition, $e_W e^{-1} = 1$, determines the economy's natural rate of unemployment. In addition, this condition will be used to simplify equations expressed in terms of deviations from steady-state values.

Equation (9a) is the labor demand curve, and (9b) is the efficiency wage-setting condition. Totally differentiating (9a) and (9b) and dividing by the original equations yields

$$\hat{W}_t = \frac{1}{\gamma}\hat{Y}_t + \frac{\phi(\gamma-1)}{\gamma}\hat{A}_t + \frac{\phi\gamma-\phi-\gamma}{\gamma}\hat{L}_t + \frac{\phi(\gamma-1)}{\gamma} e_W e^{-1}\frac{W_t}{\overline{W}_t^{e}}\hat{W}_t$$
$$-\frac{\phi(\gamma-1)}{\gamma} e_W e^{-1}\frac{W_t}{\overline{W}_t^{e}}\hat{\overline{W}}_t^{e} + \frac{\phi(\gamma-1)}{\gamma} e_u e^{-1} du_t + \hat{\overline{P}}_t, \tag{11a}$$

and

$$\frac{\phi+\gamma-\phi\gamma}{\gamma}\hat{L}_t = \frac{1}{\gamma}\hat{Y}_t + \frac{\phi(\gamma-1)}{\gamma}\hat{A}_t + \frac{\phi\gamma-\phi-\gamma}{\gamma} e^{-1}\left[e_W \frac{W_t}{\overline{W}_t^{e}}\hat{W}_t - e_W\frac{W_t}{\overline{W}_t^{e}}\hat{\overline{W}}_t^{e} + e_u du_t\right]$$
$$+ e_{WW} e_W^{-1}\frac{W_t}{\overline{W}_t^{e}}\hat{W}_t - e_{WW} e_W^{-1}\frac{W_t}{\overline{W}_t^{e}}\hat{\overline{W}}_t^{e} + e_{Wu} e_W^{-1} du_t - \hat{\overline{W}}_t^{e} + \hat{\overline{P}}_t, \tag{11b}$$

where variables with "^"'s over them represent percentage deviations from steady-state values (e.g., $\hat{W}_t = dW_t / W_t$). The above equations express the relationships between percentage deviations in W_t, percentage deviations in $\overline{W}_t^{e}$, and percentage-point deviations in u_t from their equilibrium values. (Thus, $du_t=u_t-u^*$, where u^* is the natural rate.) If small deviations of W, $\overline{W}^{e}$, and u from their steady-state values are considered, the coefficients on these variables can be treated as constants, with these constants determined by the equilibrium values of W_t, $\overline{W}_t^{e}$, e, e_W, e_u, e_{WW}, and e_{Wu}.

The Appendix demonstrates that calculating deviations in steady-state values in the production function (5) and the unemployment equation (7) and substituting these expressions into (11a) and (11b) results in the following equations for wages:

$$\hat{W}_t = \frac{\hat{M}_t + s_L^{-1} du_t + \psi\hat{\overline{P}}_t^{e}}{1+\psi}, \qquad \text{and} \tag{12a}$$

$$\hat{W}_t = \frac{-e_{WW} e_W^{-1}\hat{\overline{W}}_t^{e} + \left[s_L^{-1} - e_u e^{-1} + e_{Wu} e_W^{-1}\right] du_t + \psi\hat{\overline{P}}_t^{e} + \hat{M}_t}{1 - e_{WW} e_W^{-1} + \psi}, \tag{12b}$$

where s_L is the equilibrium value of 1-*u*. Equations (12a) and (12b) are, respectively, the labor demand curve and the efficiency wage-setting condition, expressed as relationships between wages and unemployment. By subtracting the lag of each equation, the following expressions for wage inflation are obtained:

$$\hat{W}_t - \hat{W}_{t-1} = \frac{(\hat{M}_t - \hat{M}_{t-1}) + s_L^{-1}(du_t - du_{t-1}) + \psi(\hat{\bar{P}}_t^e - \hat{\bar{P}}_{t-1}^e)}{1+\psi}, \tag{13a}$$

and

$$\hat{W}_t - \hat{W}_{t-1} = \frac{\begin{array}{l} -e_{WW}e_W^{-1}(\hat{\bar{W}}_t^e - \hat{\bar{W}}_{t-1}^e) + \left[s_L^{-1} - e_u e^{-1} + e_{Wu}e_W^{-1}\right](du_t - du_{t-1}) \\ +\psi(\hat{\bar{P}}_t^e - \hat{\bar{P}}_{t-1}^e) + (\hat{M}_t - \hat{M}_{t-1}) \end{array}}{1 - e_{WW}e_W^{-1} + \psi}. \tag{13b}$$

Equation (13a) will be called the dynamic labor demand (DLD) curve, and equation (13b) will be called the dynamic efficiency wage-setting (DEWS) condition. The dynamic efficiency wage-setting condition is a relatively complicated expression that includes the changes in the unemployment rate, expected average wages, expected prices, and nominal demand. Neither the coefficient on the change in expected average wages nor the coefficient on the change in expected prices will generally equal 1.

The DLD-DEWS framework can be used to show how wage inflation and unemployment evolve over time in response to a shock to the growth rate of nominal demand ($\hat{M}_t$). In particular, it is assumed that demand is growing at a rate of g^o prior to period 1 and that demand growth decreases to g^n in period 1 and remains at g^n indefinitely. Since wages can be adjusted each period, it is reasonable to view a period as corresponding to a year. To simulate a demand shock, it is necessary to make assumptions about the parameters in (13a) and (13b) and about the nature of inflationary expectations. For parameters, the equilibrium unemployment rate is set at

5%, equilibrium effort (e) is assumed to equal 0.8, and e_W is determined from (10). Values for e_u and e_{Wu} are determined from the effort model of Campbell (2006) with the condition that the elasticity of efficiency with respect to unemployment equals 0.05, consistent with estimates in Weisskopf (1987) and Wadhwani and Wall (1991).[13] The model of Campbell (2006) also yields a value for e_{WW}, which results in a realistic Phillips curve slope with sporadic wage adjustment (as demonstrated in the next subsection), but yields a slope that is much higher than empirical estimates if wages are adjusted each period. Thus, e_{WW} is set so that the slope of the Phillips curve equals -1, in line with Blanchard and Katz's (1997) estimates with annual data. In addition, the short-run labor supply elasticity (ψ) is assumed to equal 0 since empirical studies find that this elasticity is low.[14]

There are several ways in which inflationary expectations can be modeled. It could be assumed that inflationary expectations are rational, in which case nominal demand shocks have no systematic effect on unemployment. However, workers may choose not to acquire all available information because of the cost of information.

If information is costly, there are at least two approaches to modeling expectations. One is to assume, in the spirit of Mankiw and Reis's (2002) sticky information model, that each period a fraction of workers receives new information about the current and expected future values of all macroeconomic variables, while the rest operate with out-of-date information.[15] Let $\theta(I)$ represent the proportion of workers who receive new information in each period, with $\theta'(I) > 0$ (i.e., an increase in I means that information is updated more frequently). Suppose that workers whose information has not been updated expect wages to continue to grow at a rate of g^o. Suppose also that workers who have received new information know the true growth rate of demand, know the true fraction of workers receiving new information in each period, and know the wage expectations of workers whose information has not been updated since period 0. Then these workers will have rational expectations about average wages from the time they receive this information onwards. Thus, overall expectations can be expressed as

$$\hat{\bar{W}}_t^e = [1-(1-\theta(I))^t](\hat{\bar{W}}_t + \varepsilon_t) + (1-\theta(I))^t tg^o \,, \tag{14}$$

where ε_t is a white noise error. A second approach to modeling workers' expectations when information is costly is to assume, as in Campbell (2010) and Levine et al. (2012), that expectations are a mixture of rational and adaptive expectations, so that

$$\hat{\bar{W}}_t^e = \omega(I)(\hat{\bar{W}}_t + \varepsilon_t) + (1-\omega(I))\left[\hat{\bar{W}}_{t-1} + \sum_{i=1}^{T} \lambda_i (\hat{\bar{W}}_{t-i} - \hat{\bar{W}}_{t-i-1})\right] \text{ with } \lambda_1 + \lambda_2 + \cdots + \lambda_T = 1, \tag{15}$$

where $\omega(I)$ represents the degree to which expectations are rational, with $\omega'(I) > 0$.[16] A model providing justification for this assumption from workers' utility maximization is derived in Campbell (2013).[17] In addition, Levine et al. (2012) develop a DSGE model both under the assumption that all firms and households have rational expectations and the assumption that a proportion of households and firms have rational expectations and the rest have adaptive expectations. They find that, "All behavioural models [i.e., models with mixed rational and adaptive expectations] 'decisively', in fact very decisively, dominate the purely rational models with very large LL [log likelihood] differences of around 20." With their best specification they estimate that 17-25% of firms and 30-34% of households have rational expectations.

To demonstrate the effects of nominal demand shocks (technology shocks are also considered in Section IV), simulations are performed under both assumptions about wage expectations. In simulations with mixed rational and adaptive expectations, ω is set at 0.5 (i.e., expectations are treated as an equal mixture of rational and adaptive expectations), and it is assumed that λ_1=1 and $\lambda_2 = \lambda_3 = \cdots = \lambda_T = 0$, consistent with empirical evidence with annual data.[18] For the sticky information model, θ is set at 0.5.

Figures 1a (with mixed rational and adaptive expectations) and 1b (with sticky information) show how wage inflation and unemployment respond over time to a decrease in the growth rate of demand from 5% to 0%. The DLD and the DEWS curves are shown for the initial equilibrium and the first three periods following the reduction in demand growth.[19] Values of inflation and unemployment are denoted by dots (including values after period 3), and the initial and first five unemployment–inflation points are numbered. This demand shock initially causes a rise in unemployment and a fall in wage inflation. Over time, the economy eventually reaches a new equilibrium in which unemployment returns to the natural rate and inflation equals the new growth rate of demand. By comparing Figures 1a and 1b, it is seen that unemployment and inflation in period 1 are the same with both assumptions and that wages and unemployment adjust slowly to equilibrium in both cases. The differences are that unemployment falls below the natural rate in Figure 1a, but not in Figure 1b, and that convergence is slower in Figure 1b, since, with sticky information, some workers still expect that wages have been increasing 5% per period since period 1. Because the results are qualitatively similar, only the results with mixed rational and adaptive expectations are shown in the remainder of this study.

While the DLD-DEWS framework is one way to show the paths of wage inflation and unemployment in the transition between equilbria, there is another way to show the transition paths. If (12a) is solved for $\hat{M}_t$ and the resulting expression is substituted into (12b), the following equation is obtained for an individual firm's optimal wage:

$$\hat{W}_t = \hat{\bar{W}}_t^{\,e} + \frac{e_u - e_{Wu}}{e_{WW}} du_t .^{20} \tag{16}$$

By subtracting $\hat{W}_{t-1}$ from both sides of (16) and averaging across firms, the relationship between wage inflation, expected wage inflation, and unemployment is

$$(\hat{\overline{W}}_t - \hat{\overline{W}}_{t-1}) = (\hat{\overline{W}}_t^{\,e} - \hat{\overline{W}}_{t-1}) + \frac{e_u - e_{Wu}}{e_{WW}} du_t \,. \tag{17}$$

Equation (17) has the characteristics of a Phillips curve (PC), as the coefficient on expected inflation equals 1, the difference between the current unemployment rate and the natural rate (i.e., du_t) appears on the right-hand side with a negative sign (since $e_u > 0$, $e_{Wu} < 0$, and $e_{WW} < 0$), and the growth rate of demand is not an explanatory variable.

If expectations are a mixture of rational and adaptive expectations, there is another way to express the Phillips curve. Substituting (15) into (16) and averaging across firms yields

$$(\hat{\overline{W}}_t - \hat{\overline{W}}_{t-1}) = \sum_{i=1}^{T} \lambda_i (\hat{\overline{W}}_{t-i} - \hat{\overline{W}}_{t-i-1}) + \frac{e_u - e_{Wu}}{(1-\omega)e_{WW}} du_t \,. \tag{18}$$

While (17) and (18) are equivalent specifications, they differ in two ways: ω affects the slope of (18) but not of (17), and the Phillip curve is shifted by expected inflation in (17), but is shifted by lagged inflation in (18). (These specifications are identical if ω=0.)

Figure 2 illustrates the response of wage inflation and unemployment to a decline in the growth rate of demand from 5% to 0%, using the DLD-PC framework. (The DEWS curve is also included in Figure 2.) The Phillips curve (as expressed in (18)) is shifted by lagged wage inflation, and the DLD curve is shifted by changes in the growth rate of nominal demand and by lagged unemployment. Figure 2 shows that the DLD-PC framework and the DLD-DEWS framework both predict the same paths for wage inflation and unemployment.

While the DLD-DEWS framework and the DLD-PC framework give the same results, the Phillips curve is a much more parsimonious specification than the dynamic efficiency wage-setting condition. The DEWS condition includes the changes in nominal demand and expected prices, variables that do not appear in the Phillips curve. The unemployment variable is its level in the Phillips curve, but is its change in the DEWS condition. In addition, the DEWS condition includes the change in wage expectations, and the coefficient on this difference depends on the model's microeconomic parameters. Thus, it is likely to vary across countries and across time, and it is unlikely to equal 1. In contrast, the Phillips curve includes expected wage inflation, and the coefficient on this variable equals 1 for any set of microeconomic parameters.

The exogenous variable that shifts the DLD and DEWS curves, and thus determines the trajectory of unemployment and inflation, is the growth rate of nominal demand. In deriving the Phillips curve, however, nominal demand drops out. Thus, the Phillips curve can be viewed as the relationship between two endogenous variables as they adjust in response to a nominal demand shock. The actual values of unemployment and wage inflation depend on the interaction between the Phillips curve and the DLD curve.

In (17) and (18), wage inflation is a function of unemployment and either expected wage inflation or lagged wage inflation (i.e., a wage-wage Phillips curve). However, when economists estimate Phillips curves, the right-hand side variable is generally expected price inflation rather than expected wage inflation. While expected price inflation is the independent variable in the vast majority of Phillips curve studies, the right-hand side variable in Phelps's (1968) seminal paper is expected wage inflation, resulting in a wage-wage Phillips curve (although this equation is not empirically estimated).[21]

Even if workers' efficiency is a function of relative wages, it is still likely that researchers will find evidence for a price-price and wage-price Phillips curve, as well as for a wage-wage Phillips curve. Campbell (2009) demonstrates that a model in which efficiency depends on relative wages yields asymptotic price-price and wage-price Phillip curves when the economy is subjected to stochastic aggregate demand shocks. In this model, equations are derived for the paths of wages, prices, and unemployment in response to nominal demand shocks, and these equations are used as data in a theoretical regression of either price inflation or wage inflation on unemployment and lagged price inflation. In these regressions, the coefficient on lagged price inflation asymptotically approaches 1 as the sample size increases, and it is close to 1 even when the sample size is small.

Technology shocks ($\hat{A}$) do not appear in the dynamic labor demand curve, the dynamic efficiency wage-setting condition, or the Phillips curve, which means that these shocks leave nominal wages and unemployment unchanged in both the short run and the long run. While technology shocks do not affect wages and unemployment, these shocks immediately and permanently change prices by -ϕ times the percentage change in technology.

A Model with Overlapping Wage Contracts

The model can be extended to assume that wages are set by multi-period overlapping contracts. In the model in the previous subsection, wages can be changed each period, which implies that a period in these models corresponds to a year of actual time. However, to model short-run fluctuations it is more convenient to treat a period as a quarter. With a period corresponding to a quarter, it is reasonable to assume that only a fraction of firms adjust wages each period and that these adjustments are not synchronized. Let τ represent the proportion of firms that can change wages in each period and β represent the discount rate. Then the Appendix demonstrates that wage inflation (π_t^w) can be expressed as,

$$\pi_t^w = \frac{\beta(1-\tau)}{1-\beta\tau(1-\tau)}\pi_{t+1}^{w,e} + \frac{\tau[1-\beta(1-\tau)]}{1-\beta\tau(1-\tau)}\pi_t^{w,e} + \frac{\tau[1-\beta(1-\tau)]}{1-\beta\tau(1-\tau)}\frac{e_u - e_{Wu}}{e_{WW}}du_t\,, \tag{19}$$

where $\pi_t^{w,e}$ represents workers' expectations of wage inflation in period t and $\pi_{t+1}^{w,e}$ represents firms' expectations of next period's wage inflation at the time they set wages for period t. As before, workers' expectations of current average wages are treated as a mixture of rational and adaptive expectations. In addition, in line with the findings of Levine at al. (2012), firms' expectations of future wages are also assumed to be a mixture of rational and adaptive expectations. Because expectational lags are likely to be long with quarterly data, a geometric lag structure is used. For simplicity, the degree to which expectations are rational and the geometric lag parameter in the adaptive component are assumed to be the same for both workers and firms. In particular, the specifications for expected wage inflation are

$$\pi_t^{w,e} = \omega\pi_t^w + (1-\omega)\sum_{i=1}^{\infty}\lambda^*(1-\lambda^*)^{i-1}\pi_{t-i}^w \,, \qquad \text{and} \tag{20}$$

$$\pi_{t+1}^{w,e} = \omega\pi_{t+1}^{w,ue} + (1-\omega)[\lambda^*\pi_t^w + \sum_{i=2}^{\infty}\lambda^*(1-\lambda^*)^{i-1}\pi_{t+1-i}^w]\,, \tag{21}$$

where $\pi_{t+1}^{w,ue}$ represents firms' unbiased expectations of future wages. Substituting (20) and (21) into (19) yields the following expression for current wage inflation:

$$\pi_t^w = \frac{\beta(1-\tau)\omega\pi_{t+1}^{w,ue} + (1-\omega)[\beta(1-\tau)(1-\lambda^*) + \tau - \tau\beta(1-\tau)]\sum_{i=1}^{\infty}\lambda^*(1-\lambda^*)^{i-1}\pi_{t-i}^w + \tau[1-\beta(1-\tau)]\dfrac{e_u - e_{Wu}}{e_{WW}}du_t}{1-\omega\tau-\beta(1-\tau)(1-\omega)(\tau+\lambda^*)}\,. \tag{22}$$

The DLD-PC framework is used to simulate the economy's response to a deceleration in the growth rate of nominal demand. In simulating this shock, the quarterly discount factor (β) is set at 0.99, and the fraction of firms adjusting wages (τ) is assumed to equal 0.25, implying that wages are adjusted, on average, once each year. Values of u^*, ψ, e, e_W, e_u, and e_{Wu} are the same as before. In addition, e_{WW} is also determined from the effort model of Campbell (2006).[22] Thus, e_u, e_{Wu}, and e_{WW} are all determined from a micro-based effort model based on the utility function in (1). A value for λ* is chosen to minimize the sum of squared errors been actual wage inflation (with quarterly Employment Cost Index data) and the value predicted by the geometric lag structure with eight lags. Based on this criterion, λ* is set at 0.32. The parameter ω (measuring the extent to which expectations are rational) is set equal to 0.25, consistent with Levine et al.'s (2012) estimates of the degree of rational expectations. These estimates are also in line with Mankiw and Reis's (2002) assumption that 25% of firms receive new information each quarter. The value of π_{t+1}^{w} is calculated by solving (12a) for du_t, substituting the resulting expression into (22), and solving a difference equation in wage levels.

With these parameters, the predicted coefficient on the unemployment rate is –0.141. In addition, if it is assumed that firms do not have perfect information about current wage inflation when they set wages,[23] and estimate current inflation through a mixture of rational and adaptive expectations, then the predicted coefficient is –0.116. When Galí (2011) estimates the New Keynesian wage Phillips curve with quarterly data through the end of 2007 (with lagged year-to-year price inflation as a dependent variable), the sum of coefficients is –0.096 or –0.099, depending on the measure of wages.[24] Campbell (1997) estimates conventional wage Phillips curves with quarterly Employment Cost Index data, and finds that the sum of coefficients on unemployment lies between –0.0910 and –0.109. Thus, the predicted coefficient on the unemployment rate is close to coefficients estimated with U.S. data, and it is remarkably close if firms lack perfect information about current wage inflation. The most important determinants of this slope are τ and $(e_u - e_{Wu})/e_{WW}$, and both values are based on microeconomic evidence.[25] The value of the first comes from the assumption that the average worker's wages are adjusted once a year, and the value of the second is determined from the microeconomic-based effort model of Campbell (2006), using independent, yet similar, estimates from Weisskopf (1987) and Wadhwani and Wall (1991) about the effect of unemployment on productivity.

Figure 3 shows how unemployment and wage inflation respond to a deceleration of the growth rate of nominal demand from a 1.25% quarterly rate (i.e., a 5% annual rate) in period 0 and prior to 0% in period 1 and thereafter. The initial equilibrium is marked with “0.” In this initial equilibrium, the unemployment rate is slightly below the natural rate consistent with zero inflation because demand is rising and the sum of coefficients on expected future inflation and lagged inflation is slightly less than 1, since $\beta<1$. (If $\beta=1$, then $u=5\%$ in period 0.) In response to this disinflationary shock, wage inflation gradually declines and unemployment increases, until it reaches its maximum value in period 5. It remains close to this value in period 6 and then starts to fall significantly in period 7, until it falls to within 0.2% of the natural rate in period 14. In post-WWII recessions, the median time for unemployment to reach its maximum after the recession began (based on the NBER’s dating) is six quarters, and the median time for it to fall to within 0.2% of the Congressional Budget Office’s (CBO) estimate of the natural rate is 13.5 quarters.[26] Thus the theoretical model, based on the optimizing behavior of individuals and firms, does a very good job of explaining both the slope of the Phillips curve and the economy’s behavior in the typical postwar recession.[27]

IV. The Price-Price Phillips Curve

It is now assumed that workers’ efficiency depends on the ratio between their wages and their expectations of the price level. Only the case in which prices can be adjusted each period is considered since Nakamura and Steinsson (2008) find a great degree of heterogeneity in the frequency of price changes across sectors of the U.S. economy and since Christiano et al. (2005)

find that overlapping wage contracts are much more important than overlapping price contracts in explaining macroeconomic dynamics. If efficiency depends on wages relative to price expectations, equations (5), (8), (9a), and (9b) become

$$Q_t = A_t^{\phi} L_t^{\phi} K_0^{1-\phi} e\left[W_t/\overline{P}_t^{e}, u_t\right]^{\phi}, \tag{23}$$

$$\Pi_t = Y_t^{\frac{1}{\gamma}} \left[A_t^{\phi} L_t^{\phi} K_0^{1-\phi} e[W_t / \overline{P}_t^{e}, u_t]^{\phi}\right]^{\frac{\gamma-1}{\gamma}} \overline{P}_t - W_t L_t - rK_0, \tag{24}$$

$$\frac{d\Pi_t}{dL_t} = 0 = \frac{\phi(\gamma-1)}{\gamma} Y_t^{\frac{1}{\gamma}} A_t^{\frac{\phi(\gamma-1)}{\gamma}} L_t^{\frac{\phi(\gamma-1)}{\gamma}-1} K_0^{\frac{(1-\phi)(\gamma-1)}{\gamma}} e[W_t / \overline{P}_t^{e}, u_t]^{\frac{\phi(\gamma-1)}{\gamma}} \overline{P}_t - W_t, \tag{25a}$$

and

$$\begin{aligned} \frac{d\Pi_t}{dW_t} = 0 = {} & \frac{\phi(\gamma-1)}{\gamma} Y_t^{\frac{1}{\gamma}} A_t^{\frac{\phi(\gamma-1)}{\gamma}} L_t^{\frac{\phi(\gamma-1)}{\gamma}} K_0^{\frac{(1-\phi)(\gamma-1)}{\gamma}} e[W_t / \overline{P}_t^{e}, u_t]^{\frac{\phi(\gamma-1)}{\gamma}-1} \\ & \times e_W [W_t / \overline{P}_t^{e}, u_t] \frac{1}{\overline{P}_t^{e}} \overline{P}_t - L_t. \end{aligned} \tag{25b}$$

Combining (25a) and (25b) and taking steady-state values yields the following equilibrium condition:

$$e_W\left[\frac{W}{\overline{P}^{e}}, u\right] e^{-1}\left[\frac{W}{\overline{P}^{e}}, u\right] \frac{W}{\overline{P}^{e}} = 1. \tag{26}$$

As in Section III, this steady-state condition determines the natural rate of unemployment and will be used to simplify equations expressed in terms of deviations from equilibrium values.

The Appendix derives equations for the dynamic labor demand curve, the dynamic efficiency wage-setting condition, and the Phillips curve, and it is demonstrated that the dynamic labor demand curve and the Phillips curve are, respectively,

$$(\hat{\overline{P}}_t - \hat{\overline{P}}_{t-1}) = (1-\phi)(\hat{M}_t - \hat{M}_{t-1}) - \phi(\hat{A}_t - \hat{A}_{t-1}) + \phi(\hat{\overline{P}}_t^{e} - \hat{\overline{P}}_{t-1}^{e}) - \phi e_u e^{-1}(du_t - du_{t-1}), \tag{27}$$

and

$$(\hat{\hat{P}}_t - \hat{\hat{P}}_{t-1}) = (\hat{\hat{P}}_t^e - \hat{\hat{P}}_{t-1}) - \frac{(1-\phi)[e_{WW}\zeta^2 - (1+\psi)s_L(e_u - \zeta e_{Wu})] + \phi e_{WW}\zeta^2 e_u e^{-1} s_L}{e_{WW}\zeta^2 s_L} du_t - \phi\hat{A}_t, \tag{28}$$

[28]

where ζ equals the equilibrium real wage. The dynamic efficiency wage-setting condition is quite complex and is not reported in the main body of the text. As before, the Phillips curve is a more parsimonious specification than the DEWS condition, and the growth rate of nominal demand appears in the DEWS condition but not in the Phillips curve.

In the Phillips curve in (28), the coefficient on expected inflation equals 1 and the coefficient on the unemployment rate is negative, since $1-\phi>0$, $e_u>0$, $e_{WW}<0$, $e_{Wu}<0$, $\psi\geq 0$, and $\zeta>0$. In addition, price inflation depends negatively on technology shocks.

Figure 4 shows the paths of unemployment and price inflation in response to a deceleration in the growth of nominal demand from g^o to g^n (where g^o=5% and g^n=0%), using the dynamic labor demand curve (27) and the Phillips curve (28) for the initial equilibrium and for three periods following the deceleration in demand.[29] As before, values of unemployment and inflation are denoted by dots (including values after period 3). This deceleration initially causes unemployment to rise and inflation to fall. In the long run, the economy reaches a new equilibrium in which unemployment equals the natural rate and inflation equals the new growth rate of nominal demand.

The economy's response to a technology shock is illustrated in Figure 5 for the initial equilibrium and for four periods following the shock. In this simulation, technology decreases by 3% in period 1 and remains at this level indefinitely. (Nominal demand is assumed to remain constant.) Unemployment and inflation both initially rise, but then decrease as the economy adjusts to its new equilibrium.

If efficiency depends on real wages, technology shocks alter the equilibrium unemployment rate. A technology shock changes the equilibrium real wage, and (26) shows that a change in the equilibrium real wage results in a different natural rate of unemployment. Thus, the economy is not characterized by a fixed natural rate of unemployment in response to technology shocks if efficiency depends on real wages. As discussed in Section V, however, it is much more likely that efficiency depends on relative wages in the long run. Because the assumption that efficiency depends on real wages probably does not describe the long run, points corresponding to inflation and unemployment after period 4 are not included in Figure 5.

V. Does Workers' Efficiency Depend on Relative or Real Wages?

In the model in Section III, workers' efficiency depends on their wage relative to average wages, while their efficiency depends on the real wage in Section IV. There is little empirical evidence concerning whether efficiency is a function of relative or real wages. In theory, workers' efficiency is likely to depend more on relative wages than on real wages since decisions regarding shirking and quitting should depend on a worker's wage relative to wages elsewhere. However, there are reasons why efficiency may depend on real wages in the short run. First, in the fair wage model of Akerlof and Yellen (1990), workers may view the fair wage as a function of the real wage and thus feel that their employer has an obligation to compensate them for a rise in consumer prices. Second, even if workers are concerned about relative wages, they may use information about price inflation to predict how rapidly wages are rising at other firms, since price inflation data are more widely publicized than wage inflation data, and these series are highly correlated. Thus, it is possible that both real and relative wages affect efficiency in the short run. In the long run, it is almost certain that efficiency depends on relative wages, since what ultimately matters for workers' quit and effort decisions are their wages relative to wages elsewhere. For example, the dramatic rise in real wages since World War II does not appear to have significantly increased effort or decreased quits.

VI. Extending the Model to Include Reference Wages

The model can be extended to assume that workers' efficiency depends on the ratio between their wage and their reference wage (W_t^R), the wage to which they compare their own wages in making decisions that affect their efficiency (e.g., deciding how hard to work or how much time to spend on job search, which affects their quit propensities). Under this assumption, efficiency can be expressed as

$$e = e[W_t / W_t^R, u_t].$$

An important determinant of the reference wage is workers' expectations of the average wage, as previously assumed. However, the reference wage may also depend on workers' perception of their fair wage, which may be partly determined by last period's wage or last period's wage plus norms regarding wage increases.[30] Such a model can explain why firms may be reluctant to reduce nominal wages, even when economic conditions are very poor, and thus can explain how high unemployment can persist for long periods of time. In addition, if workers have norms about receiving annual wage increases, firms may have an incentive to raise wages during recessions, even though they could hire all the workers they need at last period's wage. For example, wages grew 1.5% in 2009,[31] and wage growth did not decelerate in 2010–2012 (with rates of wage inflation of 1.6% in 2010, 1.4% in 2011, and 1.7% in 2012), in spite of unemployment rates of at least 7.8% in these years, well above most estimates of the natural rate.

A model with efficiency depending on the ratio between a worker's wage and perceived fair wage can explain why wage growth did not decelerate in the presence of high unemployment.

In the model developed in Section III, efficiency depends on workers' wages relative to their expectations of average wages, and their expectations of average wages depend partly on lagged wages. The assumption that efficiency depends on the relationship between current wages and lagged wages or prices is critical to generating a downward-sloping Phillips curve. A model with efficiency depending on the ratio between a worker's wage and reference wage (with the reference wage partly determined by past wages) provides a second avenue through which efficiency may depend on the relationship between current and lagged wages and thus provides an additional way to obtain a downward-sloping Phillips curve.

VII. Conclusion

While the Phillips curve has been an important part of empirical macroeconomic modeling, it has been a challenge for economists to provide theoretical justification for this relationship. This study demonstrates that a Phillips curve can be derived from a model with efficiency wages and imperfect information about aggregate wages or prices. The model incorporates both a labor market and a product market, in which firms act as wage setters and price setters. The model's friction lies in the labor market, as wage or price expectations (which affect workers' efficiency) depend partly on lagged values of these variables.

The maximization problem of firms yields the dynamic labor demand curve and the dynamic efficiency wage-setting condition, and shifts of these curves trace out the dynamics of unemployment and wage or price inflation in response to shocks. If one first-order condition is substituted into the other, a third equation is obtained. This third relationship has the characteristics of a Phillips curve, as the coefficient on expected inflation equals 1 or is very close to 1, inflation depends on the level of unemployment, and nominal demand is absent. Shifts in the DLD curve and the Phillips curve produce the same intersection points as shifts in the DLD curve and the DEWS condition. However, the former is a more convenient framework because fewer variables appear in the Phillips curve than in the DEWS condition, and the coefficient on expected inflation is always close to 1 in the Phillips curve, but not in the DEWS condition.

If only a fraction of firms adjust wages each period, wage inflation depends on both lagged wage inflation and expected future wage inflation. In this case, the predicted coefficient on the unemployment rate in the Phillips curve approximates values that have been estimated with U.S. data, and the model does a very good job of predicting unemployment dynamics in postwar recessions. In the New Keynesian Phillips curve, lagged inflation is often included by making the *ad hoc* assumption that firms index wages or prices to past inflation if they cannot adjust them in the current period. In the present study, lagged inflation is an independent variable in the Phillips curve because expectations of aggregate wages or prices are assumed to be partly adaptive, in line with evidence from Levine et al. (2012).

In conventional specifications, the Phillips curve shows the combinations of inflation and unemployment that are possible, but does not predict the actual values of these variables. The present study uses a consistent framework to derive both the Phillips curve and the dynamic labor demand curve. The intersections of these curves determine the values of inflation and unemployment that result from nominal demand shocks or technology shocks in the transition between the economy’s initial equilibrium and its new equilibrium. Thus, the model developed in this study not only shows the tradeoff between inflation and unemployment, but also predicts the paths of these variables over time.

Appendix

Derivation of (12a) and (12b)

Let s_L represent the steady-state value of L_t / N and ψ represent the short-run labor supply elasticity (i.e., N' / N). From (7), du_t can be approximated by

$$du_t = \frac{-NdL_t + L_t N'\left[\dfrac{1/\overline{P}_t^e}{W^*/\overline{P}^*} dW_t - \dfrac{W_t/(\overline{P}_t^e)^2}{W^*/\overline{P}^*} d\overline{P}_t^e\right]}{N^2} \approx -s_L \hat{L}_t + s_L \psi \hat{W}_t - s_L \psi \hat{\overline{P}}_t^e . \quad \text{(A1)}$$

Solving the above equation for L_t yields

$$\hat{L}_t \approx -s_L^{-1} du_t + \psi \hat{W}_t - \psi \hat{\overline{P}}_t^e . \quad \text{(A2)}$$

Since $Y_t = Q_t$, totally differentiating (5) and dividing by the original equation results in the following expression for $\hat{Y}_t$:

$$\hat{Y}_t = \phi \hat{A}_t + \phi \hat{L}_t + \phi e_W e^{-1} \frac{W_t}{\overline{W}_t^e} \hat{W}_t - \phi e_W e^{-1} \frac{W_t}{\overline{W}_t^e} \hat{\overline{W}}_t^e + \phi e_u e^{-1} du_t . \quad \text{(A3)}$$

Substituting (A1) and the steady-state relationships, $W / \overline{W}^e = 1$ and $e_W e^{-1} = 1$ (from equation (10)), into (A3) yields

$$\hat{Y}_t = \phi \hat{A}_t + \phi(1 - e_u e^{-1} s_L) \hat{L}_t + \phi(1 + e_u e^{-1} s_L \psi) \hat{W}_t - \phi \hat{\overline{W}}_t^e - \phi e_u e^{-1} s_L \psi \hat{\overline{P}}_t^e . \quad \text{(A4)}$$

To derive an equation for labor demand, the above steady-state conditions ($W / \overline{W}^e = 1$ and $e_W e^{-1} = 1$), equation (A1), and the aggregate demand relationship, $\hat{\overline{P}}_t = \hat{M}_t - \hat{Y}_t$, are substituted into (11a). By making these substitutions, the labor demand curve can be expressed as

$$
\begin{aligned}
\hat{W}_t = {} & \frac{1-\gamma}{\gamma}\hat{Y}_t + \frac{\phi(\gamma-1)}{\gamma}\hat{A}_t + \frac{\phi(\gamma-1)}{\gamma}(1-e_u e^{-1}s_L)\hat{L}_t - \hat{L}_t \\
& + \frac{\phi(\gamma-1)}{\gamma}(1+e_u e^{-1}s_L\psi)\hat{W}_t - \frac{\phi(\gamma-1)}{\gamma}\hat{\bar{W}}_t^{e} - \frac{\phi(\gamma-1)}{\gamma}e_u e^{-1}s_L\psi\hat{\bar{P}}_t^{e} + \hat{M}_t .
\end{aligned} \tag{A5}
$$

If (A4) is substituted into (A5), the equation for the wage simplifies to

$$
\hat{W}_t = \hat{M}_t - \hat{L}_t . \tag{A6}
$$

Substituting (A2) into (A6) yields the labor demand equation,

$$
\hat{W}_t = \frac{\hat{M}_t + s_L^{-1}du_t + \psi\hat{\bar{P}}_t^{e}}{1+\psi} . \tag{A7}
$$

To derive the efficiency wage-setting condition, the steady-state conditions and the aggregate demand relationship are substituted into (11b), resulting in the expression,

$$
\begin{aligned}
\frac{\phi+\gamma-\phi\gamma}{\gamma}\hat{L}_t = {} & \frac{1-\gamma}{\gamma}\hat{Y}_t + \frac{\phi(\gamma-1)}{\gamma}\hat{A}_t + \frac{\phi\gamma-\phi-\gamma}{\gamma}\left[\hat{W}_t - \hat{\bar{W}}_t^{e} + e^{-1}e_u du_t\right] \\
& + e_{WW}e_W^{-1}\hat{W}_t - e_{WW}e_W^{-1}\hat{\bar{W}}_t^{e} + e_{Wu}e_W^{-1}du_t - \hat{\bar{W}}_t^{e} + \hat{M}_t .
\end{aligned} \tag{A8}
$$

If (A2) and (A4) are substituted into (A8), the following equation for the efficiency wage-setting condition is obtained:

$$
\hat{W}_t = \frac{-e_{WW}e_W^{-1}\hat{\bar{W}}_t^{e} + \left[s_L^{-1} - e_u e^{-1} + e_{Wu}e_W^{-1}\right]du_t + \psi\hat{\bar{P}}_t^{e} + \hat{M}_t}{1 - e_{WW}e_W^{-1} + \psi} . \tag{A9}
$$

<u>Derivation of (19):</u>[32]

Let *x* represent the wage set by firms that are able to adjust their wages. Then,

$$
\hat{\bar{W}}_t = \tau x_t + (1-\tau)\hat{\bar{W}}_{t-1} .
$$

Subtracting $\hat{\bar{W}}_{t-1}$ from both sides yields

$$\pi_t^w = \tau(x_t - \hat{\bar{W}}_{t-1}), \tag{A10}$$

where π_t^w is the rate of wage inflation. The optimal wage value of x is

$$\begin{aligned} x_t &= \sum_{j=0}^{\infty} \frac{\beta^j (1-\tau)^j}{\sum_{k=0}^{\infty} \beta^k (1-\tau)^k} \hat{W}_{t+j}^{*,e} \\ &= [1-\beta(1-\tau)] \sum_{j=0}^{\infty} \beta^j (1-\tau)^j \hat{W}_{t+j}^{*,e} \end{aligned}$$

where β is the discount factor and $\hat{W}_{t+j}^{*,e}$ is a firm's expectation of its optimal wage in future periods. Then,

$$x_t = [1-\beta(1-\tau)]\hat{W}_t^* + \beta(1-\tau)x_{t+1}^e .$$

Substituting (16) for $\hat{W}_t^*$ and subtracting $\hat{\bar{W}}_{t-1}$ from both sides of the above equation yields

$$x_t - \hat{\bar{W}}_{t-1} = \beta(1-\tau)x_{t+1}^e + [1-\beta(1-\tau)]\left[\hat{\bar{W}}_t^e + \frac{e_u - e_{Wu}}{e_{WW}} du_t\right] - \hat{\bar{W}}_{t-1}$$

$$\begin{aligned} x_t - \hat{\bar{W}}_{t-1} &= \beta(1-\tau)(x_{t+1}^e - \hat{\bar{W}}_t) + \beta(1-\tau)(\hat{\bar{W}}_t - \hat{\bar{W}}_{t-1}) \\ &\quad + [1-\beta(1-\tau)](\hat{\bar{W}}_t^e - \hat{\bar{W}}_{t-1}) + [1-\beta(1-\tau)]\frac{e_u - e_{Wu}}{e_{WW}} du_t . \end{aligned}$$

From (A10), $x_t - \hat{\bar{W}}_{t-1} = \pi_t^w / \tau$, and $x_{t+1}^e - \hat{\bar{W}}_t = \pi_{t+1}^{w,e} / \tau$. As a result,

$$\frac{1}{\tau}\pi_t^w = \beta(1-\tau)\frac{1}{\tau}\pi_{t+1}^{w,e} + \beta(1-\tau)\pi_t^w + [1-\beta(1-\tau)]\pi_t^{w,e} + [1-\beta(1-\tau)]\frac{e_u - e_{Wu}}{e_{WW}} du_t .$$

Simplifying the above expression yields the Phillips curve equation,

$$\pi_t^w = \frac{\beta(1-\tau)}{1-\beta\tau(1-\tau)}\pi_{t+1}^{w,e} + \frac{\tau[1-\beta(1-\tau)]}{1-\beta\tau(1-\tau)}\pi_t^{w,e} + \frac{\tau[1-\beta(1-\tau)]}{1-\beta\tau(1-\tau)}\frac{e_u - e_{Wu}}{e_{WW}} du_t . \tag{A11}$$

Derivation of (27) and (28)

Totally differentiating (23), dividing by the original equation, and making the substitution, $Y_t = Q_t$, yields

$$\hat{Y}_t = \phi\hat{A}_t + \phi\hat{L}_t + \phi e_W e^{-1}\frac{W_t}{\overline{P}_t^e}\hat{W}_t - \phi e_W e^{-1}\frac{W_t}{\overline{P}_t^e}\hat{\overline{P}}_t^e + \phi e_u e^{-1} du_t\,. \tag{A12}$$

If (A2) and the equilibrium condition from (26) are substituted into (A12), $\hat{Y}_t$ can be expressed as,

$$\hat{Y}_t = \phi\hat{A}_t + \phi(1+\psi)\hat{W}_t - \phi(1+\psi)\hat{\overline{P}}_t^e + \phi(e_u e^{-1} - s_L^{-1})du_t\,. \tag{A13}$$

To obtain the labor demand curve, (25a) is totally differentiated and divided by the original equation, yielding

$$\begin{aligned}\hat{W}_t = {} & \frac{1}{\gamma}\hat{Y}_t + \frac{\phi(\gamma-1)}{\gamma}\hat{A}_t + \frac{\phi\gamma-\phi-\gamma}{\gamma}\hat{L}_t + \frac{\phi(\gamma-1)}{\gamma}e_W e^{-1}\frac{W_t}{\overline{P}_t^e}\hat{W}_t \\ & -\frac{\phi(\gamma-1)}{\gamma}e_W e^{-1}\frac{W_t}{\overline{P}_t^e}\hat{\overline{P}}_t^e + \frac{\phi(\gamma-1)}{\gamma}e_u e^{-1}du_t + \hat{\overline{P}}_t.\end{aligned} \tag{A14}$$

By substituting (A2) and the equilibrium condition from (26) into (A14), wages can be expressed as,

$$\begin{aligned}\hat{W}_t = {} & \frac{1}{(\gamma+\phi-\phi\gamma)(1+\psi)}\Big\{\hat{Y}_t + \phi(\gamma-1)\hat{A}_t \\ & + \left[(\gamma+\phi-\phi\gamma)\psi - \phi(\gamma-1)\right]\hat{\overline{P}}_t^e + \left[\phi(\gamma-1)e_u e^{-1} + (\gamma+\phi-\phi\gamma)s_L^{-1}\right]du_t + \gamma\hat{\overline{P}}_t\Big\}\end{aligned} \tag{A15}$$

Substituting (A15) into (A13) yields,

$$\hat{Y}_t = \frac{\phi}{1-\phi}\hat{A}_t - \frac{\phi}{1-\phi}\hat{\overline{P}}_t^e + \frac{\phi e_u e^{-1}}{1-\phi}du_t + \frac{\phi}{1-\phi}\hat{\overline{P}}_t. \tag{A16}$$

If (A16) is substituted into the relationship, $\hat{\bar{P}}_t = \hat{M}_t - \hat{Y}_t$, the price level can be expressed as

$$\hat{\bar{P}}_t = (1-\phi)\hat{M}_t - \phi\hat{A}_t + \phi\hat{\bar{P}}_t^e - \phi e_u e^{-1} du_t. \tag{A17}$$

Equation (A17) is the labor demand curve. To obtain the efficiency wage-setting condition for the case in which efficiency depends on real wages, (25b) is totally differentiated and divided by the original equation, yielding

$$\begin{aligned}\hat{L}_t &= \frac{1}{\gamma}\hat{Y}_t + \frac{\phi(\gamma-1)}{\gamma}\hat{A}_t + \frac{\phi(\gamma-1)}{\gamma}\hat{L}_t + \frac{\phi\gamma-\phi-\gamma}{\gamma} e_W e^{-1}\frac{W_t}{\bar{P}_t^e}\hat{W}_t \\ &\quad - \frac{\phi\gamma-\phi-\gamma}{\gamma} e_W e^{-1}\frac{W_t}{\bar{P}_t^e}\hat{\bar{P}}_t^e + \frac{\phi\gamma-\phi-\gamma}{\gamma} e_u e^{-1} du_t + e_{WW} e_W^{-1}\frac{W_t}{\bar{P}_t^e}\hat{W}_t \\ &\quad - e_{WW} e_W^{-1}\frac{W_t}{\bar{P}_t^e}\hat{\bar{P}}_t^e + e_{Wu} e_W^{-1} du_t - \hat{\bar{P}}_t^e + \hat{\bar{P}}_t.\end{aligned} \tag{A18}$$

Let $\zeta = W/\bar{P}^e$ (i.e., the equilibrium value of the real wage). Then substituting (26) and (A2) into (A18) results in the equation,

$$\begin{aligned}\hat{W}_t &= \frac{1}{\kappa\gamma}\hat{Y}_t + \frac{\phi(\gamma-1)}{\kappa\gamma}\hat{A}_t + \frac{1}{\kappa}\left[\frac{\phi+\gamma-\phi\gamma}{\gamma}(1+\psi) - e_{WW} e_W^{-1}\zeta - 1\right]\hat{\bar{P}}_t^e \\ &\quad + \frac{1}{\kappa}\left[\frac{\phi+\gamma-\phi\gamma}{\gamma}(s_L^{-1} - e_u e^{-1}) + e_{Wu} e_W^{-1}\right] du_t + \frac{1}{\kappa}\hat{\bar{P}}_t,\end{aligned} \tag{A19}$$

where $\kappa = \dfrac{\phi+\gamma-\phi\gamma}{\gamma}(1+\psi) - e_{WW} e_W^{-1}\zeta$.

If (A19) is substituted into (A13), output can be expressed as

$$\hat{Y}_t = \phi\hat{A}_t + \phi(1+\psi)\frac{1}{\kappa\gamma}\hat{Y}_t + \phi(1+\psi)\frac{\phi(\gamma-1)}{\kappa\gamma}\hat{A}_t + \phi(1+\psi)\frac{1}{\kappa}\left[\kappa-1\right]\hat{\bar{P}}_t^e$$
$$+\phi(1+\psi)\frac{1}{\kappa}\left[\frac{\phi+\gamma-\phi\gamma}{\gamma}(s_L^{-1}-e_u e^{-1})+e_{Wu}e_W^{-1}\right]du_t$$
$$+\phi(1+\psi)\frac{1}{\kappa}\hat{\bar{P}}_t - \phi(1+\psi)\hat{\bar{P}}_t^e + \phi(e_u e^{-1}-s_L^{-1})du_t$$

$$\hat{Y}_t = \frac{\phi[1+\psi-e_{WW}e_W^{-1}\zeta]}{(1-\phi)(1+\psi)-e_{WW}e_W^{-1}\zeta}\hat{A}_t - \frac{\phi(1+\psi)}{(1-\phi)(1+\psi)-e_{WW}e_W^{-1}\zeta}\hat{\bar{P}}_t^e$$
$$+\frac{\phi[(1+\psi)e_{Wu}e_W^{-1}-e_{WW}e_W^{-1}\zeta e_u e^{-1}+e_{WW}e_W^{-1}\zeta s_L^{-1}]}{(1-\phi)(1+\psi)-e_{WW}e_W^{-1}\zeta}du_t$$
$$+\frac{\phi(1+\psi)}{(1-\phi)(1+\psi)-e_{WW}e_W^{-1}\zeta}\hat{\bar{P}}_t.$$

From the relationship, $\hat{\bar{P}}_t = \hat{M}_t - \hat{Y}_t$, the efficiency wage-setting condition is

$$\hat{\bar{P}}_t = \frac{(1-\phi)(1+\psi)-e_{WW}e_W^{-1}\zeta}{1+\psi-e_{WW}e_W^{-1}\zeta}\hat{M}_t - \phi\hat{A}_t + \frac{\phi(1+\psi)}{1+\psi-e_{WW}e_W^{-1}\zeta}\hat{\bar{P}}_t^e$$
$$-\frac{\phi[(1+\psi)e_{Wu}e_W^{-1}-e_{WW}e_W^{-1}\zeta e_u e^{-1}+e_{WW}e_W^{-1}\zeta s_L^{-1}]}{1+\psi-e_{WW}e_W^{-1}\zeta}du_t. \tag{A20}$$

To derive the Phillips curve, (A17) is solved for $\hat{M}_t$, which yields

$$\hat{M}_t = \frac{1}{1-\phi}\hat{\bar{P}}_t + \frac{\phi}{1-\phi}\hat{A}_t - \frac{\phi}{1-\phi}\hat{\bar{P}}_t^e + \frac{\phi}{1-\phi}e_u e^{-1}du_t. \tag{A21}$$

By substituting (A21) into (A20), the price level can be expressed as

$$\hat{\bar{P}}_t = \frac{(1-\phi)(1+\psi)-e_{WW}e_W^{-1}\zeta}{1+\psi-e_{WW}e_W^{-1}\zeta}\left[\frac{1}{1-\phi}\hat{\bar{P}}_t + \frac{\phi}{1-\phi}\hat{A}_t - \frac{\phi}{1-\phi}\hat{\bar{P}}_t^e + \frac{\phi}{1-\phi}e_u e^{-1}du_t\right]$$
$$-\phi\hat{A}_t + \frac{\phi(1+\psi)}{1+\psi-e_{WW}e_W^{-1}\zeta}\hat{\bar{P}}_t^e - \frac{\phi[(1+\psi)e_{Wu}e_W^{-1}-e_{WW}e_W^{-1}\zeta e_u e^{-1}+e_{WW}e_W^{-1}\zeta s_L^{-1}]}{1+\psi-e_{WW}e_W^{-1}\zeta}du_t$$

$$\hat{\overline{P}}_t = \hat{\overline{P}}_t^e - \frac{(1-\phi)[e_{WW}e_W^{-1}\zeta s_L^{-1} - (1+\psi)(e_u e^{-1} - e_{Wu}e_W^{-1})] + \phi e_{WW}e_W^{-1}\zeta e_u e^{-1}}{e_{WW}e_W^{-1}\zeta} du_t - \phi\hat{A}_t.$$

By multiplying the numerator and denominator of the coefficient on du_t by s_L and e, and substituting the relationship, $ee_W^{-1} = W / \overline{P}^e = \zeta$, the above equation can be rewritten as

$$\hat{\overline{P}}_t = \hat{\overline{P}}_t^e - \frac{(1-\phi)[e_{WW}\zeta^2 - (1+\psi)s_L(e_u - \zeta e_{Wu})] + \phi e_{WW}\zeta^2 e_u e^{-1} s_L}{e_{WW}\zeta^2 s_L} du_t - \phi\hat{A}_t . \qquad \text{(A22)}$$

The dynamic labor demand curve and the dynamic efficiency wage-setting condition are derived by, respectively, subtracting the lag of (A17) from (A17) and subtracting the lag of (A20) from (A20), yielding

$$\hat{\overline{P}}_t - \hat{\overline{P}}_{t-1} = (1-\phi)(\hat{M}_t - \hat{M}_{t-1}) - \phi(\hat{A}_t - \hat{A}_{t-1}) + \phi(\hat{\overline{P}}_t^e - \hat{\overline{P}}_{t-1}^e) - \phi e_u e^{-1}(du_t - du_{t-1}), \qquad \text{(A23)}$$

and

$$\begin{aligned}\hat{\overline{P}}_t - \hat{\overline{P}}_{t-1} = {} & \frac{(1-\phi)(1+\psi) - e_{WW}e_W^{-1}\zeta}{1+\psi - e_{WW}e_W^{-1}\zeta}(\hat{M}_t - \hat{M}_{t-1}) + \frac{\phi(1+\psi)}{1+\psi - e_{WW}e_W^{-1}\zeta}(\hat{\overline{P}}_t^e - \hat{\overline{P}}_{t-1}^e) \\ & - \phi(\hat{A}_t - \hat{A}_{t-1}) - \frac{\phi[(1+\psi)e_{Wu}e_W^{-1} - e_{WW}e_W^{-1}\zeta e_u e^{-1} + e_{WW}e_W^{-1}\zeta s_L^{-1}]}{1+\psi - e_{WW}e_W^{-1}\zeta}(du_t - du_{t-1}).\end{aligned} \qquad \text{(A24)}$$

The Phillips curve is derived by subtracting $\hat{\overline{P}}_{t-1}$ from both sides of (A22), giving

$$\begin{aligned}(\hat{\overline{P}}_t - \hat{\overline{P}}_{t-1}) = {} & (\hat{\overline{P}}_t^e - \hat{\overline{P}}_{t-1}) \\ & - \frac{(1-\phi)[e_{WW}\zeta^2 - (1+\psi)s_L(e_u - \zeta e_{Wu})] + \phi e_{WW}\zeta^2 e_u e^{-1} s_L}{e_{WW}\zeta^2 s_L} du_t - \phi\hat{A}_t.\end{aligned} \qquad \text{(A25)}$$

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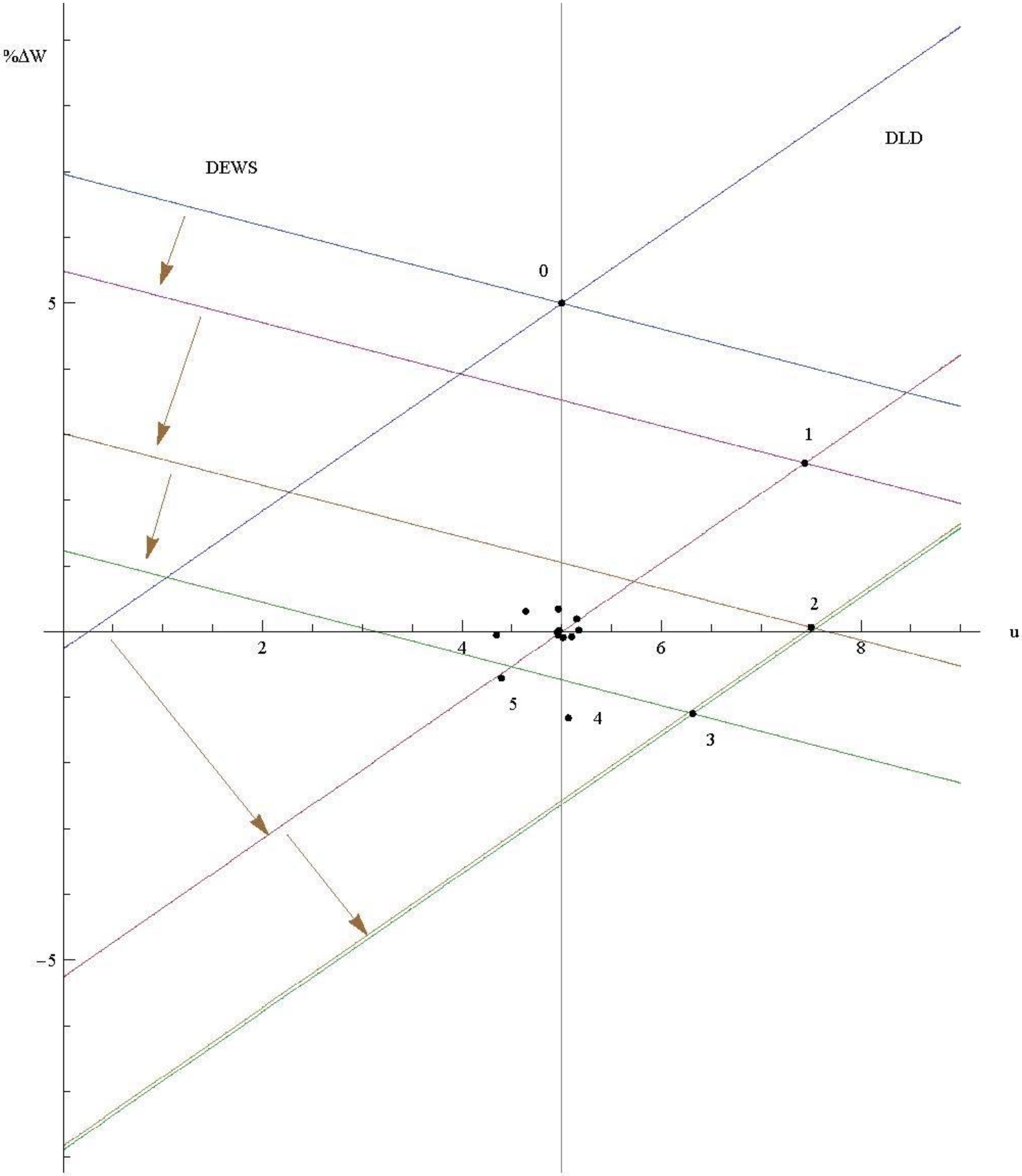

Figure 1a

Wage Inflation and Unemployment in Response to a Disinflationary Nominal Demand Shock (5% to 0%)

Wage Expectations Treated as a Mixture of Rational and Adaptive Expectations

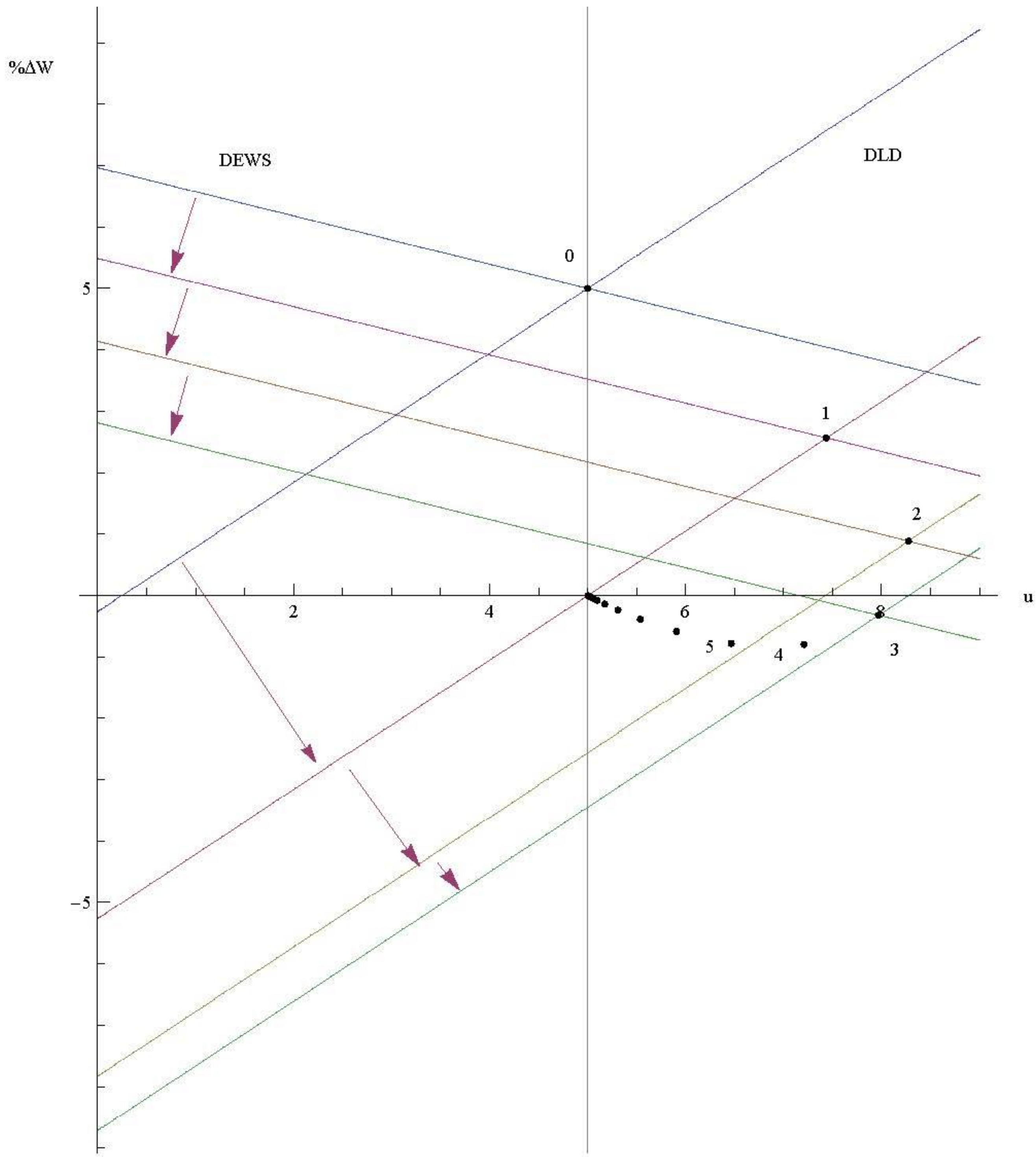

Figure 1b

Wage Inflation and Unemployment in Response to a Disinflationary Nominal Demand Shock (5% to 0%)

Wage Expectations Determined from Sticky Information Process

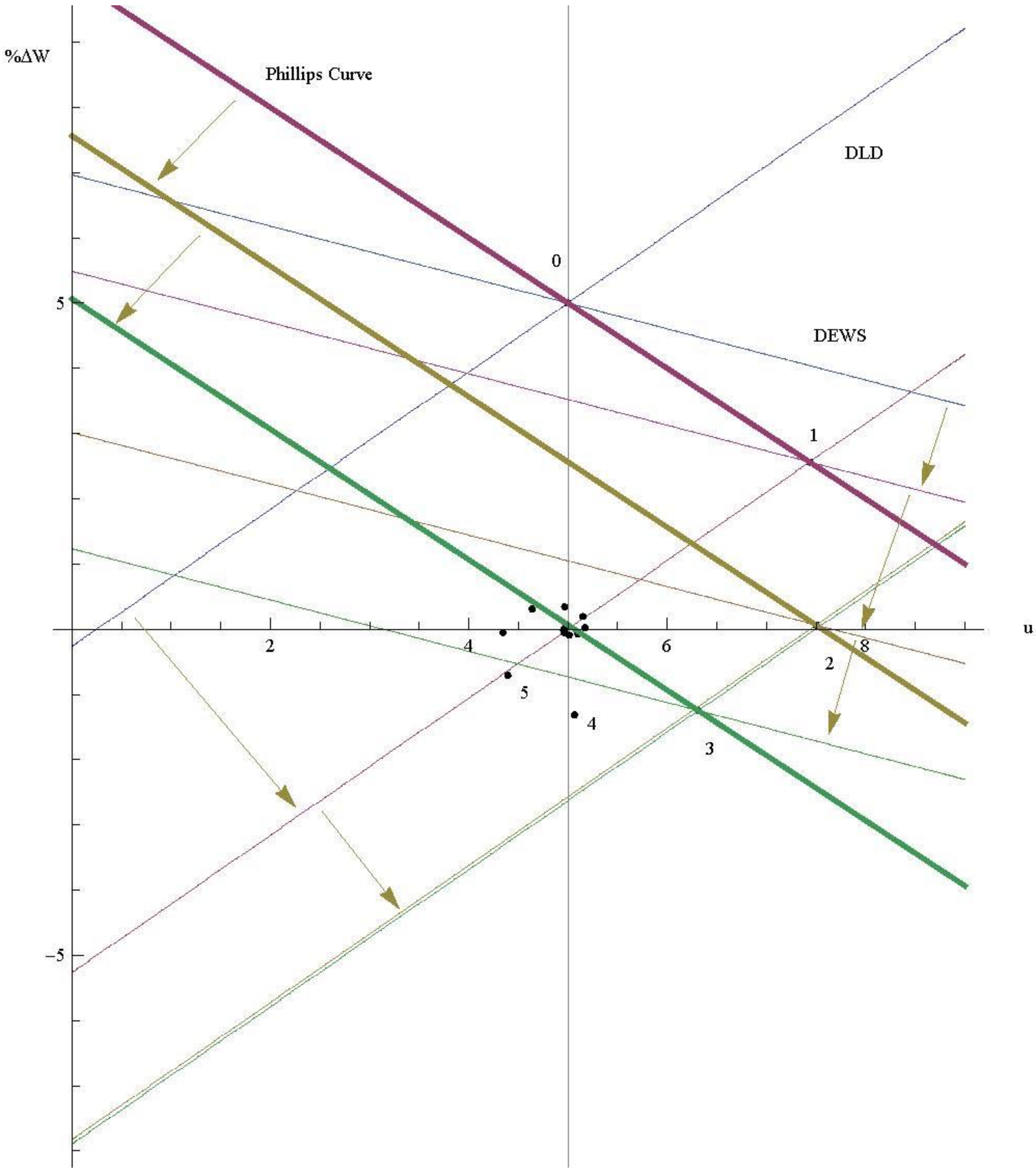

Figure 2
Wage Inflation and Unemployment in Response to a Disinflationary Nominal Demand Shock (5% to 0%)

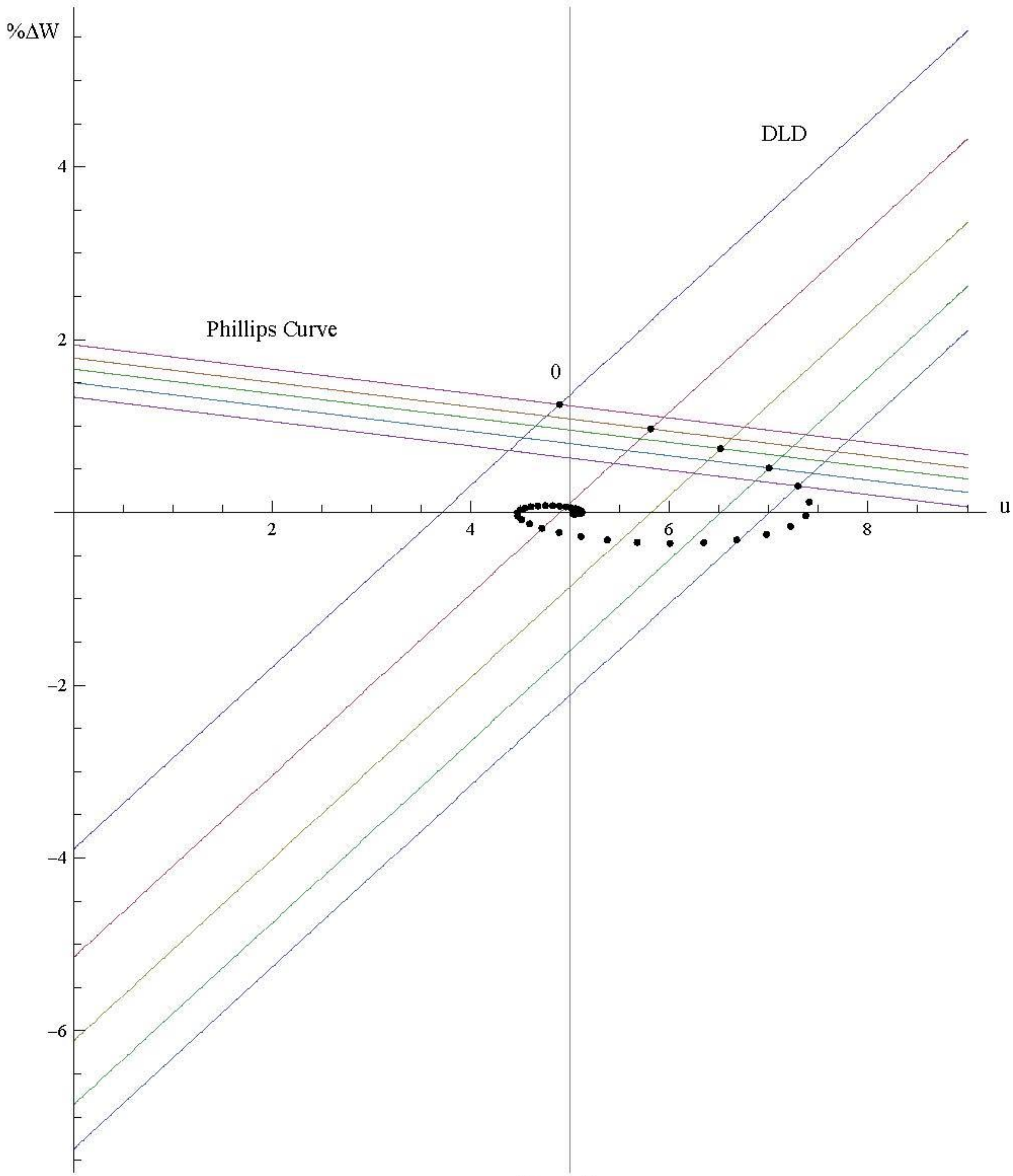

Figure 3
Wage Inflation and Unemployment in Response to a Disinflationary Nominal Demand Shock (1.25% to 0%) with Staggered Wage Setting

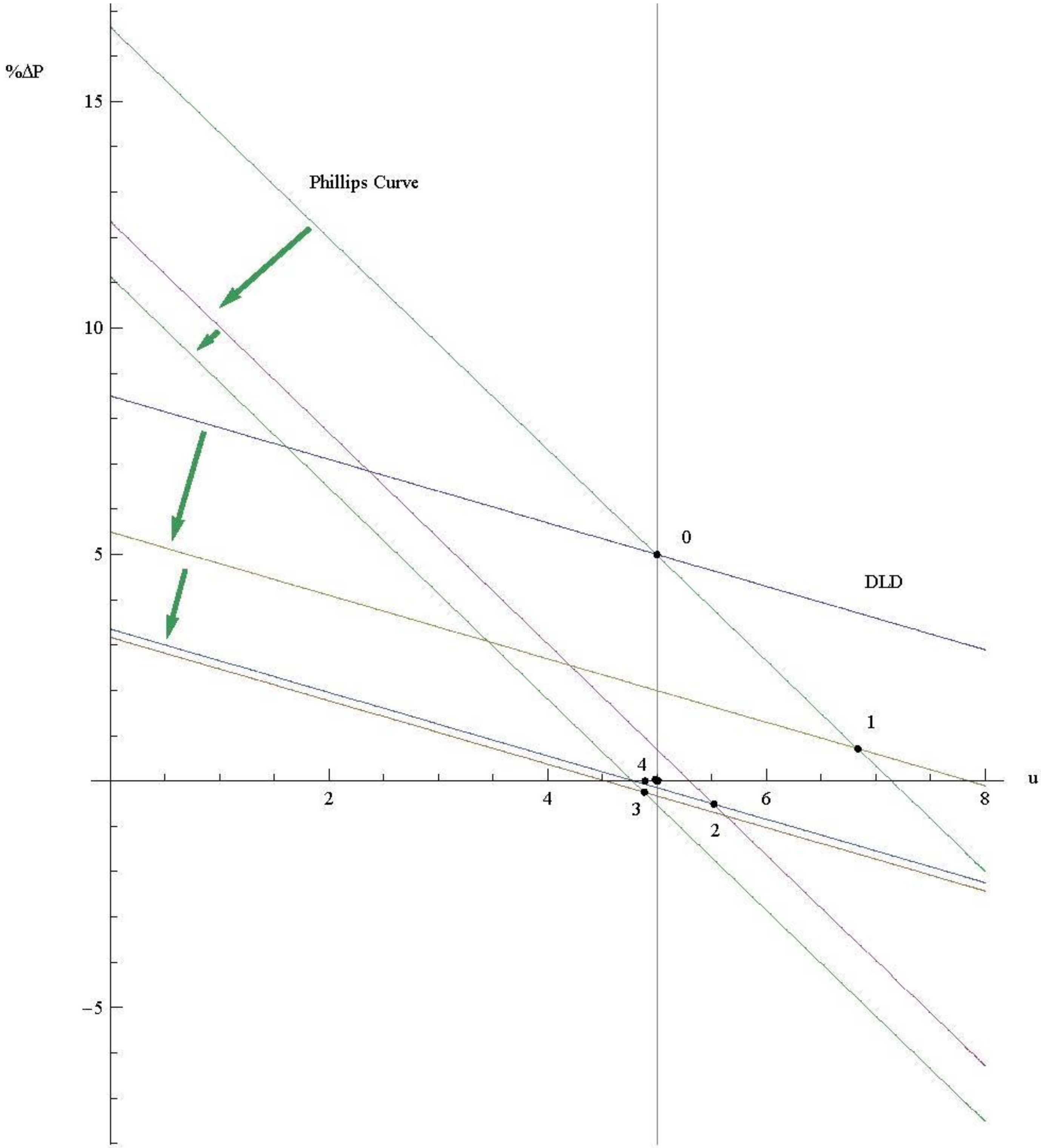

Figure 4
Price Inflation and Unemployment in Response to a Disinflationary Nominal Demand Shock (5% to 0%)

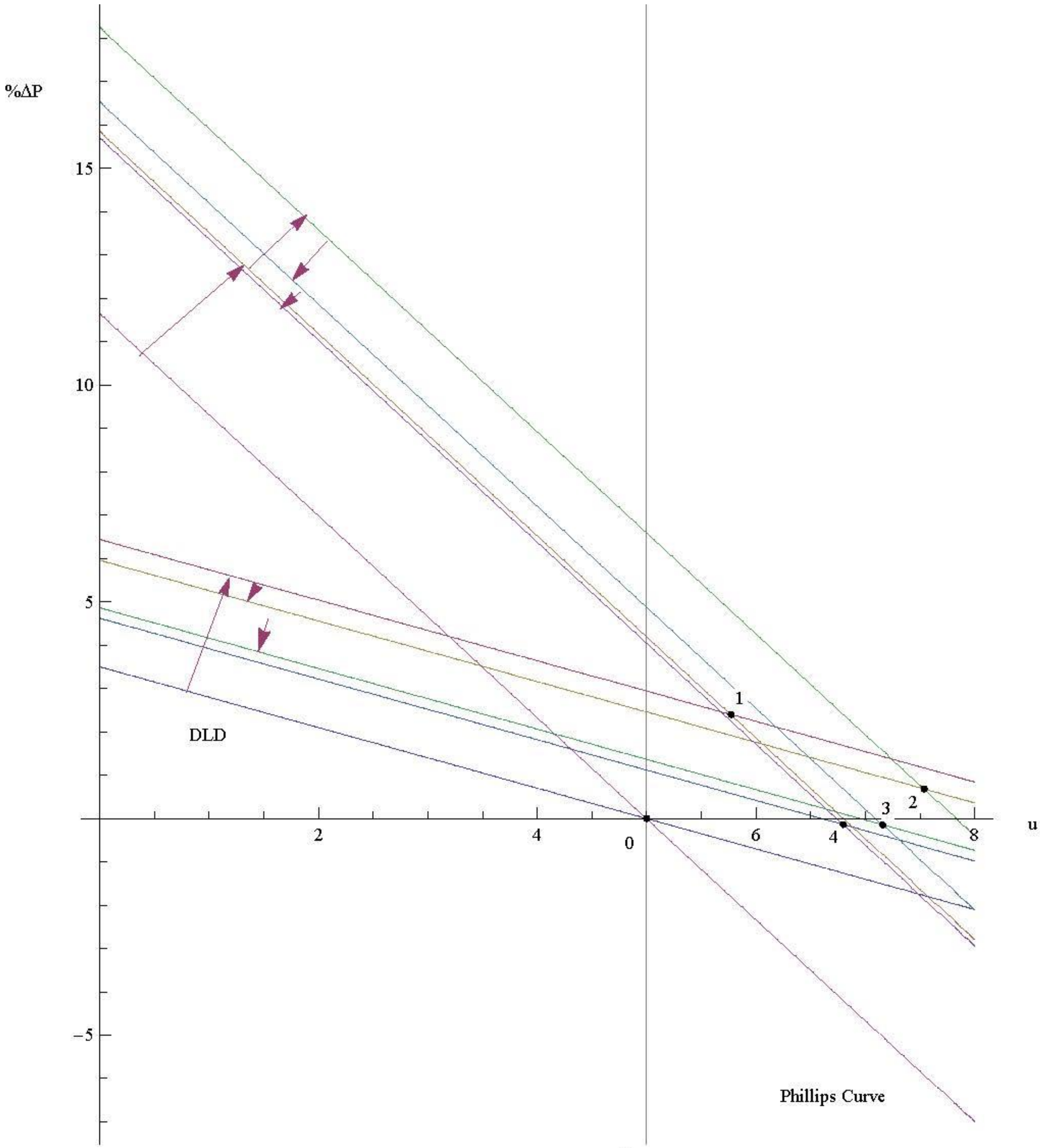

Figure 5

Price Inflation and Unemployment in Response to a -3% Productivity Shock

Footnotes

[1] Phillips (1958) finds an inverse relationship between wage inflation and unemployment with British data from 1861-1957. Samuelson and Solow (1960) show that a similar relationship can be derived between price inflation and unemployment.

[2] Studies that find econometric support for the Phillips curve include King and Watson (1994), Fuhrer (1995), Staiger, Stock, and Watson (1997), King and Morley (2007), and Lee and Nelson (2007).

[3] In a similar vein, Fuhrer (1995, p. 43) writes, "Perhaps the greatest weakness of the Phillips curve is its lack of theoretical underpinnings: No one has derived a Phillips curve from first principles, beginning with the fundamental concerns and constraints of consumers and firms."

[4] According to McCallum (1997), the Calvo-Rotemberg model of the Phillips curve has become, "the closest thing there is to a standard specification."

[5] Rudd and Whelan (2005) first regress current inflation on the discounted sum of future output gaps and find that the coefficient on this term has the incorrect sign. They then regress current inflation on both this discounted sum and lagged inflation, and they find that the sum of the coefficients on lagged inflation is highly significant and close to 0.9. These results indicate that past inflation matters a great deal even when controlling for forward-looking expectations.

[6] See Congressional Budget Office (2004) for a description of how the CBO calculates potential GDP, using both estimates of the natural rate and Okun's law in its calculations.

[7] In particular, a worker who overestimates average wages will exert less effort than is optimal, so that, on average, the future utility loss resulting from the increased probability of dismissal will exceed the current utility gain from lower effort. The opposite type of utility loss will occur if the worker underestimates average wages.

[8] This derivation is straightforward and is available from the author upon request.

[9] The model in Campbell (2006) differs from the model in the present study in that the previous study allows individuals to hold non-labor wealth, assumes that unemployed individuals receive benefits, and does not consider the utility from leisure. The first two differences are minor, and the third is not important if the first-order conditions are approximated around their steady-state values. Thus, the qualitative predictions of Campbell (2006) are valid for the model in the present study. In this previous study, the probability of dismissal is $PD = m(1-e)^2$, where m is monitoring intensity, and an individual's probability of hire equals the number of new hires in a period divided by the pool of the unemployed at the beginning of the period, under the assumption that there is a constant probability of an exogenous separation. The fair wage in Campbell (2006) corresponds to the average wage in the present study.

[10] In this model, efficiency depends positively on wages since higher wages raise workers' effort by reducing shirking and fostering better morale. Other explanations for a positive effect of wages on efficiency include the labor turnover models of Stiglitz (1974), Schlicht (1978), and Salop (1979) and the adverse selection model of Weiss (1980).

[11] The Bureau of Labor Statistics publishes figures on average wages for more than 800 occupational groups, based on data from the May CPS, but these figures are not published until the following March, a lag of ten months.

[12] Assuming a positive relationship between wages and efficiency does not guarantee that there will be excess supply of labor. Whether a firm operates on its labor supply curve or to the left of its labor supply curve (i.e., pays an efficiency wage) depends on the elasticity of output with respect to the wage, calculated at the market-clearing wage. Parameters are chosen so that firms maximize profits by operating to the left of their labor supply curves.

[13] This elasticity is based on Weisskopf's (1987) and Wadhwani and Wall's (1991) estimates of the effect of unemployment on productivity. See Campbell (2008), which also uses the same conditions to derive e_u and e_{Wu} (except that equilibrium unemployment is set at 6% in this earlier study), for a discussion of Weisskopf's and Wadhwani and Wall's findings. Campbell (2008) also discusses the assumptions about the interest rate, the proportion of workers who are dismissed, the probability of an exogenous separation, and the ratio between unemployment benefits and average wages. Similar results are obtained when the model is calibrated with quarterly data and with annual data. In the present study, $e_u = 0.8$, $e_{Wu} = -4.609$, and $e_{WW} = -3.981$.

[14] See, for example, Blundell and MaCurdy (1999) and Card (1991). Assuming that ψ=0 means that it is not necessary to model the formation of price expectations.

[15] Reis (2006) develops microfoundations for a model in which optimizing producers sporadically update their information in setting plans for the prices they charge. A similar framework could be used to model the way that workers update their information about average wages.

[16] The adaptive component equals last period's wage plus a weighted average of past wage inflation. Campbell (2008) also assumes that expectations are a mixture of rational and adaptive expectations, but uses a simpler specification for the adaptive component.

[17] In Campbell (2013), workers can estimate the average wage by observing lagged average wages at a low fixed cost and by incurring an added variable cost to acquire additional information about the mean of the current wage distribution through sampling wages at other firms and through obtaining and processing macroeconomic data. Individuals acquire the amount of information that minimizes the sum of information acquisition costs and the expected utility loss resulting from imperfect information (since the effort of workers who form incorrect expectations is non-optimal). Workers use a Kalman filtering process to predict the current average wage, and it is demonstrated that expectations are a mixture of rational and adaptive expectations if workers sample wages at other firms or if they acquire macroeconomic information and orthogonally update previous information. In the adaptive component, workers look at an infinite number of lags (i.e., $T=\infty$), with exponentially declining values of λ.

[18] Using annual Employment Cost Index data, current wage inflation was regressed on five values of lagged wage inflation. The coefficient on the first lag was close to 1, and the coefficients on further lags were close to 0.

[19] An arrow is not used to show the rightward shift of the DLD curve between periods 2 and 3 in Figure 1a since the distance between the lines is too small.

[20] This derivation makes use of the fact that $e_W e^{-1} = 1$ in equilibrium, from (10).

[21] See equation 25 on p. 698 of Phelps (1968).

[22] While the value of e_{WW} predicted by the model of Campbell (2006) results in a Phillips curve slope that is much higher than empirically estimated slopes when wages can be adjusted each period, this value of e_{WW} predicts the Phillips curve slope reasonably accurately when only a fraction of firms adjust wages in each period.

[23] This could occur if firms set wages simultaneously and if wages vary across firms (as previously assumed), so that firms cannot infer the average wage from the wage they set.

[24] Galí's (2011) estimate of the Phillip curve slope is lower when more recent data are included because of the unusual behavior of wages in the most recent recession.

[25] Varying ω does not have a large effect on the Phillips curve slope. In the range $0 \leq \omega \leq 1$ (keeping the other parameters constant), the Phillips curve slope lies between -0.117 and -0.152 (although if $\omega=1$, the Phillip curve shifts immediately to keep the economy at its natural rate). In addition, if λ^* is varied within reasonable values (0.2 to 0.5), the slope lies between -0.127 and -0.168.

[26] If the unemployment rate in the first quarter of a recession (based on NBER dating) is higher than in the previous quarter, the first quarter is treated as period 1. Otherwise, the subsequent quarter is treated as period 1. The calculations for the median time for u_t to fall to within 0.2% of u^* do not include the 1969-70 recession, since unemployment was below the natural rate for almost the entire recession, and do not include the 1980 recession, since unemployment was not close to the natural rate until after the subsequent recession. The reasons for using the criteria that unemployment falls to within 0.2% of the CBO's natural rate estimate are that the CBO's estimate may be imprecise and that, following two recessions (the ones beginning in 1960:II and 1990:III), the unemployment rate fell to within 0.2% of the CBO's estimate of u^*, subsequently rose, and did not reach u^* for more than a year.

[27] It is possible to allow ω to be different for firms (ω_f) and workers (ω_w). With the specification used in Figure 3 that $\omega_f=\omega_w=0.25$, the maximum unemployment rate is 7.4%. The maximum unemployment rate is 7.2% if $\omega_f=0.25$ and $\omega_w=1$ and is 5.1% if $\omega_f=1$ and $\omega_w=0.25$. Thus, while workers' imperfect information is critical and firms' imperfect information is irrelevant if wages are adjusted each period, firms' imperfect information is much more important than workers' imperfect information if wages are adjusted less frequently.

[28] This is the same equation for the price-price Phillips curve that is derived in Campbell (2010), except it was implicitly assumed that ζ was normalized to 1 in Campbell (2010).

[29] In this simulation, the parameters are the same as those in Figures 1a and 2. In addition, ϕ (the elasticity of output with respect to labor) is set equal to 0.7, and ζ (the equilibrium ratio between wages and prices) is assumed to equal 1.

[30] The fair wage-effort hypothesis is developed in Akerlof and Yellen (1990), and the concept of norms is discussed in Akerlof (2007).

[31] Wage inflation figures are based on fourth quarter to fourth quarter changes in the Employment Cost Index for the wages and salaries of all civilian workers.

[32] This derivation follows Romer (2012, pp. 329-331).