Stock index hedge using trend and volatility regime switch model considering hedging cost

Su, EnDer

National Kaohsiung First University of Science and Technology

9 January 2013

Stock Index Hedge Using Trend and Volatility Regime Switch Model Considering Hedging Cost

EnDer Su*

Abstract

This paper studies the risk hedging between stock index and underlying futures. The hedging ratios are optimized using the mean-variance utility function as considering the hedging cost. The trend of returns and variance are estimated by the model of regime switch on both vector autoregression (VAR) and GARCH(1,1) compared to three restricted models: VAR switch only, GARCH(1,1) switch only, and no switch. The hedge portfolio is constructed by Morgan Stanley Taiwan Index (MSTI) and Singapore Traded MSTI futures. The hedge horizon is set as a week to reduce the hedging cost and the weekly in-sample data cover from 08/09/2001 to 05/31/2007. The rolling window technique is used to evaluate the hedge performances of out-of-sample period spanning subprime, Greek debt, and post-risk durations. The subprime period indeed is evidenced very vital to achieve the hedge performance. All models perform surprisingly far above average during subprime period. The hedge ratios indeed are the tradeoff between maximum expected return and minimum variance. It is demonstrated challenging for all models to increase returns and reduce risk together. The hedge context is further classified into four hedge states: *uu*, *ud*, *du*, and *dd* (*u* and *d* denote respectively usual and down) using the state probabilities of series. The regime switch models are found to have much greater wealth increase when in *dd* state. It is decisive to hedge risk in *dd* state when volatility is extensively higher as observed recurrently in subprime period. Remarkably, the trend switch is found having larger wealth increase while the volatility switch is not found prominent between models. While the no switch model has larger utility increase in *uu* state as most observed in Greek debt or post risk period, its performance is far below average like other models.

Keywords: stock index, regime switch, hedging cost, hedging ratio

*Corresponding author, Associate Professor of Risk Management and Insurance
National Kaohsiung First University of Science and Technology
Email:suender@ccms.nkfust.edu.tw.

1. Introduction

The stock indices are fluctuated frequently by the shocks of erupted information from everywhere including economic, political, and natural disasters. The major market crises such as Asian currencies, subprime, 311 earthquake in Japan, Greek debt have all caused the panic and tumble of the global stock markets. That's why the hedge of stock portfolio is necessary.

In this paper, the MSCI Taiwan stock index is regarded as an spot position owned by international investors that is susceptible to shocks of either bad or good news to Taiwanese stock market. To hedge the spot position, the hedge tools used is the futures of MSCI Taiwan stock index coded and traded as STW on the Singapore Exchange (SGX) formed in 1999 from the merger of the Stock Exchange of Singapore (SES) and the Singapore International Monetary Exchange (SIMEX). At present, with the remove of investment quotas for foreign investors in July 2003, the foreign investors can invest in the Taiwan Stock Exchange (TWSE) openly and feely. As the end of 2012, the total market capitalization of the Taiwan stock exchange has grew 70 % compared to 2008 and amounted to US$666 billion with 797 listed companies, the daily trading value reached about US$3 billion and the market turnover rate was around 119.87%. Today, surprisingly over 80% of trading value is fulfilled by foreign investors. It becomes more and more important for foreign investors to understand how to hedge their portfolio risk.

The optimal hedging ratios i.e. amount of suitable futures to hedge spot position are calculated using the mean-variance method and the regime switch models. Since the stock markets are non-stationary, there exhibit different states of trend and volatility. For example, the trend of returns often shifts up and down swiftly and the volatility is persistent for a while in high or low state. The traditional regression method concerning only one state equation for one explanatory variable is not enough to describe the dynamic behavior of stock market. So, the regime switch vector autoregression and GARCH model is proposed here to fit to the different states in trend of returns as well as volatility. With the help of regime switch, the trend of returns is allowed to shift between different states and the volatility can be persistent either in high or low state. Once the trend of returns and volatility are predicted by model, the hedging ratios are computed consequently as taking into account the hedge cost.

The remainder of the paper proceeds as follows. In section two, the methodologies of hedging ratios and the regime switch models are discussed according to related articles. In section three, the sample of data, the estimations of regime switches, the methods of hedging ratio calculation, and the measurements of hedge performance are described in detail. In section four, the hedging ratios are obtained through rolling window technique and the hedge results are compared between different hedge models, measurements, periods, and states. The final section concludes some important remarks.

2. Literature Reviews

Investors would always ask how to find the optimal hedging ratio to reach the best hedge of portfolio risk. According to Markowitz (1952), the investors would like to pursue the minimum risk of their investment. Hence, the minimum-variance optimization method was applied by early authors like Johnson (1960), Stein (1961), and Ederington (1979) and later authors like Baillie and Myers (1991), Koutmos and Pericli (1998), Moschini and Myers (2002), Harris and Shen (2003), Poomimars et al. (2003), Alizadeh et al.(2008), Lien and Yang (2008) etc. However, the method ignores the profit or wealth effect and does not take into account the hedge cost either.

As Working (1962) pointed out the investors would chase not only the minimum risk but the expectation of profit, hence the mean-variance utility (MVU) method was adopted by authors like Heifner (1972), Kroner and Sultan (1993), Lence (1995), Gagnon, et al. (1998). The MVU method incorporating the expectation of investment return and variance into utility function with a fine degree of risk aversion. Further, Frechette (2000) described that the cost of hedging would affect the hedging profits and ratios. Thus, later authors like Haigh and Holt (2000, 2002), Liu et al. (2001), Yun and Kim (2010), Jin and Koo (2006) have derived their hedging ratios while taking into accounting the cost of hedging as well. Through their demonstration of hedging ratio formulas, the hedging cost indeed affects the hedging ratios and performances.

The hedging ratio determination requires to predict return and variance. Hamilton (1990, 1991) has found that the stock returns exhibit high and low states of volatilities and the volatility state tends to persist for a while. In this regard, Hamilton and Susmel (1994) proposed an unobserved Markov chain in ARCH to model the property of volatility regime switch. They however have recognized that the regime switch in GARCH is not feasible because the condition variance has the issue of path dependence. In other words, the conditional variance is the recursive solution of past state random variable and hence the pattern of past regimes should be accountable in estimation. To overcome the path dependence of estimation, Gray (1996) has developed a framework of regime switch GARCH to model the time-varying conditional volatilities in different regimes. Later, Klassen (2002), Haas et al. (2004), and Marcucci (2005) proposed the more advanced methods to describe the dynamic regime switch volatility process.

For the conditional variance and covariance model of multivariate assets, multivariate GARCH (MGARCH) has been used in Bollerslev et al. (1988), Ng (1991), and Hansson and Hordahl (1998). It was also applied to explain the spillover effect of contagion as in Tse and Tsui (2002) and Bae, Karolyi and Stulz (2003). Further, Ramchand and Susmel (1998), Ang and Bekaert (2002), Honda (2003), Haas et al. (2004), and Haas and Mittnik (2008) have developed a Markov-switching MGARCH model.

3. Research Objects and Methodology

The purpose of this paper is to describe the hedging ratio and effectiveness of Morgan Stanley corporation's MSCI Taiwan stock index coded with MXTW in Bloomberg. The MXTW index is created and constructed by the stock shares of 100 Taiwanese larger companies including most electronics companies and financial banks. The underlying futures contracts on the MSCI Taiwan Index coded with STW were started to trade on the Singapore Exchange (SGX) on January 9, 1997. Before long, the Taiwan Futures Exchange started to trade futures contracts on the MSCI Taiwan Index on March 27, 2006 but stopped trading for higher transaction cost comprising 0.004% tax rate of contract value plus NT$20 commission fee per contract and less demand compared to SGX.

To hedge the risk of MXTW, the STW futures are deemed as the hedge tools. They are traded based on the underlying asset of MXTW index with contract size: US$100 x SGX MSCI Taiwan Index Futures(i.e. STW) price, contract months: 2 nearest serial months and 12 quarterly months on a cycle of March, June, September and December, tick size: equal to 0.1 index point equivalent to US$10, and the daily price lime: ±7%. There is no trading tax for short or long STW but the commission fee is US$2.85 per contract per side based on Interactive Brokers.

In fact, the hedgers would take into account both the return and volatility of hedged portfolio. As a result, the hedging return, variance, and cost must be considered at the same time and the mean-variance utility should be maximized to search for the optimal hedging ratios. The hedge models are used to estimate the conditional returns and volatilities in utility function and the hedge model performances are evaluated by wealth increase, variance decrease or utility increase.

The stock markets are evidenced to be nonlinear and leptokurtotic in many documents. For example, the impact of bad and good news are asymmetric to the market and investors usually overact to the bad news while accustomed to the good news. Besides, trend of stock returns is swift often very quickly and volatility is not stable always. In this regards, the specification of the vector autoregression and GARCH with regime switch is described to estimate the trend and volatility of MXTW and STW under different states. The dynamic changes of returns and volatility are asymmetric. In fact, volatility is persistent for a while between high and low states and trend of returns is swift between up and down states. Therefore, two different sets of parameters in VAR and GARCH are established here to describe their behaviors.

In this paper, four models are proposed as follows: (1) regime switch for both VAR and GARCH, (2) regime switch for VAR only, (3) regime switch for GARCH only, and (4) no switch for VAR and GARCH. The VAR is capable to catch the trend property of data and the GARCH is to catch the volatility property of data so that the VAR and GARCH regime switch is used to catch not only both data properties but the asymmetric effect i.e. data properties in different states. The framework of VAR and GARCH regime switch model is described in detail as follows. The other three models can be constructed accordingly since they are just the restricted cases of the VAR and GARCH regime switch.

3.1 Vector autoregression with regime switch

The regime switch VAR for bivariate STW and MXTW is expressed to interpret the trend of returns and is written as

$$\mathbf{r}_t = \sum_{i=1}^{k} \boldsymbol{\theta}_i^{s_t} \mathbf{r}_{t-i} + \boldsymbol{\varepsilon}_t^{s_t}, \tag{1}$$

where S_t is a transition state variable at time *t*. It is assumed switching between two different states. In our case, the trend of returns is supposed to stay in "up" state denoted by outcome $s_t=u$ or "down" state denoted by outcome $s_t=d$ and the state probability is written as

$$p_{s_t,s_{t-1}} = P(S_t = s_t \mid S_{t-1} = s_{t-1}, \Omega_{t-1}) \ with \ s = u, d, \tag{2}$$

Hence the variance covariance of residuals is written as

$$\Sigma^{S_t} = \begin{pmatrix} \sigma_{11,S_t}^2 & \sigma_{12,S_t}^2 \\ \sigma_{21,S_t}^2 & \sigma_{22,S_t}^2 \end{pmatrix}. \tag{3}$$

3.2 Bivariate GARCH(1,1) with regime switch

It is not straight forward to express the conditional volatility by GARCH(1,1) with regime switch because the conditional volatility depends on the unobservable past latent state series. Some articles have proposed different methods to solve the conditional volatility switching under latent state series such as Gray (1996), Klaassen (2002), and Lee and Yoder (2006). In reference to them, the conditional volatility process under latent state series is solved by integrating out the latent state variables i.e. taking the expectation of conditional volatility under current or previous states. Unfortunately, the result formulus of conditional volatility are not quite explicit compared to typical GARCH model. Hence, to keep GARCH meaningful and handle path dependence as well, the regime switch is assumed changing concurrently in two sets of GARCH(1,1) with their own state outcome and probability.

Furthermore, the dynamic latent state variable is assumed to have two specific state outcomes and probabilities embedded with parameters of transition probabilities. Fortunately, the dynamic state variable can be solved iteratedly forward based on Bayes' theorem as described in following section. Hence the sum of log likelihood function can be constructed by GARCH estimated conditional volatilities.

To simplify the estimation process, the bivariate GARCH(1,1) model is applied here to estimate the conditional volatilities using the residual series as in Equation (1). On the other hand, it is complicated to solve the multivariate GARCH model due to the positive definitive requirement of multivariate variance covariance matrix and the large amount estimation of parameters. To make model parsimonious, the conditional constant correlation (CCC) MGARCH proposed by Bollerslev (1990) is adopted here. Actually, the switch between different state correlations would be able to describe the change of correlation structures as suggested by Pelletier (2006) and the dynamic process of volatility could result from different state correlations

The regime change of CCC is specified as follows. Assume the time invariant correlation between series i and j under state S_t at time t is $\rho_{ij}^{S_t}$ and its time varying variance covariance is $h_{ij,t}^{S_t}$ so that

$$h_{ij,t}^{S_t} = \rho_{ij}^{S_t} \sqrt{h_{i,t}^{S_t} h_{j,t}^{S_t}}, \tag{4}$$

where $h_{i,t}$ and $h_{j,t}$ are the dynamic variances. In our case, i and j represents either MXTW or STW denoted by 1 or 2 respectively so that the dynamic conditional variances and covariances are written as

$$\begin{aligned} h_{1,t}^{S_t} &= w_1^{S_t} + \alpha_1^{S_t} r_{t-1}^2 + \beta_1^{S_t} h_{1,t-1}^{S_t} \\ h_{2,t}^{S_t} &= w_2^{S_t} + \alpha_2^{S_t} r_{t-1}^2 + \beta_2^{S_t} h_{2,t-1}^{S_t}, \end{aligned} \tag{5}$$

$$\mathbf{H}_r^{S_t} = \begin{pmatrix} h_{1,t}^{S_t} & h_{1,t}^{S_t} h_{2,t}^{S_t} \rho_{1,2}^{S_t} \\ h_{1,t}^{S_t} h_{2,t}^{S_t} \rho_{1,2}^{S_t} & h_{2,t}^{S_t} \end{pmatrix}, \tag{6}$$

The conditional variances and covariances in matrix $\mathbf{H}_t^{S_t}$ are path dependent on the dynamic process of unknown latent random variable S_t. However, it is formed by two concurrent sets of GARCH with their own state outcome and probability. As a result, the regime switch conditional volatility is just the aggregate of two GARCH conditional volatilities weighted by their own state probability.

3.3 The maximum likelihood estimation (MLE)

Given the estimation of trend of returns and conditional volatilities, it follows that the quasi log likelihood function is written as

$$\begin{aligned} \ln L &= \sum_{t=1}^{T} \ln f(\boldsymbol{\varepsilon}_t \mid \Omega_{t-1}) = \sum_{t=1}^{T} \sum_{s_t=1}^{2} \ln f(\boldsymbol{\varepsilon}_t, S_t = s_t \mid \Omega_{t-1}) \\ &= \frac{1}{2} \sum_{t=1}^{T} \sum_{s_t=1}^{2} (k \log(2\pi) + \log(|\mathbf{H}_t^{S_t}|) + \boldsymbol{\varepsilon}_t^{S_t\prime} \mathbf{H}_t^{S_t -1} \boldsymbol{\varepsilon}_t^{S_t}) P(S_t = s_t \mid \Omega_{t-1}), \end{aligned} \tag{7}$$

where $\mathbf{H}_t^{S_t}$ is the matrix of the dynamic variances and covariances as above equation, $\boldsymbol{\varepsilon}_t^{S_t}$ is the residual vector in Equation (1), and $P(S_t \mid \Omega_{t-1})$ is the state probability.

If the random state variable S_t is observable, it is straightforward to solve parameters by MLE. However, S_t is a latent random variable. The plausible way to solve likelihood function under latent state series is to adopt Bay's theorem to describe the density of ε_t conditioned on past information Ω_{t-1} i.e. the likelihood function of $f(\boldsymbol{\varepsilon}_t \mid S_t = s_t, \Omega_{t-1})$ as the integration of joint probability of ε_t and S_t with s_t=*u*, *d*. The joint probability is formed by the multiplication of conditional probability of ε_t under latent state variable and the probability of latent state variable. Typically, the series of state random variable have the property of Markov chains: the current state estimation depends on the previous state information only. Thus, the probability of latent state variable is assumed to be the multiplication of conditional probability of latent state variable and probability of previous latent state variable.

Iteratedly one step forward from the initial condition, the likelihood functions are then constructed jointly with two concurrent state outcomes and probabilities. The details are described as follows.

To integrate out the state variable in Equation (7), the state probability must be solved accordingly. Hence, Equation (7) is rewritten as

$$
\begin{aligned}
f(\boldsymbol{\varepsilon}_t \mid \Omega_{t-1}) &= \sum_{s_t=u}^{d} f(\boldsymbol{\varepsilon}_t, S_t = s_t \mid \Omega_{t-1}) = \sum_{s_t=u}^{d} f(\boldsymbol{\varepsilon}_t \mid S_t = s_t, \Omega_{t-1}) P(S_t = s_t \mid \Omega_{t\text{-}1}) \\
&= f(\boldsymbol{\varepsilon}_t \mid S_t = u, \Omega_{t-1}) P(S_t = u \mid \Omega_{t\text{-}1}) + f(\boldsymbol{\varepsilon}_t \mid S_t = d, \Omega_{t-1}) P(S_t = d \mid \Omega_{t\text{-}1}).
\end{aligned} \tag{8}
$$

It is ought to use the joint probability of S_{t-1} and S_t in order to obtain the probability of $P(\mathrm{S}_t = s_t \mid \Omega_{t-1})$ that can be written as

$$
\begin{aligned}
P(S_t = s_t \mid \Omega_{t-1}) &= \sum_{s_t=u}^{d} P(S_t = s_t, S_{t\text{-}1} = s_{t-1} \mid \Omega_{t-1}) \\
&= \sum_{s_1=u}^{d} P(S_t = s_t \mid S_{t\text{-}1} = s_{t-1}\ \Omega_{t-1}) P(S_{t-1} = s_{t-1} \mid \Omega_{t-1}).
\end{aligned} \tag{9}
$$

Yet $P(S_t = s_t \mid \Omega_{t-1})$ must filter for time *t+1 as* $\boldsymbol{\varepsilon}_t$ is observed and is rewritten as

$$
\begin{aligned}
P(S_t = s_t \mid \boldsymbol{\varepsilon}_t, \Omega_{t-1}) &= \frac{f(\boldsymbol{\varepsilon}_t, S_t = s_t \mid \Omega_{t-1})}{f(\boldsymbol{\varepsilon}_t \mid \Omega_{t-1})} = \frac{f(\boldsymbol{\varepsilon}_t, S_t = s_t \mid \Omega_{t-1})}{\sum_{s_t=u}^{d} f(\boldsymbol{\varepsilon}_t, S_t = s_t \mid \Omega_{t-1})} \\
&= \frac{f(\boldsymbol{\varepsilon}_t \mid S_t = s_t, \Omega_{t-1}) P(S_t = s_t \mid \Omega_{t-1})}{\sum_{s_t=u}^{d} f(\boldsymbol{\varepsilon}_t \mid S_t = s_t, \Omega_{t-1}) P(S_t = s_t \mid \Omega_{t-1})}.
\end{aligned} \tag{10}
$$

To illustrate the MLE in more detail, an illustration at time t=0 to t=1 is described in the following steps:

(1) Given the information Ω_0 representing known $\boldsymbol{\varepsilon}_0$, $P(S_0 = u \mid \Omega_0) = \pi_u$, and $P(S_0 = d \mid \Omega_0) = \pi_d$, the density of log likelihood function conditional on regime s_1 at t= 1 is

$$
\begin{aligned}
f(\boldsymbol{\varepsilon}_1 \mid \Omega_0) &= \sum_{s_t=u}^{d} f(\boldsymbol{\varepsilon}_1, S_1 \mid \Omega_0) = \sum_{s_t=u}^{d} f(\boldsymbol{\varepsilon}_1 \mid S_1, \Omega_0) P(S_1 \mid \Omega_0) \\
&= f(\boldsymbol{\varepsilon}_1 \mid \mathrm{s}_1 = u, \Omega_0) P(\mathrm{s}_1 = u \mid \Omega_0) + f(\boldsymbol{\varepsilon}_1 \mid \mathrm{s}_1 = d, \Omega_0) P(\mathrm{s}_1 = d \mid \Omega_0).
\end{aligned} \tag{11}
$$

(2) The initial state probability of regime when $S_1 = s_1$ i.e. $P(S_1 = s_1 \mid \Omega_0)$ is

$$P(S_1 = s_1 \mid \Omega_0) = \sum_{s_0=u}^{d} P(S_1 = s_1, S_0 = s_0 \mid \Omega_0)$$
$$= \sum_{s_0=u}^{d} P(S_1 = s_1, S_0 = s_0 \mid \Omega_0) P(S_0 = s_0 \mid \Omega_0). \tag{12}$$

Specifically, $P(S_1 = s_1 \mid \Omega_0)$ is written further for $s_1 = u$ or d respectively as

$$P(s_1 = u \mid \Omega_0) = p_{uu}\pi_u + (1 - p_{dd})\pi_d \quad \text{and} \quad P(s_1 = d \mid \Omega_0) = (1 - p_{uu})\pi_u + p_{dd}\pi_d, \tag{13}$$

Formally, the conditional transition probability is denoted by $p_{s_t s_{t+1}} = P(S_{t+1} = s_{t+1}, S_t = s_t \mid \Omega_t)$ and the conditional state probability is denoted by $\pi_{s_t,t} = P(\mathrm{S}_t = s_t \mid \Omega_t)$.

(3) The likelihood function of $\boldsymbol{\varepsilon}_1$ is

$$f(\boldsymbol{\varepsilon}_1 \mid \Omega_0) = \sum_{s_t=u}^{d} f(\boldsymbol{\varepsilon}_1, S_1 \mid \Omega_0) = \sum_{s_t=u}^{d} f(\boldsymbol{\varepsilon}_1 \mid S_1, \Omega_0) P(S_1 \mid \Omega_0)$$
$$= f(\boldsymbol{\varepsilon}_1 \mid s_1 = u, \Omega_0) P(s_1 = u \mid \Omega_0) + f(\boldsymbol{\varepsilon}_1 \mid s_1 = d, \Omega_0) P(s_1 = d \mid \Omega_0). \tag{14}$$

(4) The state probabilities $P(s_1 = u \mid \Omega_0)$ and $P(s_1 = u \mid \Omega_0)$ is updated with known $\boldsymbol{\varepsilon}_1$ so that

$$P(S_1 = s_1 \mid \boldsymbol{\varepsilon}_1, \Omega_0) = \frac{f(\boldsymbol{\varepsilon}_1 \mid S_1 = s_1, \Omega_0) P(S_1 = s_1 \mid \Omega_0)}{\sum_{s_1=u}^{d} f(\boldsymbol{\varepsilon}_1 \mid S_1 = s_1, \Omega_0) P(S_1 = s_1 \mid \Omega_0)}. \tag{15}$$

Repeatedly iterating through steps (1)~(4) from t=2 to t=T, all sample data likelihood functions as Equation (14) and all state probabilities as Equation (15) are acquired. Due to information update, Equation (15) is also called a filter probability. As a result, the sum of log likelihood equation for the bivariate vector $\boldsymbol{\varepsilon}_t$ in our case with t=1 to T is written as

$$\sum_{s_t=u}^{d}\sum_{t=1}^{T} \log f(\varepsilon_t \mid \Omega_{t-1}) = \sum_{s_t=u}^{d}\sum_{t=1}^{T} f(\varepsilon_t, S_t \mid \Omega_{t-1}) = \sum_{s_t=u}^{d}\sum_{t=1}^{T} \log f(\varepsilon_t \mid S_t, \Omega_{t-1}) P(S_t \mid \Omega_{t-1})$$
$$= \sum_{t=1}^{T} \log\big(f(\varepsilon_t \mid s_t = u, \Omega_{t-1}) P(s_t = u \mid \Omega_{t-1}) + f(\varepsilon_t \mid s_t = d, \Omega_{t-1}) P(s_t = d \mid \Omega_{t-1}) \big). \tag{16}$$

Once the path dependence of recursive conditional volatility is handled well, the Brendt-Hall-Hall-Hausman(BHHH) or Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm can be applied to solve the above optimization problem to maximize the log likelihood of Equation (16) and find the optimal parameters including VAR parameters $\boldsymbol{\theta}_i^{S_t}$, the GARCH parameters $(w_i^{S_t}, \alpha_i^{S_t}, \beta_i^{S_t})$, the constant correlations $\rho_{ij}^{S_t}$, and the transition parameters for the transition probabilities p_{uu} and p_{dd}. Note that the cumulative probabilities of standard normal distribution of the transition parameters are equal to the transition probabilities p_{uu} and p_{dd}.

3.4 Review of Markov chains

The transition from one state to another state is controlled by the transition matrix of conditional probabilities $P = \begin{bmatrix} p_{uu} & 1-p_{uu} \\ 1-p_{dd} & p_{dd} \end{bmatrix}$, where $1-p_{uu}$ represents the conditional probability of a transition from u state to d state, and $1-p_{dd}$ represents the conditional probability of a transition from down d state to u state.

According to Markov chains, the conditional probability would convergence to a constant matrix in the end. For example, assume P is a $k \times k$ transition matrix. If λ is a real number and $X(\neq 0)$ is a $k \times 1$ column vector, the eigenvalue decomposition of P is obtained as $P = X\lambda X^{'}$. If the Markov process moves forward infinitely, the transition matrix becomes $\lim_{n\to\infty} P^{(n)} = R$ so that R is the limit of the series $P^{(1)}, P^{(2)}, \cdots, P^{(n)}, \cdots$ i.e. the unconditional state probability. The relationship of the unconditional state probabilities and conditional transition probability according to Hamilton (1989) is written as

$$\pi_u = \frac{1-p_{dd}}{2-p_{uu}-p_{dd}} \text{ and } \pi_d = \frac{1-p_{uu}}{2-p_{uu}-p_{dd}}. \tag{17}$$

3.5 Hedging ratio and measurement

To calculate hedging ratio and measure hedge effectiveness as considering hedge cost, the mean-variance utility and optimization must be adopted here.

Suppose that a hedged portfolio is constructed by long one unit of spot equal to the index points of MXTW and short Q_F units (i.e. contracts) of STW futures. The hedger's expected utility at time t is constructed by the wealth expectation and variance of hedged portfolio using information set Ω_{t-1}. Accordingly the hedging ratios are obtained by maximizing the hedger's utility as

$$\underset{Q_{F_t}}{Max}\ U_t = E(W_t \mid \Omega_{t-1}) - \frac{1}{2}\lambda Var(W_t \mid \Omega_{t-1}), \tag{18}$$

where W_t is the wealth of hedged portfolio, $E(\bullet)$ and $Var(\bullet)$ represent respectively the operators of expectation and variance, and λ is the risk aversion coefficient. According to Gagnon et al. (1988), the λ is set to 1 here for a mild risk averse hedger.

The portfolio is created by lone one spot and short Q_{F_t} futures, the wealth change of hedged portfolio at time t as considering the cost of hedging is written as

$$W_t = S_t - Q_{F_t}(F_t - F_{t-1}) - Q_{F_t} c_h, \tag{19}$$

where S and F denote spot and futures price respectively. The transaction cost of STW charging only commission fee is equal to US\$2.85 per contract per side as aforementioned. Thus for trading STW, c_h is set to US\$ 5.7 i.e. 5.7/100 index points for offset of futures position.

The spot and futures prices are known at time t-1 and thus the expectation and variance of Equation (19) are written respectively as

$$E(W_t) = E(S_t) - Q_{F_t} E(F_t) + Q_{F_t} F_{t-1} - Q_{F_t} c_n \tag{20}$$

$$Var(W_t) = \sigma^2(S_t) + Q_{F_t}^2 \sigma^2(F_t) - 2Q_{F_t} \sigma\ (S_{t,} F_t), \tag{21}$$

where $\sigma^2(\bullet)$ and $\sigma(\bullet,\bullet)$ denote respectively the operators of variance and covariance. Because returns are the first order difference of prices, it is appropriate to use returns instead of prices to express the expectation and variance so that $S_t = S_{t-1} \cdot r_{s_t}$ and $F_t = F_{t-1} \cdot r_{f_t}$. Hence, Equations (21) is rewritten as

$$\begin{aligned} Var(W_t) &= \sigma^2(S_{t-1} r_{S_t}) + Q_{F_t}^2 \sigma^2(F_{t-1} r_{F_t}) - 2Q_{F_t} \sigma(S_{t-1} r_{S_t}, F_{t-1} r_{F_t}) \\ &= S_{t-1}^2 \sigma^2(r_{S_t}) + F_{t-1}^2 Q_{F_t}^2 \sigma^2(r_{F_t}) - 2S_t F_t Q_{F_t} \sigma(r_{S_t}, r_{F_t}). \end{aligned} \tag{22}$$

To simplify expression, $\sigma^2(r_{S_t})$ is replayed by $\sigma_{r_{S_t}}^2$ and similarly for the others. Then above equation becomes

$$Var(W_t) = S_{t-1}^2 \sigma_{r_{S_t}}^2 + F_{t-1}^2 Q_{F_t}^2 \sigma_{r_{F_t}}^2 - 2S_{t-1} F_{t-1} Q_{F_t} \sigma_{r_{S_t}, r_{F_t}}. \tag{23}$$

Substituting Equation (20) and (23) into Equation (18) gives

$$\begin{aligned} &\underset{Q_{F_t}}{Max}\ U_t = \\ &E(S_t) - Q_{F_t} E(F_t) + Q_{F_t} F_{t-1} - Q_{F_t} c_n - \frac{1}{2}\lambda\left(S_{t-1}^2 \sigma_{r_{S_t}}^2 + F_{t-1}^2 Q_{F_t}^2 \sigma_{r_{F_t}}^2 - 2S_{t-1} F_{t-1} Q_{F_t} \sigma_{r_{S_t}, r_{F_t}}\right). \end{aligned} \tag{24}$$

To maximize Equation (24) and solve for Q_{F_t} , the first order derivative of Equation (24) with respect to Q_{F_t} is derived as

$$\frac{dU_t}{dQ_{F_t}} = -E(F_t) + F_{t-1} - c_n - \frac{1}{2}\lambda(2Q_{F_t} F_{t-1}^2 \sigma_{r_{F_t}}^2 - 2S_{t-1} F_{t-1} \sigma_{r_{S_t}, r_{F_t}}). \tag{25}$$

Let Equation (25) equal to zero and the solution is the optimal hedging ratio as

$$Q_{F_t}^* = \frac{R_t + \lambda S_{t-1} F_{t-1} \sigma_{r_{S_t}, r_{F_t}}}{\lambda F_{t-1}^2 \sigma_{r_{F_t}}^2} \text{ and } R_t = F_{t-1} - E(F_t) - c_n. \tag{26}$$

At current time t-1 the spot and futures prices are known in Equation (26), but the expectation of futures price at time t is not. Thus, the futures price should be predicted by the VAR model as expressed in Equation (1). The covariance $\sigma_{r_{S_t}, r_{F_t}}$ and variance $\sigma_{r_{F_t}}^2$ are supposed to be estimated by the GARCH(1,1) to acquire the optimal hedging ratio $Q_{F_t}^*$.

To compare the performance of our models, the variance of wealth in Equation (23) is assumed changing with the variable of hedging ratio only so that the variance and covariance of MXTW and STW is deemed as respectively the h-day realized variance(RV) or quadratic variation proposed by Anderson et al.(2003) and approximately realized covariance (*RC*) or cross variation in reference to Hayashi, T. and N. Yoshida (2005) as

$$RV_t = \sum_{i=1}^{h/\Delta} \mathbf{r}'_{t-h+i\Delta}\mathbf{r}_{t-h+i\Delta} \tag{27}$$

$$RC_{r_{S_t},r_{F_t}} = \sum_{j=1}^{h/\Delta} r_{S,t-h+i\Delta} r_{F,t-h+j\Delta}, \tag{28}$$

where for MXTW and STW, $\mathbf{r}_{t-h+i\Delta}$ denotes a 2×1 vector of logarithmic returns in one 1/Δ period, RV_t denotes a 2×1 vector of return variances in *h* period, and $RC_{r_{S_t},r_{F_t}}$ denotes logarithmic return covariance in *h* period. In our case with weekly data, *h* is equal to 5 and Δ is equal to 1.

In fact, Equation (26) consists of two components: one is the position of return and the other is the position of variance. The position of the return increases as the futures prices and transaction cost decrease. On the other hand, the position of the variance increases as the correlation or relative volatility between spot and futures increases. The increase of the position of variance should be accompanied by the decrease of position of retunes and vice the versa until the optimal solution of hedging position is reached. The risk aversion also reveals the tradeoff amount between the two position. It implies one unit of risk deduction for a half λ unit of return compensation.

The hedge effectiveness is measured by the three methods: incremental wealth increase (IWI), reduced variance decrease (RVD), and incremental utility increase (IUI) that are written as follows:

$$HE_{V_t} = \frac{V_t^{Hg} - V_t^{UHg}}{V_t^{UHg}} = \frac{V_t^{Hg}}{V_t^{UHg}} - 1 \tag{29}$$

$$HE_{W_t} = \frac{W_t^{Hg} - W_t^{UHg}}{W_t^{UHg}} = \frac{W_t^{Hg}}{W_t^{UHg}} - 1 \tag{30}$$

$$HE_{U_t} = \frac{U_t^{Hg} - U_t^{UHg}}{U_t^{UHg}} = \frac{U_t^{Hg}}{U_t^{UHg}} - 1, \tag{31}$$

where *W*, *V*,*U*, *Hg*, and *UHg* denote respectively wealth, variance, utility, hedge, and unhedged so that V_t^{Hg} means the variance of the hedged portfolio and the other notations are denoted similarly.

4. Empirical Results

In fact, the correlation of spots and nearby month futures is the largest compared to the distant month futures. Hence, to hedge MXTW well, the STW maturity selected is the nearby month. Both MXTW index and nearby STW futures price data are collected from the database of Taiwan Economic Journal (TEJ). The data of STW in TEJ are continued by the method of interpolation between consecutive months while the futures prices rollover between nearby and expired months.

Due to transaction cost considered in hedge position, it is not clever to hedge the spot position daily. Therefore, the hedge horizon is set as a week. The period of in-sample is set from 08/09/2001 to 05/31/2007 yielding total 288 weekly observations to estimate trend and volatility using regime switch models. The period of out-of-sample is set from 06/07/2007 to 06/26/2013 yielding total 302 weekly observations. The rolling window technique is used to analyze the model estimations and forecasts for out-of-sample period. The size of windows is fixed the same as that of in-sample period. The fixed size window then rolls one step forward (i.e. a week in our case) repeatedly through whole out-of-sample period and thus creates 302 forecasts.

Furthermore, to determine and compare the hedging ratios and performances during large risk events such as subprime, Greek debt crises. The out-of-sample data are separated into three data sets: the subprime data spanning from 06/07/2007 to 03/05/2009, the Greek debt data spanning from 12/08/2009 to 03/03/2011, and the post risk data spanning from 11/11/2011 to 05/23/2013.

4.1 Data description

Table 1 reports the basic statistical properties of weekly logarithmic returns of MXTW and STW data sets for in-sample and out-of-sample periods. The values of standard deviation of STW futures are higher than those of MXTW spots in either in-sample or out-of-sample period. It appears that the futures have higher volatility than the spots. For both datasets and periods, the excess kurtosis testes and the non-normally distributed Jarque-Bera tests are founded strong significantly with nearly zero p value. The skewness tests are significant at negative value as both spots and futures are in out-of-sample period though not significant at positive value in in-sample period. It indicates that both data sets are fat tailed with higher peakedness. The means of returns in both in-sample and out-of-sample periods are negative but the tests of mean equal to zero are insignificant to zero with large p values more than 0.4. Accordingly the trend of returns is not stable but swift and uneasy to predict.

However, the minimum, maximum, and standard deviation of logarithmic returns show that the returns are too volatile to be unaware. Thus the regime switch is used to catch the trend and volatility changes between different states. The correlation between MXTW and STW is higher enough to reach over 0.97 so that the hedge of risk position is reasonable.

Figure 1 shows the movements of indices and basis risk between MXTW and STW. The basis risk is MXTW index minus STW index and it is found frustrating between -10.64 and 17.39. This range of basis risk is equal to 28.03x$100/point=$2,803 for a futures contract so that the profit or loss might reach to $2,803*the number of futures contracts. If the futures position is large enough, the basis risk would become quite appalling. That’s why the hedging ratios should be optimized cautiously.

Table 1 Data Description

	Min	Max	Mean	Stdev	Skewness	Kurtosis	Jarque-Bera Test
In-Sample Data							
	Min	Max	Mean	Stdev	Skewness	Kurtosis	Jarque-Bera Test
MXTW	-0.1407	0.1617	1.475E-03	0.0360	0.0516	5.7153	93.146
p value			0.4869		0.7205	0.000	0.000
STW	-0.1503	0.1797	1.475E-03	0.0400	0.1399	5.6533	89.797
p value			0.5323		0.3325	0.000	0.000
Correlation	0.9782						
Out-Sample Data							
	Min	Max	Mean	Stdev	Skewness	Kurtosis	Jarque-Bera Test
MXTW	-0.1274	0.1327	-5.173E-04	0.0331	-0.3508	4.7474	46.880
p value			0.7858		0.0128	0.000	0.000
STW	-0.1298	0.1376	-5.479E-04	0.0346	-0.4648	4.9424	61.071
p value			0.7836		0.0009	0.000	0.000
Correlation	0.9745						

Note: The in-sample (08/09/2001~ 05/31/2007) and out-of-sample (06/07/2007~06/26/2013) have respectively 288 and 302 weekly observations.

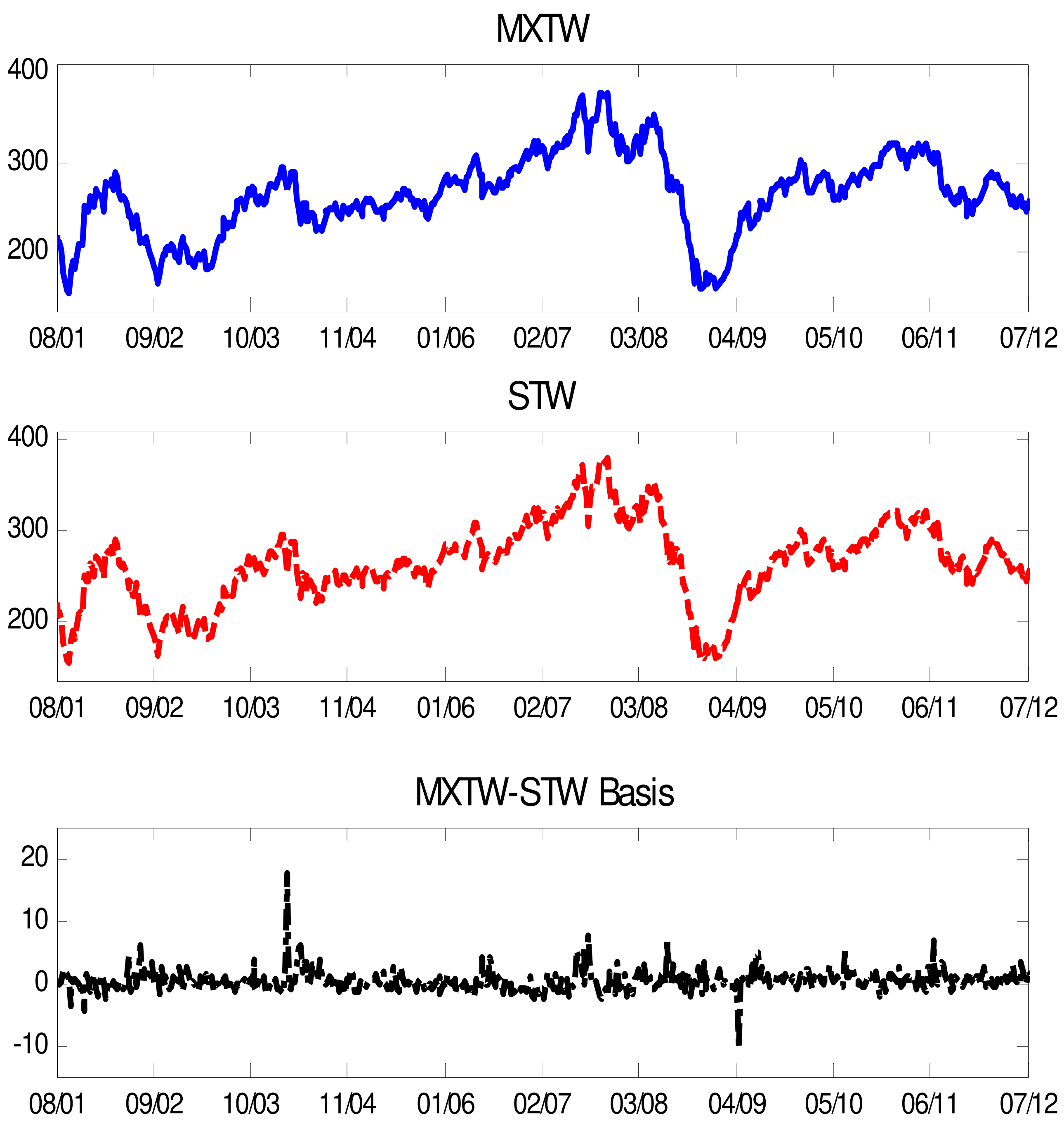

Figure 1 MXTW and STW Prices and Spreads

Table 2 reports the properties of logarithm returns of MXTW and STW series. Both return series have no significant autocorrelation nor unit root while both return series do have significant ARCH effect. Also, both price series are tested for the unit root. The result shows that both price series have significant unit root and hence should be differenced once to turn into return series for model use.

Table 2 The Tests of MXTW and STW Return Series

	Autocorrelation Test						Unit Root Test						ARCH Test	
	MXTW			STW			MXTW			STW			MXTW	STW
Lag	AC	Q-Stat	*p* value	AC	Q-Stat	*p* value	ADF(Return)			ADF(Return)			*F* test	*F* test
1	-0.003	0.0045	0.947	-0.045	1.1984	0.274	-25.336			-24.292			21.233	22.824
2	0.008	0.040	0.980	-0.029	1.7007	0.427	Critical Level			Critical Level			*p* value	*p* value
3	0.037	0.8707	0.832	0.038	2.5389	0.468	1%	5%	10%	1%	5%	10%	0.000	0.000
4	0.073	4.0262	0.402	0.065	5.0409	0.283	-3.441	-2.866	-2.569	-3.441	-2.866	-2.569	LM	LM
5	-0.015	4.1561	0.527	-0.033	5.6838	0.338	ADF(Price)			ADF(Price)			20.56226	22.04214
6	0.025	4.5381	0.604	0.006	5.7055	0.457	-1.630			-1.850			*p* value	*p* value
7	0.037	5.3464	0.618	0.057	7.6389	0.366	Critical Level			Critical Level			0.000	0.000
8	0.043	6.4556	0.596	0.052	9.288	0.319	1%	5%	10%	1%	5%	10%		
9	-0.046	7.7175	0.563	-0.059	11.405	0.249	-3.453	-2.871	-2.572	-3.453	-2.871	-2.572		

Note: The tests are performed using all 590 observations including in-sample and out-of-sample data.

4.2 Model estimation and prediction

Four models are established for trend of returns and volatility estimations, The basic regime switch model is Model 1 that can switch for both trend of returns and volatility, the other three models are the restricted forms of Model 1 with Model 2 for trend of returns or VAR switch only, Model 3 for volatility or GARCH switch only and Model 4 without switch.

Table 3 reports the estimations of regime switch models using in-sample data from 08/09/2001 to 05/31/2007. As shown overall, the correlation between MXTW and STW is large and significant up to 0.98 plus. The level of trend of returns is significant except Model 2 in state 1 and Model 3 in both states. The coefficients of ARCH and GARCH are significant in state 1. The parameters of transition probability is significant in state 1 as well. In fact, the volatility switch between high and low volatility states is mostly demonstrated but the trend switch is yet to investigate further.

According to the parameter setup, the probability of stay in state 1 i.e. p_{11} or stay in state 2 i.e. p_{22} as in Equation (13) is equal to the cumulated probability of standard normal distribution. Hence, in Model 1 the conditional transition probabilities of p_{11} and p_{22} are equal to 0.9579 and 0.4396 respectively while the unconditional state probabilities π_1 and π_2 as in Equation (17) are equal to 0.9301 and 0.0699 respectively.

Table 3 The Estimations of Regime Switch Models

	Model 1				Model 2				Model 3				Model 4	
	State 1		State 2		State 1		State 2		State 1		State 2			
	MXTW	STW	MXTW	STW	MXTW	STW	MXTW	STW	MXTW	STW	MXTW	STW	MXTW	STW
ρ	0.9883		0.9901		0.9914		0.9914		0.9882		0.9878		0.9886	
Stderr	0.0016		0.0045		0.0012		0.0012		0.0012		0.0084		0.0018	
p value	0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0000	
θ_0	0.0058	0.0065	-0.0337	-0.0406	-0.0008	-0.0006	0.0441	0.0330	0.0049	0.0055			0.0049	0.0052
Stderr	0.0017	0.0017	0.0101	0.0104	0.0020	0.0021	0.0065	0.0066	0.0035	0.0037			0.0017	0.0019
p value	0.0006	0.0002	0.0009	0.0001	0.6798	0.7679	0.0000	0.0000	0.1615	0.1436			0.0047	0.0060
θ_1	0.5307	1.1837	0.2525	1.2881	0.1943	0.8730	-1.2424	-1.5367	0.2218	0.8640			-0.1573	0.3470
Stderr	0.2897	0.3007	0.4366	0.5900	0.2513	0.2858	0.6999	0.7291	0.2977	0.3305			0.2138	0.2493
p value	0.0682	0.0001	0.5636	0.0300	0.4401	0.0025	0.0771	0.0361	0.4569	0.0095			0.4624	0.1652
θ_2	0.8557	1.4133	-0.0426	0.3404	0.4077	0.9964	0.3547	1.5132	0.5183	1.0763			0.2340	0.6891
Stderr	0.2727	0.2897	0.2644	0.2880	0.2346	0.2698	0.6483	0.8403	0.3530	0.3862			0.1417	0.1463
p value	0.0019	0.0000	0.8721	0.2383	0.0835	0.0003	0.5848	0.0729	0.1432	0.0057			0.1000	0.0000
θ_3	0.1771	0.3368	1.5200	1.5200	0.2328	0.4412	-1.1702	-1.1337	0.0526	0.2520			0.1085	0.2442
Stderr	0.2381	0.2571	0.4199	0.5831	0.1988	0.2343	0.2299	0.2251	0.3335	0.3663			0.2627	0.2938
p value	0.4577	0.1914	0.0004	0.0097	0.2426	0.0608	0.0000	0.0000	0.8747	0.4920			0.6800	0.4067
θ_4	-0.5808	-1.2461	-0.1087	-0.9373	-0.1163	-0.7887	0.5158	0.8631	-0.2626	-0.9132			0.0491	-0.4659
Stderr	0.2619	0.2725	0.3582	0.4972	0.2191	0.2520	0.5360	0.5725	0.2640	0.2975			0.1853	0.2221
p value	0.0275	0.0000	0.7619	0.0606	0.5962	0.0020	0.3368	0.1329	0.3208	0.0024			0.7913	0.0369
θ_5	-0.9086	-1.4706	-0.4441	-0.9472	-0.4252	-1.0151	0.1608	-0.9287	-0.5389	-1.0988			-0.2413	-0.6954
Stderr	0.2478	0.2631	0.1598	0.1955	0.2010	0.2373	0.6003	0.7350	0.3340	0.3603			0.1355	0.1461
p value	0.0003	0.0000	0.0059	0.0000	0.0354	0.0000	0.7890	0.2075	0.1079	0.0025			0.0760	0.0000
θ_6	-0.3152	-0.5027	-1.1166	-1.3663	-0.2109	-0.4455	1.0636	0.8208	-0.1498	-0.3757			-0.1162	-0.2751
Stderr	0.2166	0.2345	0.1461	0.3288	0.1590	0.1964	0.2659	0.2486	0.3066	0.3359			0.2235	0.2524
p value	0.1469	0.0330	0.0000	0.0000	0.1859	0.0241	0.0001	0.0011	0.6255	0.2644			0.6035	0.2767
w	0.0000	0.0000	0.0014	0.0013	0.0002	0.0802			0.0000	0.0000	0.0037	0.0046	0.0005	0.0005
Stderr	0.0000	0.0000	0.0008	0.0008	0.0001	0.0146			0.0000	0.0000	0.0055	0.0068	0.0001	0.0001
p value	0.3694	0.5508	0.0783	0.1040	0.0416	0.0000			0.2608	0.3176	0.2508	0.2513	0.0001	0.0001
α	0.0224	0.0190	0.6691	0.1272	0.5718	0.0003			0.0201	0.0174	0.7070	1.0000	0.3521	0.4035
Stderr	0.0074	0.0051	0.3409	0.2070	0.1183	0.0001			0.0070	0.0052	1.1795	1.3501	0.0799	0.0809
p value	0.0027	0.0003	0.0508	0.5394	0.0000	0.0115			0.0022	0.0005	0.2747	0.2298	0.0799	0.0809
β	0.9582	0.9642	0.3627	0.7280	0.1235	0.5333			0.9633	0.9672	0.0000	0.0275	0.3498	0.3527
Stderr	0.0164	0.0124	0.1134	0.0954	0.0234	0.1093			0.0176	0.0134	1.3373	1.2278	0.0929	0.0746
p value	0.0000	0.0000	0.0016	0.0000	0.0000	0.0000			0.0000	0.0000	0.5000	0.4911	0.0929	0.0746
Pm	1.7275		-0.1521		2.2176		0.6136		1.6721		-0.3276			
Stder	0.1782		0.3200		0.2988		0.8637		0.2143		0.4388			
p value	0.0000		0.6350		0.0000		0.4781		0.0000		0.4560			
LogL	1689.32				1663.71				-1678.01				1616.62	
AIC	3466.64				3401.42				3416.03				3275.24	
BIC	3627.81				3536.95				3525.92				3352.16	
p	0.9579		0.4396		0.9867		0.7303		0.9527		0.3716			
π	0.9301		0.0699		0.9531		0.0469		0.9301		0.0699			

Notes: 1. Model 1 is switch for trend and volatility, Model 2 is switch for trend only, Model 3 is switch for volatility only, and Model 4 is no switch. 2. The estimated parameters are the coefficient of correlation ρ in Equation (4), the coefficients of VAR $\boldsymbol{\theta}$ in Equation (1), and the coefficients of GARCH (w, α, β) in Equation (5). 3.*Pm* is the parameters of transition probability and the cumulative probability of standard normal distribution of *Pm* is the transition probability p that is p_{11} for stay in state 1 or p_{22} for stay in state 2. 4. π is the unconditional state probability as in Equation (17). 5. In Model 2 the GARCH estimates are the same for State 1 and 2, and in Model 3 the VAR estimates are the same for State 1 and 2. The estimates are listed in state 1 but not in state 2.

Similarly, Table 3 lists the results of transition and state probabilities for Model 2 and 3. As the conditional transition and unconditional state probabilities reveal, state 1 is more likely to stay in than state 2 and the transition to state 1 is more possible than state 2. Observing stock prices for a long time, trends are more likely to go up than go down and volatilities are more likely to stay tranquil as usual than turbulent.

Besides, for all models, state 2 appears to have higher level of trend and volatility. It means to stay in state 2 is not as usual. On the other hand, state 1 with less trend and volatility levels is more significant and possible than state 2. Consequently, state 1 with higher p_{11} is assumed to be the usual (u) state and state 2 is the down (d) state.

The one-week ahead prediction is obtained, once the model parameters are estimated. Then using technique of rolling over out-of-sample period step by step is able to obtain 302 one-week ahead predictions. Figure 2 exhibits the predictions of indices, volatility, covariance, and hedge ratios for out-of-sample period. Obviously, the volatilities and covariances are found much higher during subprime period around the second part of 2008 and the hedge ratios are noticed more volatile as well. The extracted index predicitions for all models show higher than the actual index from 08/2008 to 12/2008 within subprime period.

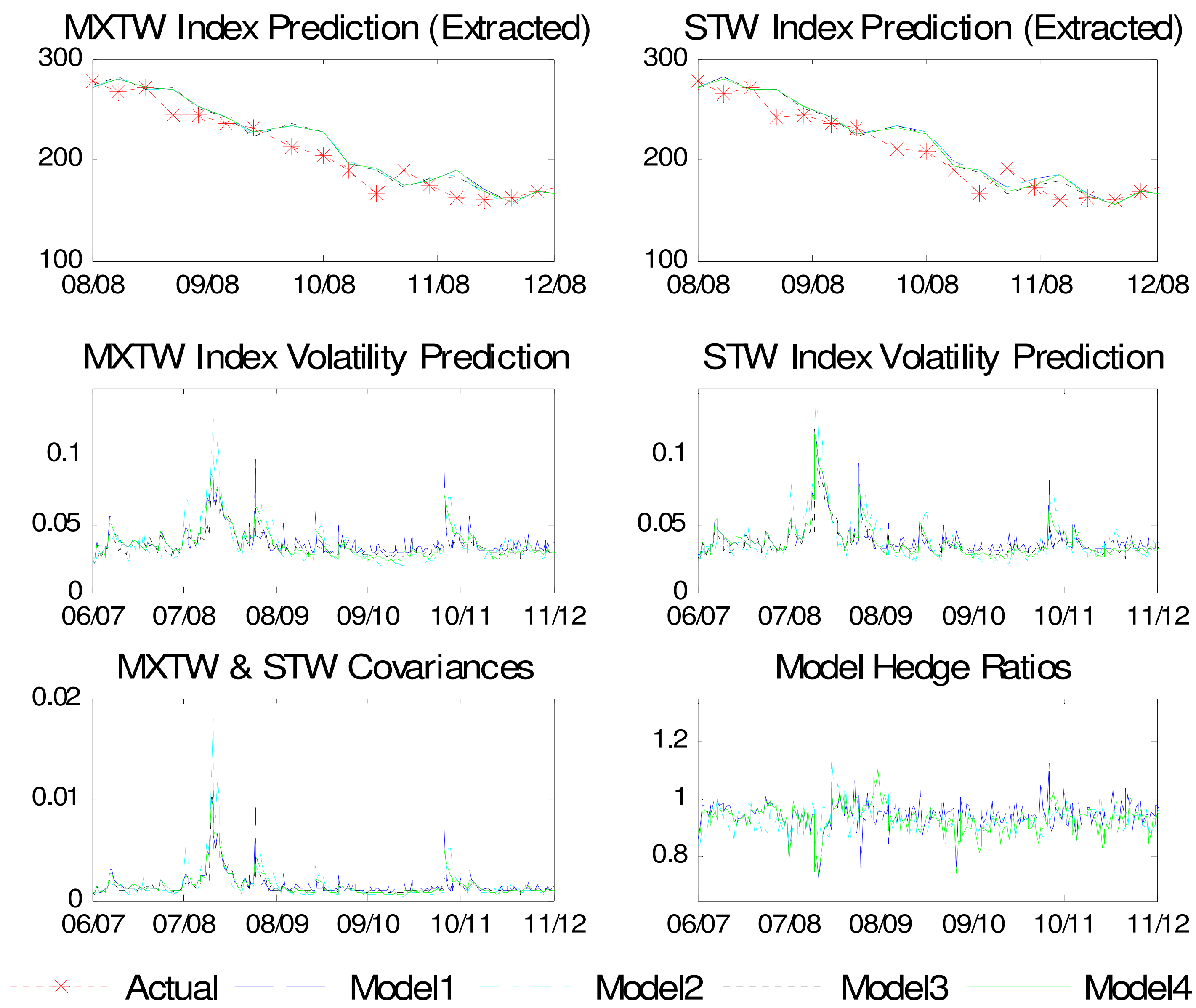

Figure 2 The Estimated Indices, Volatilities, Covariances, and Hedged Ratios

4.3 Hedge performance

Table 4 reports the hedge performances using the 302 one-week ahead predictions over out-of-sample period. In comparison of three periods: subprime, Greek debt, and post risk, surprisingly the hedge performances over the subprime period have much the best advantages for all four models with IWI over 0.81%, RVD below 83%, and IUI reaching up to 59% far above the average that has IWI above 0.10%, RVD below 82.66%, and IUI above 26.41% calculated through the whole out-of-sample period. The subprime period is found very crucial to acquire better hedge performance.

Table 4 Hedge Performance Comparison of Models

Panel A. Whole out-of-sample period

	Mean(W_0)	Mean(Var_0)	Mean(U_0)	Mean(HR_1)	Std(HR_1)	IWI	RVD	IUI
Model 1	278.06	79.31	238.40	94.543%	4.601%	0.112%	-84.038%	26.397%
Model 2	278.06	79.31	238.40	92.916%	4.287%	0.119%	-84.279%	26.443%
Model 3	278.06	79.31	238.40	98.402%	8.048%	0.114%	-82.654%	26.417%
Model 4	278.06	79.31	238.40	93.061%	4.788%	0.108%	-84.473%	26.832%

Panel B. Subprime period

	Mean(W0)	Mean(Var0)	Mean(U0)	Mean(HR1)	Std(HR1)	IWI	RVD	IUI
Model 1	287.36	146.71	214.00	93.794%	5.578%	0.815%	-84.348%	60.310%
Model 2	287.36	146.71	214.00	92.911%	5.571%	0.841%	-84.178%	59.814%
Model 3	287.36	146.71	214.00	95.393%	4.715%	0.837%	-83.287%	59.613%
Model 4	287.36	146.71	214.00	93.430%	5.264%	0.818%	-84.629%	60.890%

Panel C Greek debt period

	Mean(W0)	Mean(Var0)	Mean(U0)	Mean(HR1)	Std(HR1)	IWI	RVD	IUI
Model 1	287.48	93.49	265.76	94.075%	3.673%	-0.096%	-95.502%	7.891%
Model 2	287.48	93.49	265.76	91.294%	3.283%	-0.077%	-95.268%	7.864%
Model 3	287.48	93.49	265.76	96.574%	6.997%	-0.080%	-94.982%	7.805%
Model 4	287.48	93.49	265.76	90.649%	3.961%	-0.079%	-95.450%	7.895%

Panel D Post risk period

	Mean(W0)	Mean(Var0)	Mean(U0)	Mean(HR1)	Std(HR1)	IWI	RVD	IUI
Model 1	271.26	38.81	251.85	95.101%	3.263%	-0.053%	-85.711%	7.604%
Model 2	271.26	38.81	251.85	93.560%	3.323%	-0.048%	-85.756%	7.594%
Model 3	271.26	38.81	251.85	102.155%	4.235%	-0.043%	-83.813%	7.508%
Model 4	271.26	38.81	251.85	92.338%	3.648%	-0.048%	-85.692%	7.589%

Notes: 1.W_0, Var_0, and U_0 represent respectively no hedge wealth, variance of wealth, and utility. HR_1, W_1, Var_1, and U_1 represent respectively hedging ratio, wealth, variance of wealth, and utility. 2. IWI, RVD, and IUI are defined in Equations(29), (30), and (31) respectively.

The hedge performances over Greek or post risk periods are far below the average except RVD. The whole out-of-sample period shows that the regime switch models including Model 1 to 3 have better performance in IWI while no regime switch i.e. Model 4 is desirable in RVD and IUI. Generally for out-of-sample period, Model 2 is superior to Models 1 and 3 in IWI, RVD, and IUI. Besides, for the subprime period, it turns to favor Model 2 with trend switch. Thus, using trend switch alone expresses more appropriate than using both trend and volatility switches or volatility switch alone. The Greek debt and post risk periods appears to prefer Model 4 with no regime switch; however, all models perform far below the average.

4.4 Hedge measurements in different states

One essential advantage to use regime switch model is to classify the hedge context into different hedge states to measure hedge effectiveness. For a univariate series, the hedge states can be classified into usual (u) i.e. state 1 and down(d) i.e. state 2. If the state probabilities are greater than 0.5, they are deemed as state d otherwise state u. Thus, for the case of two univariate MXTW or STW, it turns out to have four hedge states: uu, ud, du, or dd. The hedge performances are evaluated in the four hedge states to explore which hedge state performs the best.

The state probabilities for univariate MXTW or STW are estimated similar to bivariate series except that the matrix $\boldsymbol{\varepsilon}$ in Equation (1) is reviewed as a number. Again using rolling over technique, the dynamic process of total 302 state probabilities is found for out-of-sample period.

Figure 3 exhibits the dynamic process of state probabilities for both MXTW and STW. Clearly, the state probabilities switch very often for both series during 06/2007 to 11/2009 covering the period of subprime. After that, both of them become very calm from 07/2010 to 03/2011.

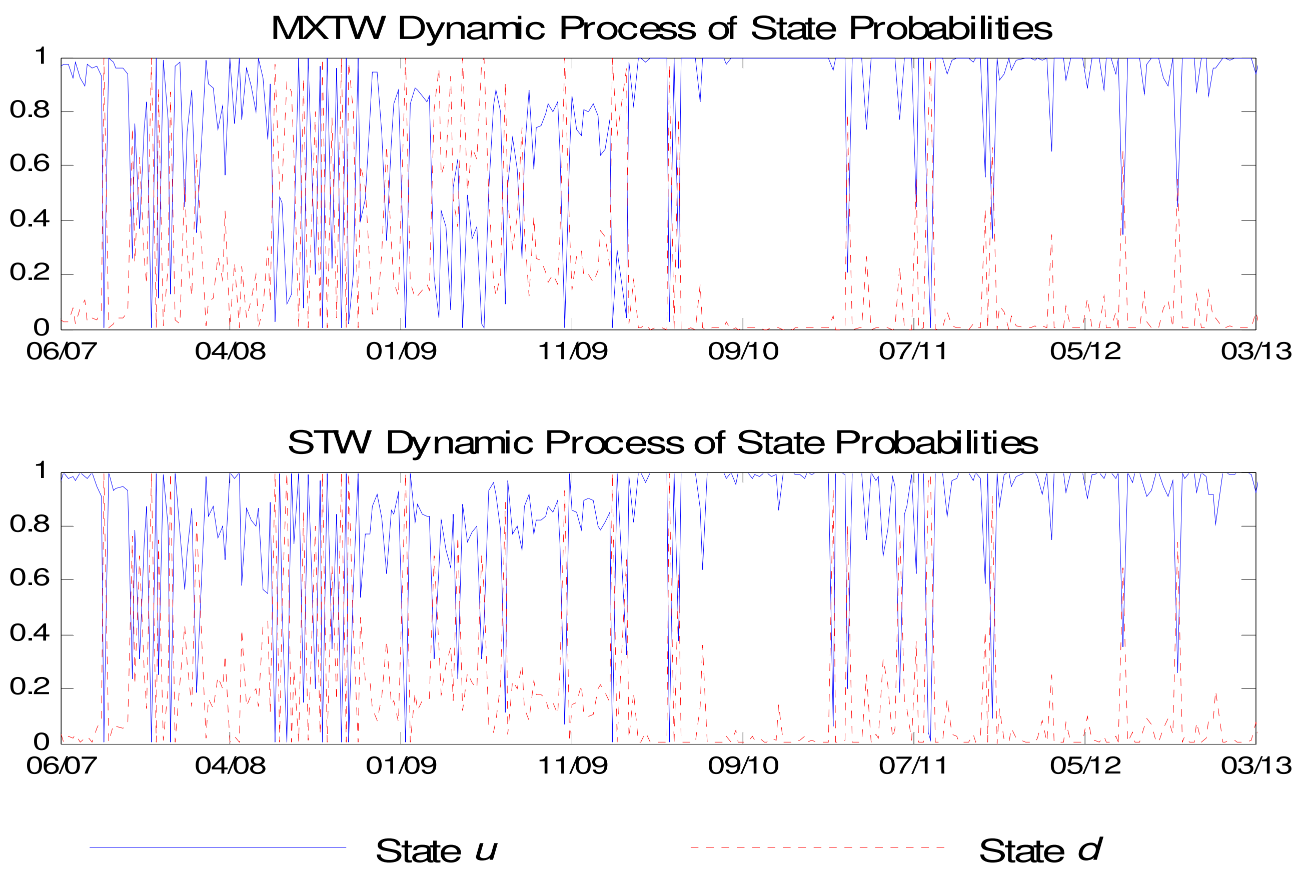

Figure 3 The Dynamic Process of State Probabilities

Table 5 reports the hedge performances evaluated in four different hedge states. Evidently for all models, the hedges in dd states have the most astonishing performance having IWI above +1.4%, RVD below -86%, and IUI above +40% compared to the average of whole out-of-sample period having IWI above 0.10%, RVD below 82.66%, and IUI above 26.41%.

The *uu* states are the most frequently observed incidents 246/302 during out-of-sample period. However, all models perform below the average in *uu* state having IWI, RVD, and IUI respectively below+0.009%, above -84%, and below+23%. All models in *du* state have the largest IUI above 50% but lower negative IWI below -0.078%. All models in *ud* state perform the worst having the lowest IWI below -2.10% and IUI lower than +12% though having the most reduced RVD below -96%. Undoubtedly, hedgers have to avoid the *ud* states and pursue much more important hedge opportunities in *dd* and *du* hedge states.

Table 6 reports the number of hedge states in a certain hedge period over total 302 occurrences. The no regime switch model appears to perform better in most seen *uu* state as occupying 93.67% in post risk period and 90.63% in Greek debt period but the results are below average like the other models. However, Figure 2 has showed that the volatilities and covariances are frustrated the most extensively in subprime period. Thus the most required hedge moment is not in *uu* state but in subprime period as occupying up to 19.54% in *dd* state. The hedgers have to catch this decisive 19.54% opportunity to establish appealing hedge results.

Table 5 Hedge Measurements in Different Hedge States

Panel A. Model 1 with trend and volatility switches

State	Nos.	Mean(W0)	Mean(Var0)	Mean(U0)	Mean(HR1)	Std(HR1)	IWI	RVD	IUI
all	302	278.06	79.31	238.40	94.543%	4.601%	0.112%	-84.038%	26.397%
uu	246	282.20	247.94	68.52	94.785%	3.638%	0.008%	-83.172%	22.660%
ud	5	286.79	251.72	70.15	94.647%	3.110%	-2.357%	-96.652%	11.269%
du	19	234.45	172.77	123.37	89.975%	9.459%	-0.092%	-84.668%	50.712%
dd	32	270.76	202.00	137.52	95.377%	5.812%	1.420%	-88.349%	43.053%

Panel B. Model 2 with trend switch

State	Nos.	Mean(W0)	Mean(Var0)	Mean(U0)	Mean(HR1)	Std(HR1)	IWI	RVD	IUI
all	302	278.06	79.31	238.40	92.916%	4.287%	0.119%	-84.279%	26.443%
uu	246	282.20	247.94	68.52	93.324%	4.025%	0.008%	-83.543%	22.538%
ud	5	286.79	251.72	70.15	89.963%	3.711%	-2.189%	-96.381%	11.365%
du	19	234.45	172.77	123.37	90.490%	4.849%	-0.078%	-84.163%	53.474%
dd	32	270.76	202.00	137.52	91.688%	5.204%	1.454%	-88.116%	42.770%

Panel C. Model 3 with volatility switch

State	Nos.	Mean(W0)	Mean(Var0)	Mean(U0)	Mean(HR1)	Std(HR1)	IWI	RVD	IUI
all	302	278.06	79.31	238.40	98.402%	8.048%	0.114%	-82.654%	26.417%
uu	246	282.20	247.94	68.52	98.672%	7.340%	0.008%	-81.871%	22.455%
ud	5	286.79	251.72	70.15	96.462%	5.170%	-2.350%	-96.381%	11.259%
du	19	234.45	172.77	123.37	93.499%	6.134%	-0.102%	-83.221%	57.716%
dd	32	270.76	202.00	137.52	99.536%	12.672%	1.441%	-86.200%	40.658%

Panel D. Model 4 with no switch

State	Nos.	Mean(W0)	Mean(Var0)	Mean(U0)	Mean(HR1)	Std(HR1)	IWI	RVD	IUI
all	302	278.06	79.31	238.40	93.061%	4.788%	0.108%	-84.473%	26.832%
uu	246	282.20	247.94	68.52	93.433%	4.404%	0.009%	-83.727%	22.862%
ud	5	286.79	251.72	70.15	88.496%	4.041%	-2.346%	-96.173%	11.175%
du	19	234.45	172.77	123.37	91.131%	7.613%	-0.156%	-84.950%	54.877%
dd	32	270.76	202.00	137.52	92.068%	5.075%	1.412%	-88.102%	43.150%

Note: "all" is the all out-of-sample data covering all hedge states.

Table 6 The Number of Hedge States in Different Hedge Periods

		uu		*ud*		*du*		*dd*
Period	Nos.	Percentage	Nos.	Percentage	Nos.	Percentage	Nos.	Percentage
Subprime	62	71.26%	0	0.00%	8	9.20%	17	19.54%
Greek Debt	58	90.63%	1	1.56%	1	1.56%	4	6.25%
Post Risk	74	93.67%	2	2.53%	0	0.00%	3	3.80%
Other	52	72.22%	2	2.78%	10	13.89%	8	11.11%
Total Nos.=302	246	81.46%	5	1.66%	19	6.29%	32	10.60%

Remarkably, the regime switch models such as Models 1 and 2 appear to perform better than no switch model under *dd* state in terms of IWI and RVD according to Table 5. After all, the most important hedge is to acquire hedging wealth or reduce variations in *dd* state as the results are indeed far above average.

5. Conclusions

The logarithm returns of MXTW and STW are demonstrated having the higher perakness and correlation and the Jarque-Bera test indicates the nonlinear property in series. The regime switch models are proposed to better predict the nonlinear expected returns and volatilities of MXTW and STW.

The higher volatility and covariance of returns, and frustration of hedge ratios are found intensively occurring in subprime period. In fact, the most demanded hedging period is the subprime period. At this moment, all four models would have surprising far above average hedge performance in terms of wealth or utility increase while they perform far below average except risk decreasing in Greek debt or post risk period.

As the hedge context are classified into different states, it is found that all four models in *du* and *dd* states have the most advantageous performance in terms of IWI and IUI while the *ud* state is not desirable. In fact, the subprime period is found consisted of 19.54% *dd* states occupying much the most amount in all periods. Thus, the hedgers must catch this crucial opportunity as found recurrently in subprime period to perform the hedge.

The hedge ratios are constructed by two components: the position of expected return and the position of variance. As a matter of fact, the stock markets are found having the property of tradeoff between return and variance. It seems hard to increase the return and reduce the variance at the same time. The four hedge models are either effective in IWI, RVD, or IUI but not all of them. The regime switch models are found very outstanding at the IWI or RVD but not IUI especially when both series are in down state as occurring very often in subprime period. The use of trend switch to catch the wealth increase appears more appropriate than the volatility switch to reduce the risk.

Noticeably, the no switch model appears to perform the best when both MXTW and STW are in usual state as most observed in both Greek debt and post risk periods but the results are found below the average calculated through out-of-sample period. Consequently, it is found advantageous to use trend switch model to catch the wealth increase and use no switch model to acquire the utility increase.

References

Alizadeh, A.H., N.K. Nomikos, and P.K. Pouliasis (2008). A Markov regime switching approach for hedging energy commodities. *Journal of Banking and Finance*, 32(9), 1970-1983.

Andersen, T., T. Bollerslev, F. Diebold, and P. Labys (2003). Modeling and forecasting realized volatility, *Econometrica*, 71, 579–625.

Ang, A. and G. Bekaert (2002). International asset allocation with regime shifts. *Review of Financial Studies*, 11, 1137-1187.

Babula, R.A., F.J. Ruppel, and D.A. Bessler (1995). U.S. corn exports: The role of the exchange rate. *Agricultural Economics*, 13, 75-88.

Bae, K.H., G. A. Karolyi, and R. M. Stulz (2003). A new approach to measuring financial contagion. *The Review of Financial Studies*, 16, 717–763.

Baillie, R.T. and R.J. Myers (1991). Bivariate GARCH estimation of the optimal commodity futures hedge. *Journal of Applied Econometrics*, 6, 109-124.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rate: A multivariate generalized ARCH approach. *Review of Economics and Statistic*, 72, 498-505.

Bollerslev, T, R. F. Engle, and J. M. Wooldridge (1988). A capital asset pricing model with time-varying covariances. *Journal of Political Economy*, 96(1), 116-131.

Ederington, L. H. (1979). The hedging performance of the new futures markets. *Journal of Finance*, 34(1), 157-170.

Frechette, D.L. (2000). The demand for hedging and the value of hedging opportunities. *American Journal of Agricultural Economics*, 82 (4), 897-907.

Gagnon, L., G.J. Lypny, and T.H. McCurdy (1998). Hedging foreign currency portfolios. *Journal of Empirical Finance*, 5, 197-220.

Gray, S.(1996). Modeling the conditional distribution of interest rates as a regime-switching process. *Journal of Financial Economics*, 42, 27-62.

Haas, M., S. Mittnik, and M. S. Paolella (2004). A new approach to Markov switching GARCH models. *Journal of Financial Econometrics*, 2(4), 493-530.

Haas, M. and S. Mittnik (2008). Multivariate regime switching GARCH with an application to international stock markets. CFS Working Paper Series 2008/08, Center for Financial Studies.

Haigh, M.S. and M.T. Holt (2000), Hedging multiple price uncertainty in international grain trade. *American Journal of Agricultural Economics*, 82(4), 881-896.

Haigh, M. S. and M.T. Holt (2002). Hedging foreign currency, freight and commodity futures portfolios-A note. *The Journal of Futures Markets*, 22 (12), 1205-1221.

Hamilton, J.D. (1990). Analysis of time series subject to changes in regime. *Journal of Econometrics*, 45, 39-70.

Hamilton, J.D. (1991). A quasi-Bayesian approach to estimating parameters for mixtures of normal distributions. *Journal of Business and Economic Statistics*, 9, 27-39.

Hamilton, J.D. and R. Susmel (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, 64, 307-333.

Hansson, B. and P. Hordahl (1998). Testing the conditional CAPM using multivariate GARCH. *Applied Financial Economics*, 8, 377-388.

Harris, R.D.F. and J. Shen (2003). Robust estimation of the optimal hedging ratio. *Journal of Futures Markets*, 23(8), 799-816.
Hayashi, T. and N. Yoshida (2005). On covariance estimation of non-synchronously observed diffusion processes. *Bernoulli*, 11, 359–379.
Heifner, R. G. (1972). Optimal hedging levels and hedging effectiveness in cattle feeding. *Agricultural Economic Research*, 24, 25-35.
Honda, T. (2003). Optimal portfolio choice for unobservable and regime-switching mean returns. *Journal of Economic Dynamics and Control*, 28, 45-78.
Jin, H.J. and W.W. Koo (2006). Offshore hedging strategy of Japan-based wheat traders under multiple sources of risk and hedging costs. *Journal of International Money and Finance*,25(2), 220-236.
Johnson, L. (1960). The theory of hedging and speculation in commodity futures. *Review of Economic Studies*, 27, 139–151.
Klaassen, F. (2002). Improving GARCH volatility forecasts. *Empirical Economics*, 27(2), 363-394.
Koutmos, G. and A. Pericli (1998). Dynamic hedging of commercial paper with T-bill futures. *Journal of Futures Markets*, 18(8), 925-938.
Kroner, K.F. and J. Sultan (1993). Time varying distributions and dynamic hedging with foreign currency futures. *Journal of Financial and Quantitative Analysis*, 28, 535-551.
Lee H., J.K. Yoder, R.C. Mittelhammer, and J.J. McCluskey (2006). A random coefficient autoregressive Markov regime switching model for dynamic futures hedging. *The Journal of Futures Markets*, 26, 103-129.
Lence, S.H. (1995). The economic value of minimum-variance hedges. *American Journal of Agricultural Economics*, 77(2), 353-364.
Lien, D and L. Yang (2008). Asymmetric effect of basis on dynamic futures hedging: Empirical evidence from commodity markets. *Journal of Banking and Finance*, 32(2), 187-198.
Liu, K.E., J. Geaun, and L.F. Lei (2001). Optimal hedging decisions for Taiwanese corn traders on the way of liberalization. *Agricultural Economics*, 25, 303-309.
Marcucci, J. (2005). Forecasting stock market volatility with regime-switching GARCH models. *Studies in Nonlinear Dynamics & Econometrics*, 9(4), Article 6.
Markowitz, H. M. (1952). Portfolio selection. *Journal of Finance*, 7(1), 77-91.
Moschini, G. and R. Myers (2002). Testing for constant hedging ratios in commodity markets: A multivariate GARCH approach. *Journal of Empirical Finance*, 9, 589-603.
Ng, L. (1991). Tests of the CAPM with time-varying covariances: A multivariate GARCH approach. *The Journal of Finance*, 46, 1507-1521.
Pelletier,D. (2006).Regime switching for dynamic correlations. *Journal of Econometrics*, 131(1-2), 445-473.
Poomimars P, J. Cadle, and M. Theobald (2003). Futures hedging using dynamic models of the variance/covariance structure. *Journal of Futures Markets*, 23(3), 241-260.
Ramchand, L. and R. Susmel (1998). Cross correlations across major international markets. *Journal of Empirical Finance*, 5, 397-416.
Stein, J. (1961). The simultaneous determination of spot and futures prices. *American Economic Review*, 51, 1012–1025.
Tse, Y. K. and K. C. Tsui (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business and Economic Statistics*, 20, 351-362.
Working, H. (1962). New concepts concerning futures markets and prices. *American Economic Review*, 52(3), 431-459.
Yun, W.C. and H.J. Kim (2010). Hedging strategy for crude oil trading and the factors influencing hedging effectiveness. *Energy Policy*, 38(5), 2404-2408.