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A TEST OF THE STABLE PARETIAN HYPOTHESIS FOR THE DISTRIBUTION OF INCOME*

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I. INTRODUCTION

Determination of the distribution of income has long interested economists (see Champernowne (1953); Lydall (1959); Pasinetti (1980); and Ring (1985)). Recently Mandelbrot (1960, 1961, 1963, 1983) proposed that the distribution of income might be described by a class of mathematical processes called Stable Paretian functions. These functions have many of the desirable properties of the commonly used Gaussian distributions, but they have infinite variance, which has implications for making projections. Mandelbrot's hypothesis applies only to very high income families. His ideas are of interest to the Army because children in high income families have a low propensity to enlist in the military, so an increasingly affluent population could have an effect on Army recruiting. This paper tests the Stable Paretian Hypothesis for high income families.

II. METHOD

A "weak form of the law of Pareto" states that, for a yearly income U there exist two constants 'C' and 'a' such that for very large values of income u, the probability P that income U exceeds u is given by

$$P = Cu^{-a} \qquad (2.1)$$

The so-called "strong form of the law of Pareto" states that equation (2.1) holds for all values of income u. The strong form of the Pareto law has been rejected empirically. The inclusion of self-employed persons and managers with wage earners, and the differing methods of reporting income by occupational category make it extremely unlikely that any single law could ever explain the entire distribution of income. For example, a lognormal distribution holds for a large range of incomes, but does not hold for very high incomes.

The distribution of income for all families in the U.S. is shown in Figure 1, where families are ranked by increasing income levels. The 45 degree line would mean a perfectly even distribution of income. The actual distribution of income is shown by the dashed Lorenz curve. In 1982 the lowest 40% of families earned only 16% of the total income, while the highest 20% earned almost 43% of the national income. To test the Stable Paretian Hypothesis we first calculate the negative of the distribution function for a given Census group, add +1 and plot the result on double-logarithmic graph paper. We then measure the slope at the high income part of the graph. An example is shown in Figure 2. We plotted similar graphs for households that included 18 to 21 year old dependents (Census P-60 Reports, 1979, 1980, 1981, 1982). The Stable Paretian Hypothesis asserts that the absolute value of the slope of the high income of the double-logarithmic graph in Figure 2 will be between 1 and 2.

Table 1 shows the results for high income (over $50,000) families that had 18 - 21 year olds that the Census Bureau calls "other relative of householder," i.e., the youths were neither the spouse nor the head of the household. Every coefficient in Table 1 is much greater than two, a novel result that means that for these age groups and income ranges the distributions are neither lognormal nor Stable Paretian. The significance of the Stable Paretian Hypothesis test is this: it allows us to assume the distribution of income will be given by a stationary income distribution curve for each household age range given by the Census. We could thus forecast the distribution of incomes of interest to Army recruiters. The results of these forecasts have been reported elsewhere (see Dale (1985)).

III. CONCLUSIONS

The distribution of income for high income families with 18 to 21 year old dependents is not Stable Paretian as defined by Mandelbrot. A possible alternative is given by Lydall (1959), who assumes a structured hierarchy of incomes. In any case forecasters may not need to deal with infinite variances, and may assume only that the underlying distributions are stationary.

FOOTNOTES

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#100016. The views expressed in this paper are solely those of the author and not necessarily those of any of the aforementioned individuals, the U.S. Army Research Institute, or the Department of Defense.

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TABLE 1

PARETO-INCOME DISTRIBUTION EXPONENTS

18 - 21 YEAR OLDS NOT HEADING HOUSEHOLDS

YEAR	MALES	FEMALES
1982	3.23	3.56
1981	3.97	3.69
1980	3.64	3.86
1979	3.82	3.74

Raw Data Source: Bureau of the Census.

DISTRIBUTION OF INCOME
ALL FAMILIES — 1982 DATA

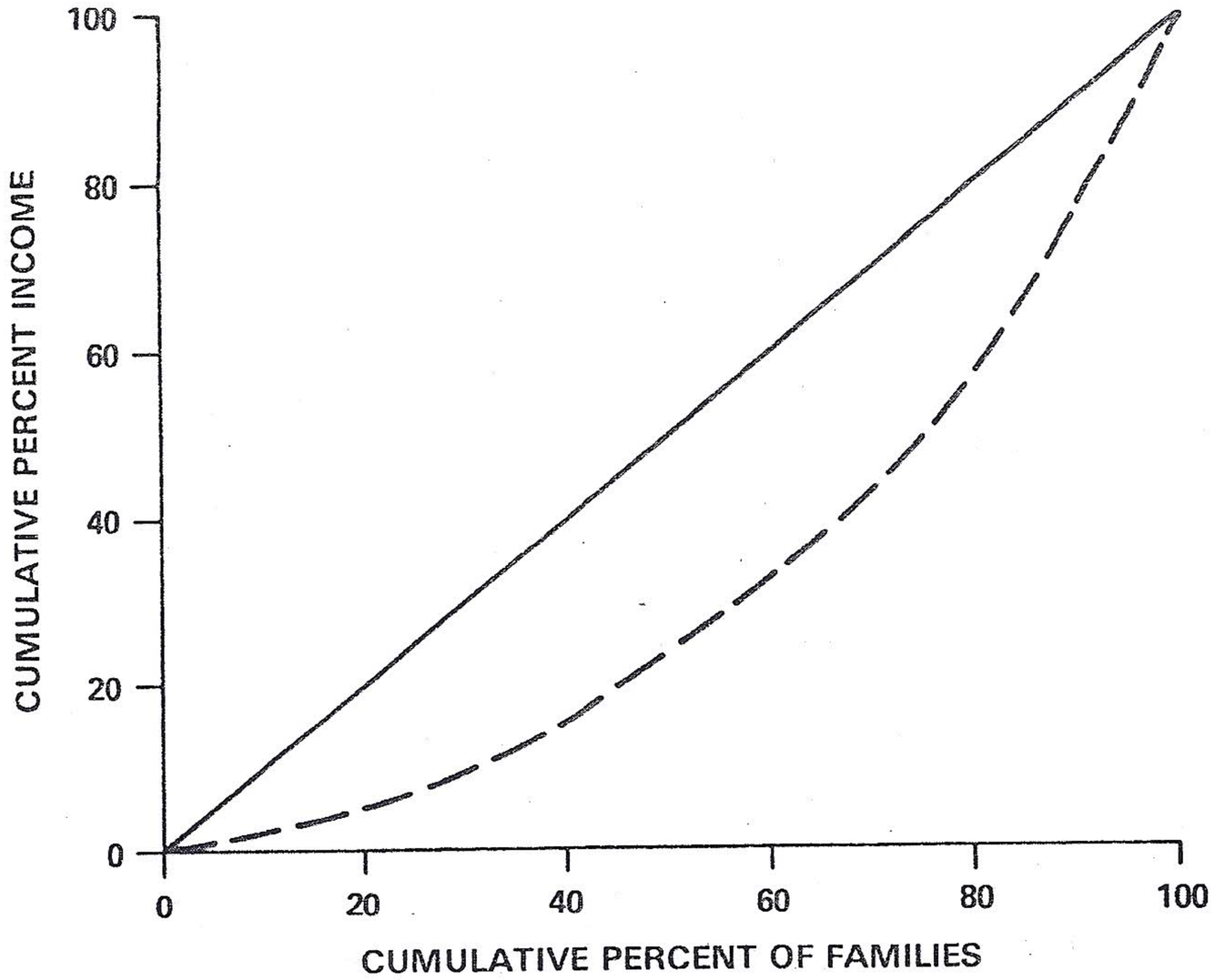

FIGURE 1

STABLE PARETIAN HYPOTHESIS
MALE HOUSEHOLDERS — 35 TO 44 YEARS — 1981 DATA

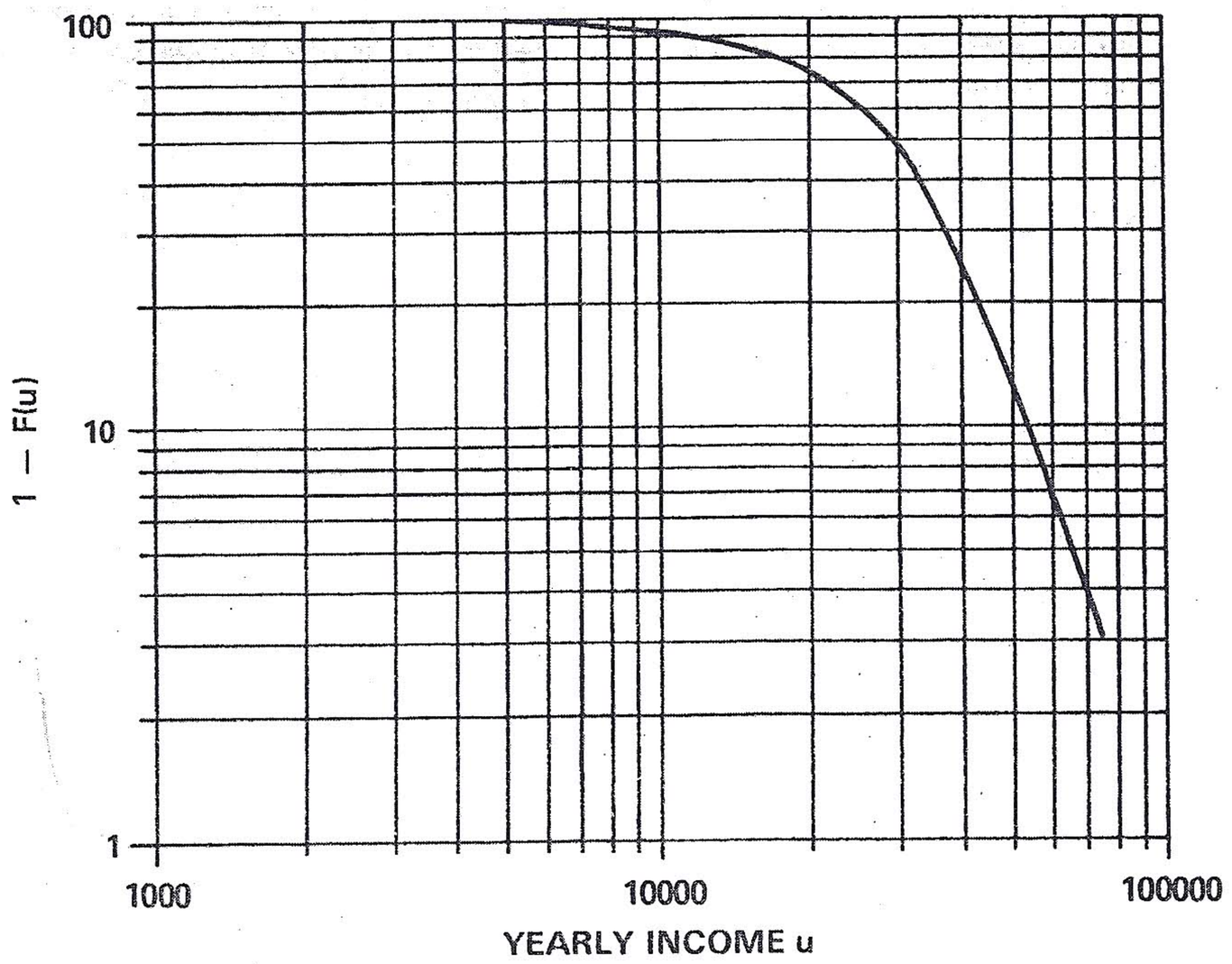

FIGURE 2