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Alikhanov, Murat and Taylor, Leon

KIMEP University

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Murat Alikhanov[1]
and
Leon Taylor[2]

Abstract

Most macroeconomic models, such as the IS-LM, assume equilibrium in money markets. Since money demand is an inverse function of velocity, an inaccurate estimate of velocity will lead to errors in calculating the monetary and general equilibria. This note suggests a way to gauge the potential error in estimating velocity. The algorithm arises from the quantity equation of exchange, which one may prefer to an ad hoc *model of velocity.*

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JEL Classifications: E47; E52

I. Introduction

Fluctuations in the turnover rate of a unit of money – velocity -- complicate the central bank's forecast of the economic impact of a change in monetary policy. Monetarists contend that when velocity is stable, a change in money supply leads to a predictable change in nominal income. But when data are scarce and economic institutions are changing rapidly – for example, early in the post-Soviet transition to markets – income velocity can be hard to estimate (Citrin, 1995). Forecasts of the effects of monetary changes, in a given scenario, may benefit from a way to gauge the magnitude of possible errors in estimating velocity. This note suggests such an algorithm.

A typical monetary forecast begins with the identity that total spending equals total receipts by the factors of production.[3] This is the quantity equation of exchange, $MV = PQ$. The left-hand side multiplies the money supply M by velocity V; the right-hand side

[1] *Department of Economics, KIMEP University; muratstan@gmail.com*
[2] *Department of Economics, KIMEP University; ltaylor@kimep.kz. Taylor is the corresponding author.* *We thank participants in a seminar at the Department of Economics at KIMEP in the fall of 2012 -- especially Gerald Pech, for correcting an error in an early draft.*

[3] The factors include entrepreneurs.

multiplies the price *P* of a typical bundle of goods by the number of bundles *Q*. When *V* and *Q* are constants, a change in *M* induces a proportional change in *P*, which simplifies the bank's forecasting. Indeed, when the economy produces at full capacity, then *Q* may be constant; but in general, a constant *V* is harder to justify. Marshall (1923) suggested that velocity may be slow to change because habit determines the share of income that people spend.

In truth, velocity is often volatile in the short run. For the velocity of the currency in Kazakhstan, the tenge, the ratio of the standard deviation to the mean varied from .044 in 2005 to .213 in 2009 (Table 1).[4] The ratio was more than twice as high in the period 2009-2011 (.197) -- which contained an economic slowdown and a 25% devaluation of the tenge -- as in the period 2000-2008 (.076).

Table 1

M1 velocity statistics for the tenge

Year	Standard Deviation	Velocity mean	Ratio
2000	0.267	3.985	0.067
2001	0.408	3.993	0.102
2002	0.359	4.134	0.087
2003	0.227	3.290	0.069
2004	0.212	2.822	0.075
2005	0.113	2.585	0.044
2006	0.139	2.422	0.057
2007	0.270	2.112	0.128
2008	0.129	2.321	0.056
2009	0.385	1.805	0.213
2010	0.362	1.890	0.192
2011	0.356	1.911	0.186

Notes: Column 2 gives the standard deviation of velocity; Column 3, the mean of velocity, calculated as the annual average of quarterly estimates; and Column 4, the ratio of the standard deviation to the mean. Appendix B lists data used in the computations. Source of raw data: The National Bank of Kazakhstan

Modeling income velocity is often difficult. For example, its link to lagged money volatility is not always clear. Friedman (1984) argued, in effect, that velocity would fall when economic uncertainty increased, since people would hold money as a precaution.[5]

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[4] Research into the tenge supply often uses *M2* or *M3*. But *M1* is more typical than these measures are of research into the variance of velocity.

[5] For broad perspectives, see Friedman (1970, pp. 227-9) -- and Mascaro & Meltzer (1983), which "develops a general equilibrium model in which variability or risk affects the choice of portfolios" (p. 488).

Money volatility would relate indirectly to income velocity.[6] Hall & Noble (1987) tested for Granger causality in United States data and concluded that the log of *M1* velocity was "caused" partly by its own lags and by lags of the volatility of money growth. Other studies indicated that these results might vary with the period studied, since the monetary environment evolves over time due to changes in such factors as regulation and inflation (Brocato & Smith, 1989; Mehra, 1987 and 1989). The results in Mehra (1989) were also sensitive to specification of the equation – *e.g.*, in levels or in first differences. In addition, Granger-causality estimates often depend on the lengths of the lags specified, concluded Thornton & Batten (1985). Thornton (1995) turned up evidence supporting Friedman's hypothesis for three of nine industrial countries studied, but only in certain time periods. Thornton concluded that "the Friedman hypothesis would appear to have little general applicability" (p.290).

II. Analysis

For a way to estimate velocity that is not *ad hoc*, begin with the quantity equation of exchange:[7]

$$V = \frac{PQ}{M}. \tag{II.1}$$

At times, we may have uncertain estimates for the three right-hand variables in Equation II.1. For example, we may lack reliable monthly data for these variables (in particular, for *Q*) when estimating monthly velocity. Or the analyst may base her prediction of velocity on assumed values of the independent variables – assumptions that may not come true. In either event, Equation II.1 may estimate *V* imprecisely. It would be useful to have an estimate of *V*'s volatility.

Consider *P*, *Q* and *M* as random variables. Then a Taylor series and a well-known property of variance (Larsen & Marx, 2006, p.238; Appendix A below) give a first-order approximation of the variance of V:

[6] From this perspective, the increased volatility of the tenge in the period of 2009-11 may have reflected uncertainty about the optimal amount of money to hold following the global financial crisis of 2008.

[7] The velocity of money increases in nominal income (*PQ*), since people will spend a given money supply faster when they can afford more purchases; and velocity falls with an increase in money, since there are more tenge now to finance aggregate purchases of a given size.

$$\mathrm{var}(V) \approx \frac{Q^2}{M^2}\mathrm{var}(P) + \frac{P^2}{M^2}\mathrm{var}(Q) + \frac{P^2Q^2}{M^4}\mathrm{var}(M). + 2\frac{QP}{M^2}\mathrm{cov}(P,Q)$$
$$-2\frac{PQ^2}{M^3}\mathrm{cov}(P,M) - 2\frac{P^2Q}{M^3}\mathrm{cov}(M,Q).$$

(II.2)

In some short-run cases, *P*, *Q* and *M* may be independent of one another -- each subject to random factors, such as measurement error, which need not affect the other two variables. The covariances then are zero, and the last three terms in Equation II.2 will disappear.

Equation II.2 applies what we will call the Larsen-Marx algorithm. (The two mathematicians had developed it to help interpret dental X-rays (Larsen & Marx, 2006).) Given the long-run variances of *P, Q* and *M,* the equation can forecast the variance of velocity in a scenario specifying the former three variables.

For example, suppose that the National Bank of Kazakhstan considers an increase in the money supply equal to the forecasted annual rate of growth in *Q*, 7.5%. The Bank assumes that *P* would not change. In addition to the levels of *P*, *Q,* and *M1*, one might assume for these variables their average annual variances for the period 2000-2011 (Table 2). By Equation II.1, the predicted value of M1 velocity is 7.7. By Equation II.2, the predicted standard deviation of velocity is 1.39, or .18 of the mean. This ratio is 70% higher than the average for 2000-2011, so the Bank may wish to act on its scenario forecast with caution. If velocity follows a normal distribution, then the 95% confidence interval that is implied for it is about (4.9, 10.5).

Table 2

Forecasting example

Variables	Level	Variance	Standard deviation
Price level	228.7	1,647.0	40.6
Output	128,036.7	979,119,830.8	31,290.9
M1 money	3,819,483.9	1,301,845,692,972.7	1,140,985.0
Velocity	7.7	1.9	1.4

The covariances among *M, P* and *Q* play a critical role. Keynesian monetary policy assumes that the short-run correlation between prices and money is low enough to permit an infusion of money to affect real GDP rather than the price level. But in Kazakhstan, using annual data for 2000 through 2011, the simple correlations of the Consumer Price Index, the money supply (*M0* or *M1*), and output in Kazakhstan all exceed .97. For monthly data, the correlation between the CPI and *M1* also exceeds .97. The three variables may each relate to a time trend, or they may be cointegrated; but the point is that their covariances cannot be ignored.

III. Conclusions

The Larsen-Marx algorithm may be most useful when applied to short-run monetary relationships, since these may be harder to estimate than long-run ones. Yilmaz, Oskenbayev & Kanat (2010) find that a model of M2 demand in Kazakhstan, based on an output proxy, the interest rate, and on foreign exchange rates, has the expected coefficient signs in the long run but not in the short. The algorithm may indicate how severe the misspecification in short-run estimates may be.

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V. Appendix A

V.1 The Case of Independent Random Variables

In a Taylor series, a first-order expansion approximates a function *g* around some point ($\mu_1, \mu_2, \ldots, \mu_n$):

$$g(W_1, W_2, ..., W_n) \approx g(\mu_1, \mu_2, ..., \mu_n) + \sum_{i=1}^{n} \frac{\partial g}{\partial W_i}(W_i - \mu_i).$$

where the derivatives are evaluated at the point ($\mu_1, \mu_2, \ldots, \mu_n$).
For velocity, such a Taylor series would be

$$V(M, P, Q) \approx V(\mu_m, \mu_p, \mu_q) + \frac{\partial V}{\partial M}(M - \mu_m) + \frac{\partial V}{\partial P}(P - \mu_p) + \frac{\partial V}{\partial Q}(Q - \mu_q), \tag{V.1.1}$$

where μ_m, μ_p and μ_q are arbitrary constants. Note that $V(\mu_m, \mu_p, \mu_q)$ is also a constant.
A well-known result concerning the variance of a linear sum of independent random variables W_i with finite means is that

$$Var(\sum_{i=1}^{n} a_i W_i) = \sum_{i=1}^{n} a_i^2 Var(W_i) \tag{V.1.2}$$

where a_i is a constant. Applying Equation V.1.2 to Equation V.1.1 gives us

$$Var(V) \approx Var\left(\frac{\partial V}{\partial M}(M-\mu_m) + \frac{\partial V}{\partial P}(P-\mu_p) + \frac{\partial V}{\partial Q}(Q-\mu_q) \right)$$
$$= \left(\frac{\partial V}{\partial M} \right)^2 Var(M-\mu_m) + \left(\frac{\partial V}{\partial P} \right)^2 Var(P-\mu_p) + \left(\frac{\partial V}{\partial Q} \right)^2 Var(Q-\mu_q)$$
$$= \left(\frac{\partial V}{\partial M} \right)^2 Var(M) + \left(\frac{\partial V}{\partial P} \right)^2 Var(P) + \left(\frac{\partial V}{\partial Q} \right)^2 Var(Q),$$

(V.1.3)

where the last line uses Equation V.1.2 again:

$$Var(X-\mu) = Var(X) + Var(\mu) = Var(X),$$

since μ is a constant. Note that the -1 coefficient of the variance of μ is squared.

V.2 The General Case

When covariances are not zero, then the general version of Equation V.1.2 is

$$Var(\sum_{i=1}^{n} a_i W_i) = \sum_{i=1}^{n} a_i^2 Var(W_i) + 2\sum_{j<k} a_j a_k Cov(W_j, W_k),$$

(V.2.1)

where we have used the result

$$Cov(a_i W_i, a_j W_j) = E(a_i W_i a_j W_j) - E(a_i W_i)E(a_j W_j) = a_i a_j E(W_i W_j) - a_i E(W_i) a_j E(W_j)$$
$$= a_i a_j \left[E(W_i W_j) - E(W_i)E(W_j) \right] = a_i a_j Cov(W_i, W_j).$$

Equation II.2 specifies Equation V.2.1.

VI. Appendix B

Table 3

Dataset for simulation

Time	P	Q	M0	M1
2000	105.1	24,697.9	95,844.1	162,832.9
2001	114.0	28,477.7	112,141.3	202,735.2
2002	121.4	31,042.1	135,651.4	228,326.6
2003	131.3	35,045.2	192,612.3	352,616.1
2004	140.8	41,639.6	290,824.4	524,762.3
2005	150.0	50,504.2	388,599.3	732,620.9
2006	159.8	63,707.8	506,327.0	1,052,436.7
2007	174.0	73,659.0	720,892.9	1,520,003.7
2008	188.8	84,930.4	763,243.5	1,722,722.4
2009	200.2	84,678.0	789,508.7	2,340,956.5
2010	213.5	101,818.4	1,015,448.1	2,854,529.7
2011	228.7	119,103.9	1,196,024.8	3,553,008.3

P is the Consumer Price Index. It is averaged from monthly data. *Q* is real output. It equals nominal gross domestic product divided by the CPI. Each annual estimate of *Q* sums the four quarterly estimates. The money supplies *M0* and *M1* are measured in millions of tenge. They are averaged from monthly data. Source of raw data: The National Bank of Kazakhstan