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MULTINOMIAL PROBIT MODELS OF MILITARY ENLISTMENTS: A COMPARISON OF ALTERNATIVE SOLUTION ALGORITHMS

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I. INTRODUCTION

Qualitative response models have gained wide acceptance in recent years. Amemiya (1981) and Maddala (1983) surveyed the rapidly growing literature in a number of areas while Daganzo (1979) discussed the particular use of multinomial probit models in economics. The increasing popularity of these models has led to the development of sophisticated algorithms to solve them. The relative efficiency and accuracy of various algorithms is a subject of considerable importance to the Army, because the results derived from probit models may be used increasingly by Army decision-makers.

II. SOLUTIONS TO PROBIT MODELS

The effects of economic variables on Army enlistments have been studied using traditional econometric techniques by a number of researchers (see, for example, Ash et. al. 1983, and Dale and Gilroy 1983 a, b, c). More recently the Fifth Quadrennial Review of Military Compensation (QRMC) has begun its Congressional mandate to examine a number of military compensation issues. These issues include special pays, the retirement system, the value of military discount rates (see Cylke, et. al. 1983 and Black 1983), and recruiting and retention issues. Important considerations in the latter areas include the effects of unemployment and relative wages on enlistment and reenlistment decisions.

The present research expands upon the seminal work of Daula, et. al. (1982) and Baldwin, et. al. (1982). They have developed sophisticated multinomial probit models for enlistment and reenlistment decisions, and concluded that wage and unemployment effects are much larger than had been previously believed. The importance of these results has drawn attention to the comparative accuracy and reliability of various solution algorithms.

The present work used cross-sectional data from the National Longitudinal Surveys (NLS) to measure the effects of wages, unemployment and other variables on the enlistment decision. The NLS sample used consisted of 4807 males, age 18 to 25, who were interviewed about their income and other demographic variables for calendar year 1981. The results were weighted to reflect the population of the entire country (see Center for Human Resource Research 1982).

The decision to enlist in the military depends upon a person's employment opportunities in the civilian sector, his place of residence, race, intelligence (since the military has enlistment standards), and the ratio of potential military to civilian earnings. Civilian earnings, however, are only observed for persons who did not enlist. Therefore, to avoid sample selection bias, we estimated a separate civilian earnings equation (Table 1). We then used the estimated prospective civilian earnings for each person in the sample in an equation with a person's military/civilian status as the limited dependent variable. The resulting equation was estimated using multinomial probit techniques (Table 2).

Four popular ways to solve probit models are the Newton algorithm, the Davidon-Fletcher-Powell (DFP) method, the Method of Moments (MM), and the Berndt, Hall, Hall, and Hausman (BHHH 1974) algorithm. Greene (1982) states in his software documentation that "BHHH and Newton ... are likely to be very expensive but not very accurate" (pg. 17-9). It is, therefore, important to compare these techniques, since future decisions may be based on the outcome. Daula, et. al., and Baldwin, et. al, for example, used a form of the BHHH algorithm.

An IBM System 370 Model 3081 computer was used throughout this research. The expense and relative "inaccuracy" of cross-sectional studies may surprise econometricians who normally deal with time-series data, and who are accustomed to estimating equations that may cost 25¢ each to run on a computer and that typically produce $\bar{R}^2$ values above .8 or .9. As Table 1 shows, $\bar{R}^2$ values for cross-sectional studies are usually much smaller than for time-series data, and the cost of handling large data sets is much higher.

III. RESULTS

Table 1 shows that the results from the civilian earnings equation are in accord with a priori expectation. Earnings are likely to be higher for those who score well on an intelligence test, and are likely to be lower for nonwhites, for residents of the South, and for men who live in areas of high unemployment. The estimated civilian earnings from this equation were used in the second stage probit estimations of Table 2.

Greene (1982) pointed out that Newton's method of solution or the Method of Moments (MM) should normally be the algorithm of choice for limited dependent models, because they are generally the least costly. The Davidon-Fletcher-Powell (DFP) method may require considerably fewer calculations but more frequent use of the data. Finally, the Berndt, Hall, Hall and Hausman (BHHH) algorithm may be theoretically as good as the others, but it can be very unstable and extremely costly to use.

Results are shown in Table 2. Each equation was estimated with and without a cyclical unemployment term. This approach was inspired by the results obtained by Baldwin et. al. (1982), who reported that adding such a term lowered the level elasticity of unemployment from 2.30 to 0.93. Since this result implies that military enlistment rates could still fall sharply in an improving economy, but not nearly as sharply as an elasticity of 2.30 would indicate, we tried a similar estimation procedure in the present work.

We obtained positive coefficients for residents of the South, people who did well on intelligence tests, and nonwhites. We obtained insignificant coefficients on the pay and unemployment variables. We did not calculate unemployment elasticities for our estimates, since the coefficients were not only insignificant, but negative in sign. In addition, we did not obtain Baldwin, et. al.'s large change in the unemployment coefficient when we added a cyclical unemployment term and used the BHHH algorithm. Nevertheless we obtained some interesting results.

All of the algorithms were internally stable. Adding a cyclical unemployment term changed the coefficients only slightly. All produced perverse signs on the ratio of military to civilian pay, but these coefficients were all statistically insignificant.

Three of the algorithms also produced almost identical results. Newton, DFP, and the Method of Moments produced results that agreed with each other, but differed dramatically from the BHHH method. The BHHH method converged on a completely different set of estimates, although the signs on all terms except the constant agreed with the other methods.

Computer costs are another striking difference. The Method of Moments costs as little as $6.48 to estimate an equation. This is much higher than comparable time-series estimates, but still only about one-tenth of the DFP costs. These results are summarized in Table 3. Clearly the Method of Moments is the most accurate and cost-effective method of solving the probit model. Canned software packages such as LIMDEP, which was used here, are becoming increasingly available that allow the user to choose any algorithm he wants, so these results should be kept in mind by any researcher with limited research time and a finite computer budget.

IV. CONCLUSIONS

This paper has compared several algorithms for solving a multinomial probit model for military enlistments. The objective was to compare Newton's method, the Davidon-Fletcher-Powell (DFP) algorithm, the Method of Moments (MM), and the Berndt, Hall, Hall, and Hausman (BHHH) algorithm. All the algorithms were internally stable, with only a small change in coefficients when a cyclical unemployment term was added to the equations. The BHHH algorithm, however, did converge to a somewhat different set of solutions than the others.

Table 3 shows that the Method of Moments was by far the least expensive method of solution, in terms of computer time and run cost. The BHHH algorithm was about twice as expensive, and its inability to converge on the same solution as the other methods leaves little reason to recommend using it. Newton's method was much more expensive than the Method of Moments, while the Davidon-Fletcher-Powell (DFP) method was by far the slowest and most costly to use.

In conclusion, econometricians who undertake analyses using large data bases for cross-sectional studies will gain very little by using some of the recently developed algorithms such as BHHH. These new methods represent elegant and impressive theoretical achievements, but in practice they do not justify their enormous cost.

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TABLE 1

CIVILIAN EARNINGS EQUATION FOR TWO-STAGE MULTINOMIAL PROBIT MILITARY ENLISTMENT MODELS

Variable	Coefficient (Standard Error)
Constant	-492.01 (102.28)
Age	33.60 (5.19)
Intelligence	8.61 (31.32)
Nonwhite	-5.34 (37.04)
South	-19.38 (27.74)
Local Unemployment Level	-0.91 (4.14)
F - Ratio for Regression	8.93
$\bar{R}^2$	.09
Clock Time of Estimation (Seconds)	70
CPU Time (Seconds)	8.14
Approximate Cost	$5.16

Dependent Variable: 1981 civilian earnings from all sources.

TABLE 2

PROBIT MODEL FOR MILITARY ENLISTMENTS
SOLUTION WITH NEWTON'S ALGORITHM

Variable	Coefficient (Standard Error) Without Cyclical Unemployment	With Cyclical Unemployment
Constant	909.46 (796.75)	893.48 (815.66)
Relative Pay	-.033 (.089)	-.033 (.089)
Intelligence	1135.7 (514.6)	1137.1 (514.9)
South	881.9 (461.6)	882.7 (461.7)
Nonwhite	602.3 (615.5)	602.8 (615.6)
Local Unemployment Level	-19.59 (68.26)	-17.54 (71.84)
Cyclical Unemployment		-62.53 (681.7)
$\bar{R}^2$	.01	.01
Clock Time of Estimation (Seconds)	4431	2561
CPU Time (Seconds)	56.31	48.37
Approximate Cost	$38.81	$34.50

Dependent Variable: In Military = 1; Civilian = 0.

TABLE 3

SUMMARY COMPARISON OF SOLUTION ALGORITHMS

Algorithm	Average Clock Time (Seconds)	Average CPU Time (Seconds)	Average Run Cost
Newton	3496	52.34	$36.66
DFP	8295	84.81	$60.23
MM	452	9.12	$6.48
BHHH	1343	16.23	$11.56

DFP = Davidon-Fletcher-Powell.

MM = Method of Moments.

BHHH = Berndt, Hall, Hall, and Hausman.