Rising RD Intensity and Economic Growth

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RISING R&D INTENSITY AND ECONOMIC GROWTH

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Abstract

Over the past decades, private R&D spending in the US and other developed countries has been growing faster than GDP. At the same time, the growth rates of per capita and aggregate output have been rather stable, possibly declining slightly.

This paper proposes a growth model that can account for the observed phenomenon by explicitly describing competition among technological leaders and followers in individual markets in a way that is consistent with existing studies on firms' motivation to invest in R&D. The model shows the possibility that the unsustainable trend of rising R&D intensity persists for a very long time.

JEL-Classification: O3, O4, L1

Keywords: Endogenous Growth, Research & Development, Market Structure

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1 INTRODUCTION

It has been observed that for at least the past sixty years, US growth rates have been remarkably stable while R&D efforts seem to have been growing much faster than GDP. Figure 1 shows the growth rate of US per capita GDP and the ratio of non-federal R&D expenditure to GDP, henceforth referred to as R&D intensity, since the 1950s. It is evident that while the former has no apparent trend, the latter has increased substantially from 0.63% in 1953 to 1.95% in 2007, i.e. by more than a factor of three. Different authors have looked at other measures of R&D intensity, most frequently numbers of R&D scientists and engineers as a share of the labour force[1], but the same basic picture emerges. Moreover, while data for other developed countries are not available as far back as for the US, the recent trends observed elsewhere are similar. While Figure 2 replicates the well-known trends of R&D employment in the G5 documented by Jones (1995a) , Figure 3 shows that the since the early 1980s, R&D intensity in the G7 countries, which accounted for over 80% of worldwide R&D in 2004, [2] has also been growing on average.

The implications of rising R&D intensity can only be fully appreciated if its causes are known and understood. In particular, as the current trend of fast growing R&D efforts must eventually come to an end, it would be important to be able to make predictions about when the current trend will be reversed, how this will happen, what the new growth path will look like and whether there will be a slow transition or an abrupt change.

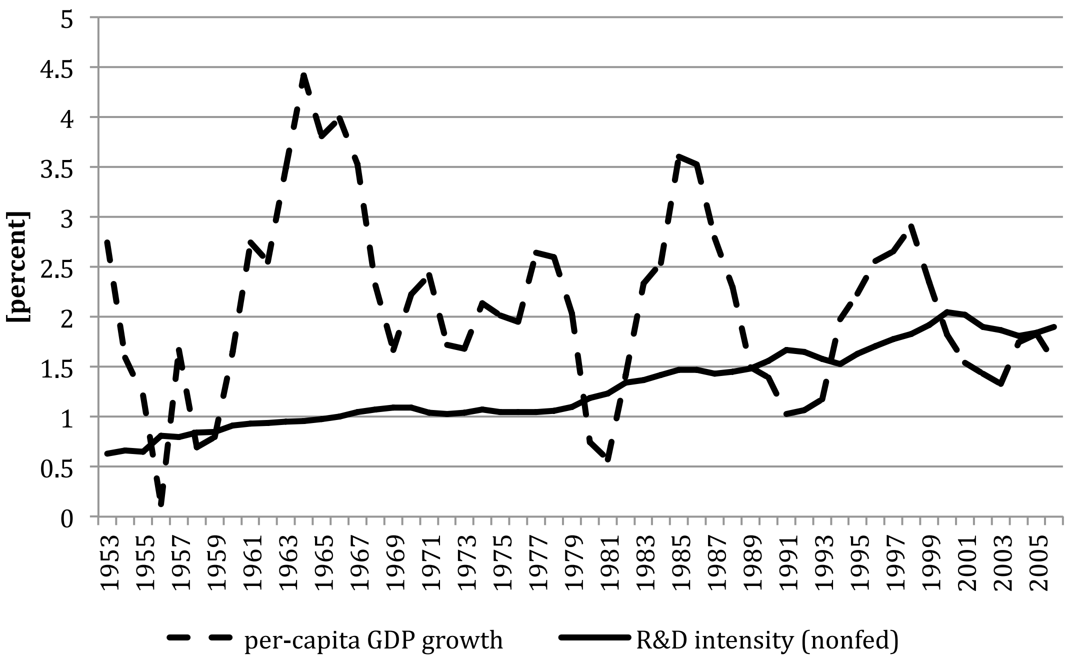
This paper shows how a standard model of semi-endogenous growth due to R&D investment can be modified to account for these empirical patterns. The central idea is that incumbent firms protect their competitive position by innovating at a rate that is high enough to prevent market entry by competitors. R&D investment is therefore determined by the

[1] See for example Jones (1995b) or Kortum (1997) .

[2] National Science Board (2008) , Chapter 4, p 4-35.

competitive conditions in individual markets, in particular the ease of entry, rather than expected future profits. This means that, in contrast to many of the workhorse models in growth theory, there is no a priori reason why R&D expenditure should grow at the same rate as GDP, at least as long as its share in firms' cost is low. The main contribution of this paper is to show a simple way in which rising R&D intensity can be explained as an endogenous model feature, and how the observation of almost constant growth rates in excess of what would be possible on the balanced growth path (BGP) can persist for such a long time. Our model emphasizes the difference between the growth rate of innovation and imitation costs as the driving factor behind growing R&D intensity. This difference can be calibrated using existing data and used to project the transition of the current high-growth economy to a low-growth balanced growth path.

Figure 1: US per-capita growth and Intensity of non-federally funded R&D



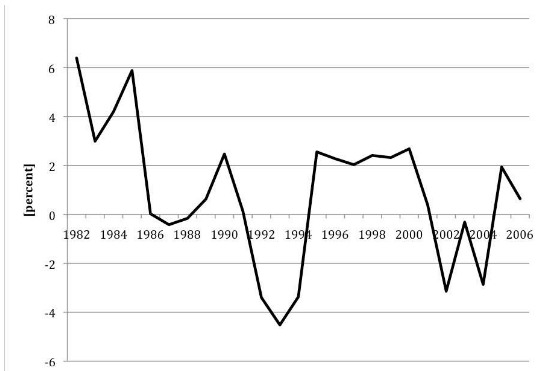
Notes: per-capita GDP growth: five-year averages; *data source*: Bureau of Economic Analysis
R&D intensity: ratio of non-federally funded R&D expenditure to GDP; *data source*: National Science Foundation.

The paper is organized as follows. The following section briefly reviews the previous literature on the subject. Section 3 starts out by arguing that existing endogenous growth models are not well suited to explaining rising R&D intensities and discusses alternative motives to innovate that have not received much attention in the theoretical literature on economic growth. Section 4 describes a conceptually simple modification of existing standard growth models that can not only make them consistent with the data, but also bring them better in line with existing micro evidence on the effectiveness of the patent system. The model is calibrated and simulated in section 5, and its predictions about the future of economic growth are discussed, including the inevitable transition to a different growth path. Some brief remarks on welfare and policy implications are made in section 6. The last discusses possible extensions of the model and concludes.

Figure 2: R&D employment as share of labour force in the G5

Notes: Scientists and engineers in R&D as share of civilian labour force. *Source*: Jones (2002) . The employment data is available online at http://www.stanford.edu/~chadj/upslow1.asc; labour force data from OECD and BLS.

Figure 3: Growth rate of R&D intensity in the G7



Notes: Average change in R&D intensity over previous year, weighted by GDP. *Data sources*: France, Germany, Italy, Japan, UK, US: Eurostat (R&D in business and enterprise sector), Canada: OECD (Manufacturing sector only); growth rate excludes Canada before 1988, Germany before 1992. Average growth for time span shown: 0.9%.

2 PREVIOUS LITERATURE

Several papers have studied specific implications of rising R&D intensity. Jones (1995a) has argued that rising R&D efforts and constant growth rates are inconsistent with many of the models of endogenous growth developed in the late 1980s and early 1990s, as these models exhibit scale effects in R&D, i.e. rising R&D leads to rising growth rates. He advocates models of "semi-endogenous" growth, which avoid these problems.

Segerstrom (1998) and Kortum (1997) investigate how the apparently inconsistent observations of rising R&D employment and stable patent creation rates can be reconciled. Both articles conclude that it must be becoming increasingly hard to innovate as technological knowledge increases. However, neither paper addresses the question why R&D employment is rising faster than the labour force. Solving his model only for a growth path along which R&D intensity is constant, Segerstrom (1998) conjectures that the disequilibrium dynamics that lead to a rising R&D intensity could be a consequence of increasing subsidies or growing market size.

Jones (2002) explores how the fact that R&D intensity is increasing affects our assessment of the scope for sustained growth in the long term. Since this variable cannot continue to rise at the current rate forever – R&D expenditure is bounded above by GDP – at some point in the future the growth of R&D intensity must slow down and eventually stop. In models where semi-endogenous growth results from R&D efforts, this reduction of R&D growth leads to a drop of the overall growth rate. Only after R&D intensity and other variables that currently grow at an unsustainably high rate have stabilized can the economy be on a balanced growth path.

In a growth accounting exercise, the author attributes more than 50% of the post-war US per-capita growth to the effects of rising R&D intensity and another about 30% to a different transitory effect, the increase in educational attainment. Only the remaining 20%, corresponding to an annual per-capita growth rate of 0.4%, can persist in the long term when the economy is on a balanced growth path.[3]

[3] Ibid., p. 228, table 2, $\gamma = 0.333$ case.

To describe the present situation in which stocks and flows grow at constant but possibly unsustainable rates resulting in a constant rate of output growth, Jones (2002) introduces the concept of a constant growth path. If the constant growth path cannot be followed forever, at some point the growth rates of certain variables will have to adjust, and the economy may eventually converge to a balanced growth path.

While the apparent tension between a constant growth rate and rising R&D intensity has been investigated in a number of papers, typically with the intention to make a specific point as in the articles mentioned above or to falsify existing concepts as in Jones (1995b) , relatively little effort has been made to understand the causes of growing R&D intensity and to explain why the US economy has embarked on a growth path that appears to be driven by transition dynamics while at the same time showing no sign of convergence towards what could be considered a more sustainable balanced growth path.[4] The main objective of this paper is to present a model that explains not just how a transition from a high-growth to a low-growth scenario takes place, but also explains why the US economy has experienced rising R&D intensities.

One paper that has addressed this issue Pintea and Thompson (2007) . These authors construct a model that can explain how rising educational attainment and rising R&D intensity can coincide with falling growth rates. They argue that a one-time increase of the

[4] One possible explanation is the increasing economic integration of the US and other OECD countries with the rest of the world, resulting in rapid market growth. While probably being a relevant aspect of the explanation, this is unlikely to be quantitatively the most important one. To see why, consider the following simple calculation. According to IMF data, the G7 countries accounted for about 55% of world GDP in 2007. If in the 1950s the G7 countries had made up the whole world market, and today they were perfectly integrated with the remaining 45% of the world, and if all the world's R&D were performed in the G7 countries, this would explain an increase of R&D intensity by 82%. This is far less than the actual rise of US R&D intensity by 210% from 1953 to 2007.

difficulty to innovate, following a drop in the effectiveness of learning-by-doing, has triggered innovative monopolists to attempt to raise R&D employment. The resulting increase in demand for skilled labour resulted in higher skill premium and – as a supply response – higher educational attainment. At the same time, growth rates declined as a result of the adverse change in the environment affects an increasing number of firms. While producing dynamics that resemble US post-war developments in R&D intensity and productivity growth, these outcomes critically rely on a distinct specification of the knowledge production function, which triggers profit-maximizing firms to *increase* R&D spending in response to a drop in its effectiveness. In contrast to Jones (2002) , Pintea and Thompson do predict a reduction of the growth rate once the rise in R&D intensity comes to an end; according to their model, the growth rate has already been dropping, possibly overshooting, as the US economy is making the transition to a new balanced growth path, characterized by a somewhat lower per-capita growth rate.

3 MOTIVES TO INVEST IN R&D

The models of R&D and growth most frequently employed in the literature are descendants of two distinct but related approaches, Romer's (1990) model of an expanding variety of goods and the framework introduced by Grossman and Helpman (1991) and Aghion and Howitt (1992) known as the quality ladder or Schumpeterian model. While differing in several important ways, both frameworks rely on an R&D sector as the source of innovation and growth. They also share the central assumptions that (1) new inventions are patented, giving inventors the right to become the exclusive supplier of the new product (possibly for a limited time), which results in monopoly rents, and that (2) there is free entry into the R&D sector, which implies that in equilibrium, engaging in R&D does not yield a positive expected economic profit. In other words, R&D expenditure at any point in time is determined by the expected discounted value of future monopoly rents resulting from innovation. These models predict that if the production sector is growing at a constant rate, monopoly rents grow at the same rate. Thus, R&D expenditure should also grow at the same rate as the production sector. Note that this is not only true along a balanced growth path, but whenever the growth rate of output is stable. It is therefore hard to see how this type of model could be consistent with a constant growth rate and at the same time a low but rising R&D intensity.[5, 6]

In this framework, the driving force behind any R&D investment is the prospect of earning monopoly rents from selling the invention in the future, a prospect which is guaranteed by a system of patent protection. Innovation is driven by a competitive R&D sector comprised of entrepreneurs who want to become a monopolist, either by creating a new market or capturing an existing one.

It has been noted, however, that market leaders are often innovative firms.[7] One potential motive for firms at the technological frontier to engage in R&D could be that it is optimal to make this kind of investment to benefit from lower costs or larger sales in the future. In this scenario, R&D expenditures are still closely linked to the appropriable returns from R&D.

Both in the case of competitive R&D and in the case of innovation by a market leader, the protection of monopoly rents by a legal system that effectively enforces patents is crucial. Yet, while this assumption is convenient and theoretically appealing, it may not be entirely realistic. In their study of the consequences of patents for imitation costs for a small sample of American firms, Mansfield et al. (1981) find that the effectiveness of the patent system in

[5] The same basic mechanism is at work in more complex models of growth and R&D, such as the model of increasing quality and varieties presented in Young (1998) .

[6] There is a possibility that increasing R&D investments are being made in anticipation of future returns that have not yet materialized. However, given the length of the period for which data is available this explanation seems highly unlikely.

[7] See, for example, Segerstrom (2007) .

protecting innovators is less than impressive. While the costs of imitating a product may be substantial, they are only increased by 11% if the product was patented for the median innovation. Moreover, 60% of patented successful innovations were imitated within four years.

Table 1: Effectiveness of alternative means of protecting the comparative advantage

		Levin et al. (1987) [a]		Cohen et al. (2000) [b]	
		Processes	Products	Processes	Products
Patents	to prevent duplication	3.52 (0.06)	4.33 (0.07)	23.30 (0.83)	34.83 (0.94)
	to secure royalty income	3.31 (0.06)	3.75 (0.07)		
Other legal				15.39 (0.63)	20.71 (0.73)
Secrecy		4.31 (0.07)	3.57 (0.06)	50.59 (1.03)	51.00 (0.96)
Lead time		5.11 (0.05)	5.41 (0.05)	38.43 (0.96)	52.75 (0.92)
Moving quickly down the learning curve		5.02 (0.05)	5.09 (0.05)		
Sales or service efforts		4.55 (0.07)	5.59 (0.05)	30.73 (0.88)	42.74 (0.91)
Complementary mfg				43.00 (0.95)	45.61 (0.88)

[a] Range: 1 (not effective at all) to 7 (very effective); average of 650 responses reported; standard errors in parentheses
[b] Range: effectiveness in percent; average of 1118 (products) and 1087 (processes) responses reported; standard errors in parentheses
Sources: Levin et al. (1987) , table 1, and Cohen et al. (2000) , tables 1 and 2.

If patents are insufficient to protect the return to innovation, firms must look for other means of ensuring that they can recover their R&D investment. In a survey of more than 600 firms performing R&D, Levin et al. (1987) asked what means of protecting the competitive advantage resulting from innovation were most effective. The results of their study, as well as a more recent survey by Cohen et al. (2000) are reported in Table 1. In both studies, patents and other legal mechanism get consistently lower ratings than alternative means. Lead time is regarded as very effective, at least for product innovations, in both studies. Secrecy gets much higher ratings in the more recent survey, possibly reflecting a change in the importance of this instrument since the 1980s.

The relative effectiveness of lead time, as well as to a certain extent the possible importance of secrecy and the need to quickly move down the leaning curve, seem to suggest that ongoing innovation at a high enough rate are quite effective in ensuring an innovator's leading position in the market. What is interesting about this conjecture is that if the attempt to stay ahead of the competition is an important motive for performing R&D, the scale of firms' R&D investment is not only determined by the return to R&D and thus future market size, but also by technological parameters such as the rate and cost of imitation, as well as the market structure. It may therefore help to explain why firms have apparently chosen to increase their R&D expenditure faster than their revenues could be expected to grow as a result.

The following section presents a model that includes these aspects and shows how the appropriability of the returns from innovations may change as R&D becomes an increasingly important part of GDP and also firms' expenditures.

4 A MODEL

4.1 Idea

Our model is built around a production sector in which a fixed number of intermediate goods are used to produce final output. The quality of these intermediate goods can be improved through R&D, in a similar fashion as in popular Schumpeterian models of quality ladders. While intermediate goods are still supplied by only one firm at a time, we assume that there is no effective patent protection, so competitors can enter the market at any time if they are willing to invest in imitating the current technology in order to be able to offer a competitive product.

Our key assumption is that the cost getting to the technological frontier increases with the rate of innovation that the incumbent chooses. In other words, the current supplier of an intermediate good can choose to be more innovative in order to make it harder for other firms to enter the market and compete with him.[8]

Thus, the rate of innovation and consequently the growth rate is determined by the ease of entry into the market for intermediate goods. If this growth rate is higher than what would be possible on a balanced growth path, R&D intensity must increase over time to sustain it. Such a situation of excessive growth can persist as long as R&D expenditures are small. Eventually, as R&D spending gets large enough compared to firms' revenues, profits drop, reducing the incentive for competitors to enter the market. Under this reduced competitive pressure, incumbents can eventually lower the rate of innovation, starting a transition towards the economy's balanced growth path.

In the following subsection, we first develop the model for the case of an uncontested monopoly, before introducing competition in 4.3.[9]

[8] This notion is related to the concept of pre-emptive patenting due to Gilbert and Newbery (1982) . In contrast to their analysis, which is mostly concerned with patenting possible substitutes to prevent market entry by competitors, we focus on the possibility to deter entry by innovating at a higher rate to maintain a technological advantage over the competition.

[9] In what follows, we will stay as close to the generic quality ladder model (without scale effects) as possible, using specifications and notation familiar from the growth literature. While attempting to keep the model simple, in introducing competition from potential entrants, we will derive the cost of entry from a plausible generalization of a standard R&D production function rather than using an ad-hoc specification. This is mainly done to show that our model results from a natural modification of the standard version, and also to convince the reader that the new ingredients added here can be derived from broadly accepted assumptions. The resulting mathematical complexity is, to the extent possible, hidden in the appendix.

4.2 The monopoly case

Our model is based on a quality ladder framework similar to that of Aghion and Howitt (1992) .

Households maximize the present value U of the utility they derive from consumption,

$$U = \int_0^\infty e^{-\rho t} u(c) dt , \tag{1}$$

where ρ is the discount rate and u is the utility flow as a function of per-capita consumption c. Assuming log utility, $u(c) = \ln c$, and free access to capital markets, the solution to the households' optimization problem relates the interest rate r in the economy to the growth rate of consumption in the usual manner,

$$r = \dot{c}/c + \rho . \tag{2}$$

Output Y is produced according to the production function

$$Y = A L_Y^{1-\alpha} \sum_{j=1}^{J} \left(q_j X_j \right)^{\alpha} , \tag{3}$$

where L_Y is the quantity of labour employed in final good production. A large fixed number J of different intermediate goods are used to produce output. The productivity of intermediate good $j = 1 \ldots J$ depends on the quantity used X_j and its quality q_j. As usual, $1-\alpha$ stands for the labour share and A is a positive constant.

We assume that each intermediate good is supplied by a single firm that possesses the technology to convert any quantity of output into the same quantity of the intermediate good at no further cost. The solution is standard. At a price of p_j, demand for good j in the competitive final goods sector is $X_j = \left(\alpha A q_j^{\alpha} p_j^{-1} \right)^{1/1-\alpha} L_Y$. A monopolistic profit-maximizing supplier thus chooses to price this good at $1/\alpha$. In this case, he sells a total of

$$X_j = \left(\alpha^2 A q_j^{\alpha} \right)^{1/1-\alpha} L_Y \tag{4}$$

units and enjoys a profit flow of

$$\pi(L_Y, q_j) = \bar{\pi} L_Y q_j^{\alpha/1-\alpha}, \tag{5}$$

where $\bar{\pi} = \frac{1-\alpha}{\alpha}\left(\alpha^2 A\right)^{1/1-\alpha}$ is a positive constant.

We will start from the assumption that each intermediate good supplier enjoys an uncontested monopoly in his market. In such a setting, the monopolist's motivation to engage in R&D results from the fact that a higher product quality q_j results in higher demand and thus larger profit flows, as can be seen from equation (5).

The supplier of intermediate good j can improve the quality of his product by investing in R&D according to the innovation production function

$$\dot{q}_j = \chi L_j^{\lambda} q_j^{\phi}. \tag{6}$$

Here, L_j is the quantity of labour employed in the development quality improvements of good j and $\chi > 0$, $\lambda \in (0,1)$ and $\phi < 1$ are constants. The knowledge production function (6) is virtually the same as the one used in Jones (1995a) . While the difficulty of achieving a given absolute quality improvement may either rise or fall with the current quality q_j, $\phi < 1$ implies that implementing a given relative improvement unambiguously gets harder as q_j rises. Moreover, $\lambda < 1$ makes sure that at any point in time, there are diminishing returns to R&D employment. The combined effect of these two factors, $\frac{1-\phi}{\lambda}$, will be referred to as the burden of knowledge.[10] A higher value of this term implies that to maintain a given positive rate of quality improvement, R&D employment must rise faster.

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[10] See Jones (2009) for micro-evidence on this burden as well as for a model that provides microfoundations for equation (6).

As pointed out by Jones (2005, 1995a) , this specification of the innovation function with $\frac{1-\phi}{\lambda} > 0$ ensures the absence of scale effects, i.e. in the long term the growth rate of the economy must be determined by the growth rate of employment rather than its level.

Firms choose the quality improvement of their product at any point in time to maximize the discounted value of their future profit flows net of R&D expenditure,

$$\max \int_0^\infty e^{-rt} \left(\pi(L_Y, q_j) - wL_j \right) dt , \tag{7}$$

subject to the knowledge production function (6). The evolution of the interest rate r and the wage rate w, which is the same as in the final goods sector, are beyond the firm's control. As firms are owned by households, the stream of net profit income that results from the monopolists' activity accrues with consumers.

In equilibrium, all labour L is employed either in final goods production or R&D,

$$L = L_Y + \sum_{j=1}^{J} L_j . \tag{8}$$

We assume that the labour force grows at the rate n.

The following proposition shows the existence and some stability properties of a balanced growth path. Here and in what follows, a hat ^ always marks the growth rate of a variable.

Proposition 1

(i) The economy in which R&D is performed by monopolists has a symmetric balanced growth path, along which all sectors grow at the same rate $g = n + \frac{\alpha}{1-\alpha}\hat{q} = n + \frac{\alpha}{1-\alpha}\frac{\lambda}{1-\phi}n$ ($\hat{q}_j = \hat{q}_k \equiv \hat{q}$ for all sectors *j, k*) as aggregate output.

(ii) If the R&D disadvantage from having a better technology grows faster with q_j than the profits associated with a better intermediate good, i.e. $1-\phi > \frac{\alpha}{1-\alpha}$, a small industry starting out at an arbitrary technological level converges to the balanced growth path.

(iii) The balanced growth path is stable with respect to deviation of the economy-wide productivity level.

Proof:

See appendix A.1.

□

The idea is simple: For symmetry among industries, the knowledge production technology (6) can be aggregated and implies $\hat{q} = \frac{\lambda}{1-\phi} n$. The growth rate of output is then determined by (3) and (4). It follows from equation (5) that profits grow at the same rate as output. R&D spending grows at the rate $n + \hat{w}$, where the rate of wage growth $\hat{w} = g - n$ is determined by the marginal product of labour $(1-\alpha) Y/L_Y$ in the final goods sector. Thus, the ratio of R&D expenditures to profits remains constant.

As one would expect, the balanced growth rate depends on the rate of population growth and the burden of knowledge. The assumption of a monopolistic market structure does not change the growth properties of the model compared to the benchmark model with competition in R&D.

As argued above, the observation of a rising R&D intensity in the US and other countries since the 1950s is at odds with the type of balanced growth just described. If this simple quality ladder model with monopoly suppliers of intermediate goods is to explain the empirical evidence, it must be in the context of a transition path.

Assume that all intermediate goods producers $j = 1 ... J$ are at the same quality level that is either higher or lower than the balanced-growth level under the initial population and market size. In this case, the dynamics of the model are driven by two differential equations: Equation (6) determines how fast intermediate good quality improves in each market, and the first-order condition of (7) with respect to R&D employment L_j pins down the evolution of R&D intensities. Assuming symmetry, the dynamic equations ruling each individual market also determine the transition path for the economy as a whole.

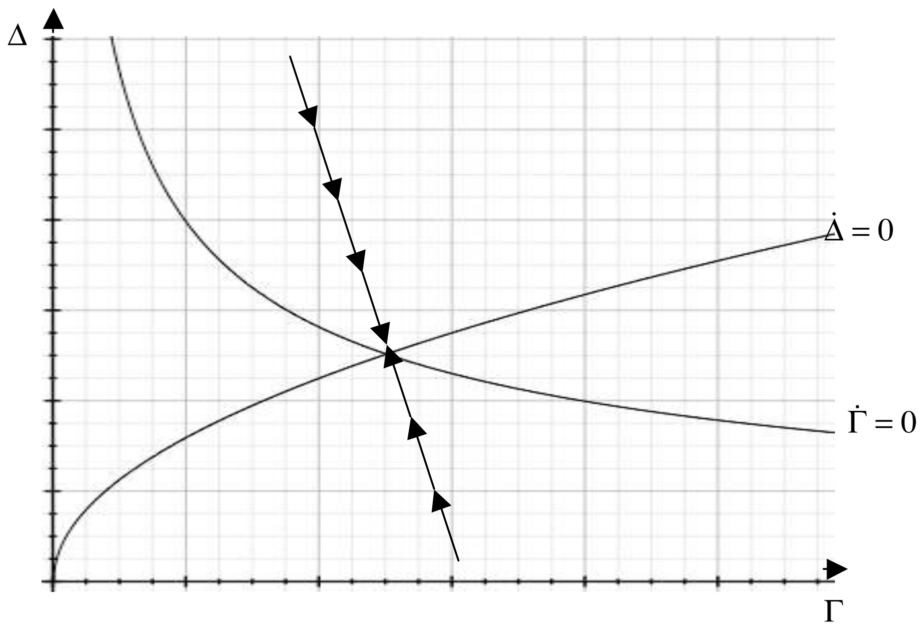
Letting Γ be R&D intensity and Δ the ratio of current to steady-state product quality, the dynamic system that characterizes the transition path of the economy can be written as

$$\dot{\Delta}/\Delta = -\Theta_1 + \Theta_2 \Delta^{\phi-1}\Gamma^{\lambda} \tag{9}$$

$$\dot{\Gamma}/\Gamma = \Theta_3 - \Theta_4 \Delta^{\phi-1}\Gamma^{\lambda-1} \tag{10}$$

for constants $\Theta_i > 0$, $i = 1...4$ as long as the R&D sector is small (see appendix A.1). This dynamic system is saddle-path stable, with a downward sloping transition path in the (Γ,Δ) space, as depicted in Figure 4.

Figure 4: Transition dynamics in the monopoly case



For the economy to experience a rise in R&D intensity during the transition to the balanced growth path, it has to start out with a higher stock of knowledge about the production of intermediate goods than in the long-term equilibrium. As it approaches the steady state, growth rates increase. Since the data suggest that per-capita growth rates have been rather stable or even declining over the past decades, the transitional dynamics implied by this model do not seem to provide a convincing explanation for the increase in R&D spending. Moreover, it is not obvious why the current stock of knowledge should be above its steady-state value.

4.3 *The Possibility of Entry*

4.3.1 *The Strategy of the Incumbent*

Now assume that competitors can enter the market with equivalent products if they spend an amount $C(z)>0$ to acquire the necessary technical knowledge using an imitation approach, where z is a variable controlled by the incumbent that affects the difficulty of catching up. Assume that increasing z raises a competitors costs, $C'(z)>0$, but that it also reduces the incumbent's profit flow and thus its present value of monopoly profits $V(z)$, i.e. $V'(z)<0$.

Entrants cannot surpass the incumbent's technological level by imitating. Once they catch up, they have to enter the market and use the same process of innovation at the same cost as the incumbent. At this point, price competition would drive firm profits down and the best choice for the two firms is to merge. Assume that in this case, the owner of the entrant firm acquires a share $\sigma \in (0,1)$ of the merged firm. Given the symmetry between incumbent and entrant, $\sigma = \frac{1}{2}$ is a plausible value.

If it is optimal to deter entry, the incumbent has to chose z such that $\sigma V(z) \le C(z)$, i.e. a possible entrant's share of the (optimal) value of the monopoly supplier is worth no more than the cost of getting into the market and acquiring this share.

As long as the incumbent does not deter other firms from entering the market, entry will happen repeatedly, each time driving down the share of the original shareholders in the operation, so that their claims to profit flows approach zero. As deterring entry yields a positive discounted present value of profits $\sigma^{-1}C(z)$, allowing entry cannot be optimal.

In the following, we will model a scenario in which the incumbent's strategy variable *z* is the rate of technological progress.

4.3.2 Choosing the Rate of Innovation

Assume that the entrant can acquire the necessary knowledge using a technology similar to the leader's R&D innovation production function (6),

$$\dot{\tilde{q}}_j = \tilde{\chi}\tilde{L}_j^{\tilde{\lambda}}\tilde{q}_j^{\tilde{\phi}}e^{\eta \cdot age(\tilde{q}_j)}. \tag{11}$$

A tilde ~ marks variables and constants referring to the entrant's imitation technology. Apart from potential differences in the values of the constants, equation (11) differs from (6) in including the term $e^{\eta \cdot age(\tilde{q}_j)}$, where η is a positive constant and $age(q)$ stands for the time that has passed since the quality level q was first introduced by the incumbent. It captures the effect that older technologies that have been available for a longer time can be understood and replicated more easily, which is a form of intra-industry knowledge spillovers.

Even though we will refer the entrant's accumulation of technological knowledge as imitation, it should not be interpreted as pure mindless copying of existing technologies. According to the survey by Levin et al. (1987) , the single most effective way of learning about new processes and products of competitors is independent R&D.[11] Equation (11) simply states that it is easier to follow in somebody's footsteps than to blaze a trail. Any firm that wants to challenge the monopolist's position will eventually have to innovate once it reaches the incumbent's technological level.

We assume that initially, potential entrants have negligible knowledge of the technology employed by the incumbent. Even so, the fact that old technologies can be replicated much

[11] Ibid., p. 806, table 6.

more easily allows an entrant to acquire the skills necessary to produce a good of the same quality as the incumbent at finite costs in finite time. In this case, the cost of entry is a function of the incumbent's technology level, his rate of innovation, and the overall wage.

Lemma 1

Consider the case where that the growth rate of the economy is close to constant, and that the initial technological knowledge of an entrant approaches zero. Then, the lower bound of the cost C of matching an incumbent's technology q_j can be written as

$$C = \Theta\left(1 - \tilde{\phi} + \frac{\eta}{\hat{q}_j}\right)^{-\frac{1}{\lambda}} q_j^{\frac{1-\tilde{\phi}}{\lambda}} w, \tag{12}$$

where w is the wage rate at the time technological parity is reached, $\hat{q}_j$ is the growth rate of the incumbent's technology and Θ is a positive constant.

Proof:

See appendix A.2.

□

The incumbent can directly affect the entrant's cost at any point in time by changing the rate of innovation $\hat{q}_j$. A higher rate of quality growth means that any previous technology has been available and known for a shorter period of time, which according to (11) makes it harder to imitate. Thus, choosing a higher $\hat{q}_j$ leads to a higher relative cost increase for the entrant than for the incumbent.

In order to retain his monopoly[12], the incumbent must choose the optimal R&D intensity according to (7) subject to the additional constraint

$$V \equiv \int_0^\infty e^{-rt}\left(\pi(L_Y, q_j) - wL_j\right)dt \le \sigma^{-1}C\,. \qquad (13)$$

If imitation costs are high and grow at least at the growth rate of the profit flow so that this constraint never binds, the monopolist behaves exactly like in the unconstrained situation described above.

If, however, the parameters of the model are such that constraint (13) binds at any point in time, the incumbent's R&D expenditure is driven not only by future profits, but also by the R&D costs of potential competitors. This case occurs if imitation costs rise more slowly with the level of technology than innovation costs, $\frac{1-\tilde{\phi}}{\tilde{\lambda}} < \frac{1-\phi}{\lambda}$.[13] Note that according to (12), if $\hat{w}$ and $\hat{q}_j$ are constant, the cost of entering the market grows at the rate $\frac{1-\tilde{\phi}}{\tilde{\lambda}}\hat{q}_j + \hat{w}$. In this case, equation (6) implies that the incumbent's R&D expenditures grow at the rate $\frac{1-\phi}{\lambda}\hat{q}_j + \hat{w}$. It therefore becomes cheaper and cheaper over time to enter the market.

Analyzing the balanced growth path of this model is not particularly interesting. As in all semi-endogenous growth models of this type, growth is eventually constained by population growth and the burden of knowledge. A balanced growth path along which the market-entry constraint (13) does not bind is identical to the path described in proposition 1. If (13) is binding, as will be seen below, the resulting balanced growth path is characterized by the same growth rates of all variables, but a higher level of technology, resulting in higher R&D costs and zero profits of incumbents, making entry unattractive.

[12] Equation (13) implements the simple entry game described above. Alternative assumptions are possible, but as long as entry is triggered by the ratio between the monopolist's (potential) net profits and the costs of entry, our results remain the same.

[13] It does not seem implausible that it is harder to accelerate R&D with additional resources than imitation.

In what follows, we will characterize the growth dynamics of an economy in which the market entry constraint is binding. In such a scenario, it is possible to observe rising R&D shares in the presence of stable aggregate growth. However, as R&D shares must eventually stabilize, the growth rate of the economy must change over time. To analyze the growth properties of the economy along such a path, which is characterized by time-varying growth rates and convergence towards a balanced growth path, without getting caught up in inter-industry short-term dynamics, we introduce the concept of a medium-term growth path.

Definition

The economy is said to follow a medium-term growth path (MTGP[14]) starting at time s if (i) all industries are at the same technological level at s and (ii) the market entry constraint (13) is binding for any time $t \geq s$.

Requirement (i) makes sure that the model works as if there were only one intermediate good industry. This makes it possible to abstract from the effects of individual industries converging to the general productivity level. Note, however, that this definition does not necessarily identify a unique growth path for the economy, as initial conditions affect the shape of the MTGP. Requirement (ii) lets us focus on the part of an economy's growth experience for which R&D investment is driven by potential competition in each industry.

Proposition 2

[14] The term "medium-term growth path" was chosen to emphasize that observed growth rates may be different form the growth rates possible in the (very) long run, while at the same time short-term adjustments like the convergence of individual industries to the economy-wide productivity level are ignored.

A MTGP along which monopolists are choosing R&D effort to prevent entry has the following properties:

(i) It is characterized by a transition from high growth to low growth, $\lim_{t\to-\infty}\gamma_t = \frac{\tilde{\lambda}}{1-\tilde{\phi}}n > \frac{\lambda}{1-\phi}n = \lim_{t\to\infty}\gamma_t$, where γ_t is the rate of GDP growth at time *t*.

(ii) If $\beta\frac{1-\phi}{\lambda} + (1-\beta)\frac{1-\tilde{\phi}}{\tilde{\lambda}} > \frac{\alpha}{1-\alpha}$, a small industry locally converges to the MTGP technology level of the rest of the economy. β is the ratio of the current R&D share to its long-term level.

Proof:

See appendix A.3.

□

Proposition 2 (i) is the analogue of the same part of proposition 1 for the case of possible entry. The growth rate of the economy drops from a high level that is determined by the ease of imitation to a lower level that is consistent with a constant R&D share. This long-term growth rate is the same that would result under alternative assumptions regarding market structure in the R&D sector on a balanced growth path. Part (ii) of proposition 2, which corresponds to proposition 1 (ii), shows that individual industries are more likely to converge to the general productivity level at a later point in time when the R&D share is higher. Note that there is no stability result corresponding to proposition 1 (iii), as different initial condition with regards to the general productivity level simply characterize different MTGPs.

In a MTGP setting, the monopolist's only control variables are the price of the intermediate good and R&D spending. The price is pinned down by the solution to the static profit maximization problem. While clearly reducing profits, adjusting the price does not deter entry, as other firms are attracted by the maximum net profit they could earn rather than the current monopolist's actual profit flow. Thus, the only way for the monopolist to discourage entry is to adjust R&D spending.

Increased R&D spending has two effects. First, it reduces the monopolist's net profit flow, making entry less attractive. Assuming that the growth rate of technology $\hat{q}$ is constant over time and that the share of labour employed in the R&D sector is very small, so that L_Y grows approximately at the rate of population growth even if R&D employment $L_{R\&D} = \sum_{j=1}^{J} L_j$ grows at a higher rate, the present value of the net profit flow t periods in the future, $NP(t)$ can be written as

$$NP(t) = e^{-rt}\left(\bar{\pi} L_Y q_j^{\alpha/1-\alpha} - wL_j\right) = \bar{\pi} L_{Y,0} q_{j,0} e^{(n+\alpha/1-\alpha\hat{q}-r)t} - w_0 L_{j,0} e^{(\hat{w}+1-\phi/\lambda\hat{q}-r)t},$$

where a subscript 0 denotes the current value of this variable and $\hat{w}$ is the growth rate of wages. Remember that equation (6) implies that R&D employment must rise at the rate $1-\phi/\lambda\,\hat{q}$. If $\hat{q}$ is above the (sustainable) unconstrained optimum, R&D intensity rises over time. If the R&D sector is small enough, however, per-capita consumption still grows at approximately the same rate $\alpha/1-\alpha\,\hat{q}$ as productivity in the final goods sector, so we can use equation (2) to rewrite $NP(t)$ as

$$\begin{aligned} NP(t) &= \bar{\pi} L_{Y,0} q_{j,0} e^{(n+\alpha/1-\alpha\hat{q}-(\alpha/1-\alpha\hat{q}+\rho))t} - w_0 L_{j,0} e^{(\alpha/1-\alpha\hat{q}+1-\phi/\lambda\hat{q}-(\alpha/1-\alpha\hat{q}+\rho))t} \\ &= \bar{\pi} L_{Y,0} q_{j,0} e^{(n-\rho)t} - w_0 L_{j,0} e^{(1-\phi/\lambda\hat{q}-\rho)t} \end{aligned} \quad (14)$$

Thus, a change of the growth rate affects the present value of the net profit flow only through the increase in R&D expenditure, not through the effect on future rents earned in the market for intermediate goods.

The second effect of higher R&D expenditures is that they increase the growth rate of intermediate good quality, $\hat{q}$, which, according to equation (12), makes it more costly for another firm to replicate the current technology and enter the market.

As long as total R&D spending is very low compared to the monopolist's profit flow, this second effect is quantitatively more important.[15] In this case, the rate of innovation and consequently the growth rate are mainly determined by equation (12), and thus by the incumbent's effort to improve product quality at a rate that is high enough to make it too costly for competitors to enter the market.

A situation like this can persist as long as the R&D sector is small compared to the rest of the economy. As R&D employment is growing faster than the labour force in order to maintain a high rate of innovation, R&D expenditure eventually becomes a relevant cost factor and net profits as a share of total GDP begin to drop. This effect reduces competitors' incentive to enter the intermediate goods market, thus allowing the incumbent to reduce the rate of innovation. Asymptotically, R&D intensity stabilizes, monopoly rents net of R&D expenditures drop, and the economy converges to a balanced growth path characterized by the same growth rate g as in the uncontested monopoly case. If the initial R&D intensity is low enough, these medium-term dynamics can persist for a very long time, even if the growth rated during this period are substantially above the level that is sustainable in the long term.

5 TRANSITIONAL DYNAMICS

5.1 Calibrating the Model

As mentioned above, Jones (2002) argues that the bulk of US growth is due to this sort of transitional dynamics. In a growth accounting exercise he attributes about half of the 2% annual per capital GDP growth in the US to rising R&D intensity, 30% to improved human capital, and only 20% (i.e. about 0.4% of growth per annum) to sustainable steady state

[15] Equation (14) seems to suggest that even for a low initial level of R&D expenditures, the R&D cost effect of increasing quality growth can be large. Remember, however, that this equation is an approximation that is only valid as long as (a) the R&D sector is small and (b) $\hat{q}$ is constant. Since the intermediate goods supplier can leave the market at any time, the present value net profits can never be negative.

growth.[16] Our model is able to explain why R&D effort might be rising faster than GDP, and can also be used to predict when and how this growth episode must end.

The following calibrations are based on the idea that the evolution of the growth rate of an economy can be fully determined by the observation of the R&D share and growth rate at one point in time: The two state variables of the model are productivity and population. If the growth rate is always above its long-term level, the R&D share is growing monotonically, so there is a bijection between the R&D share and L. Once L is known, the other state can be obtained from equation (6) using the current growth rate. It is thus possible to obtain the current state of the economy without having to calibrate level variables like q or L. However, the values of several important model parameters must still be determined.

According to data provided by the National Science Foundation, the ratio of non-federal R&D spending to GDP has risen from 0.63% in 1953 to 1.95% in 2007. While this corresponds to a substantial rise of the R&D share of 2.1% per year, total R&D expenditures as a share of GDP are still quite small. Since our model predicts a profit share in GDP of $\alpha(1-\alpha)$, which for a capital share of $\alpha = 1/3$ works out to about 22%, the ratio of R&D expenditures to profits is still low enough make the simplifying assumption that net profits NP have grown at the same rate as GDP a reasonable approximation.

We can write the growth rate of the share of R&D expenditures in GDP as

$$\hat{s} = \widehat{\left(wL_{R\&D}\right)} - \hat{Y} = \left(\hat{w} + \tfrac{1-\phi}{\lambda}\hat{q}\right) - \left(\tfrac{\alpha}{1-\alpha}\hat{q} + n\right) = \tfrac{1-\phi}{\lambda}\hat{q} - n \,. \tag{15}$$

If we assume that $1-\tilde{\phi}$ is small compared to $\eta/\hat{q}$, which is not unreasonable considering that plausible values of $\tilde{\phi}$ lie between -1 and 1 and that the rate η at which innovations become easier to imitate is likely to be much higher than the growth rate, we can write the growth rate of imitation cost compared to GDP approximately as

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[16] Ibid., p 228, table 2, $\gamma = 0.33$ case.

$$\hat{i} = \hat{C} - \hat{Y} \approx \left(\frac{\hat{\hat{q}}}{\tilde{\lambda}} + \frac{1-\tilde{\phi}}{\tilde{\lambda}} \hat{q} + \hat{w} \right) - \left(\tfrac{\alpha}{1-\alpha} \hat{q} + n \right) = \frac{\hat{\hat{q}}}{\tilde{\lambda}} + \frac{1-\tilde{\phi}}{\tilde{\lambda}} \hat{q} - n \,. \qquad (16)$$

If monopolists are deterring entry by setting the growth rate of the product quality accordingly, entry costs C are growing at the same rate as net profits and GDP, so $\hat{i}$ is zero. Relating the rate of quality growth to the growth rate of productivity γ, $\hat{q} = {}^{1-\alpha}\!/\!_{\alpha}\gamma$, we can rewrite equations (15) and (16) as

$$\hat{s} = \tfrac{1-\phi}{\lambda} \tfrac{1-\alpha}{\alpha} \gamma - n \qquad (17)$$

$$\hat{s} = -\tfrac{\hat{\gamma}}{\tilde{\lambda}} + \tfrac{1-\alpha}{\alpha} \gamma \left(\tfrac{1-\phi}{\lambda} - \tfrac{1-\tilde{\phi}}{\tilde{\lambda}} \right). \qquad (18)$$

These equations determine the R&D burdens of innovators and imitators, $\tfrac{1-\phi}{\lambda}$ and $\tfrac{1-\tilde{\phi}}{\tilde{\lambda}}$, as functions of observable variables, if we are able to find a value for $\hat{\gamma}/\tilde{\lambda}$. Since the change of the per-capita growth rate has been close to zero since the 1950s, for this term to play an important role in equation (18), $\tilde{\lambda}$ would have to be very small. We will thus assume that $\hat{\gamma}/\tilde{\lambda} = 0$. Setting $\alpha = 1/3$, and using the US post-war averages of R&D intensity growth, per-capita output growth, and labour force growth, $\hat{s} = 2.1\%$, $\gamma = 2\%$, and $n = 1.7\%$, we find that $\tfrac{1-\phi}{\lambda} = 0.95 \approx 1$ and $\tfrac{1-\tilde{\phi}}{\tilde{\lambda}} = 0.425 \approx 0.4$. It is therefore significantly easier to speed up the imitation process compared to increasing the rate of R&D output. Allowing for a negative $\hat{\gamma}/\tilde{\lambda}$ would lead to a somewhat higher value of $\tfrac{1-\tilde{\phi}}{\tilde{\lambda}}$.

These crude calculations suggest that cost of innovating has grown roughly in proportion to the quality of intermediate goods, while the cost of imitation has grown at less than half this rate. Recasting this result in terms of the revenue stream earned from innovation, whose value is proportional to $q^{\alpha/1-\alpha}$ (see equation (5)), the cost of imitation has grown at the 80% of the rate of profit growth, while the cost of innovating has grown twice as fast, which is equivalent to saying that R&D employment must have grown twice as fast as per-capita GDP.

So, according to these values, while the cost of imitating an idea of a given value is about 20% lower today than in 1955, the cost of originally inventing the final 1% of it has grown by a factor of about 2.8.

5.2 The Growth Path

Based on the calibrations of $\frac{1-\phi}{\lambda}$ and $\frac{1-\tilde{\phi}}{\tilde{\lambda}}$ obtained above, we now investigate the properties of the resulting growth path numerically. This path is determined by the requirement that at any point in time the growth rate of all discounted future rents in the market for intermediate goods be the same as the growth rate of the minimal imitation cost, taking into account the time-variable interest rate as defined in (2) and the fact that resources spent on intermediate goods and R&D are not available for consumption. Initial conditions regarding R&D intensity and GDP growth then fully determine the future growth path for a given set of model parameters. To avoid having to calibrate the parameters η and $\tilde{\phi}$ that determine the growth rate of imitation costs in equation (12), we continue to assume $1-\tilde{\phi} \ll \eta/\hat{q}_j$, so that cost of entering the market for an intermediate good grows at the rate $\frac{1}{\lambda}\hat{\bar{q}} + \frac{1-\tilde{\phi}}{\tilde{\lambda}}\hat{q}_j + \hat{w}$.

The numerical simulations are based on a discrete-time version of the model with a period length of one year. Details can be found in appendix A.4.

Table 2 lists the parameters that need to be calibrated. Most of the choices are pretty straightforward: Values of $\frac{1-\phi}{\lambda}$ and $\frac{1-\tilde{\phi}}{\tilde{\lambda}}$ that match US data were determined in the previous subsection; labour force growth is assigned the US post-war average of 1.7% per year; the discount rate and the labour share are set to the values most commonly used in the literature. The only parameter for which we cannot easily obtain a value is the elasticity of imitation output with respect to labour input, $\tilde{\lambda}$. We therefore use three values within the admissible interval of $(0,1)$, $\tilde{\lambda} \in \{1/4, 1/2, 3/4\}$.

Table 2: Calibration

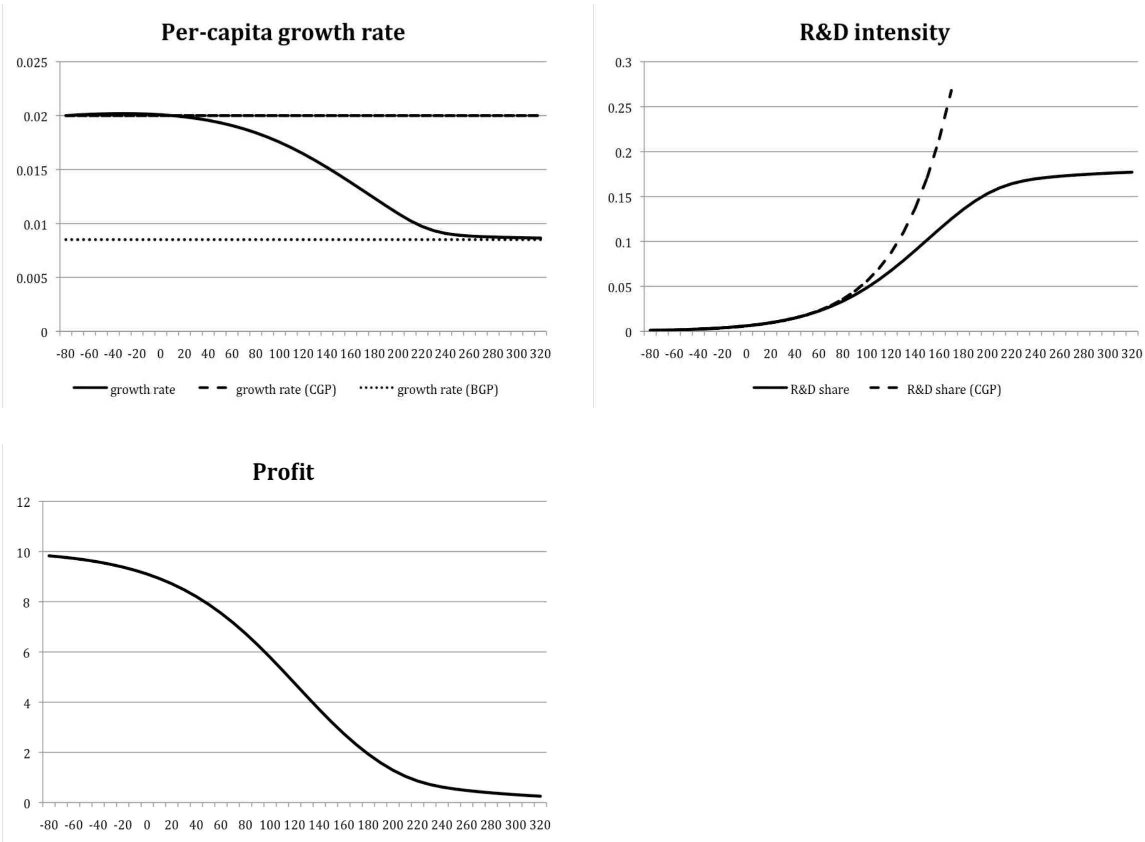
Parameter		Value
ρ	discount rate	4% p.a.
α	capital share	1/3
$\frac{1-\phi}{\lambda}$	burden of knowledge: innovator	1
$\frac{1-\tilde{\phi}}{\tilde{\lambda}}$	burden of knowledge: imitator	0.4
n	population growth	1.7% p.a.
$\tilde{\lambda}$	elasticity of imitation output w.r.t. labour	1/4, 1/2, 3/4

Table 3: Transition paths

	$\tilde{\lambda} = 1/4$	$\tilde{\lambda} = 1/2$	$\tilde{\lambda} = 3/4$
Per-capita growth rate			
$t = 0$	2.00%	2.00%	2.00%
$t = 50$	1.96%	1.93%	1.91%
$t = 100$	1.83%	1.76%	1.72%
$t = 200$	0.94%	1.10%	1.12%
$t = \infty$	0.85%	0.85%	0.85%
$t(1.425)$	166 years	155 years	149 years
R&D intensity (ratio to extrapolated trend in parentheses)			
$t = 0$	0.60% (100%)	0.60% (100%)	0.60% (100%)
$t = 50$	1.80% (98.3%)	1.79% (97.3%)	1.77% (96.7%)
$t = 100$	5.01% (89.3%)	4.75% (84.5%)	4.58% (81.5%)
$t = 200$	17.8% (33.8%)	15.2% (28.9%)	13.8% (26.3%)
$t = \infty$	18.2% (0%)	18.2% (0%)	18.2% (0%)
$t(\text{ratio} = 1/2)$	177 years	163 years	156 years
Present value of net profit as share of GDP			
$t = 0$	9.06	9.12	9.15
$t = 50$	7.76	7.94	8.03
$t = 100$	5.38	5.86	6.12
$t = 200$	0.23	1.33	1.95
$t = \infty$	0	0	0
$t(4.83)$	109 years	120 years	127 years

Table 3 presents some characteristics of the growth path that is determined by an initial annual growth rate of per-capita GDP of 2% and an initial R&D intensity of 0.6%, roughly corresponding to the respective values for the US economy in 1950. The period length is one year. For the three different levels of $\tilde{\lambda}$ under consideration, the table shows the values of the growth rate, R&D intensity, and the ratio of profits to GDP at five points in time, as well as a measure of the half-life of the deviation of each variable from its long-term value.

Figure 5: Transition dynamics



Generally, for lower $\tilde{\lambda}$, growth rates remain high for a longer time and R&D intensities grow faster. This is because for a low $\tilde{\lambda}$, a given reduction of the growth rate of technology *ceteris paribus* has a bigger impact on the amount of labour required to catch up to the technological leader, thus having a larger effect on imitation costs. At the same time, having a high growth rate for a longer time means that R&D expenditures grow faster, consequently exhausting the monopolists' profits sooner. Eventually, the economy therefore approaches its balanced growth path sooner for lower $\tilde{\lambda}$.

The trajectories of the growth rate, R&D intensity, and profits are shown in Figure 5 for the intermediate case of $\tilde{\lambda} = 1/2$ over a time span of 400 years, starting in period -80 (as above, period 0 is calibrated to resemble 1950). Where relevant, the evolution of the corresponding variable along the balanced growth path (BGP) and the constant growth path[17] (CGP) along which the original growth rate is maintained are also plotted.

What is interesting about this scenario is the fact that our model is able to generate a growth rate above the balanced growth path level that is maintained almost unchanged for a long period of time. In fact, per-capita GDP growth remains remarkably close to 2% (between 1.97% and 2.02%) for 130 years, which is in line with the US growth experience since the late 19th century.[18] This era, closely resembling a constant growth path as described in Jones (2002) is followed by a relatively pronounced transition period. 80% of the drop of the growth rate takes place between periods 80 and 220. After that, the economy follows the balanced growth path.

6 WELFARE AND POLICY

A full investigation of welfare implications and optimal policy in the context of our model are beyond the scope of this paper. However, a few brief remarks are in place.

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[17] See Jones (2002) .

[18] This scenario assumes that entry has always been a threat for the whole time span under consideration. Since the burden of knowledge grows faster for innovators than imitators, it is possible that before some point in the past, the optimal rate of innovation chosen by an unconstrained monopolist was high enough to prevent entry. In this case, before this specific point in time, the R&D share would be constant, resulting in a lower rate of growth before the threat of competition raises growth.

First of all, since an unconstrained monopolist internalizes the full cost of R&D but only the part of the social return he can appropriate, R&D effort is too low if there is no threat of entry.

With entry in the long run, on the other hand, once the economy is on the balanced growth path, the allocation is remarkably similar to what would result in a model of creative destruction with a competitive R&D sector, in the sense that the present value of monopoly rents and the present value of R&D expenditures are the same. The result that whether there is too much or too little R&D in this case depends on parameters formally carries over to this case. Note, however, that the interpretation of R&D expenditure may be somewhat different in our case. While one factor leading to inefficiencies in the decentralised economies with competitive R&D sectors results from the duplication of efforts, in our model where innovation is still coordinated by a monopolist, this reason for inefficiency would likely not arise. However, as our specification assumed diminishing returns to R&D employment, a similar source of wasteful overinvestment in innovation still exists.

As the economy moves towards the balanced growth path, increasing R&D intensity, at least initially it gets closer to the amount of R&D investment a social planner would choose. As we have seen in the previous subsection, however, the transition may take a very long time.

Close to the balanced growth path, when R&D spending is essentially determined by future rents, taxes or subsidies have the usual effect on innovation. Remember though that on the initial constant growth path, the rate of innovation is driven by the ease of entry. Subsidizing the market leader's R&D is likely to mostly result in somewhat higher profit flows. To effectively influence the incumbent's behaviour, it would be necessary to affect the cost of entry, for example by subsidizing R&D expenditures of imitators or using other means of encouraging competition.

7 CONCLUSIONS

The purpose of this paper was to provide an explanation of the observation that, while per capita growth has been quite stable for a long time in the US and other comparable countries, R&D intensity has grown over time.

We have shown that a model in which monopoly suppliers can more easily maintain their market position if they innovate at a higher rate is capable of generating the dynamics we observe. Moreover, the model has the interesting feature that an unsustainably high growth rate accompanied by a rising R&D intensity can persist for a long time, resembling the constant growth path described by Jones (2002) . Eventually, however, the economy must embark on a transition towards a balance growth path. This transition is driven by falling profits of R&D intensive firms.

We have chosen our model to be as similar as possible to the generic quality ladder model, and we have introduced extensions to or modifications of the standard framework in a simple and stylized way. Presumably, the basic mechanisms at work would carry over to a more complex and realistic specifications.

For a model like this to better match the data, it would for example be desirable to choose a production function that disentangles the demand elasticity that monopolists are facing from the labour share. This would make it possible to calibrate monopoly rents independently, which is important as it is the amount of these rents what eventually drives the transition to the balanced growth path. Some back-of-the-envelope calculations suggest that the share of monopoly rents in GDP of over 20% and the resulting firm value along the initial constant growth path are somewhat too high.

Moreover, the result that each market is served by a monopolist who retains his position indeterminately is unrealistic. However, it should not be too difficult to make modifications that allow for more general and realistic market structures.[19] In a setting, for example, where a market leader engages in Cournot competition with followers, the market share and thus the oligopoly rents of the leader are still sensitive to his technological advantage, giving him an incentive to innovate at a rate fast enough to stay ahead.[20] In addition to this, one could allow for stochastic R&D outcomes, introducing the possibility of the technological leader being replaced in the case of a bad realization.

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[19] See Klette and Kortum (2004) and Segerstrom (2007) for examples of growth models that attempt to model the behaviour of market participants more realistically.

[20] See, for example, Denicolò and Zanchettin (2008) for a quality-ladder model that allows for Cournot competition.

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APPENDIX

A.1 The Uncontested Monopoly Case

Setting up the Hamiltoninan corresponding to the firm's optimization problem (objective (7) subject to (6)) yields – after elimination of the co-state and some further manipulation – the following two equations that determine the monopolist's behaviour:

$$(1-\lambda)\hat{L}_j = r - \hat{w} - \chi\lambda\frac{\alpha}{1-\alpha}\overline{\pi}\left[w^{-1}L_j^{\lambda-1}q_j^{\phi-1+\frac{\alpha}{1-\alpha}}L_Y\right] \qquad (19)$$

$$\hat{q}_j = \chi L_j^{\lambda} q^{\phi-1} \qquad (20)$$

If there is a symmetric balanced growth path, equation (20) (which is the same as (6)) implies

$$\hat{q}_j = \frac{\lambda}{1-\phi}n\,. \qquad (21)$$

Then, we have from (4) that $\hat{X}_j = n + \frac{\alpha}{1-\alpha}\frac{\lambda}{1-\phi}n$, which can be used in the production function (3) to get

$$\hat{Y} = n + \frac{\alpha}{1-\alpha}\frac{\lambda}{1-\phi}n\,. \qquad (22)$$

As the capital inputs grow at the same rate as output, employment in the production of each variety grows at the rate n,

$$\hat{L}_j = n\,, \qquad (23)$$

and so does R&D employment. The growth rate of per-capita consumption is therefore constant:

$$\hat{c} = \hat{Y} - \hat{L} = \frac{\alpha}{1-\alpha}\frac{\lambda}{1-\phi}n \qquad (24)$$

Using this in (2) determines the interest rate

$$r = \frac{\alpha}{1-\alpha}\frac{\lambda}{1-\phi}n + \rho . \tag{25}$$

Finally, wages are pinned down by the marginal product of labour in production, $w = (1-\alpha)Y/L_Y$, so

$$\hat{w} = \frac{\alpha}{1-\alpha}\frac{\lambda}{1-\phi}n . \tag{26}$$

For firm behaviour to be consistent with this balanced growth path, the term in square brackets on the right hand side of (19) must be constant; only then is a constant growth rate of L_j possible. Using equations (26), (23) and (21), it is easily verified that this is indeed the case on the balanced growth path.

To see whether a single small intermediate goods industry converges to the balanced growth path, along which the rest of the economy is already moving, define the variables Γ and Δ by $L_j = \Gamma L$ and $q_j = \Delta q$, where $q = q_k$, $k \neq j$, is the quality level in all other industries. Using these definitions in (19) and (20) yields the dynamic system

$$\hat{\Delta} = -\Theta_4 + \Theta_5 \Gamma^{\lambda} \Delta^{\phi-1} \tag{27}$$

$$\hat{\Gamma} = \Theta_6 - \Theta_7 \Gamma^{\lambda-1} \Delta^{\phi-1+\frac{\alpha}{1-\alpha}} \tag{28}$$

for positive constants Θ_i, $i = 4...7$ as long as $\frac{\rho}{1-\lambda} > n$. For $1-\phi > \frac{\alpha}{1-\alpha}$, this system is qualitatively the same as the saddle-path stable system shown in Figure 4.

To show that the economy converges to a balanced growth path when all monopolists start out at the same quality level above or below the balanced-growth value, the same basic approach as for the single-industry case above can be used. The main difference is that now wage growth changes along the transition path, $w = (1-\alpha)Y/L_Y$, which must be taken into account when rewriting equation (19) in terms of variables Γ and Δ to get (9) and (10).

A.2 The Case of Possible Entry

To determine the optimal cost an imitator must incur in order to catch up to the technological level of the incumbent, we proceed in three steps: First, we derive the optimal path of R&D employment during the catch-up process solving the firm's dynamic problem. Second, we determine the trajectory of technological knowledge that results from an efficient employment path. Finally, we solve the static problem of finding the right level of R&D employment, which is the same as solving for the optimal duration of the catch-up period. In doing all this, we will assume a constant rate of innovation by the incumbent, $\hat{q}_j$, as well as symmetry between industries, $\hat{q}_i = \hat{q}_k =: \hat{q}$ ($i, k \in \{1 ... J\}$) and $\hat{c} \approx \hat{w}$.[21]

Before we begin, it is helpful to eliminate the function age from equation (11). Noting that for a constant rate of technological progress $\hat{q}$, we have $\tilde{q}_j \exp\left(\hat{q} \cdot age(\tilde{q}_j)\right) = q_j$, so that we can rewrite (11) as

$$\dot{\tilde{q}}_j = \tilde{\chi} \tilde{L}_j^{\tilde{\lambda}} \tilde{q}_j^{\tilde{\phi} - \eta / \hat{q}} q_j^{\eta / \hat{q}} . \tag{29}$$

Step 1:

The entrant has to invest in R&D for a time T in order to catch up to the leader at $t = 0$. To do so efficiently, he solves the dynamic problem

$$\min_{\tilde{L}_j} \int_{-T}^{0} e^{-rt} \tilde{L}_j(t) w(t) \, dt \tag{30}$$

subject to (29). From the first-order conditions of the Hamiltonian we get, after eliminating the co-state and solving for employment,

$$\hat{\tilde{L}}_j = \frac{\eta + r - \hat{w}}{1 - \tilde{\lambda}} = \frac{\eta + \rho}{1 - \tilde{\lambda}} . \tag{31}$$

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[21] While these assumptions strictly speaking only hold along a balanced growth path, any changes in the relevant variables along a transition path happen slowly enough for these conditions to be fulfilled almost exactly at a horizon of years or even decades.

Step 2:

With a constant growth rate of R&D employment and the incumbent's technology, we can rewrite equation (29) as

$$\dot{\tilde{q}}_j = \tilde{\chi}\tilde{L}_{j,0}^{\tilde{\lambda}} q_{j,0}^{\eta/\hat{q}} \tilde{q}_j^{\tilde{\phi}-\eta/\hat{q}} \exp\left((\tilde{\lambda}\tfrac{\eta+\rho}{1-\tilde{\lambda}} + \tfrac{\eta}{\hat{q}}\hat{q})t \right), \tag{32}$$

where an subscript 0 is used to mark the value of variables at time $t = 0$ when the entrant reaches the technological level of the incumbent and is ready to compete.

(32) is a Bernoulli equation. Writing it as $\dot{\tilde{q}}_j = \Lambda_1 \tilde{q}_j^{\Lambda_2} e^{\Lambda_3 t}$ for constants Λ_i, $i = 1,2,3$, it is easily verified that

$$\tilde{q}_j(t) = \left[(1-\Lambda_2)\frac{\Lambda_1}{\Lambda_3}\left(e^{\Lambda_3 t} - e^{\Lambda_3 0}\right) + \tilde{q}_{j,0}^{1-\Lambda_2} \right]^{\frac{1}{1-\Lambda_2}} \tag{33}$$

is a solution. To find out how long it takes to catch up, we determine the point in time $-T$ when the imitator's technological knowledge is zero, $\tilde{q}_j(-T) = 0$. Using (33) and solving for the variable of interest, we obtain

$$-T = \frac{1}{\Lambda_3}\ln\left(1 - \tilde{q}_{j,0}^{1-\Lambda_2} \frac{\Lambda_3}{\Lambda_1(1-\Lambda_2)} \right). \tag{34}$$

Note that admissible values of the constants Λ_i are such that the argument of the logarithm is between zero and one, leading to a (weakly) positive, possibly infinite duration of the catch-up process.

Step 3:

We still need to determine the duration of the catch-up process or, alternatively, the level of employment $\tilde{L}_{j,0}$. To do so, we minimize the firms R&D expenditure

$$\min_{\tilde{L}_{j,0}} \int_{-T}^{0} w_0 \tilde{L}_{j,0} \exp\left((\hat{w} + \frac{\eta+\rho}{1-\tilde{\lambda}} - r)t \right) dt, \tag{35}$$

where $-T$ is determined by (34). Filling in the proper parameters for Λ_i and using $\tilde{q}_{j,0} = q_{j,0}$, we can derive

$$-T = \left(\tilde{\lambda} \frac{\eta + \rho}{1 - \tilde{\lambda}} + \eta \right)^{-1} \ln \left(1 - \tilde{L}_{j,0}^{-\tilde{\lambda}} \tilde{q}_{j,0}^{1-\tilde{\phi}} \frac{\tilde{\lambda} \frac{\eta+\rho}{1-\tilde{\lambda}} + \eta}{\tilde{\chi}\left(1 - \tilde{\phi} + \frac{\eta}{\hat{q}}\right)} \right). \tag{36}$$

Remember that the argument of the logarithm must not be negative, so we also have the constraint

$$\tilde{L}_{j,0} \geq \left[\tilde{q}_{j,0}^{1-\tilde{\phi}} \frac{\tilde{\lambda} \frac{\eta+\rho}{1-\tilde{\lambda}} + \eta}{\tilde{\chi}\left(1 - \tilde{\phi} + \frac{\eta}{\hat{q}}\right)} \right]^{\frac{1}{\tilde{\lambda}}}. \tag{37}$$

Solving the integral in (35), using (36) and simplifying as much as possible, we arrive at

$$\min_{\tilde{L}_{j,0}} w_0 \tilde{L}_{j,0}^{1-\tilde{\lambda}} q_{j,0}^{1-\tilde{\phi}} \left(\tilde{\chi}(1 - \tilde{\phi} + \frac{\eta}{\hat{q}}) \right)^{-1}. \tag{38}$$

This shows that the objective function is monotonically decreasing in $\tilde{L}_{j,0}$. Thus using the lowest admissible value for this variable as given by (37), we finally arrive at the infimum amount of imitation costs (12).

This optimal cost is achieved only for an infinite duration of the catch-up process. However, due to the properties of (36) for values of the argument of $\ln$ close to zero, very small increases of the cost above its infimum can buy substantial reductions of the (finite) R&D duration.

A.3 Proof of Proposition 2

(i) For $t \to \infty$, the growth rate γ of the economy cannot be higher than the growth rate on a balanced growth path, $\frac{\lambda}{1-\phi} n$, because the R&D share would grow without bounds. It cannot be lower either, because then net profits would eventually grow at the rate γ, while the cost of imitation grows at a lower rate, making market entry possible at some point in time.

For $t \to -\infty$, the growth rate is given by the equality between the present value of monopoly profits (R&D costs are not relevant yet, L_Y=L) and entry costs:

$$\int_t^\infty e^{-rs}\bar{\pi} L_Y q_j^{\alpha/1-\alpha} ds = 2\Theta\left(1-\tilde{\phi}+\tfrac{\eta}{\hat{q}}\right)^{-\frac{1}{\lambda}} q_j^{\frac{1-\tilde{\phi}}{\tilde{\lambda}}} w$$

Using equation (2), $w \sim q^{\frac{\alpha}{1-\alpha}}$, the fact that consumption grows at the same rate as GDP, and substituting Ω for constants, we get

$$L_Y = \Omega \frac{q_j^{\frac{1-\tilde{\phi}}{\tilde{\lambda}}}}{\left(1-\tilde{\phi}+\frac{\eta}{\hat{q}}\right)^{\frac{1}{\lambda}}}.$$

At any point in time, there is a level of q_j such that $\hat{q}$ is constant. In this case, q_j must grow at the rate $\frac{\tilde{\lambda}}{1-\tilde{\phi}}n$. It is easy to show that q_j converges to this level.

Assume that the productivity in industry j, q_j, is changed by a factor Δ. Then, if everything else stays the same, monopoly profits change by a factor of $\Delta^{\frac{\alpha}{1-\alpha}}$ at any point in time, R&D costs change by $\Delta^{\frac{1-\phi}{\lambda}}$, and entry costs by $\Delta^{\frac{1-\tilde{\phi}}{\tilde{\lambda}}}$. The ratio of net profits to entry costs, which is unity along the MTGP, is thus $\Phi = \Delta^{\frac{\alpha}{1-\alpha}-\frac{1-\tilde{\phi}}{\tilde{\lambda}}} - \beta\Delta^{\frac{1-\phi}{\lambda}-\frac{1-\tilde{\phi}}{\tilde{\lambda}}}$, where β is the ratio of R&D costs to monopoly rents. For the industry to locally converge to the productivity level of the economy, Φ needs to drop when q_j rises, resulting in a reduced future growth of productivity in this industry. We thus have

$$\left.\frac{d\Phi}{d\Delta}\right|_{\Delta=1} < 0 \Rightarrow \frac{\alpha}{1-\alpha} < \beta\frac{1-\phi}{\lambda} + (1-\beta)\frac{1-\tilde{\phi}}{\tilde{\lambda}}$$

A.4 The Solution Algorithm

The model is solved for periods $t = 0..T$. For $t > T$, all variables are assumed to be at their asymptotic balanced growth path levels. The solution algorithm for this perfect-foresight model can be described as follows:

(i) guess $\hat{q}$ for periods $t = 0..T$;

(ii) for each of these periods, find

  a. an index of output resulting from $\hat{q}$,

  b. R&D intensity, using the initial value for period 0 and the R&D technology (6),

  c. net profits as a share of GDP, using R&D intensity,

  d. the interest rate according to (2), resulting from consumption growth (i.e. the growth of output net of R&D expenditure),

  e. discounted future profits, using period profits and interest rates for all future periods,

  f. imitation costs, using the fact that they are proportional to discounted future profits in the first period (because (13) is binding) and grow at a rate $\frac{1}{\tilde{\lambda}}\hat{\hat{q}} + \frac{1-\tilde{\phi}}{\tilde{\lambda}}\hat{q}_j + \hat{w}$ thereafter;

(iii) check whether the ratio of imitation costs to discounted future profits is the same for every period; if this is the case $\hat{q}$ was chosen such that (13) is binding every period and we are done; if not, update the guess of $\hat{q}$ and start over with (ii).