A note on NIG-Levy process in asset price modeling: case of Estonian companies

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Abstract. The purpose of this note is two folds. First, to correct mistakes relating to terminology and analysis of results in Teneng [7]. Second, to extend results by showing returns of companies trading on Tallinn Stock Exchange between 01 January 2008 and 01 January 2012 cannot be modeled by NIG distribution; both in cases where closing prices can and cannot be modeled by NIG distribution. Thus, the NIG-Levy process cannot be used to forecast the future prices of these assets.

Keywords: NIG, Levy process, Jumps, forecasting, goodness of fits.

JEL Classification: C51, C44
AMS Classification: 60G50, 60G15

1 Introduction

Of late, many processes have been suggested to replicate price trajectories. Famous among is Brownian motion, known to have serious modeling limitations like light tails, inability to effectively capture and model jumps, stochastic volatility and a host of others [1, 5].

Levy process based models, of which Brownian motion is a special case, seek to eliminate shortcomings of Brownian motion based models. They clearly can distinguish between large and small jumps, and do not necessarily have continuous paths [4]. Two major classes of Levy processes are of interest here: Jump diffusion and infinite activity Levy processes. For jump diffusion cases, jumps are considered rare events and in any finite interval, there are only finitely many jumps. Opposite case applies to infinite activity models i.e. in any finite interval, there are infinitely many jumps. Infinite activity models are our focus, and particularly the NIG case. NIG case was implemented with daily closing data of some companies trading on Tallinn Stock Exchange in [7] and extended for their returns in this note.

The Black-Scholes model makes use of the exponential of Brownian motion with drift [1, 4, 5]. This ensures its probability distribution is lognormal. This is especially interesting when considering log returns which are not the case in this note. I work simply with one period returns. One period returns have little difference with log returns but for long time scales [3]. As well, lognormal model has continuous paths but cannot model jumps; one of the reasons why Levy process based models are attractive here.

This note is intended as a complement to [7]. The theory and model selection procedure are the same, but applied to return data here. I fit one period returns of companies trading on the Tallinn Stock Exchange between 01 January 2008 and 01 January 2012 with the NIG distribution and select good models following the procedure in [7]. I observe that none of these returns can be appropriately modeled with NIG-distribution; a pre-condition for forecasting with NIG-Levy process [4]. Hence, NIG-Levy process cannot be used to forecast the closing prices of any of these assets.

Path properties of Levy processes are studied in [4]. NIG-Levy process has been demonstrated to model and forecast closing prices of indexes [6]. This clearly confirms that classes of asset returns exhibit unique characteristics even though most exhibit general stylized facts [2, 3]. This is particularly important as most practitioners classify stock market data as financial data and apply general set of modeling criteria without considering the uniqueness of each asset class. Failing to model and forecast stock prices is a clear indication that the returns of stocks (companies) exhibit unique stylized facts which can differentiate them from returns of other asset classes.

Section two recaps the definition of NIG- Levy process and outlines model selection strategy. Corrections to [7] and results of modeling returns with NIG-distribution are the subject of section 3.

2 NIG Levy process and model selection strategy

Definition 1 (Levy process). *A real valued stochastic process $X_t = X(t,\omega), t \geq 0, \omega \epsilon \Omega$ defined on a probability space (Ω, F, P) is called a one dimensional Levy process if it satisfies the conditions: $X_0 = 0$ a.s., X has inde-*

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pendent and stationary increments with stochastically continuous paths, and X has paths continuous from the right with left-sided limits. In fact, for any time t > s, the distribution of the increments $(X_t - X_s)$ depends only on the length of the interval (t-s) and $(X_t - X_s)$ is independent of $(X_u, u \leq s)$. Standard and linear Brownian motion, Poisson and compound Poisson processes are examples of Levy processes [4]

Definition 2 (NIG-Levy process). *NIG-Levy process with parameters $\alpha, \beta, \mu, \delta$ and condition $-\alpha < \beta < \alpha$ denoted $NIG(\alpha, \beta, \delta, \mu)$ can be defined as follows:*
Consider a bivariate Brownian motion (u_t, v_t) starting at point $(u, 0)$ and having constant drift vector (β, γ) with $\gamma > 0$ and let z denote the time at which v_t hits the line $v = \delta > 0$ for the first time (u_t, v_t are assumed independent). Then letting $\alpha = \sqrt{\beta^2 + \gamma^2}$, the law of u_z is $NIG(\alpha, \beta, \delta, \mu)$ [4]. This distribution has probability density of the form

$$f_{NIG}(x:\alpha,\beta,\delta,\mu) = \frac{\alpha\delta}{\pi} e^{\left(\delta(\alpha^2-\beta^2)^{1/2}-\beta(x-\mu)\right)} \frac{K_1\left(\alpha(\delta^2+(x-\mu)^2)^{1/2}\right)}{(\delta^2-(x-\mu)^2)^{1/2}} \tag{1}$$

where $K_1(x) = \frac{x}{4}\int_0^\infty e^{\left(t+\frac{y^2}{4t}\right)} t^{-2}\, dt$ is a modified Bessel function of the third kind. $\delta > 0$ is scale, $\beta \geq 0$ is symmetry, μ is location and $\alpha > 0$ is tail heaviness.

Modeling with NIG-distribution entails estimating the parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{\mu})$ of the NIG-distribution and performing classical goodness of fits test. This is just density modeling i.e. checking whether empirical and theoretical distribution functions are comparable using any known classical goodness of fits test like Kolmogorov-Smirnov. Our interest in this note is on returns. This is particular because if returns can be modeled with NIG-distribution, then we can make accurate forecast of future prices by drawing return values from the probability distribution of returns. These drawn return values can be added to the stock price. With these, we can compare forecasted price with known price and check RMSE values both in-sample and out of sample. This will equip us with other forecasting method. Our results can then be compared to standard Random Walk. Below, we outline model selection strategy.

Käärik and Umbleja proposed method for selecting best models

1. choose a suitable class of distributions (using general or prior information about the specific data) ;
2. estimate the parameters (by finding maximum likelihoods);
3. estimate goodness of fit;
   a) visual estimation,
   b) classical goodness-of-fit tests (Kolmogorov-Smirnov, chi-squared with equiprobable classes),
   c) probability or quantile-quantile plots.

3 Corrections to [7] and returns modeling

3.1 Corrections

In [7], the closing prices of assets trading on the Tallinn stock exchange from 01 January 2008 to 01 January 2012 were fitted with the normal inverse Gaussian (NIG) distribution[2]. Interpretation of results concluded Baltika and Ekpress Grupp were suitable candidates for NIG-Levy asset model. Unfortunately, there were mistakes in terminology and analysis. First correction deals with the definition of general Levy process i.e. the independence criteria (pg. 2). It is suppose to read

$$for\ all\ s,t \geq 0, X_{s+t} - X_s\ is\ independent\ of\ X_u, u \leq t \tag{1}$$

i.e. independent increments. Second correction is related to analysis of data (Pg. 4). From Table 1 in this note, we can clearly see that Ekpress Grupp has a very small Kolmogorov-Smirnov (KS) test p-value (0.012). This means there is no significance at five percent level and we reject it. As well, the graphs were not very clear and I have included an updated version (Figure 1) to display correctly the goodness of fits. Thus, following model selection procedure outlined above, only Baltika can have its closing prices modeled with NIG-distribution, despite my being able to estimate NIG-distribution parameters for it and other companies.

[2] See [6, 7] for exposition on NIG distribution.

Company	alpha	beta	dealt	mu	skew	kurtosis	KS p	KS d
Arco Vara	468.9	468.86	0.03	0.02	0.38	-1.53	$<10^{-5}$	0.23
Baltika	7.06	6.62	0.22	0.52	1.67	-1.53	0.06	0.06
Ekpress Grupp	2.68	2.15	0.49	0.85	1.70	2.53	0.012	0.07
Harju Elekter	3.2	-2.07	0.72	2.95	-0.82	-0.05	0.0003	0.09

Table 1 Estimated NIG Parameters, Skews, Kurtoses, KS test results for NIG models (Daily closing prices)

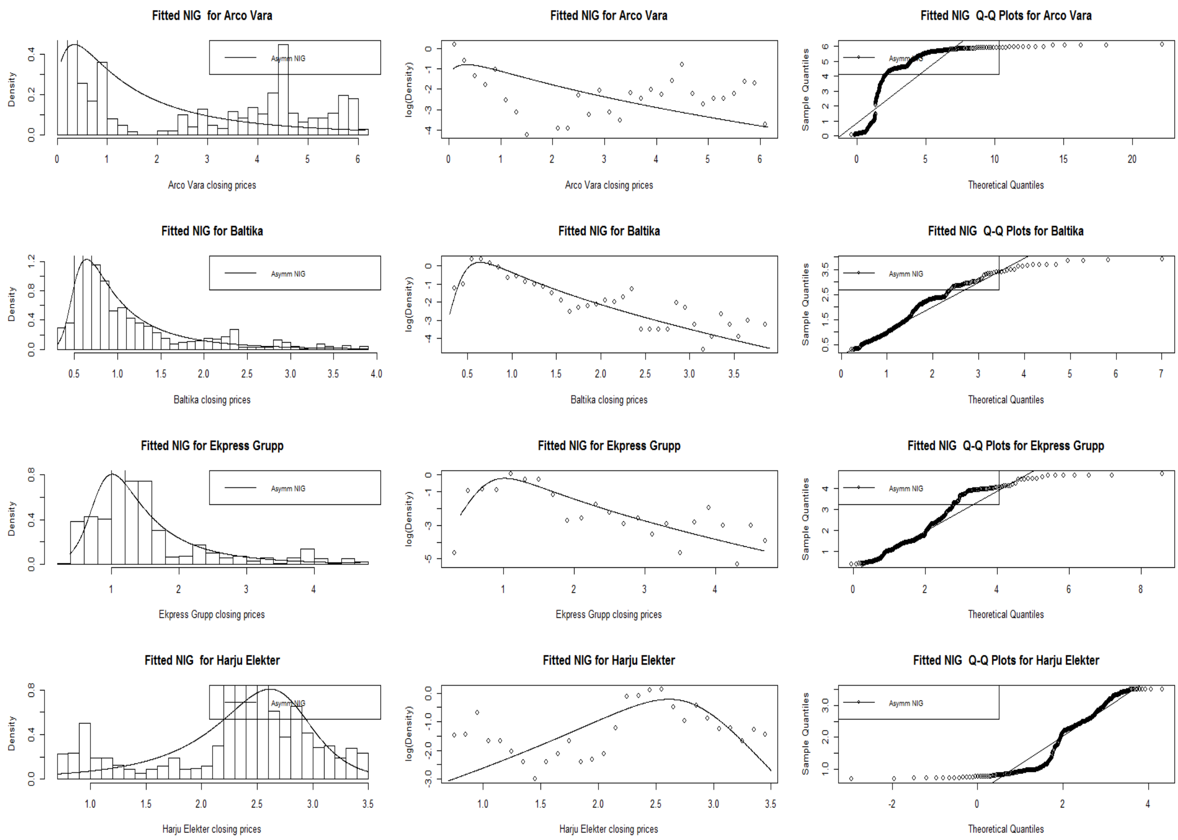

Figure 1 Fitted NIG density, log densities and Q-Q plots for Baltika, Arco Vara, Harju Elekter and Ekpress Grupp (Daily closing prices)

3.2 Modeling returns

For closing prices to be forecast-able with NIG-Levy process, the returns should be NIG distributed [4]. In this note, one period returns are fitted with NIG-distribution and classical goodness of fits test performed to validate the model. I start by looking at skews and kurtosis which clearly suggest using a heavy tail distribution. Then estimating NIG-distribution parameters from return data (i.e. alpha, beta, delta, mu). Having these estimated parameters seem to suggest distribution is a candidate for good model. Looking at plots (Figure 2) especially log density plots, they are not good fits. Classical goodness of fits test considering five percent level of significance also confirm bad model. Results of these are displayed in Table 2 and plots of these are displayed in Figure 2 (only four of them could be plotted due to poor estimates of parameters). Following procedure outlined above, I reject all return models. This is despite result above that closing prices of Baltika can be modeled with NIG-distribution. Hence, these companies cannot have their returns modeled with NIG-distribution. Not being able to have the returns modeled with NIG-distribution means we cannot predict future asset price values with NIG-Levy process [4]. In other words, the precondition of forecasting with NIG-Levy process (i.e. asset returns being NIG-distributed) is not met by any of these companies trading on Tallinn Stock Exchange between 01 January 2008 and 01 January 2012.

Company	alpha	beta	delta	mu	skew	kurtosis	KS p	KS d
Arco Vara	0.96	0.756	0.015	-0.0017	31.47	994.45	$<10^{-5}$	0.1433
Baltika	20.923	0.858	0.032	-0.003	-0.06	2.07	0.002	0.082
Ekpress Grupp	15.59	0.7032	0.024	-0.002	0.253	2.308	0.0002	0.095
Harju Elekter	14.32	0.774	0.015	-0.0008	0.247	3.098	$<10^{-5}$	0.128
Viisnurk	6.001	0.227	0.011	$7.5e^{-05}$	-0.168	4.05	$<10^{-5}$	0.18
Olympic Entertainment Grupp	19.58	1.565	0.238	-0.003	0.017	4.52	0.006	0.0762
Silvano Fash-ion Grupp	9.203	0.516	0.0189	-0.0006	0.195	4.19	$<10^{-5}$	0.1088
Tallink	22.19	2.27	0.02	-0.0022	0.817	5.7	$<10^{-5}$	0.1385
Tallinna Kaubamaja	21.17	-0.384	0.007	-0.0004	-0.844	10.42	$<10^{-5}$	0.1108
Trigon	1.097	-1.097	$5.5e^{-12}$	$6.03e^{-21}$	0.28	7.45	$<10^{-5}$	0.3145
Nordecon	24.34	2.48	0.025	-0.0037	0.13	2.053	0.002	0.0841

Table 2 Estimated NIG Parameters, Skews, Kurtoses, KS test results for NIG distribution models (returns)

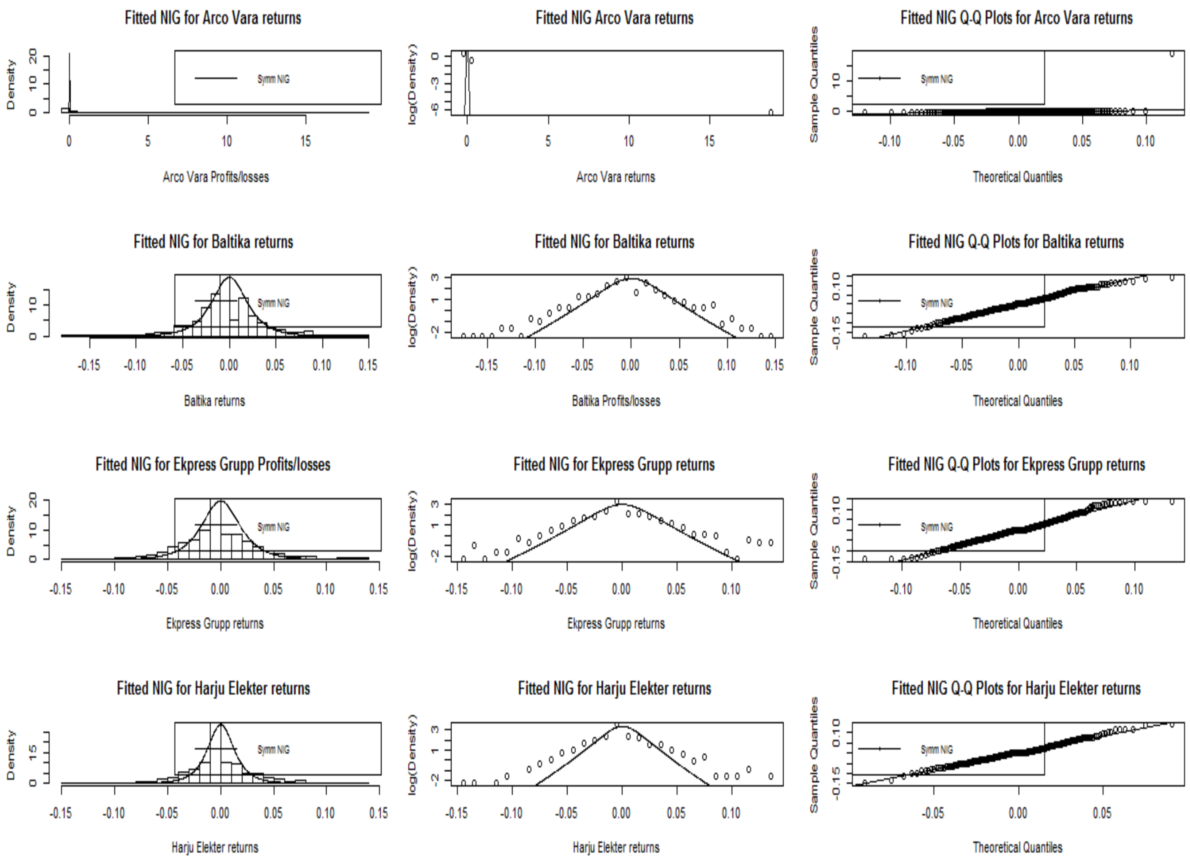

Figure 2 Fitted NIG density, log densities and Q-Q plots for Baltika, Arco Vara, Harju Elekter and Ekpress Grupp (returns)

4 Conclusion

Results of Teneng [7] have been updated to conclude that closing prices of Baltika (company trading on the Tallinn Stock Exchange between 01 January 2008 and 01 January 2012) can be modeled with NIG distribution, but its returns cannot. Other companies considered can have neither their closing prices nor their returns modeled by NIG-distribution. These lead to the conclusion that none of these companies can have their future prices forecasted with NIG-Levy process. This clearly confirms that classes of asset returns exhibit unique characteristics even though most exhibit general stylized facts. This is particularly important as most practitioners classify stock market data as financial data and apply general set of modeling criteria without considering the uniqueness of each asset class. Failing to model and forecast stock prices is a clear indication that the returns of stocks (companies) exhibit unique stylized facts which can differentiate them from returns of other asset classes.

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