Satisficing decision procedure and optimal consumption-leisure choice

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Satisficing Decision Procedure and Optimal Consumption-Leisure Choice.

Abstract
The paper argues that when a consumer searches for a lower price, a satisficing decision procedure equalizes marginal costs of search with its marginal benefit. The consumer can maximize the utility of his consumption-leisure choice with regard to the equality of marginal values of search. Therefore, the satisficing decision procedure results in the optimizing consumer behavior. The paper formulates the analytical relationship between savings on purchase, willingness to pay, and the time horizon of the consumption-leisure choice, which can be verified by careful psychological field studies.

JEL Classification: D11, D83.

Introduction
The discussion between the search-satisficing concept and the neoclassical paradigm has a long story. In 1957 H.Simon revived the Scottish word satisficing to denote decision making “that sets an aspiration level, searches until an alternative is found that is satisfactory by the aspiration level criterion, and selects that alternative”. The confrontation between two approaches had reached its peak in 1977 when H.Simon presented his Richard T. Ely Lecture. Then, the discussion went into decline, but from time to time researchers in different fields animated it (see for example Slote (1989), Schwartz et al. (2002), Fellner et al. (2006)). As a result, the theory of consumer behavior has accepted the strict distinction between “maximizers” and “satisficers” (Lewer et al. (2009)). Unfortunately, opponents have forgot the fact that H.Simon himself paid attention to the possibility of matching the satisficing and optimizing procedures. In 1972 he wrote:

“A satisficing decision procedure can be often turned into a procedure for optimizing by introducing a rule for optimal amount of search, or, what amounts to the same thing, a rule for fixing the aspiration level optimally.” (Simon (1972), p.170)

This note tries to restore the methodological equilibrium. The rule for optimal amount of search is derived from the reserve maximization model, which emphasizes the role of the need to save for daily expenses and purchases (Malakhov (2011b)). This paper shows how a satisficing
decision procedure results in an optimal search-stopping rule and in an optimal consumption-leisure choice.

**Satisficing Price Decision and Optimal Search-Stopping Rule**

Let us start with the famous distinction between an optimizing model and a satisficing model. In 1978 H. Simon wrote:

“In an optimizing model, the correct point of termination is found by equating the marginal cost of search with the (expected) marginal improvement in the set of alternatives. In a satisficing model, search terminates when the best offer exceeds an aspiration level that itself adjusts gradually to the value of the offers received so far” (Simon (1978, p.10)).

Suppose a consumer who reserves the labor income $wL_0$ for the purchase of an item $Q = 1$. He begins to search for a cheaper price from the starting price of the search $P_S > wL_0$ and he concludes the search at the satisficing purchase price $P_P < wL_0$.

Let us analyze the intersection of two curves, the expenditures $P(S)$ curve and the labor income $wL(S)$ curve, where $T$ is the time horizon of the consumption-leisure choice, the value $w \times \partial L/\partial S$ is negative ($w \times \partial L/\partial S < 0$), because the best offer $P_P$ exceeds the aspiration level $wL_0$ and the value $\partial P/\partial S$, which is exposed at the moment of purchase by the tangent dotted line, is also negative, now due to the diminishing marginal efficiency of the search $S$ (Fig.1):

![Fig.1](image1.png)

If the value $P_P$ is equal to the *disposable* labor income $wL(S)$ at the moment of purchase, the slope ($-w$) gives us the value of the labor time $L$ on horizontal axis. However, it also gives us the value $P_0$ on the vertical axis, which is equal to the *potential* labor income (Fig.2), or

$$P_0 = w(L+S) \quad (1)$$

![Fig.2](image2.png)
The Fig. 2 shows that the absolute value of the decrease in the potential labor income at the moment of purchase is greater than the absolute value of the decrease in the disposable labor income, or \( w > |w \times \partial L/\partial S| \). This consideration attracts attention to the core function \( L(S) \). Indeed, when we take the values \( \partial P/\partial S < 0, \partial^2 P/\partial S^2 > 0 \), we simply follow the assumption of the diminishing marginal efficiency of the search. However, the behavior of the \( L(S) \) function is not so clear.

When the search \( S \) “squeezes out” the labor \( L \) and the leisure \( H \) from the time horizon \( T \), the \( \partial L/\partial S \) rate directly depends on the value \( \partial H/\partial S \). However, the value \( \partial H/\partial S \) can be determined by a very simple rule. If we take the differential \( dH(S) \), we can see that the absolute rate of the decrease in leisure time is equal to its share in the time horizon, or \( |\partial H/\partial S| = H/T \) and \( H/T = - \partial H/\partial S \). From here we get the value of the propensity to search \( \partial L/\partial S \). It is negative because the labor and the search represent alternative sources of income. We can also get its derivatives, which are very important for our analysis:

\[
L(S) = T - H(S) - S; \\
\partial L/\partial S = -\partial H/\partial S - 1; \\
dH(S) = dS \frac{\partial H}{\partial S} = -dS \frac{H}{T} \\
\frac{\partial L}{\partial S} = \frac{\partial H}{\partial S} = 1 - \frac{H}{T} = 1 - \frac{H - T}{T} = -\frac{L + S}{T} \\
\partial L/\partial S = -\frac{L + S}{T} \Rightarrow \partial^2 L/\partial S^2 = -\frac{\partial L/\partial S + 1}{T} < 0 \text{ } (2)
\]

If at the moment of purchase the marginal loss in labor income is equal to the marginal benefit of the search, we have:

\[\frac{\partial L}{\partial S} = -\frac{L + S}{T} \Rightarrow \partial^2 L/\partial S^2 = -\frac{\partial L/\partial S + 1}{T} < 0 \]

\[1\text{ Here, the value of the propensity to search is limited by } -1 < \partial L/\partial S < 0. \text{ When the propensity to search becomes } \partial L/\partial S < -1, \text{ the value } \partial H/\partial S \text{ becomes positive. The ‘price of time’ } \mu = Q \times \partial P/\partial S \text{ (see Aguiar and Hurst 2007, p.1536), here the value } w \times \partial L/\partial S \text{, becomes greater than the wage rate. The marginal value of leisure } MU_H \text{ becomes negative and it produces the Veblen effect (Malakhov 2012b).} \]
\[
\frac{\partial P}{\partial S} = w \frac{\partial L}{\partial S} = w \frac{H - T}{T} = -w \frac{L + S}{T} \quad (3)
\]

If we re-arrange the Equations (1) and (3), we get that the value of potential labor income is equal to the value of the time horizon times the absolute value of the price reduction at the moment of purchase (Fig.3), or

\[
P_0 = -T \times \frac{\partial P}{\partial S} = w \times (L + S) \quad (4)
\]

Now we can proceed to the indirect proof of the correspondence of the optimal search-stopping rule to the satisficing choice.

Let us take the “apple of discord” between psychologists and economists and to presuppose that at the moment of purchase the absolute value of the marginal decrease in the labor income for the given wage rate \(w\) is still less than the marginal benefit of the search, or

\[
w \frac{\partial L}{\partial S} < \frac{\partial P}{\partial S} \quad (5)
\]

Here, the optimizing approach requires continuing the search. However, the Equation (4) tells us that this case should result in the hypothetical value \(P'_0\), where, for the given wage rate \(w\):

\[
P'_0 = w(L' + S') < P_0 = w(L + S) \quad (6)
\]

Due to the rule \(\frac{\partial^2 P}{\partial S^2} < 0\), the inequality \((L' + S') < (L + S)\) produces the following inequalities: \(L' > L\) and \(S' < S\). And we can see that our assumption is false, because either the hypothetical amount of search \(S'\) results in a greater purchase price and it should be less than the actual amount of search \(S\), or the actual amount of search \(S\) should result in the value \(P' < P_p\).

We can graphically confirm these considerations, if we take the \([P'_0, L']\) line, which also has the \((-w)\) slope (Fig.4):

\[
\text{Fig.4}
\]
The same indirect proof can be used when it is supposed that at the purchase price level the marginal costs of search are decreasing already faster than its marginal benefit. The only difference is that this case can be eliminated from the analysis by definition, because it requires recognition that the chosen price is not satisficing. Indeed, we can reproduce the set of inequalities, which describe the dissatisfying choice, when a high price corresponds to unexpectedly low savings on purchase:

\[
\frac{w \partial L}{\partial S} > \frac{\partial P}{\partial S} \quad (7)
\]

\[
dS \left| \frac{w \partial L}{\partial S} \right| > dS \left| \frac{\partial P}{\partial S} \right| \quad (8)
\]

\[
dwL(S) > dP(S)
\]

Now we can say that \textit{when the consumer chooses the satisficing price, his decision automatically equalizes the marginal loss in the labor income with the marginal benefit of the search.}

The Fig.4 provides us with another interesting consideration. Let us pay attention to the situation when the same amount of search \( S \) results in a price \( P' < P_p \), i.e., when the best offer, an unexpected price discount, for example, significantly exceeds the aspiration level – the case that really challenges the optimizing approach. Here we realize that the absolute value of the actual price reduction \( |\partial P/\partial S| \) is greater than its planned value \( |\Delta P/\Delta S| \). It seems that if the consumer accepts this price, he doesn’t equalize marginal costs of the search \( |w \times \partial L/\partial S| \) with its marginal benefit \( |\partial P/\partial S| \).

However, this decision changes not only the value of the marginal benefit of the search but it also changes both the propensity to search and the marginal loss in the labor income. It happens because the choice of the lower price decreases, as Fig.4 shows and the Equation (1) proves it, the value \( T \) of the time horizon of the consumption-leisure choice.

The time horizon of the consumption-leisure choice, i.e., \textit{the time to next purchase}, depends on products’ lifecycles. The lower price can exhibit the coming expiration date for pork sausages, for example.
If we remember the Friedman’s metaphor, we should say that billiards is played by two people. The seller doesn’t bother about consumer’s marginal values of search, but he either cut the price for yesterday’s “fresh” sausages, or he offers packed pork sausages with extended shelf life. In addition, if the consumer buys yesterday’s “fresh” sausages, he should quickly eat them, i.e., to cut leisure time $H$, reserved for the consumption.

If the consumer doesn’t accept this lower price because he estimates it as too high price for the shorter shelf life, we meet again inequalities (7) and (8) of the dissatisfying choice. So, the producer should support the shorter shelf life by the corresponding price discount $|P' - P|$(Fig.5):

![Diagram](image)

Now, the new search path $P'(S)$ meets the decrease in the labor income $wL'(S)$ at the point $(S;P')$. The $P'(S)$ curve becomes steeper due to the price discount and the $wL'(S)$ curve becomes steeper due to the shorten time horizon $T'$. So, for the same amount of the search $S$ we have

$$\frac{\partial P'}{\partial S} = w \frac{\partial L'}{\partial S} = -w \frac{L' + S}{T'} \quad (9)$$

The time horizon $T'$ is cut here not only by the decrease in the labor time $L$ but also by the decrease in time of consumption, i.e., leisure time $H$, or $\Delta T = \Delta L + \Delta H$. So, the absolute value of the propensity to search $|\partial L/\partial S|$ becomes greater. In addition, the $[P_b; T]$ line also becomes steeper than the initial $[P; T]$ line because the absolute value of the equilibrium price reduction $|\partial P/\partial S|$ becomes greater.

By the way, the Fig.5 explains why sellers of high-quality products with long-term lifecycles and guarantees can leave the market because, like it happens with ‘lemons’, sellers of low-quality products without guarantees or with short “shelf lives” reduce prices.

This example provides us with the deeper understanding of the phenomenon of the visual disparity between the satisficing approach and the optimizing model. Let us come back to the “apple of discord”, i.e., to the inequality of marginal values of search, or $|w \times \partial L/\partial S| < |\partial P/\partial S|$ and, as a result, to the inequality $w(L + S) < P_0$. 
According to J. Stiglitz, only high-search costs individuals visit high-price stores (Stiglitz 1979). That assumption implicitly envisaged individuals with high wage rates. However, there are many situations when individuals with low wage rates make shopping in high-price stores. If we analyze the value of propensity to search $\partial L/\partial S$, we can see that its absolute value $|\partial L/\partial S|$ can be raised to the level of the equality with the absolute value of price reduction $|\partial P/\partial S|$ in two ways. One way is to search more intensively and to increase the $(L + S)$ value with the help of the $\partial^2 L/\partial S^2 < 0$ rate. But there is another way. If the individual cuts the time horizon $T$, he also increases the absolute value of the marginal costs of search $|w \times \partial L/\partial S|$. Why the decrease in the time horizon happens here?

Let us imagine two low-wage rate individuals who enter the high-price suit store. The first visitor does not accept high prices while the second visitor makes the purchase. We understand that they have different reservation prices. The first consumer decides either to search for a lower price or to wait sales. And the second consumer is more impatient. He accepts the high price, which is **satisficing** for him. His impatience increases the reservation level and it also **hastens the moment of consumption**. Moreover, the impatience intensifies the consumption. When the first visitor finally buys the chosen suit at a lower price at sales he will put it on occasionally. However, when the impatient consumer buys the high-price suit he will often wear it because he likes it more than his friend.

We can check all these considerations by the equation (3). The both visitors have the same potential labor income $w(L + S)$. But they have different time horizons because they have either different intensities of consumption $Q/H$ or different attitudes to fashion products. The first consumer can wait for sales while the second cannot. On the other hand, when the impatient consumer intends to wear new fashion suit often and in current season, he decreases both the total time of its consumption, i.e., the leisure time, and the time horizon. Evidently, in this case the suit either becomes shabby soon or it goes out of fashion. But the decrease in the time horizon increases the absolute value of the marginal costs of search $|w \times \partial L/\partial S|$. And this new value of $|w \times \partial L/\partial S|$ meets the high value of the price reduction $|\partial P/\partial S|$. **The high price becomes satisficing for the impatient consumer** while it stays above the reservation level of the second consumer, who does not equalize marginal costs of search with its marginal benefit and who is still followed by the inequality $|w \times \partial L/\partial S| < |\partial P/\partial S|$

This example shows how the fashion can increase the reservation level and at the same time it cuts the “shelf life” of the chosen item. However, there are markets where all consumers are impatient as well as intensive in consumption. The fish market represents a good example of this kind of the total impatience.
General relationship between savings on purchase, the time horizon of the consumption-leisure choice, and the potential labor income

Our analysis discovers the general relationship between savings on purchases \( \Delta P/\Delta S \), the time horizon of the consumption-leisure choice, and the potential labor income:

\[
-\frac{\Delta P}{\Delta S} = \frac{P}{T} \tag{10}
\]

This relationship can be illustrated by the paradox of little pre-purchase search for big-ticket items. In 1994 Grewal and Marmorstein wrote:

“Previous studies have consistently found that most consumers undertake relatively little pre-purchase search for durable goods and do even less price-comparison shopping... (when) prices of the more expensive products tend to exhibit the greatest variation across stores. Given the aforementioned evidence regarding the price variation of big-ticket items, it appears that many consumers engage in considerably less price search than is predicted by the economics-of-information theory.”

Indeed, in 1979 Kapteyn et al. had demonstrated that purchase decisions concerned durables had been satisficing rather than maximizing (Kapteyn et al. 1979). R. Thaler (1987) documented that anomaly in the following manner:

“One application of marginal analysis is optimal search. Search for the lowest price should continue until the expected marginal gain equals the value of the search costs. This is likely to be violated if the context of the search influences the perception of the value of the savings. In Thaler (1980), I argued that individuals were more likely to spend 20 minutes to save $5 on the purchase of a clock radio than to save the same amount on the purchase of a $500 television.”

We can check the results of that experiment in order to show that there was no anomaly and that that case did not conflict with the marginal approach.

Suppose an individual who is ready to give up 10 hours of leisure to get (i.e., to work and to search for) a big-ticket item \( Q_{bti} \) and only 2 hours of leisure to get a cheap item \( Q_{ci} \). We can substitute the marginal \( \partial P/\partial S \) value by the \( \Delta P/\Delta S \) value, which is much easier for our individual to plan and to compare with the value of the wage rate \( w \). If we take the value \( \Delta P \) as the constant for both items and, when \( S_0 = 0 \); \( \Delta S = S \), we have:

\[
\frac{\partial P}{\partial S} = \frac{\Delta P}{\Delta S} = w \frac{\partial L}{\partial S} = w \frac{H - T}{T} = -w \frac{L + S}{T},
\]

\[
\Delta P = -w \frac{L_{bti} + S_{bti}}{T} \Delta S_{bti} = -w \frac{L_{ci} + S_{ci}}{T} \Delta S_{ci}; \quad \tag{11}
\]

\[
\Delta P = -w \frac{10}{T} \Delta S_{bti} = -w \frac{2}{T} \Delta S_{ci};
\]

\[
5 \Delta S_{bti} = \Delta S_{ci}.
\]
When the individual finally makes these both purchases, he realizes that he has spent five times more on the search for the cheap item than on the search for the big-ticket item.

**Conclusion**

Finally, we should pay attention to the price equivalent of the potential labor income. Obviously, it represents some willingness to pay, but it might also correspond to the monopoly price from the P. Diamond’s paradox (Diamond 1971). Let us suppose that a consumer has no liquidity constraint, for example, due to his strong precautionary motive (Carroll 2001). Then, we can solve the static optimization problem of his consumption-leisure choice not with regard to the budget constraint but with regard to the equality of the marginal values of search, or

$$\Lambda = U(Q, H) + \lambda (w - Q \frac{\partial P}{\partial L} \frac{\partial S}{\partial S})$$  (12)

If we take the value $\frac{\partial P}{\partial S}$ as the value given by a particular local market, either by a convenient store or a supermarket, we can solve the optimization problem $U(Q, H)$ of the consumption-leisure choice with regard to this particular market. And the solution of this optimization problem gives us another illustration to the value of the potential labor income:

$$\frac{\partial U}{\partial H} \frac{\partial U}{\partial Q} = \text{MRS}(H \text{ for } Q) = -\frac{w}{\frac{\partial P}{\partial S}} \frac{\partial L}{\partial S} - \frac{w}{T \times \frac{\partial P}{\partial S}} \frac{w}{P}$$  (13)

The MRS $(H \text{ for } Q)$ is determined not by the purchase price but by the price equivalent of the potential labor income. The latter represents the trade-off between price reduction at the moment of purchase and the time horizon of the consumption-leisure choice. As we have tried to demonstrate, this trade-off can adjust marginal values of the search to any choice when it is really satisficing. And this trade-off, as we can see, optimizes the consumption-leisure choice.²

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² The concept of the potential labor income $w(L + S)$ has another interpretation. The Eq. (13) tells us that it corresponds to the equilibrium price. Indeed, while the monetary loss during the search is equal to the value $dS*w^*\frac{\partial L}{\partial S}$, the costs of search or transaction costs are equal to the value $dS^*w$. So, the equilibrium price includes both labor costs and transaction costs.
Related Literature


