A Search for Business Cycles with Spectral Analysis

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I. INTRODUCTION

In recent years, Army enlistment rates have been highly correlated with unemployment rates, so the condition of the economy is a topic of great interest to the Army.[1] Unfortunately the present state of knowledge makes it extremely difficult to project the course of the economy, especially at turning points. This paper describes the use of the well-established technique of spectral analysis to search for underlying periodicities in economic activity.

There has been considerable debate in recent years about whether well-defined business cycles even exist. At one extreme are those who assert that so-called business cycles are caused by exogenous shocks. Proponents of this view cite the work of Adelman and Adelman (1959), McCulloch (1975), and Adelman (1960, 1965).

At the other extreme from the 'random shock' school of business cycles are those who believe that business cycles may be endogenous, i.e., the economy has a natural tendency to go through periods of recession and depression. Recent theoretical justification of this view has been given by Grandmont (1983a, b), Day (1982), and Varian (1979).

Between these extreme points of view about the existence or nonexistence of business cycles there are a myriad of theories. It is a difficult task simply to formulate testable hypotheses to enable comparisons to be made among the various alternatives,[2] so comparatively little empirical research has been done in the area of economic fluctuations.[3] We use spectral analysis in this paper because it is a widely accepted technique, and it allows us to try to replicate some of the earlier work that found some evidence for the existence of business cycles.

II. SPECTRAL ANALYSIS

The technique of spectral analysis was used by Howrey (1968) and more recently by Haustein and Neuwirth (1982). Paradoxically, both those reports found evidence for business cycles less than ten years in length, but their original objective was to search for cycles of longer duration. We designed the present work specifically to test for short term cycles, which are within the time horizon of most Army budget planners.

We will briefly describe here the technique of spectral analysis because with a few exceptions economists have not used the method very much over the past decade, so many researchers may be unfamiliar with it. Also, the discussion will clarify our terminology, which can vary widely.

To use spectral analysis requires that a data series be stationary. A time series is stationary if the mean and variance are finite and constant, and if the autocorrelation between values of the process at two time periods depends only on the distance between the two time points and not on the time period itself.

Most economic series of any interest are not stationary. GNP, for example, not only does not stay constant over time, but the variance tends to increase as the level increases. A first step in creating a stationary series is to stabilize the variance, frequently by using a logarithmic transformation (see Vandaele (1978)).

A second step in creating a stationary series is to stabilize the mean. One way to try to do this is to take first differences of the series. This has the advantage that first differences of logarithms may be interpreted as percentage changes (Nelson (1973), pg. 59).

Once a time series has been made stationary, it can always be put into the form (Granger and Newbold (1977)):

$$X_t = \int_0^{\pi} \cos \omega t \; du(\omega) - \int_0^{\pi} \sin \omega t \; dv(\omega) \qquad (2.1)$$

where the components of the integrals are uncorrelated. Intuitively it is clear that a stationary series can be put into a form which is a combination of an infinite number of trigonometric frequency (i.e., reciprocal of the period) components.

If we now define the complex random variable

$$dz(\omega) = \tfrac{1}{2}\{du(\omega) + i \; dv(\omega)\} \qquad (2.2)$$

then (2.1) may be rewritten in exponential form

$$X_t = \int_{-\pi}^{\pi} e^{it\omega} \; dz(\omega). \qquad (2.3)$$

Similarly, we may define the covariance sequence of the series, and write

$$\lambda_\tau = \text{cov}(X_t, X_{t-\tau}) = \int_{-\pi}^{\pi} e^{it\omega} \; s(\omega) d\omega. \qquad (2.4)$$

Equations (2.3) and (2.4) are called the spectral representation of the series and the covariance of the series. We may interpret $s(\omega)d\omega$, (the 'power spectrum') as the contribution to the variance of X_t attributable to the component $X_t(\omega)$ of X_t with frequencies in the range $(\omega, \omega + d\omega)$. Thus, a plot of the power spectrum versus frequency will show which frequencies make the greatest contribution to the variance of the underlying time-series. Those frequencies are sometimes interpreted as the 'hidden periodicities' in the series.

One problem that arises in practical applications is that in practice it is only possible to obtain finite lengths of records. It is thus necessary to estimate the average power in a finite band of frequencies. It can be shown, however, that the variance of the power increases as the width of the frequency band over which the power is measured is reduced. In practice the method of analysis used is called 'window closing.' A wide frequency bandwidth is used and the power spectrum is computed. The bandwidth is then narrowed and the new spectrum is examined for comparison purposes.

Several different shapes and sizes of windows have been suggested, but in practice changing the bandwidth ('window closing') is much more important than the shape of the window ('window carpentry').

Finally, the data series is sometimes subjected to a filtering process, so that a particular frequency range may be examined more closely. For example, some studies have looked for long cycles, which correspond to low frequencies. A business cycle with a period of 20 years corresponds to a frequency of 1/20, or .05 cycle/year. Conversely a short business cycle corresponds to a high frequency. A business cycle of 4 years means a frequency of 1/4, or .25 cycle/year.

Researchers who are looking for evidence of long cycles, or low frequency waves, sometimes take moving averages of their data. The averaging tends to smooth out the high frequency, short-term cycles and concentrates the power spectrum in the low frequency cycles. In the jargon of spectral analysis, a moving average acts as a 'low pass' filter, allowing primarily low frequency, long-period components into the power spectrum. Since we are interested in searching for high frequencies, not low frequencies, we did not use a moving average filter.

A filter of more interest to us is the differencing filter. Differencing helps remove trend, as we already noted, and it also helps remove low-frequency, long-period business cycle components. Thus a differencing filter acts as a 'high pass' filter, allowing us to concentrate on high frequency, relatively short duration cycles. Since differencing was used to make our time series stationary, no additional filtering was necessary.

All of the ideas discussed above are illustrated in the next section.

III. UNIVARIATE SPECTRA

Spectral densities for two economic series of interest, GNP and the M1 money supply, both in 1972 dollars, are shown in Figures 1 through 5. In both series we used first differences of the logarithms, to help remove trend and stabilize the variance of the original series.

The poor resolution of a very wide window length is apparent from a quick examination of Figures 1 and 3. There appears to be a peak on the GNP chart (Figure 1) between 10 and 20 quarters, but it is certainly questionable exactly where it is. Similarly, there appears to be a peak on the M1 money supply chart (Figure 3) somewhere between 40 and 70 months, but again it is not clear exactly where it is.

Narrowing the window lengths helps to increase the resolution of the charts. The 5-quarter GNP window length in Figure 2 shows two well-defined peaks, at 8.5 and 16 quarters. Similarly, the 3-month M1 window length in Figure 4 shows peaks at 36 months and 63 months. Converting the monthly money supply figures to quarterly figures and using a window length of 5 quarters (Figure 5) gives well-defined M1 peaks at 12 quarters and 21 quarters, in good agreement with the monthly data.

It would be very desirable to have a quantitative measure of whether or not the peaks in Figures 2, 4 and 5 are significant in a statistical sense. For this reason statistical tests have been used on the 'periodogram' of a time series, where the periodogram is defined in terms of squared values of cosine and sine components of the original time series (Granger and Newbold 1977, pg. 11). In practice periodograms sometimes exhibit spurious 'cycles', so they are no longer widely used.[4] Nevertheless some software packages contain statistical tests for whether or not a given periodogram was generated from a white noise series. Also, the individual frequency bands are assumed to obey exponential probability distributions. If the highest peak is statistically significant, then its corresponding frequency is said to be a 'hidden periodicity.'

Feller (1971, pg. 77) notes that

> 'The fallacy of this reasoning was exposed by R. A. Fisher ... who pointed out that the maximum among n independent observations does not obey the same probability distribution as each variable taken separately. The error of treating the worst case statistically as if it had been chosen at random is still common in medical statistics.'

Nevertheless, we will use the same approach here that economists usually take in spectral analysis, i.e., we will simply ignore these statistical problems and associate periodogram peaks with hidden periodicities.[5]

White noise tests corresponding to the underlying data series that were used for Figures 1 through 5 are shown in Table 1. The 21 quarter, 36 month and 63 month money supply cycles were significant, but none of the GNP cycles were significant in terms of the Fisher test.[6] We therefore associate only the money supply peaks we discussed earlier with hidden periodicities in the series.

IV. MULTIVARIATE SPECTRA

Spectral analysis frequently involves pure empiricism, i.e., no attempt is made to explain the nature of the hidden cycles. When the researcher has reason to suspect a cause and effect relationship between two series, however, he may use multivariate methods.

A vast body of literature has discussed the possible cause and effect relationship between GNP and the money supply (Sims (1972), Granger (1969), Gordon (1977)). We will examine the relationship between these variables by using the methods of cross-spectral analysis. If a relationship can be established, this would lend support to the existence of the GNP cycles that could not be verified statistically (Table 1).

To use cross-spectral methods requires that both series be individually stationary. They must also be jointly stationary, which means that the relationship between them is time invariant. The measured relationship between the series is variously called the 'coherence function,' or the 'coherency function,' or the 'coherence function squared,' or the 'coherency function squared.' For our purposes it is sufficient to note that we pick the term 'coherency squared,' since it is interpreted in exactly the same way as the square of a correlation coefficient (Granger and Newbold (1977, pg. 58)).

One complication in cross-spectral analysis is that totally unrelated series can exhibit a high degree of correlation (Nelson and Kang (1984)). One way to minimize this problem is by a process called 'prewhitening,' which removes the relationships 'within' each series so that analysis can be made of the relationships 'between' the series.

Prewhitening begins by identifying and estimating univariate Box-Jenkins models for each series (Dale and Bailey (1982)). These models are then used to transform each series to convert it into white noise. The coherence is then computed between the two white noise series.

One technical point must be mentioned here. In related methods, called Box-Jenkins transfer function techniques (Box and Jenkins (1976)), one series is prewhitened, and the same transformation is applied to the other series (Dale (1984)). Use of the same transformation keeps the 'phase relationship,' i.e., lag structure, unchanged between the series (Hannen (1970)). Our interest here, however, is merely to examine the relationship between GNP and the money supply, without trying to determine a lag structure, so we prewhitened each series separately.

Figure 6 shows the coherency squared between GNP and the money supply, before and after prewhitening the series. The prewhitening clearly removed a great deal of spurious coherence at high frequencies. There was high coherence at four of the nonzero low frequencies, corresponding to periods of 20.7, 10.4, 6.9, and 4.7 quarters. Note that this only means that there is coherence between the corresponding frequencies of those series, but it says nothing about the distance between their corresponding peaks and troughs. Determination of lag structures involves techniques that need not concern us here.

Comparison of Figure 6 and Table 1 does not give any clear support for the existence of the GNP cycles that were not statistically significant. We conclude that there have not been well-defined business cycles during the period January 1947 through September 1983.

V. CONCLUSIONS

Spectral analyses of real GNP and the real money supply M1 did not give any clear indications of the existence of business cycles (Table 1). We can draw one or more of the following conclusions:

1. Well defined business cycles simply do not exist. Our inability to replicate the work of others (Table 2) gives support to those who assert that business cycles are simply due to random shocks to the economy.

2. Well-defined business cycles may exist, but spectral analysis isn't very helpful in searching for them.

3. Business cycles may be naturally irregular in nature, in accordance with recently developed theories (Day (1982, 1983), Benhabib and Day (1981), Mensch (1981), Mandelbrot (1983), Jensen and Urban (1984)). In this view, the fact that we can't detect business cycles shouldn't bother us (Day (1983, pg. 201)). The present work is an early confirmation that theories of irregular business cycles may be correct, and that radical changes to the use of econometric models may be necessary.

4. Economic data is simply too imprecise to be usable in sophisticated time-series analyses. There is such a high correlation between adjacent values of economic series that spectra of many series look approximately the same.

The last conclusion may be the most significant, in the light of new, competing theories of business fluctuations. The sophistication of these new theories is clearly outrunning the ability of existing data bases to be used to choose between them.

FOOTNOTES

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[1]See Dale and Gilroy (1983a, b, 1984, 1985).

[2]It should be noted that some extremely anti-mathematical groups, such as the Austrian School (Rothbard (1975, 1976), Egger (1984)), or the "Econoclasts" (Henderson (1978), Schumaker (1973)), would deny that econometric techniques can ever be used either to confirm or disprove their theories.

[3]One notable exception to the lack of empirical research may be found in the work of Moore (1983).

[4]The terminology of spectral analysis can be very confusing. For example, a plot of a periodogram is sometimes called a "spectrogram." The completely different spectral density, which is the Fourier transform of the autocovariance function (Jenkins and Watts (1968, pg. 209)), is called the similar sounding "spectrum."

[5]Other examples of tests for periodic behavior may be found in Dale (1981), and Dale and Workman (1980, 1981).

[6]Delong (1982, pg. 181) calls the white noise test "Fisher's Kappa," but he refers to a set of tables (Fuller (1976)) that call the test "Fisher's Xi." We avoid confusion in Table 1 by simply saying "Fisher's White Noise Test."

REFERENCES

Adelman, I. (1960), "Business Cycles-Endogenous or Stochastic?," Economic Journal, 70, No. 280, 783-796.

Adelman, I. (1965), "Long Cycles-Fact or Artifact?," American Economic Review, 55, 444-463.

Adelman, I., and Adelman, F. (1959), "The Dynamic Properties of the Klein-Goldberger Model," Econometrica, 27, No. 4, 596-625.

Benhabib, J. and Day, R. H. (1981), "Rational Choice and Erratic Behaviour," Review of Economic Studies, 459-471.

Box, G. E. P. and Jenkins, G. M. (1976), Time Series Analysis: Forecasting and Control, San Francisco: Holden-Day.

Dale, C., (1981), "Brownian Motion in the Treasury Bill Futures Market," Business Economics, XVI, No. 3, 47-54.

Dale, C. (1984), "Economic Early Warning Indicators for Army Recruiting," Working Paper MPPRG 84-4, U.S. Army Research Institute, Alexandria, Virginia, May.

Dale, C., and Bailey, V. B. (1982), "A Box-Jenkins Model for Forecasting U.S. Merchandise Exports," Journal of International Business Studies, Spring/Summer, 101-108.

Dale, C., and Gilroy, C. (1983a), "The Effects of the Business Cycle on the Size and Composition of the U.S. Army," Atlantic Economic Journal, XI, No. 1, 42-53.

Dale, C., and Gilroy, C. (1983b), The Economic Determinants of Military Enlistment Rates, Technical Report TR587, U.S. Army Research Institute, Alexandria, Virginia.

Dale, C., and Gilroy, C. (1984), "Determinants of Military Enlistments: A Macroeconomic Time-Series View," Armed Forces and Society, 10, No. 2, 192-210.

Dale, C., and Gilroy, C. (1985), Enlistments in the All-Volunteer Force: A Military Personnel Supply Model and its Forecasts: Comment," American Economic Review, forthcoming.

Dale, C., and Workman, R. (1980), "The Arc Sine Law and the Treasury Bill Futures Market," Financial Analysts Journal, November/December, 71-74.

Dale, C., and Workman, R. (1981), "Measuring Patterns of Price Movements in the Treasury Bill Futures Market," Journal of Economics and Business, 33, 81-87.

Day, R. H. (1982), "Irregular Growth Cycles," American Economic Review, 72, No. 3, 406-414.

Day, R. H. (1983), "The Emergence of Chaos from Classical Economic Growth," The Quarterly Journal of Economics, May, 201-213.

Delong, D. M. (1982), "The SPECTRA Procedures," in SAS/ETS User's Guide, Cary, North Carolina: SAS Institute, Inc.

Dolan, E. G., ed. (1976), The Foundations of Modern Austrian Economics, Kansas City: Sheed Andrews and McMeel.

Egger, J. B. (1984), "Shifting Triangles: A Modern Austrian Theory of the Business Cycle," presented to the meetings of the Eastern Economic Association, New York City, March 16.

Feller, W. (1971), An Introduction to Probability Theory and its Applications, Vol. II, New York: John Wiley & Sons.

Fuller, Wayne A. (1976), Introduction to Statistical Time Series, New York: John Wiley & Sons.

Gordon, R. J. (1977), "World Inflation and Monetary Accommodation in Eight Countries," Brookings Papers on Economic Activity, 2, 409-468.

Grandmont, J.-M. (1983a), "On Endogenous Competitive Business Cycles," Report No. 8316, Centre D' Études Prospectives D'Économie Mathématique Appliquées A La Planification (CEPREMAP), Paris, France.

Grandmont, J.-M. (1983b), "Periodic and Aperiodic Behavior in Discrete Onedimensional (sic) Dynamical Systems," Report No. 8317, CEPREMAP, Paris, France.

Granger, C. W. J. (1969), "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," Econometrica, 37, 424-438.

Granger, C. W. J. and Newbold, P. (1977), Forecasting Economic Time Series, New York: Academic Press.

Hannen, E. J. (1970), Multiple Time Series, New York: John Wiley & Sons.

Haustein, H. D., and Neuwirth, E. (1982), "Long Waves in World Industrial Production, Energy Consumption, Innovations, Inventions, and Patents and their Identification by Spectral Analysis," Working Paper WP-82-9, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Henderson, H. (1978), Creating Alternative Futures, New York: Berkley Windhover Books.

Howrey, P. E. (1968), "A Spectrum Analysis of the Long-Swing Hypothesis," International Economic Review, 9, 228-252.

Jenkins, G. M., and Watts, D. G. (1968), Spectral Analysis and its Applications, San Francisco: Holden-Day.

Jensen, R. V., and Urban, R. (1984), "Chaotic Price Behavior in a Nonlinear Cobweb Model," manuscript, Departments of Applied Physics and Economics, Yale University.

Mandelbrot, B. (1983), The Fractal Geometry of Nature, New York: W. H. Freeman and Company.

McCulloch, J. H. (1975), "The Monte Carlo Cycle in Business Activity," Economic Inquiry, XIII, 303-321.

Mensch, G. O. (1981), "A Bi-Equilibrium Model of Bi-Valued Technical Progress Embodied in Innovative Industrial Investments in U.S. Industry Between 1900 and 1934," Working Paper No. 53, Economics Department, Case Western Reserve University, December.

Moore, G. H., (1983), Recession-Recovery Watch, Bimonthly Reports, New York: Columbia University Center for International Business Cycle Research.

Nelson, C. R. (1973), Applied Time Series Analysis for Managerial Forecasting, San Francisco: Holden-Day.

Nelson, C. R., and Kang, H. (1984), "Pitfalls in the Use of Time as an Explanatory Variable in Regression," Journal of Business and Economic Statistics, 2, No. 1, 73-82, January.

Rothbard, M. (1975), America's Great Depression, Kansas City: Sheed and Ward.

Rothbard, M. (1976), "Praxeology: The Methodology of Austrian Economics," in Dolan (1976), 19-39.

Schumaker, E. F. (1973), Small is Beautiful, New York: Harper & Row.

Sims, C. A. (1972), "Money, Income and Causality," American Economic Review, 62, 540-552.

Vandaele, W. (1978), Time Series and Box-Jenkins Models for Business Decisions, Lexington, Mass: Data Resources, Inc.

Varian, H. R. (1979), "Catastrophe Theory and the Business Cycle," Economic Inquiry, XVII, No. 1, 14-28.

FIGURE 1

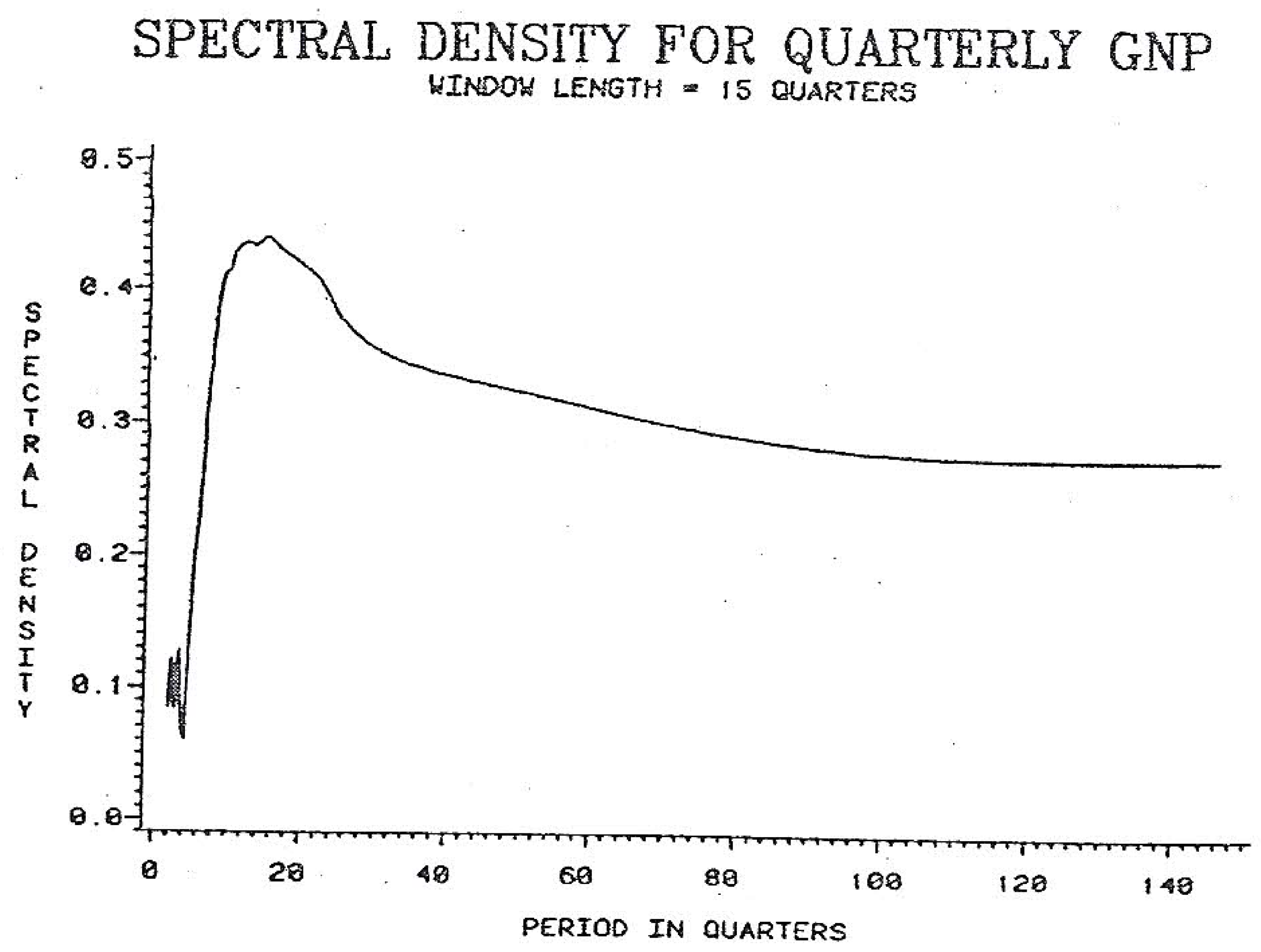

FIGURE 2

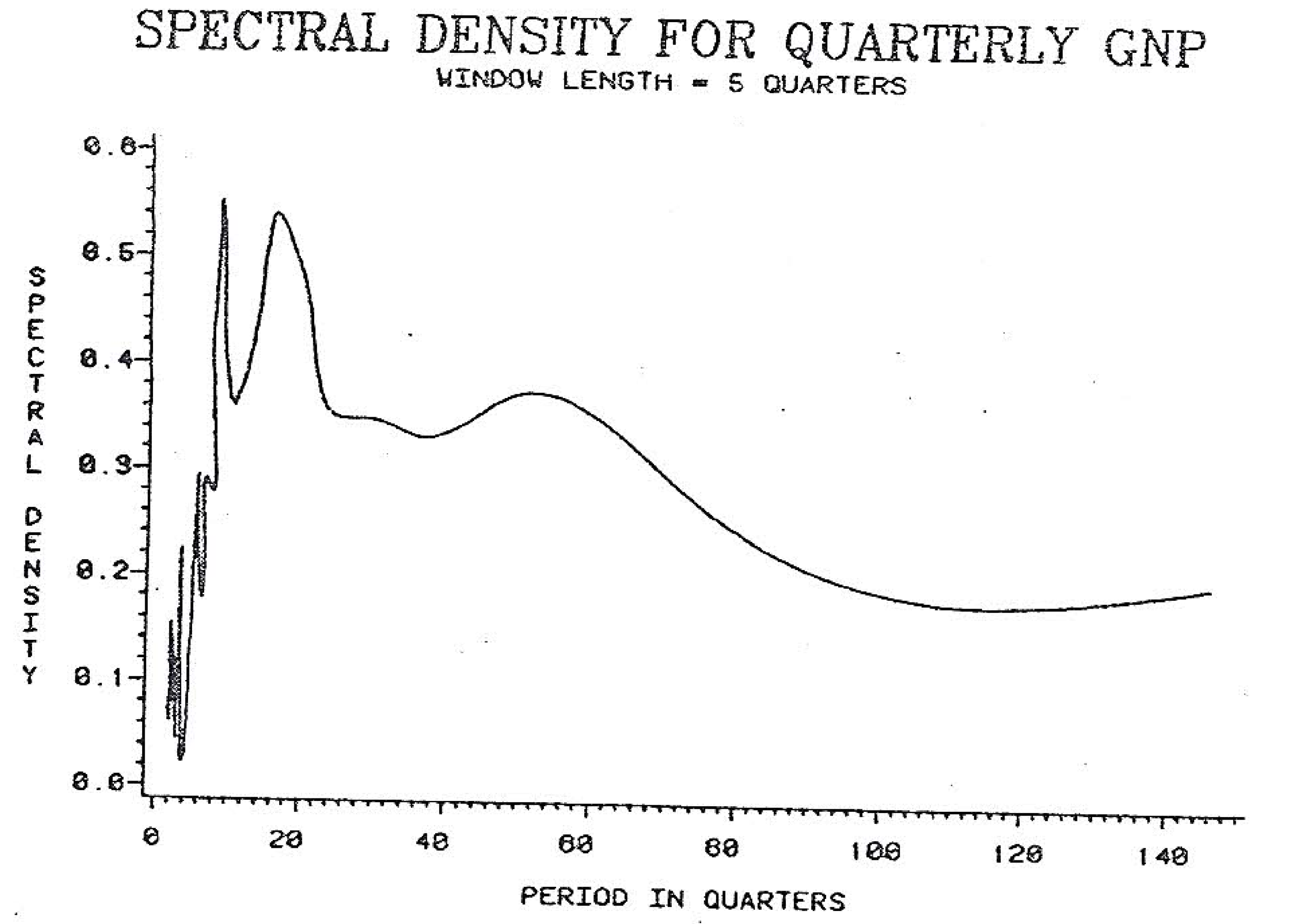

FIGURE 3

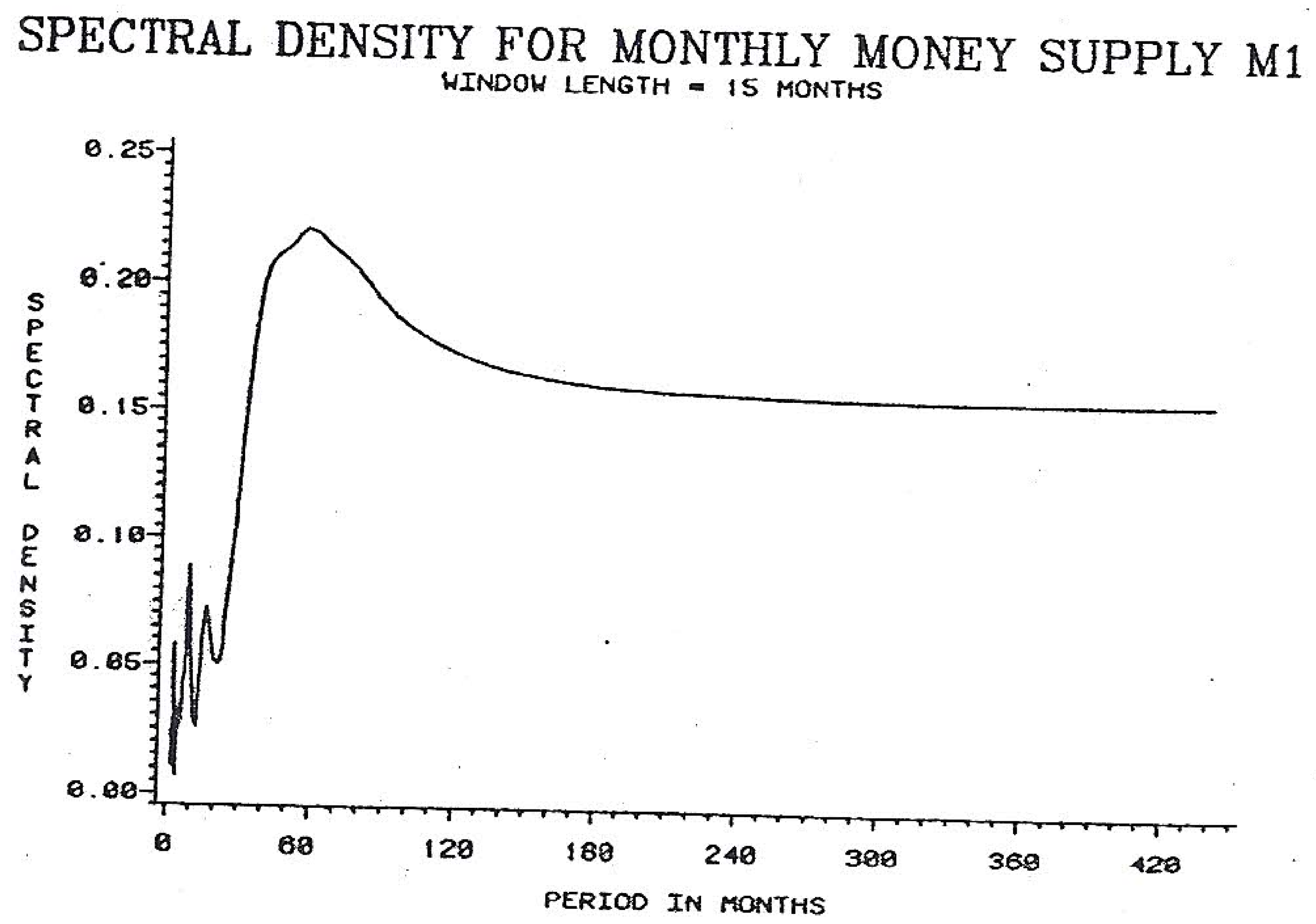

FIGURE 4

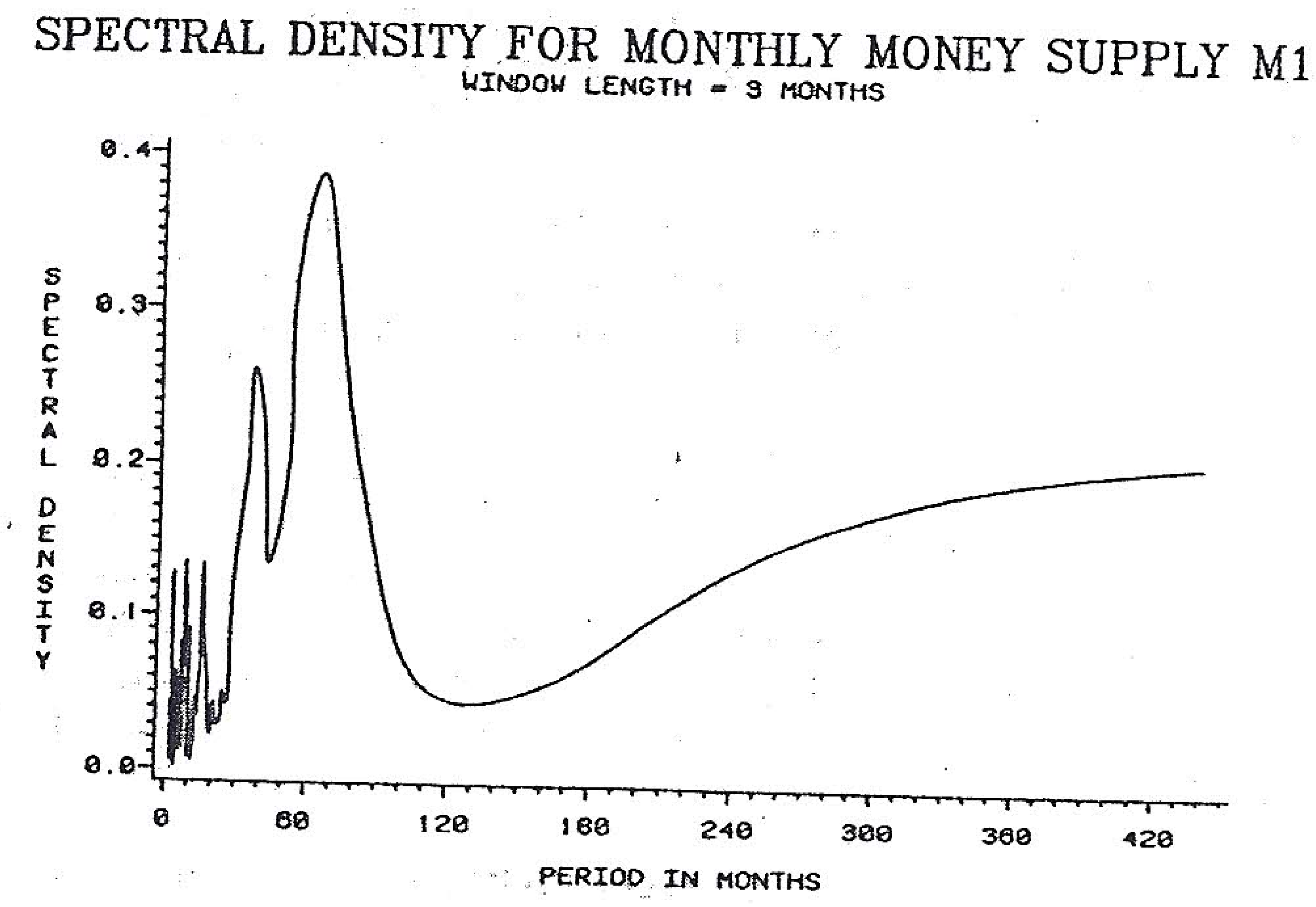

FIGURE 5

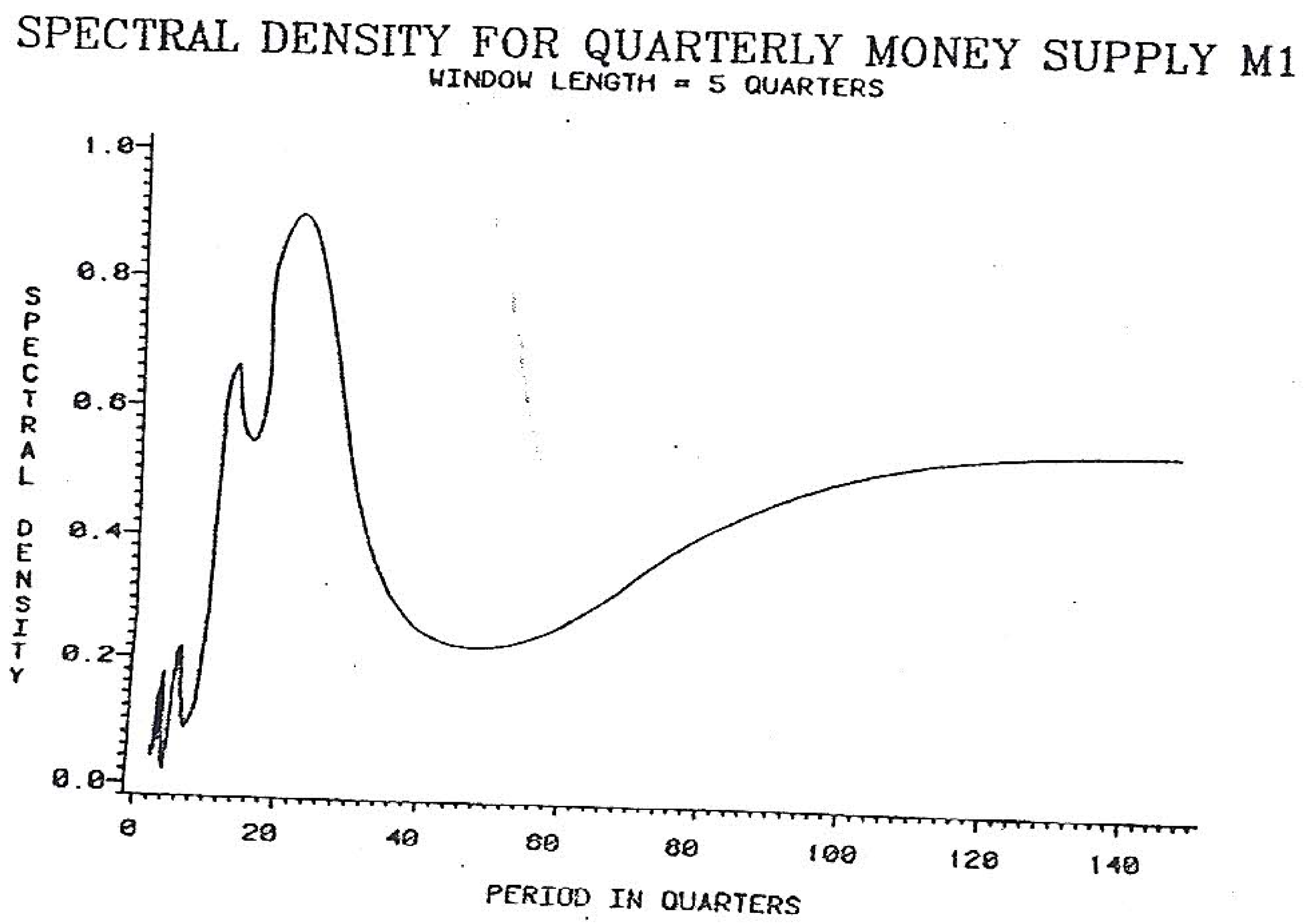

FIGURE 6

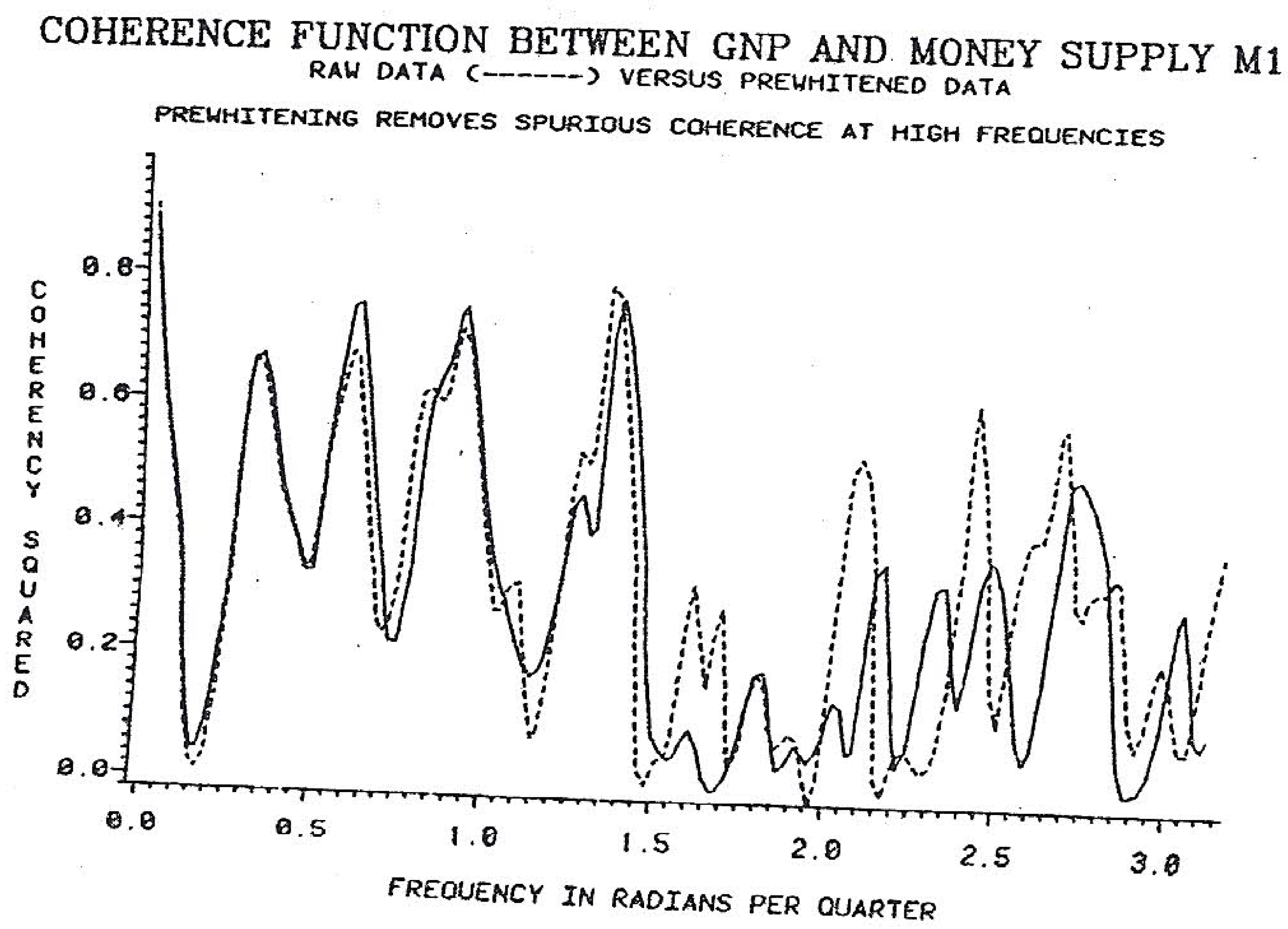

TABLE 1

FISHER'S WHITE NOISE TEST

Monthly or Quarterly Data, January 1947 through September 1983

SERIES	FISHER STATISTIC
QUARTERLY GNP:	
8.5 QUARTER CYCLE	4.23
16 QUARTER CYCLE	4.01
MONTHLY MONEY SUPPLY M1:	
36 MONTH CYCLE	7.49*
63 MONTH CYCLE	11.14***
QUARTERLY MONEY SUPPLY M1:	
12 QUARTER CYCLE	4.68
21 QUARTER CYCLE	6.96**

*Statistically significant at .10 level.
**Statistically significant at .05 level.
***Statistically significant at .01 level.

TABLE 2

COMPARISON OF MEASURED LENGTHS OF BUSINESS CYCLES

STUDY	PERIOD	CYCLE LENGTH (years)
Adelman and Adelman (1959)	0-35 years[a]	b
Howrey (1968)	1869-1955	3.4, 5.6
Haustein and Neuwirth (1982)	1740-1981	7, 13, 20, 32, 40
Dale (This Study)	1947-1983	2.1,[c] 4.[c]

Notes: a. Simulations.
b. Nonconstant cycle lengths.
c. Not statistically significant.