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Chu, Angus C. and Cozzi, Guido and Furukawa, Yuichi

University of Liverpool, University of St. Gallen, Chukyo University

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# A Schumpeterian Analysis of Monetary Policy, Innovation and North-South Technology Transfer

Angus C. Chu, University of Liverpool

Guido Cozzi, University of St. Gallen

Yuichi Furukawa, Chukyo University

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## Abstract

This study analyzes the cross-country effects of monetary policy on innovation and international technology transfer. We consider a scale-invariant North-South quality-ladder model that features innovative R&D in the North and adaptive R&D in the South. To model money demand, we impose cash-in-advance constraints on these two types of R&D investment. We find that an increase in the Southern nominal interest rate causes a permanent decrease in the rate of international technology transfer, a permanent increase in the North-South wage gap, and a temporary decrease in the rate of Northern innovation. An increase in the Northern nominal interest rate causes a temporary decrease in the rate of Northern innovation, a permanent decrease in the North-South wage gap, and an ambiguous effect on the rate of international technology transfer depending on the relative size of the two economies. We also calibrate the model to China-US data and find that the cross-country welfare effects of monetary policy are quantitatively significant. Specifically, permanently decreasing the nominal interest rate to zero in China leads to a welfare gain of 3.37% in China and a welfare gain of 1.25% in the US. Permanently decreasing the nominal interest rate to zero in the US leads to welfare gains of 0.33% in the US and 1.24% in China.

*JEL classification:* O30, O40, E41, F43

*Keywords:* monetary policy, economic growth, R&D, North-South product cycles, FDI

Chu: angusccc@gmail.com. University of Liverpool Management School, University of Liverpool, UK.

Cozzi: guido.cozzi@unisg.ch. Department of Economics, University of St. Gallen, Switzerland.

Furukawa: you.furukawa@gmail.com. School of Economics, Chukyo University, Nagoya, Japan.

# 1 Introduction

In this study, we analyze the cross-country effects of monetary policy on innovation and technology transfer via foreign direct investment (FDI). To introduce money demand, we incorporate cash-in-advance (CIA) constraints on R&D investment into a North-South version of the R&D-based growth model. Specifically, we consider the scale-invariant North-South quality-ladder model in Dinopoulos and Segerstrom (2010). The North-South R&D-based growth model originates from the seminal study by Grossman and Helpman (1991).<sup>1</sup> The model in Dinopoulos and Segerstrom (2010) is a recent vintage of this class of models and has the advantage of being free of scale effects by featuring semi-endogenous growth.<sup>2</sup> The Dinopoulos-Segerstrom model features innovative R&D in the North and adaptive R&D in the South. Multinational firms invest in adaptive R&D in the South in order to transfer the production of the highest quality products from the North to the South. Given that innovative and adaptive R&D investments are costly and subject to liquidity constraints in reality,<sup>3</sup> we follow Chu and Cozzi (2013) to impose a CIA constraint on innovative R&D in the North, and we also impose a CIA constraint on adaptive R&D in the South. Within this growth-theoretic framework, we derive the following results.

An increase in the nominal interest rate in the South causes a permanent decrease in the rate of technology transfer from the North to the South, a permanent increase in the North-South wage gap, and a temporary decrease in the rate of innovation in the North. Intuitively, an increase in the Southern nominal interest rate raises the cost of adaptive R&D and reduces the equilibrium rate of international technology transfer. As a result, less products are manufactured by Southern firms and more products are produced by Northern firms. The higher demand for production labor in the North reduces R&D labor, which in turn decreases the rate of Northern innovation in the short run. Finally, given that the increase in the Southern nominal interest rate has a direct negative effect on the demand for Southern R&D labor, it depresses the wage rate in the South relative to the North.

An increase in the nominal interest rate in the North causes a temporary decrease in the rate of Northern innovation, a permanent decrease in the North-South wage gap, and an ambiguous effect on the rate of technology transfer from the North to the South depending on the relative size of the two economies. Specifically, we find that if the Southern population size is sufficiently large (small), then an increase in the nominal interest rate in the North would cause a permanent decrease (increase) in the rate of technology transfer from the North to the South. Intuitively, an increase in the Northern nominal interest rate raises the cost of innovative R&D. As a result, the rate of innovation decreases in the short run. Given that the increase in the Northern nominal interest has a direct negative effect on the demand for Northern R&D labor, it depresses the wage rate in the North relative to the South. As for

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<sup>1</sup>See Dinopoulos and Segerstrom (2010) for a review of the subsequent development in this literature and Iwaisako *et al.* (2011) and Tanaka and Iwaisako (2013) for recent studies.

<sup>2</sup>See Jones (1999) for a discussion of scale effects in R&D-based growth models. The semi-endogenous-growth version of the quality-ladder model originates from Segerstrom (1998) and Li (2003).

<sup>3</sup>For example, Brown and Petersen (2011) and Brown *et al.* (2012) provide evidence that firms tend to smooth R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. See also Berentsen *et al.* (2012) and Chu and Cozzi (2013) for a discussion of other empirical studies that support the presence of liquidity constraints on R&D investment.

the effects on the rate of international technology transfer, there are two opposing effects. On the one hand, it reduces the long-run level of aggregate quality, which reduces the difficulty of adaptive R&D due to increasing difficulty of R&D in the semi-endogenous growth model.<sup>4</sup> This is a positive effect on international technology transfer. On the other hand, the increase in the Northern nominal interest rate also reduces the incentives for adaptive R&D because there are less benefits from FDI due to the smaller North-South wage gap. This negative effect on technology transfer via adaptive R&D labor in the South is relatively strong when the Southern labor force is large. Therefore, the overall effect of the Northern nominal interest rate on technology transfer would be negative (positive) if the Southern population size is sufficiently large (small).

We also calibrate the model to China-US data in order to conduct a quantitative investigation on the cross-country effects of monetary policy. We find that permanently decreasing the nominal interest rate to zero in China would reduce the wage gap between China and the US by about 4% (percent change) and also increase the flow of technology transfer from the US to China by about 6% (percent change). Furthermore, it leads to a long-run welfare gain that is equivalent to a permanent increase in consumption of 3.37% in China and a welfare gain of 1.25% in the US. The welfare gains mostly come from higher real wages in the two countries as a result of increased quality from innovation. On the other hand, permanently decreasing the nominal interest rate to zero in the US would raise the wage gap between the two countries by about 3% and surprisingly decrease the flow of technology transfer from the US to China by about 2%. The welfare gains for the US and China are 0.33% and 1.24% respectively. In this case, the welfare gain in the US is relatively small because the increase in real wage is partly offset by a decrease in interest income, which is an important component of income in the US. Overall, the cross-country welfare effects of monetary policy are quantitatively significant.

In the literature on inflation and economic growth, Stockman (1981) and Abel (1985) analyze a CIA constraint on capital investment in a monetary version of the Neoclassical growth model. Subsequent studies in this literature explore the effects of monetary policy in variants of the capital-based growth model. This study instead associates more closely with a related literature on inflation and *innovation-driven* growth. In this literature, Marquis and Reffett (1994) analyze the effects of monetary policy via a CIA constraint on consumption in a variant of the variety-expanding model in Romer (1990).<sup>5</sup> In contrast, we analyze monetary policy in a Schumpeterian quality-ladder model as in Chu and Cozzi (2013) and Chu and Lai (2013);<sup>6</sup> however, the present study differs from all these studies by considering an open-economy two-country model. Chu, Cozzi, Lai and Liao (2013) also analyze the effects of monetary policy in an open-economy Schumpeterian model, but they consider an environment with two Northern economies without North-South product cycles and technology transfer via FDI. To our knowledge, this is the first study that analyzes the effects of monetary policy in the presence of North-South product cycles and technology transfer via

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<sup>4</sup>See Venturini (2012) for empirical evidence based on US manufacturing industry data that supports the semi-endogenous growth model with increasing R&D difficulty.

<sup>5</sup>Chu, Lai and Liao (2013) provide an analysis of the CIA constraint on consumption in a hybrid growth model in which economic growth in the long run is driven by both variety expansion and capital accumulation.

<sup>6</sup>See also Chu and Ji (2013) and Huang *et al.* (2013), who analyze the effects of monetary policy in a Schumpeterian model with endogenous market structure.

FDI. Within this novel monetary growth-theoretic framework, we discover some interesting effects of monetary policy on innovation and international technology transfer.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 solves the steady-state equilibrium. Section 4 analyzes the effects of monetary policy. The final section concludes.

## 2 A North-South monetary Schumpeterian model

The North-South semi-endogenous growth model is based on Dinopoulos and Segerstrom (2010). In the Dinopoulos-Segerstrom model, multinational firms employ Northern R&D labor to invest in innovative R&D that improves the quality of products manufactured in the North. In order to take advantage of the lower production cost in the South, the multinational firms then employ Southern R&D labor to invest in adaptive R&D that transfers the production of the highest quality products from the North to the South. After the manufacturing process of a product is transferred to the South, the multinational firm faces the possibility of the product being imitated by domestic firms in the South. To introduce money demand, we modify the Dinopoulos-Segerstrom model by incorporating CIA constraints on innovative R&D in the North and adaptive R&D in the South. Then, we analyze the effects of the nominal interest rates in the two countries on innovation and international technology transfer.

### 2.1 Households

In each country, there is a representative household. The lifetime utility function of the household in the North is given by

$$U^N = \int_0^{\infty} e^{-(\rho - g_L)t} \ln c_t^N dt, \quad (1)$$

where  $c_t^N$  denotes per capita consumption in the North at time  $t$ , and the parameter  $\rho > 0$  determines subjective discounting. The population size in the North is  $L_t^N$ , which increases at an exogenous population growth rate  $g_L > 0$ . To ensure that lifetime utility is bounded, we impose the following parameter restriction:  $\rho > g_L$ . For simplicity, we make a common assumption that  $\{\rho, g_L\}$  are the same in the two countries. Total population in the world is  $L_t = L_t^N + L_t^S$ . We use  $s \equiv L_t^S/L_t$  to denote the share of world population in the South and  $1 - s \equiv L_t^N/L_t$  to denote the share of world population in the North.

The household in the North maximizes (1) subject to the following asset-accumulation equation:

$$\dot{A}_t^N + \dot{M}_t^N = (i_t^N - g_L)A_t^N - g_L M_t^N + i_t^N B_t^N + W_t^N + T_t^N - P_t^N c_t^N.$$

$P_t^N$  is the price of consumption goods denominated in units of domestic currency in the North.  $A_t^N$  is the nominal value of financial assets owned by each member of the household, and  $i_t^N$  is the nominal interest rate in the North.  $M_t^N$  is the nominal value of domestic

currency held by each member of the household.  $B_t^N$  is the nominal value of domestic currency borrowed by R&D entrepreneurs to finance their R&D investment in the North, and the rate of return on  $B_t^N$  is the domestic nominal interest rate  $i_t^N$ .<sup>7</sup> There is a constraint on how much money that each person can lend to R&D entrepreneurs, and the constraint is  $B_t^N \leq M_t^N$ .<sup>8</sup> Each member of the household supplies one unit of labor to earn a nominal wage  $W_t^N$ .  $T_t^N$  is the nominal value of a lump-sum transfer (or tax if  $T_t^N < 0$ ) from the government to each person in the North.

For convenience, we reexpress the asset-accumulation equation in real terms (denominated in units of consumption goods).<sup>9</sup>

$$\dot{a}_t^N + \dot{m}_t^N = (r_t^N - g_L)a_t^N - (\pi_t^N + g_L)m_t^N + i_t^N b_t^N + w_t^N + \tau_t^N - c_t^N. \quad (2)$$

$a_t^N$  is the real value of financial assets per capita, and  $r_t^N = i_t^N - \pi_t^N$  is the real interest rate in the North.  $\pi_t^N$  is the inflation rate of  $P_t^N$  in the North.  $m_t^N$  is the real value of domestic currency per capita.  $b_t^N$  is the real value of domestic currency borrowed by domestic R&D entrepreneurs, and the constraint becomes  $b_t^N \leq m_t^N$ .  $w_t^N$  is the real wage rate.  $\tau_t^N$  is the real value of lump-sum transfer from the government.

We follow Dinopoulos and Segerstrom (2010) to assume that there is a global financial market. In this case, the real interest rates in the two countries must be equal such that  $r_t^N = r_t^S = r_t$ .<sup>10</sup> From standard dynamic optimization, the familiar Euler equation is<sup>11</sup>

$$\frac{\dot{c}_t^N}{c_t^N} = \frac{\dot{c}_t^S}{c_t^S} = r_t - \rho, \quad (3)$$

which implies that the growth rate of consumption is the same across countries.

## 2.2 Consumption goods

Consumption goods are produced by perfectly competitive firms that aggregate a unit continuum of intermediate goods  $Y_t(j)$  using the following CES aggregator:

$$C_t = \left\{ \int_0^1 [Y_t(j)]^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}}, \quad (4)$$

where  $\sigma > 1$  is the elasticity of substitution between intermediate goods. The resource constraint on  $C_t$  is

$$C_t = c_t^N L_t^N + c_t^S L_t^S = [c_t^N(1-s) + c_t^S s] L_t, \quad (5)$$

where  $c_t^N L_t^N$  is total consumption in the North and  $c_t^S L_t^S$  is total consumption in the South.  $P_t^N$  is the price of consumption goods denominated in units of currency in the North.  $P_t^S$  is

<sup>7</sup>It can be easily shown as a no-arbitrage condition that the rate of return on  $B_t^N$  must be equal to  $i_t^N$ .

<sup>8</sup>In the case of an additional CIA requirement on consumption, the CIA constraint in the North becomes  $P_t^N c_t^N + B_t^N \leq M_t^N$ . Given that we focus on the case of inelastic labor supply for tractability, the CIA constraint on consumption would have no effect on the equilibrium allocations.

<sup>9</sup>Derivations are available upon request.

<sup>10</sup>The nominal interest rates in the two countries would be different if inflation rates differ across countries.

<sup>11</sup>The representative household in the South also performs an analogous dynamic optimization.

the price of consumption goods denominated in units of currency in the South. Given zero transportation cost, the law of one price holds such that  $P_t^N = \epsilon_t P_t^S$ , where  $\epsilon_t$  is the nominal exchange rate. For convenience, we will express all variables in real terms denominated in units of consumption goods that have the same value in the two countries. From profit maximization, we derive the conditional demand function for  $Y_t(j)$  as

$$Y_t(j) = p_t(j)^{-\sigma} C_t \quad (6)$$

for  $j \in [0, 1]$ .  $p_t(j)$  is the price of  $Y_t(j)$ .

### 2.3 Intermediate goods

There is a unit continuum of differentiated intermediate goods  $j \in [0, 1]$ . Some of these intermediate goods are produced in the North, and each of these Northern industries is temporarily dominated by a quality leader until the arrival of the next innovation.<sup>12</sup> The production function of intermediate goods manufactured by a quality leader in the North is

$$Y_t(j) = z^{n_t(j)} L_{y,t}^N(j) \equiv Y_t^N(j), \quad (7)$$

where the parameter  $z > 1$  is the step size of a quality improvement, and  $n_t(j)$  is the number of quality improvements that have occurred in industry  $j$  as of time  $t$ . The firm employs  $L_{y,t}^N(j)$  units of labor in the North for production. Given  $z^{n_t(j)}$ , the marginal cost of production for the industry leader is  $w_t^N / z^{n_t(j)}$ .<sup>13</sup> We follow Dinopoulos and Segerstrom (2010) to assume that new quality leaders are always able to charge the unconstrained monopolistic price because the closest competitors choose to immediately exit the market in equilibrium.<sup>14</sup> In this case, the monopolistic price charged by industry leaders is

$$p_t(j) = \frac{\sigma}{\sigma - 1} \frac{w_t^N}{z^{n_t(j)}} \equiv p_t^N(j). \quad (8)$$

To take advantage of the lower labor cost in the South (i.e.,  $w_t^N > w_t^S$ ), industry leaders in the North invest in adaptive R&D in the South in order to shift the manufacturing process to the South. If the adaptive R&D project of a Northern leader is successful, then a Southern affiliate of the Northern leader would start producing the intermediate goods. The production function of intermediate goods manufactured by the foreign affiliate of a Northern quality leader is

$$Y_t(j) = z^{n_t(j)} L_{y,t}^F(j) \equiv Y_t^F(j). \quad (9)$$

The Southern affiliate employs  $L_{y,t}^F(j)$  units of labor in the South for production, and the marginal cost of production is  $w_t^S / z^{n_t(j)}$ . Given the marginal cost of production, the unconstrained monopolistic price is given by

$$p_t(j) = \frac{\sigma}{\sigma - 1} \frac{w_t^S}{z^{n_t(j)}} \equiv p_t^F(j). \quad (10)$$

<sup>12</sup>This is known as the Arrow replacement effect in the literature; see Cozzi (2007a) for a discussion.

<sup>13</sup>It is useful to note that we here adopt a cost-reducing view of quality improvement.

<sup>14</sup>See Dinopoulos and Segerstrom (2010) for a detailed discussion.

The Southern affiliate produces the intermediate goods until the arrival of the next innovation in the North or until the current innovation is imitated by other firms in the South. When the next innovation arrives, the manufacturing process shifts back to the North. To ensure that this return of production to the North occurs, we follow Dinopoulos and Segerstrom (2010) to assume  $w_t^S > w_t^N/z$ , so that new quality leaders are able to drive out Southern affiliates of previous quality leaders.

Technologies of Southern affiliates may be imitated by other Southern firms subject to an exogenous imitation rate  $\phi$ . When this imitation occurs, the intermediate goods are produced by competitive firms in the South. The production function of intermediate goods produced by competitive firms in the South is

$$Y_t(j) = z^{n_t(j)} L_{y,t}^S(j) \equiv Y_t^S(j), \quad (11)$$

and the perfectly competitive price is given by the marginal cost of production:

$$p_t(j) = \frac{w_t^S}{z^{n_t(j)}} \equiv p_t^S(j). \quad (12)$$

Southern competitive firms produce the intermediate goods until the next innovation arrives at which point the manufacturing process shifts back to the North.

Let's define the aggregate quality index across industries  $j \in [0, 1]$  as

$$Q_t \equiv \int_0^1 q_t(j) dj,$$

where  $q_t(j) \equiv [z^{n_t(j)}]^{-\sigma}$ . Then, we can derive the labor demands for an average-quality product produced by a Northern leader as

$$\tilde{L}_{y,t}^N = Q_t \left( \frac{\sigma}{\sigma - 1} w_t^N \right)^{-\sigma} C_t, \quad (13)$$

by a Southern affiliate as

$$\tilde{L}_{y,t}^F = Q_t \left( \frac{\sigma}{\sigma - 1} w_t^S \right)^{-\sigma} C_t, \quad (14)$$

and by Southern competitive firms as

$$\tilde{L}_{y,t}^S = Q_t (w_t^S)^{-\sigma} C_t. \quad (15)$$

Using these expressions, we can then express the labor demand for product  $j$  as

$$L_{y,t}^o(j) = \frac{q_t(j)}{Q_t} \tilde{L}_{y,t}^o, \quad (16)$$

where  $o = \{N, F, S\}$ . The amount of monopolistic profit earned by a Northern leader is

$$\Pi_t^N(j) = \frac{w_t^N}{\sigma - 1} \frac{q_t(j)}{Q_t} \tilde{L}_{y,t}^N, \quad (17)$$

and the amount of monopolistic profit earned by a Southern affiliate is

$$\Pi_t^F(j) = \frac{w_t^S}{\sigma - 1} \frac{q_t(j)}{Q_t} \tilde{L}_{y,t}^F. \quad (18)$$



## 2.4 Innovative and adaptive R&D

Innovative R&D is performed by a continuum of competitive entrepreneurs in the North. If an R&D entrepreneur employs  $L_{r,t}^N(j)$  units of Northern labor to engage in innovative R&D in industry  $j$ , then she is successful in inventing the next higher-quality product in the industry with an instantaneous probability given by

$$\varphi_t^N(j) = \frac{L_{r,t}^N(j)}{\gamma q_t(j)}, \quad (19)$$

where the parameter  $\gamma > 0$  inversely measures innovation productivity.  $q_t(j)$  captures the effect of increasing innovation difficulty, and it removes the scale effect in the innovation process of the quality-ladder model as in Segerstrom (1998). The expected benefit from investing in innovative R&D is  $v_t^N(j)\varphi_t^N(j)dt$ , where  $v_t^N(j)$  is the real value of the expected discounted profits generated by an innovation and  $\varphi_t^N(j)dt$  is the entrepreneur's probability of having a successful innovation during the infinitesimal time interval  $dt$ . To facilitate the wage payment to R&D labor in the North, the entrepreneurs need to borrow domestic currency from the domestic household, and the cost of borrowing is determined by the nominal interest rate  $i_t^N$  in the North. Therefore, the total cost of innovative R&D is  $(1 + i_t^N) w_t^N L_{r,t}^N(j)dt$ . Free entry implies

$$v_t^N(j)\varphi_t^N(j)dt = (1 + i_t^N) w_t^N L_{r,t}^N(j)dt \Leftrightarrow v_t^N(j) = (1 + i_t^N) w_t^N \gamma q_t(j), \quad (20)$$

where the second equality uses (19).

Adaptive R&D is performed by Northern industry leaders and their Southern affiliates. If the Southern affiliate of a Northern leader in industry  $j$  employs  $L_{r,t}^F(j)$  units of Southern labor to engage in adaptive R&D, then the Northern firm is successful in shifting the production to the Southern affiliate with an instantaneous probability given by

$$\varphi_t^F(j) = \frac{L_{r,t}^F(j)}{\alpha q_t(j)}, \quad (21)$$

where the parameter  $\alpha > 0$  inversely measures adaptation productivity.  $q_t(j)$  captures the effect of increasing adaptation difficulty, and it removes the scale effect in the adaptation process as in Dinopoulos and Segerstrom (2010). The expected *net* benefit for the Northern leader to invest in adaptive R&D is  $[v_t^F(j) - v_t^N(j)]\varphi_t^F(j)dt$ , where  $v_t^F(j)$  is the real value of the expected discounted profits generated by the Southern affiliate and  $\varphi_t^F(j)dt$  is the probability of having a successful adaptation during the infinitesimal time interval  $dt$ . To facilitate the wage payment to R&D labor in the South, the Southern affiliate needs to borrow domestic currency from the domestic household, and the cost of borrowing is determined by the nominal interest rate  $i_t^S$  in the South. Therefore, the total cost of adaptive R&D is  $(1 + i_t^S) w_t^S L_{r,t}^F(j)dt$ . Given that the net benefit of adaptive R&D is linear in  $L_{r,t}^F(j)$ , the Southern affiliate engages in a positive finite amount of adaptive R&D if and only if the following equilibrium condition holds:

$$[v_t^F(j) - v_t^N(j)]\varphi_t^F(j)dt = (1 + i_t^S) w_t^S L_{r,t}^F(j)dt \Leftrightarrow v_t^F(j) - v_t^N(j) = (1 + i_t^S) w_t^S \alpha q_t(j), \quad (22)$$

where the second equality uses (21). Finally, Southern affiliates face the risk of imitation (with an exogenous probability  $\phi > 0$ ) by other firms in the South.

## 2.5 Stock market

The no-arbitrage condition that determines the value of  $v_t^N(j)$  is given by<sup>15</sup>

$$r_t = \frac{\Pi_t^N(j) - (1 + i_t^S) w_t^S L_{r,t}^F(j) + \dot{v}_t^N(j) - \varphi_t^N(j)v_t^N(j) + \varphi_t^F(j) [v_t^F(j) - v_t^N(j)]}{v_t^N(j)}. \quad (23)$$

This condition equates the real interest rate  $r_t$  to the asset return per unit of asset. The asset return is the sum of (a) monopolistic profits net of adaptive R&D expenditure, (b) any potential capital gain  $\dot{v}_t^N(j)$ , (c) the expected capital loss  $-\varphi_t^N(j)v_t^N(j)$  from creative destruction, and (d) the expected change in asset value  $\varphi_t^F(j) [v_t^F(j) - v_t^N(j)]$  when adaptive R&D is successful. Using (22), we simplify (23) to a more familiar expression given by

$$r_t = \frac{\Pi_t^N(j) + \dot{v}_t^N(j) - \varphi_t^N(j)v_t^N(j)}{v_t^N(j)}. \quad (24)$$

The no-arbitrage condition that determines the value of  $v_t^F(j)$  is given by

$$r_t = \frac{\Pi_t^F(j) + \dot{v}_t^F(j) - [\varphi_t^N(j) + \phi]v_t^F(j)}{v_t^F(j)}. \quad (25)$$

This condition equates the real interest rate  $r_t$  to the asset return per unit of asset. The asset return is the sum of (a) monopolistic profits in the South, (b) any potential capital gain  $\dot{v}_t^F(j)$ , (c) the expected capital loss  $-\varphi_t^F(j)v_t^F(j)$  from creative destruction, and (d) the expected capital loss  $-\phi v_t^F(j)$  from imitation.

From (19), the expected benefit from innovative R&D in industry  $j$  is  $v_t^N(j)\varphi_t^N(j) = v_t^N(j)L_{r,t}^N(j)/[\gamma q_t(j)]$ , which appears to be decreasing in  $q_t(j)$ . However, (24) implies that  $v_t^N(j)$  is linearly increasing in  $\Pi_t^N(j)$  which in turn is linearly increasing in  $q_t(j)$  as shown in (17); as a result,  $v_t^N(j)\varphi_t^N(j)$  is in fact independent of  $q_t(j)$ . Therefore, we follow the standard treatment in this class of models to focus on the symmetric equilibrium in which  $\varphi_t^N(j) = \varphi_t^N$ .<sup>16</sup> Similarly, from (21), the expected benefit from adaptive R&D in industry  $j$  is  $[v_t^F(j) - v_t^N(j)]\varphi_t^F(j) = [v_t^F(j) - v_t^N(j)]L_{r,t}^F(j)/[\alpha q_t(j)]$ , which is also independent of  $q_t(j)$  as implied by (25) and (18). Therefore, we focus on the symmetric equilibrium in which  $\varphi_t^F(j) = \varphi_t^F$ .

## 2.6 Monetary authority

The monetary policy instrument in the North (South) is the domestic nominal interest rate  $i_t^N$  ( $i_t^S$ ), which is exogenously chosen by the Northern (Southern) monetary authority. Given  $i_t^N$  ( $i_t^S$ ), the inflation rate in the North (South) is endogenously determined according to the Fisher identity  $\pi_t^N = i_t^N - r_t$  ( $\pi_t^S = i_t^S - r_t$ ), where  $r_t$  is the global real interest rate. Then, the growth rate of the nominal money supply per capita in the North (South) is endogenously

<sup>15</sup>It is useful to note that the following  $\Pi_t^N(j)$  refers to the profit after the arrival of the next innovation.

<sup>16</sup>See Cozzi (2007b) for a discussion on the possibility of multiple equilibria in the Schumpeterian growth model. Cozzi *et al.* (2007) provide theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian growth model.

determined by  $\dot{M}_t^N/M_t^N = \pi_t^N + \dot{m}_t^N/m_t^N$  ( $\dot{M}_t^S/M_t^S = \pi_t^S + \dot{m}_t^S/m_t^S$ ). Finally, the Northern (Southern) monetary authority returns the seigniorage revenue as a lump-sum transfer that has a real value of  $\tau_t^N = (\dot{M}_t^N + g_L M_t^N)/P_t^N$  ( $\tau_t^S = (\dot{M}_t^S + g_L M_t^S)/P_t^S$ ) to each member of the domestic household in the North (South).

## 2.7 Decentralized equilibrium

The equilibrium is a time path of allocations  $\{c_t^N, c_t^S, C_t, Y_t^N(j), Y_t^F(j), Y_t^S(j), L_{y,t}^N(j), L_{y,t}^F(j), L_{y,t}^S(j), L_{r,t}^N(j), L_{r,t}^F(j), L_{r,t}^S(j)\}_{t=0}^{\infty}$ , a time path of prices  $\{w_t^N, w_t^S, p_t^N(j), p_t^F(j), p_t^S(j), v_t^N, v_t^F, \epsilon_t\}_{t=0}^{\infty}$  and a time path of monetary policies  $\{i_t^N, i_t^S\}_{t=0}^{\infty}$ . Also, at each instance of time,

- the representative household in the North maximizes lifetime utility taking  $\{r_t, i_t^N, w_t^N\}$  as given;
- the representative household in the South maximizes lifetime utility taking  $\{r_t, i_t^S, w_t^S\}$  as given;
- competitive consumption-good firms produce  $C_t$  to maximize profit taking  $\{p_t^N(j), p_t^F(j), p_t^S(j)\}$  as given;
- quality leaders in the North choose  $p_t^N(j)$  and produce  $Y_t^N(j)$  to maximize profit taking  $w_t^N$  as given;
- affiliates in the South choose  $p_t^F(j)$  and produce  $Y_t^F(j)$  to maximize profit taking  $w_t^S$  as given;
- competitive intermediate goods firms produce  $Y_t^S(j)$  to maximize profit taking  $\{p_t^S(j), w_t^S\}$  as given;
- competitive R&D entrepreneurs in the North employ  $L_{r,t}^N(j)$  to do innovative R&D taking  $\{i_t^N, w_t^N, v_t^N\}$  as given;
- quality leaders in the North and their affiliates in the South employ  $L_{r,t}^F(j)$  to do adaptive R&D taking  $\{i_t^S, w_t^S, v_t^F\}$  as given;
- the market-clearing condition for consumption goods holds;
- the market-clearing conditions for labor hold in both countries.

## 3 Steady-state equilibrium

In this section, we proceed to solve the steady-state equilibrium in the following steps. First, we derive the steady-state number of each type of industries and the steady-state expression of the quality index. Then, we derive the steady-state labor market conditions in the two countries. Finally, we put all these conditions together to derive the steady-state equilibrium rates of technology transfer and innovation.

### 3.1 Industry composition and quality dynamics

In the intermediate goods sector, there are three types of industries in which intermediate goods are produced respectively by Northern quality leaders, Southern affiliates, and Southern competitive firms. We use  $\{\theta^N, \theta^F, \theta^S\}$  to denote the steady-state measure of these three types of industries. To solve for these three endogenous variables, we use the following conditions. First, the measure of all industries adds up to one.

$$\theta^N + \theta^F + \theta^S = 1. \quad (26)$$

In the steady state, the flows in and out of each type of industry must be equal. The flow into industries  $\theta^S$  dominated by Southern competitive firms is  $\theta^F \phi$  given by the measure of industries in which Southern affiliates' technologies are imitated. The flow out of industries  $\theta^S$  dominated by Southern competitive firms is  $\theta^S \varphi^N$  given by the measure of these competitive industries experiencing the arrival of new innovations in the North. Therefore, the second condition is

$$\theta^F \phi = \theta^S \varphi^N. \quad (27)$$

The flow into industries  $\theta^F$  dominated by Southern affiliates is  $\theta^N \varphi^F$  given by the measure of industries in the North experiencing successful R&D adaptation. The flow out of industries  $\theta^F$  dominated by Southern affiliates is the sum of (a)  $\theta^F \varphi^N$  given by the measure of these industries experiencing the arrival of new innovations in the North and (b)  $\theta^F \phi$  given by the measure of industries in which Southern affiliates' technologies are imitated. Therefore, the third condition is

$$\theta^N \varphi^F = \theta^F (\varphi^N + \phi). \quad (28)$$

Solving (26), (27) and (28) yields

$$\theta^N = \frac{\varphi^N}{\varphi^N + \varphi^F}, \quad (29)$$

$$\theta^F = \frac{\varphi^N}{\varphi^N + \phi} \frac{\varphi^F}{\varphi^N + \varphi^F}, \quad (30)$$

$$\theta^S = \frac{\phi}{\varphi^N + \phi} \frac{\varphi^F}{\varphi^N + \varphi^F}. \quad (31)$$

The aggregate quality index across industries  $j \in [0, 1]$  is

$$Q_t \equiv \int_0^1 q_t(j) dj = \int_0^1 \lambda^{n_t(j)} dj, \quad (32)$$

where  $\lambda \equiv z^{\sigma-1}$  is a composite parameter that is increasing in the quality step size  $z$ . This quality index can be decomposed into the following three components:

$$Q_t = Q_t^N + Q_t^F + Q_t^S = \int_{\theta^N} q_t(j) dj + \int_{\theta^F} q_t(j) dj + \int_{\theta^S} q_t(j) dj. \quad (33)$$

Lemma 1 provides the steady-state expression for the share of each of these three components of aggregate quality.

**Lemma 1** *In the steady state, the three components of aggregate quality can be expressed as*

$$\frac{Q_t^N}{Q_t} = \frac{\lambda\varphi^N}{\lambda\varphi^N + \varphi^F}, \quad (34)$$

$$\frac{Q_t^F}{Q_t} = \frac{\lambda\varphi^N}{\lambda\varphi^N + \phi} \frac{\varphi^F}{\lambda\varphi^N + \varphi^F}. \quad (35)$$

$$\frac{Q_t^S}{Q_t} = \frac{\phi}{\lambda\varphi^N + \phi} \frac{\varphi^F}{\lambda\varphi^N + \varphi^F}. \quad (36)$$

**Proof.** See Appendix A. ■

### 3.2 Northern labor market

The market-clearing condition for labor in the North is given by

$$L_t^N = \int_{\theta_t^N} L_{y,t}^N(j) dj + \int_0^1 L_{r,t}^N(j) dj. \quad (37)$$

The amount of labor employed for production by Northern quality leaders is

$$\int_{\theta_t^N} L_{y,t}^N(j) dj = \int_{\theta_t^N} \frac{q_t(j)}{Q_t} \tilde{L}_{y,t}^N dj = \frac{Q_t^N}{Q_t} \tilde{L}_{y,t}^N, \quad (38)$$

where the first equality uses (16). The amount of labor employed for innovative R&D is

$$\int_0^1 L_{r,t}^N(j) dj = \gamma\varphi_t^N \int_0^1 q_t(j) dj = \gamma\varphi_t^N Q_t, \quad (39)$$

where the first equality uses (19) and the symmetry condition  $\varphi_t^N(j) = \varphi_t^N$ . We define  $x_t^N$  as the average quality per Northern worker such that

$$x_t^N \equiv \frac{Q_t}{L_t^N}.$$

Finally, substituting (34), (38) and (39) into (37) yields the steady-state Northern labor-market condition expressed in per-capita terms given by

$$1 = \frac{\lambda\varphi^N}{\lambda\varphi^N + \varphi^F} \frac{\tilde{L}_{y,t}^N}{L_t} \frac{1}{1-s} + \gamma\varphi^N x^N, \quad (40)$$

where we also have used  $L_t^N = (1-s)L_t$ .

### 3.3 Southern labor market

The market-clearing condition labor in the South is given by

$$L_t^S = \int_{\theta_t^S} L_{y,t}^S(j) dj + \int_{\theta_t^F} L_{y,t}^F(j) dj + \int_{\theta_t^N} L_{r,t}^F(j) dj. \quad (41)$$

The amount of labor employed for production by Southern competitive firms is

$$\int_{\theta_t^S} L_{y,t}^S(j) dj = \int_{\theta_t^S} \frac{q_t(j)}{Q_t} \tilde{L}_{y,t}^S dj = \frac{Q_t^S}{Q_t} \tilde{L}_{y,t}^S, \quad (42)$$

where the first equality uses (16). The amount of labor employed for production by Southern affiliates is

$$\int_{\theta_t^F} L_{y,t}^F(j) dj = \int_{\theta_t^F} \frac{q_t(j)}{Q_t} \tilde{L}_{y,t}^F dj = \frac{Q_t^F}{Q_t} \tilde{L}_{y,t}^F, \quad (43)$$

where the first equality also uses (16). The amount of labor employed for adaptive R&D by Southern affiliates is

$$\int_{\theta_t^N} L_{r,t}^F(j) dj = \alpha \varphi_t^F \int_{\theta_t^N} q_t(j) dj = \alpha \varphi_t^F \frac{Q_t^N}{Q_t} Q_t, \quad (44)$$

where the first equality uses (21) and the symmetry condition  $\varphi_t^F(j) = \varphi_t^F$ . Substituting (34)-(36) and (42)-(44) into (41) yields the steady-state Southern labor market condition expressed in per-capita terms given by

$$1 = \frac{\varphi^F}{\lambda \varphi^N + \varphi^F} \left( \frac{\phi}{\lambda \varphi^N + \phi} \frac{\tilde{L}_{y,t}^S}{L_t^S} + \frac{\lambda \varphi^N}{\lambda \varphi^N + \phi} \frac{\tilde{L}_{y,t}^F}{L_t^S} + \alpha \lambda \varphi^N \frac{Q_t}{L_t^S} \right), \quad (45)$$

where  $Q_t/L_t^S = x^N L_t^N/L_t^S = x^N(1-s)/s$  and

$$\frac{\phi}{\lambda \varphi^N + \phi} \frac{\tilde{L}_{y,t}^S}{L_t^S} + \frac{\lambda \varphi^N}{\lambda \varphi^N + \phi} \frac{\tilde{L}_{y,t}^F}{L_t^S} = \underbrace{\left[ \frac{\phi}{\lambda \varphi^N + \phi} \left( \frac{\sigma}{\sigma - 1} \right)^\sigma + \frac{\lambda \varphi^N}{\lambda \varphi^N + \phi} \right]}_{\equiv \Phi(\phi)} \frac{\tilde{L}_{y,t}^F}{L_t^S} \frac{1}{s},$$

which uses (14), (15) and  $L_t^S = sL_t$ . It is useful to note that  $\Phi(\phi)$  is increasing in  $\phi$ .

### 3.4 Innovation and technology transfer

We first derive the growth rate of the quality index. Differentiating (32) with respect to time yields

$$\dot{Q}_t = \int_0^1 \left[ \lambda^{n_t(j)+1} - \lambda^{n_t(j)} \right] \varphi_t^N dj = (\lambda - 1) \varphi_t^N Q_t. \quad (46)$$

Then, taking the log of  $x_t^N = Q_t/L_t^N$  and differentiating with respect to time yields

$$\frac{\dot{x}_t^N}{x_t^N} = \frac{\dot{Q}_t}{Q_t} - \frac{\dot{L}_t^N}{L_t^N} = (\lambda - 1) \varphi_t^N - g_L. \quad (47)$$

In the steady state,  $x_t^N$  is stationary implying that the steady-state arrival rate of innovation is

$$\varphi^N = g_L / (\lambda - 1), \quad (48)$$

which is determined by exogenous parameters in this semi-endogenous growth model. As discussed in Dinopoulos and Segerstrom (2010), the law of motion in (47) implies that any increase (decrease) in the steady-state level of  $x^N$  must be associated with a temporary increase (decrease) in  $\varphi_t^N$  during the transition path. Therefore, if a parameter increases (decreases)  $x^N$  in the long run, it must have increased (decreased)  $\varphi_t^N$  in the short run.

Using (24) and (25), one can show that the balanced-growth values of assets are<sup>17</sup>

$$v_t^N(j) = \frac{\Pi_t^N(j)}{\rho + \varphi^N}, \quad (49)$$

$$v_t^F(j) = \frac{\Pi_t^F(j)}{\rho + \varphi^N + \phi}. \quad (50)$$

Substituting (17) and (49) into (20) yields the following steady-state innovative R&D condition:

$$(\sigma - 1)(\rho + \varphi^N) (1 + i^N) \gamma = \frac{\tilde{L}_{y,t}^N}{Q_t} = \frac{1}{(1 - s)x^N} \frac{\tilde{L}_{y,t}^N}{L_t}, \quad (51)$$

where the second equality is obtained by multiplying  $\tilde{L}_{y,t}^N/Q_t$  by  $1 = (L_t/L_t)(L_t^N/L_t^N)$ . Similarly, substituting (17), (18), (49) and (50) into (22) yields the following steady-state adaptive R&D condition:

$$(\sigma - 1)(\rho + \varphi^N + \phi) [(1 + i^S) \alpha + (1 + i^N) \gamma \omega] = \frac{\tilde{L}_{y,t}^F}{Q_t} = \frac{1}{(1 - s)x^N} \frac{\tilde{L}_{y,t}^F}{L_t}, \quad (52)$$

where  $\omega \equiv w_t^N/w_t^S$  is the relative wage between the two countries. Using (6)-(10) and (16), we derive

$$\frac{\tilde{L}_{y,t}^F}{L_t} = \omega^\sigma \frac{\tilde{L}_{y,t}^N}{L_t}. \quad (53)$$

Substituting (51) and (52) into (53) yields the following steady-state relative-wage condition:

$$\frac{\rho + \varphi^N}{\rho + \varphi^N + \phi} \omega^\sigma - \omega = \frac{(1 + i^S) \alpha}{(1 + i^N) \gamma}, \quad (54)$$

which is an implicit function determining the steady-state equilibrium value of the relative wage  $\omega(i^N, i^S)$ . It can be shown using (54) that  $\omega(i^N, i^S)$  is decreasing in  $i^N$  and increasing in  $i^S$ . Given  $\sigma > 1$ , it is easy to show that  $\omega > 1$ . Then, to ensure that  $z > \omega$ ,<sup>18</sup> we impose the following parameter restriction:

$$\frac{\rho + \varphi^N}{\rho + \varphi^N + \phi} z^\sigma - z > \frac{(1 + i^S) \alpha}{(1 + i^N) \gamma}. \quad (P1)$$

<sup>17</sup>Derivations are available upon request.

<sup>18</sup> $z > \omega$  is equivalent to  $w^S > w^N/z$ .

Substituting (51) into (40) to eliminate  $\tilde{L}_{y,t}^N/L_t$  yields the *Northern steady-state condition* given by

$$1 = \gamma x^N \left[ (\sigma - 1) (\rho + \varphi^N) \frac{\lambda \varphi^N}{\lambda \varphi^N + \varphi^F} (1 + i^N) + \varphi^N \right], \quad (55)$$

which is the same as in Dinopoulos and Segerstrom (2010) except for the addition of the Northern nominal interest rate  $i^N$ . The Northern steady-state condition contains two endogenous variables  $\{x^N, \varphi^F\}$ <sup>19</sup> and is positively sloped in the  $(x^N, \varphi^F)$  space with a positive  $x^N$ -intercept. Substituting (52) into (45) to eliminate  $\tilde{L}_{y,t}^F/L_t$  yields the *Southern steady-state condition* given by

$$1 = \frac{x^N \varphi^F (1 - s)/s}{\lambda \varphi^N + \varphi^F} \left\{ (\sigma - 1) (\rho + \varphi^N + \phi) \left[ (1 + i^S) \alpha + (1 + i^N) \gamma \omega(i^N, i^S) \right] \Phi(\phi) + \alpha \lambda \varphi^N \right\}, \quad (56)$$

which is the same as in Dinopoulos and Segerstrom (2010) except for the additions of  $\{i^N, i^S\}$ . The Southern steady-state condition also contains two endogenous variables  $\{x^N, \varphi^F\}$  and is negative sloped in the  $(x^N, \varphi^F)$  space with no intercepts. Finally, (55) and (56) are the two conditions that implicitly solve for the steady-state equilibrium values of  $\{x^N, \varphi^F\}$ . Graphically,  $x^N$  and  $\varphi^F$  are determined by the intersection of the North curve and the South curve in Figure 1.

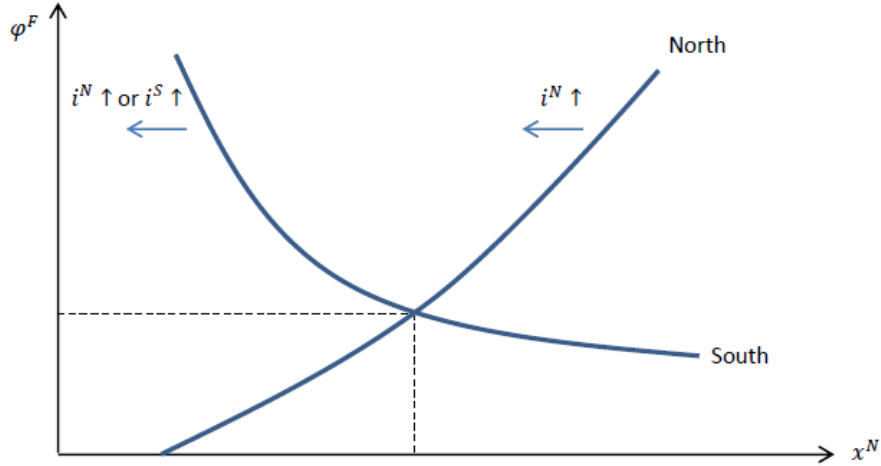


Figure 1: The steady-state equilibrium

### 3.5 Social welfare

In this section, we derive the steady-state level of social welfare in each country, which we will use to simulate the welfare effects of monetary policy in the quantitative analysis. Imposing

<sup>19</sup>Recall that  $\varphi^N = g_L/(\lambda - 1)$  in the steady state.



balanced growth on (1) yields the steady-state welfare of the Northern household given by

$$U^N = \frac{1}{\rho - g_L} \left( \ln c_0^N + \frac{g_c}{\rho - g_L} \right), \quad (57)$$

where  $g_c = g_L/(\sigma - 1)$  is determined by exogenous parameters due to semi-endogenous growth. Therefore, the steady-state welfare is determined by the balanced-growth level of consumption. Substituting the lump-sum transfer  $\tau_t^N$  from the government into (2) yields

$$c_t^N = (r_t - \dot{a}_t^N/a_t^N - g_L)a_t^N + i_t^N b_t^N + w_t^N.$$

Therefore, the balanced-growth level of consumption  $c_0^N$  is given by the sum of (a) asset income  $(\rho - g_L)a_0^N$ , (b) interest income  $i^N b_0^N$ ,<sup>20</sup> and (c) wage income  $w_0^N$ . An analogous derivation applies to the steady-state welfare of the Southern household. To determine  $a_0^N$  and  $a_0^S$ , we need to impose an assumption on the distribution of assets. Following Dinopoulos and Segerstrom (2010), we assume that the asset from innovative R&D in the North is owned by the Northern household whereas the asset from adaptive R&D in the South is owned by the Southern household. Under this assumption, we show in Lemma 2 that the balanced-growth levels of consumption can be expressed as  $c_0^N = w_0^N I^N$  and  $c_0^S = w_0^S I^S$ , where  $\{I^N, I^S\}$  denote income as a ratio of real wages because asset income and interest income are proportional to  $\{w_0^N, w_0^S\}$ .

**Lemma 2** *The balanced-growth level of consumption can be expressed as*

$$c_0^N = w_0^N I^N = (\Psi L_0^N x^N)^{\frac{1}{\sigma-1}} I^N, \quad (58)$$

$$c_0^S = w_0^S I^S = \frac{(\Psi L_0^N x^N)^{\frac{1}{\sigma-1}}}{\omega} I^S, \quad (59)$$

where  $L_0^N$  is exogenous and  $\{\Psi, I^N, I^S\}$  are given by

$$\begin{aligned} \Psi &= \underbrace{\frac{\lambda\varphi^N}{\lambda\varphi^N + \varphi^F} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}}_{\text{Northern leaders}} + \underbrace{\frac{\lambda\varphi^N}{\lambda\varphi^N + \phi\lambda\varphi^N + \varphi^F} \frac{\varphi^F}{\lambda\varphi^N + \varphi^F} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}}_{\text{Southern affiliates}} \omega^{\sigma-1} + \underbrace{\frac{\phi}{\lambda\varphi^N + \phi} \frac{\varphi^F}{\lambda\varphi^N + \varphi^F} \omega^{\sigma-1}}_{\text{Southern competitive firms}}, \\ I^N &= \underbrace{(\rho - g_L)(1 + i^N)\gamma x^N \left(\frac{\lambda\varphi^N}{\lambda\varphi^N + \varphi^F} + \frac{\lambda\varphi^N}{\lambda\varphi^N + \phi\lambda\varphi^N + \varphi^F} \frac{\varphi^F}{\lambda\varphi^N + \varphi^F}\right)}_{\text{asset income}} + \underbrace{i^N \varphi^N \gamma x^N}_{\text{interest income}} + \underbrace{1}_{\text{wage income}}, \\ I^S &= \underbrace{(\rho - g_L)(1 + i^S)\alpha x^N \left(\frac{\lambda\varphi^N}{\lambda\varphi^N + \phi\lambda\varphi^N + \varphi^F} \frac{\varphi^F}{\lambda\varphi^N + \varphi^F}\right) \frac{1-s}{s}}_{\text{asset income}} + \underbrace{i^S \varphi^F \alpha x^N \frac{\lambda\varphi^N}{\lambda\varphi^N + \varphi^F} \frac{1-s}{s}}_{\text{interest income}} + \underbrace{1}_{\text{wage income}}. \end{aligned}$$

**Proof.** See Appendix A. ■

<sup>20</sup>Interest income  $i^N b^N$  appears in the budget of the household because together with R&D labor income (captured by wage income  $w^N$ ), it represents the cost of R&D that is paid to the household.

The intuition of the above expressions can be explained as follows. Recall that real wages are given by  $w_0^N = (\Psi L_0^N x^N)^{\frac{1}{\sigma-1}}$  and  $w^S = (\Psi L_0^N x^N)^{\frac{1}{\sigma-1}} / \omega$ ; therefore, the term  $\Psi$  captures the quality contributions of Northern leaders, Southern affiliates, and Southern competitive firms to consumption through the real wage. As for the terms  $I^N$  and  $I^S$ , they represent the contributions of the different sources of income to consumption.

## 4 Monetary policy

In this section, we explore the effects of monetary policy. Section 4.1 analyzes the effects of the nominal interest rates in the two countries on the equilibrium rates of innovation and international technology transfer. In Section 4.2, we calibrate the model to provide a quantitative analysis.

### 4.1 Comparative statics

In this section, we explore the effects of monetary policy. An increase in the Southern nominal interest rate  $i^S$  affects only the Southern steady-state condition in (56). Specifically, it shifts the South curve to the left in Figure 1. As a result, both  $\varphi^F$  and  $x^N$  decrease along with an increase in  $\omega$  as implied by (54). Intuitively, an increase in the nominal interest rate  $i^S$  in the South raises the cost of adaptive R&D and reduces the equilibrium rate of international technology transfer  $\varphi^F$ . The decrease in the number of products manufactured by Southern affiliates implies more products being produced by Northern firms. The higher demand for production labor causes a reallocation of labor in the North from R&D to production. The decrease in innovative R&D in the North decreases the rate of innovation in the short run and leads to a lower average quality per worker  $x^N$  in the long run. Finally, given that the increase in  $i^S$  has a direct negative effect on the demand for Southern R&D labor, it depresses the wage rate in the South relative to the North. We summarize these results in Proposition 1.

**Proposition 1** *An increase in the nominal interest rate in the South leads to (a) a permanent decrease in the rate of technology transfer from the North to the South, (b) a permanent increase in the North-South wage gap, and (c) a temporary decrease in the rate of innovation in the North.*

**Proof.** See Appendix A. ■

An increase in the Northern nominal interest rate  $i^N$  affects both the Northern and Southern steady-state conditions in (55) and (56). Specifically, it shifts both the South curve and the North curve to the left in Figure 1. As a result, the effect on  $\varphi^F$  is ambiguous, and  $x^N$  decreases along with a decrease in  $\omega$  as implied by (54). Intuitively, an increase in the nominal interest rate  $i^N$  in the North raises the cost of innovative R&D. As a result, the rate of innovation decreases in the short run, and the average quality per worker  $x^N$

decreases in the long run. Given that the increase in  $i^N$  has a direct negative effect on the demand for Northern R&D labor, it depresses the wage rate in the North relative to the South.

As for the effect of  $i^N$  on the rate of international technology transfer  $\varphi^F$ , there are two opposing effects. To see this, we use  $\varphi_t^F(j) = \varphi_t^F$  and integrate (21) over  $\theta_t^N$  to derive

$$\varphi_t^F = \frac{1}{\alpha Q_t^N} \int_{\theta_t^N} L_{r,t}^F(j) dj = \frac{1}{\alpha x_t^N} \int_{\theta_t^N} L_{r,t}^F(j) dj \frac{Q_t}{Q_t^N (1-s)L_t}, \quad (60)$$

where the second equality uses  $x_t^N = Q_t/L_t^N$  and  $L_t^N = (1-s)L_t$ . In the steady state,  $Q_t^N/Q_t$  is given by (34), and hence, (60) can be reexpressed as

$$\frac{\lambda \varphi^N \varphi^F}{\lambda \varphi^N + \varphi^F} = \frac{1}{\alpha x^N} \int_{\theta^N} L_{r,t}^F(j) dj \frac{1}{(1-s)L_t}, \quad (61)$$

where the left-hand side is monotonically increasing in  $\varphi^F$ . From (61), we see that the Northern nominal interest rate  $i^N$  affects  $\varphi^F$  via the quality level per worker  $x^N$  and the number of adaptive R&D workers  $\int_{\theta^N} L_{r,t}^F(j) dj$ . On the one hand, an increase in  $i^N$  reduces  $x^N$  and has a positive effect on  $\varphi^F$  by decreasing the difficulty of adaptive R&D. On the other hand, the increase in  $i^N$  also reduces the incentives for adaptive R&D by changing the asset values. To see this, we combine (49) and (50) to derive

$$\frac{v_t^F(j)}{v_t^N(j)} = \frac{\rho + \varphi^N}{\rho + \varphi^N + \phi} \frac{\Pi_t^F(j)}{\Pi_t^N(j)} = \frac{\rho + \varphi^N}{\rho + \varphi^N + \phi} \left( \frac{w_t^N}{w_t^S} \right)^{\sigma-1}, \quad (62)$$

where the second equality uses (17)-(18) and then (13)-(14). Recall that the increase in  $i^N$  reduces the relative wage  $\omega = w_t^N/w_t^S$ ; therefore, it also reduces  $v_t^F(j)/v_t^N(j)$ . In other words, the decrease in the North-South wage gap makes adaptive R&D less attractive relative to innovative R&D. This leads to a decrease in adaptive R&D in the South, which in turn has a negative effect on the rate of international technology transfer  $\varphi^F$ . This negative effect of  $i^N$  via the number of adaptive R&D workers in the South is relatively strong when the Southern population size  $s$  is large. Therefore, the overall effect of  $i^N$  on  $\varphi^F$  would be negative if  $s$  is sufficiently large, and vice versa. We summarize these results in Proposition 2.

**Proposition 2** *An increase in the nominal interest rate in the North leads to (a) a temporary decrease in the rate of innovation in the North, (b) a permanent decrease in the North-South wage gap, and (c) a permanent decrease (increase) in the rate of technology transfer to the South if Southern population size is sufficiently large (small).*

**Proof.** See Appendix A. ■

## 4.2 Quantitative analysis

In this section, we provide a quantitative analysis on the effects of monetary policy. For the parameter values, we either set them to conventional values in the literature or calibrate

them using empirical moments from aggregate data of China and the US. In the above qualitative analysis, we obtain the realistic pattern of production shifting back to the North upon the arrival of new innovations by imposing  $z > \omega$  using the parameter restriction in (P1). However, it is unlikely for  $z > \omega$  to hold in the data; for example, the China-US wage gap (approximated by the relative GDP per worker) is 6.354 in 2010. Although the literature does not provide a precise empirical estimate of the quality step size  $z$ , it is unlikely to exceed this value. Nevertheless, we do observe a pattern of offshoring and reshoring in reality. For example, in a recent survey, the Boston Consulting Group (2011) document that "[t]ransportation goods such as vehicles and auto parts, electrical equipment including household appliances, and furniture are among seven sectors that could create 2 to 3 million jobs as a result of manufacturing returning to the U.S." The reason is that despite the much lower wages in China, there are other costs associated with production in China that our model does not capture. Therefore, although our calibrated parameter values imply  $z < \omega$ , we carry out the simulation by assuming that whenever a higher-quality product is invented in the North, it must dominate the market initially and be produced in the North until its manufacturing process is transferred to the South. This assumption allows the model to deliver a realistic pattern of offshoring and reshoring between the US and China.

For the discount rate  $\rho$ , we set it to a conventional value of 0.03. For the population growth rate  $g_L$ , we set it to the average population growth rate of 0.0114 in the US from 1991 to 2011.<sup>21</sup> For the relative Southern population size  $s$ , we set it to 0.811 based on data from the Penn World Table on the population size of China and the US in 2010. For the quality step size  $z$ , we follow Acemoglu and Akcigit (2012) to consider a value of 1.05. For the imitation rate  $\phi$ , we calibrate it by matching the relative wage  $\omega$  from the model to the data. We use data from the Penn World Table on the relative GDP per worker between the US and China as an approximated value of  $\omega$ , and this value is 6.354 in 2010. In the model, it is  $\alpha/\gamma$  (rather than the individual values of  $\alpha$  and  $\gamma$ ) that determines the values of variables in equilibrium.<sup>22</sup> We calibrate  $\alpha/\gamma$  by using the R&D share of GDP in the US relative to the R&D share of GDP in China.<sup>23</sup> According to the OECD Research and Development Statistics, the average R&D share of GDP is 0.0110 in China and 0.0257 in the US from 1991 to 2011.<sup>24</sup> For the substitution elasticity  $\sigma$ , we calibrate it by using the innovation arrival rate  $\varphi^N$ , and we follow Acemoglu and Akcigit (2012) to consider 3 years as the expected duration of time between arrivals of any two consecutive innovations in an industry. The calibrated value of  $\sigma$  is 1.7, which is slightly less than the estimate in Broda and Weinstein (2006).<sup>25</sup> Finally, we calibrate  $i^S$  and  $i^N$  using average inflation rates in China and the US, and  $\pi^S$  is 4.841% and  $\pi^N$  is 2.625% from 1991 to 2011 according to the World Bank Development Indicators. Under these calibrated parameter values, the

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<sup>21</sup>We consider the population growth rate in the US (instead of China) because it determines the steady-state rate of innovation that is driven by R&D in the US.

<sup>22</sup> $x^N$  is the only variable affected by  $\gamma$ , but the equilibrium value of  $\gamma x^N$  is independent of  $\gamma$ . Given that it is the value of  $\gamma x^N$  that matters, we simply normalize  $\gamma$  to one when reporting the value of  $x^N$ .

<sup>23</sup>Unfortunately, our model is unable to match the absolute R&D share of GDP in each country. As discussed in Acemoglu and Akcigit (2012), R&D-based growth models usually imply too much R&D in equilibrium compared to the data; see their footnote 26.

<sup>24</sup>The data series for China is available only from 1991 to 2011.

<sup>25</sup>In Broda and Weinstein (2006), the median estimate of the elasticity of substitution is 2.2 for three-digit industries based on data from 1990 to 2001.

equilibrium values of  $\{x^N, \varphi^F\}$  are respectively 1.790 and 0.0609. We provide a summary of the calibration in Table 1.

Parameters	$\rho$	$z$	$\phi$	$\alpha/\gamma$	$\sigma$	$i^S$	$i^N$	$g_L$	$s$
	0.03	1.050	0.131	10.378	1.698	0.095	0.073	0.0114	0.811
Variables	$r$	$g_c$	$\omega$	$R\&D^{N/S}$	$\varphi^N$	$\pi^S$	$\pi^N$	$x^N$	$\varphi^F$
	0.0464	0.0164	6.354	2.341	0.330	4.841%	2.625%	1.790	0.0609

Given these calibrated parameter values, we consider the following experiments: (a) decreasing the Southern nominal interest rate  $i^S$  to zero, and (b) decreasing the Northern nominal interest rate  $i^N$  to zero. The results are reported in Table 2. We find that a permanent decrease in the nominal interest rate in China to 0% would reduce the wage gap  $\omega$  by about 4% (percent change) and increase technology transfer  $\varphi^F$  by about 6% (percent change). It also leads to a long-run welfare gain of 3.37% in China and a welfare gain of 1.25% in the US.<sup>26</sup> In this case, the welfare gains in the two countries mostly come from the increase in real wages as a result of increased quality from innovation. A permanent decrease in the nominal interest rate in the US to 0% would raise the wage gap  $\omega$  by about 3% and decrease technology transfer  $\varphi^F$  by about 2%. Here  $\varphi^F$  decreases despite an increase in adaptive R&D because of the increase in the difficulty index  $x^N$ . The effect of  $x^N$  on  $\varphi^F$  dominates because  $s$  is not sufficiently large despite the rather large population in China. The decrease in  $i^N$  also leads to a welfare gain of 0.33% in the US and a welfare gain of 1.24% in China. In this case, the welfare gain in the US is relatively small because the increase in real wage in the North is offset by a decrease in interest income, which is an important component of income due to the relatively large amount of assets owned by the Northern household. In contrast, the real wage is by far the most important component of income for the Southern household. To conclude, we find that the cross-country welfare effects of monetary policy are quantitatively significant.

$i^S$	$\omega$	$x^N$	$\varphi^F$	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.095	6.354	1.790	0.0609	-	-	-	-
0	6.089	1.797	0.0648	1.225%	5.483%	1.245%	3.371%
$i^N$	$\omega$	$x^N$	$\varphi^F$	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.073	6.354	1.790	0.0609	-	-	-	-
0	6.569	1.839	0.0598	4.530%	1.206%	0.328%	1.237%

## 5 Conclusion

In this study, we have analyzed the effects of monetary policy via CIA constraints on R&D investment in a Schumpeterian economy with North-South product cycles. We show that monetary policy affects innovation, technology transfer and the allocation of manufacturing

<sup>26</sup>Welfare changes are all expressed in the usual equivalent variation in consumption.

activities across countries. Furthermore, calibrating the model to China-US data, we find that the cross-country welfare gains from decreasing the nominal interest rates are quantitatively significant. For example, decreasing the nominal interest rate to zero in China would lead to a long-run welfare gain of 1.25% in the US, whereas decreasing the nominal interest rate to zero in the US would lead to a long-run welfare gain of 1.24% in China. These results highlight the quantitative significance of cross-country spillover effects of monetary policy.

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## Appendix A

**Proof of Lemma 1.** As in Dinopoulos and Segerstrom (2010), the dynamics of the quality indices is given by

$$\begin{aligned}\dot{Q}_t^N &= \int_{\theta_t^N} [\lambda^{n_t(j)+1} - \lambda^{n_t(j)}] \varphi_t^N dj + \int_{\theta_t^F + \theta_t^S} \lambda^{n_t(j)+1} \varphi_t^N dj - \int_{\theta_t^N} \lambda^{n_t(j)} \varphi_t^F dj \\ &= (\lambda - 1) \varphi_t^N Q_t^N + \lambda \varphi_t^N (Q_t^F + Q_t^S) - \varphi_t^F Q_t^N,\end{aligned}$$

$$\begin{aligned}\dot{Q}_t^F &= \int_{\theta_t^N} \lambda^{n_t(j)} \varphi_t^F dj - \int_{\theta_t^F} \lambda^{n_t(j)} \varphi_t^N dj - \int_{\theta_t^F} \lambda^{n_t(j)} \phi dj \\ &= \varphi_t^F Q_t^N - \varphi_t^N Q_t^F - \phi Q_t^F,\end{aligned}$$

$$\begin{aligned}\dot{Q}_t^S &= \int_{\theta_t^F} \lambda^{n_t(j)} \phi dj - \int_{\theta_t^S} \lambda^{n_t(j)} \varphi_t^N dj \\ &= \phi Q_t^F - \varphi_t^N Q_t^S.\end{aligned}$$

Let's define  $Q_t^{FS} \equiv Q_t^F + Q_t^S$ , which implies  $\dot{Q}_t^{FS} = \varphi_t^F Q_t^N - \varphi_t^N Q_t^{FS}$ . Setting  $\dot{Q}_t^N/Q_t^N = \dot{Q}_t^{FS}/Q_t^{FS}$  yields (34), using  $Q_t^{FS} = Q_t - Q_t^N$ . Setting  $\dot{Q}_t^F/Q_t^F = \dot{Q}_t^S/Q_t^S$  yields  $Q_t^S/Q_t = (Q_t^F/Q_t) [\phi / (\lambda \varphi_t^N)]$ , noting  $Q_t^{FS}/Q_t^N = (1 - Q_t^N/Q_t) / (Q_t^N/Q_t) = \varphi_t^F / (\lambda \varphi_t^N)$ . Applying this to  $Q_t^F/Q_t + Q_t^S/Q_t = 1 - Q_t^N/Q_t$  and using (34), equations (35) and (36) follow. ■

**Proof of Lemma 2.** Time arguments are omitted for convenience. Using  $\tau^N = (\dot{M}^N + g_L M^N)/P^N$  and  $\dot{M}^N/M^N = \pi^N + \dot{m}^N/m^N$ , we derive  $\tau^N = (\pi^N + g_L) m^N + \dot{m}^N$ . Substituting this condition into the balanced-growth version of (2) yields

$$c^N = (\rho - g_L) a^N + i^N w^N \varphi^N \gamma x^N + w^N, \quad (\text{A1})$$

where we have used  $r^N = \rho + g_L / (\sigma - 1)$ ,  $\dot{a}^N/a^N = g_L / (\sigma - 1)$  and  $m^N = b^N = \int_0^1 w^N \varphi^N \gamma q(j) dj / L^N = w^N \varphi^N \gamma x^N$ . Following Dinopoulos and Segerstrom (2010), we assume that the Northern household finances innovative R&D in equilibrium. That is,  $L^N a^N = \int_{\theta^N + \theta^F} v^N(j) dj$ . Given that  $v^N(j) = (1 + i^N) w^N \gamma q(j)$  from (20), we have

$$a^N = (1 + i^N) \gamma x^N w^N \left( \frac{\lambda \varphi^N}{\lambda \varphi^N + \varphi^F} + \frac{\lambda \varphi^N}{\lambda \varphi^N + \phi} \frac{\varphi^F}{\lambda \varphi^N + \varphi^F} \right), \quad (\text{A2})$$

which uses Lemma 1. Using (A1) and (A2), we can show that  $c^N = w^N I^N$ , where  $I^N$  is defined in Lemma 2. By incorporating (8), (10) and (12) into the aggregate price index  $\left\{ \int [p_t(j)]^{1-\sigma} dj \right\}^{1/(1-\sigma)} = 1$ , we can show that the real wage in the North is  $w^N = (\Psi L^N x^N)^{\frac{1}{\sigma-1}}$ , which uses Lemma 1 and  $x^N = Q/L^N$ .  $\Psi$  is defined in Lemma 2, and we have derived (58). Applying analogous derivations to the Southern asset condition, one can also derive (59) by noting that  $m^S = b^S = \int_{\theta^N} w^S \varphi^F \alpha q(j) dj / L^S$  and  $L^S a^S = \int_{\theta^F} [v^F(j) - v^N(j)] dj$ , which comes from the assumption that the Southern household finances adaptive R&D and that  $v^F(j) - v^N(j) = (1 + i^S) w^S \alpha q(j)$  from (22). ■



**Proof of Proposition 1.** It is easy to graphically show from (54) that  $\omega$  increases with  $i^S$ , proving (b). Given this, an increase in  $i^S$  leads to a downward shift in the South curve (56), whereas it has no effect on the North curve (55). Applying a simple graphical analysis to Figure 1, we find that an increase in  $i^S$  leads to permanent decreases in  $\varphi^F$  and  $x^N$ . This proves (a) and also (c) because a permanent decrease in  $x^N$  must be associated with a temporary decrease in the innovation rate  $\varphi_t^N$  below its steady-state level  $\varphi^N = g_L/(\lambda - 1)$  given the dynamics in (47). ■

**Proof of Proposition 2.** Graphical analysis with (54) implies that  $\omega$  decreases with  $i^N$ , proving (b). An increase in  $i^N$  leads to a downward shift in both the North and South curves, (55) and (56), given that we can easily show from (54) that  $(1 + i^N)\omega$  increases with  $i^N$ . Thus, an increase in  $i^N$  leads to a decrease in  $x^N$ , implying a temporary decrease in the innovation rate  $\varphi_t^N$  given the dynamics in (47) and proving (a). As for (c), we solve (55) and (56) for  $\varphi^F$  to obtain<sup>27</sup>

$$\varphi^F = \lambda\varphi^N \frac{s}{1-s} \frac{(1+i^N) + \frac{1}{\sigma-1} \frac{\varphi^N}{\rho+\varphi^N}}{(1+i^N)\omega^\sigma \Phi(\phi) - \frac{1}{\sigma-1} \frac{\varphi^N}{\rho+\varphi^N} \left( \frac{s}{1-s} - \frac{\alpha\lambda}{\gamma} \right)}. \quad (\text{A3})$$

Differentiating (A3) with respect to  $i^N$ , we find that  $d\varphi^F/di^N > (<) 0$  holds if the following inequality holds:<sup>28</sup>

$$\omega^\sigma \Phi(\phi) \left( \frac{\sigma \frac{(1+i^S)\alpha}{\gamma} - \frac{\varphi^N}{\rho+\varphi^N} \omega}{\frac{\sigma}{\sigma-1} \frac{(1+i^S)\alpha}{(1+i^N)\gamma} + \omega} \right) > (<) \frac{\varphi^N}{\rho + \varphi^N} \left( \frac{s}{1-s} - \frac{\alpha\lambda}{\gamma} \right). \quad (\text{A4})$$

Given that the right-hand side of (A4) is monotonically increasing in  $s$ ,  $d\varphi^F/di^N > (<) 0$  becomes more likely to hold as  $s$  decreases (increases). Given that  $s$  has an upper bound  $\tilde{s}$ ,<sup>29</sup> which ensures  $\varphi^F > 0$ , we can show that the inequality  $<$  in (A4) must hold as  $s \rightarrow \tilde{s}$  implying that  $d\varphi^F/di^N < 0$  for a sufficiently large  $s$ . As  $s \rightarrow 0$ , the right-hand side of (A4) becomes negative. Therefore,  $d\varphi^F/di^N > 0$  holds if the left-hand side of (A4) is positive, which is guaranteed by  $\sigma\alpha/\gamma > z\varphi^N/(\rho + \varphi^N)$  given that  $z > \omega$ . ■

<sup>27</sup>Here we have used the following condition derived from (54):

$$(1+i^S)\alpha + (1+i^N)\gamma\omega = \frac{\rho + \varphi^N}{\rho + \varphi^N + \phi} (1+i^N)\gamma\omega^\sigma.$$

<sup>28</sup>Here have used the following condition derived from (54):

$$\frac{d(1+i^N)\omega^\sigma}{di^N} = \omega^\sigma \left( \frac{\frac{\sigma-1}{\sigma}\omega}{\frac{(1+i^S)\alpha}{(1+i^N)\gamma} + \frac{\sigma-1}{\sigma}\omega} \right).$$

<sup>29</sup>This is defined by

$$(1+i^N)\omega^\sigma \Phi(\phi) = \frac{1}{\sigma-1} \frac{\varphi^N}{\rho + \varphi^N} \left( \frac{\tilde{s}}{1-\tilde{s}} - \frac{\alpha\lambda}{\gamma} \right).$$