Input-Output-based Measures of Systemic Importance

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Non-Technical Summary

One of the remarkable consequences of the recent financial crisis has been the shift in emphasis, in economic analysis as well as policy practice, from the *micro*-prudential to the *macro*-prudential approach to banking regulation and supervision (Hanson et al. (2011)). The micro-prudential approach focuses on controlling the risk of individual banks, taking the rest of the system as given. Since banks are funded largely by deposits, an instrument protected in most countries by a collectively funded safety net, the micro-prudential regulator/supervisor needs to control that bank managers do not take advantage of such protection to assume an unduly high level of risk (*moral hazard*). However, in the presence of risk externalities, i.e. when risk-taking behavior of individual institutions affects other institutions and the system as a whole, another source of social inefficiency arises, requiring the prudential regulator/supervisor to take a macro-prudential approach. Banks need to be controlled not only in relation to their propensity to undertake high risk due to the protection they enjoy on their liability side, but also in relation to the risk they transmit to other banks with which they are connected by a reciprocal web of exposures. To guide policy it becomes necessary, in this context, to measure the systemic importance of individual banks, i.e. the degree to which they propagate systemic risk by exerting influence on the rest of the system.

A large body of literature has developed to define and measure systemic risk and to analyze its implication for the conduct of prudential regulation and supervision (for a comprehensive overview see Bisias et al. (2012)). Broadly speaking, there are two classes of indicators of systemic importance proposed in the recent literature. The first and more widely used class of indicators relies, directly or indirectly, on asset prices distributions and correlations across institutions, assets and time. The second approach, with a rapidly rising literature is that of network analysis

Our contribution here consists in proposing an alternative method for measuring the transmission of risk within banking systems, which logically belongs to the second class mentioned in the preceding paragraph, namely that based on balance sheet measures of interconnection. Our approach is new and old at the same time. It is new because, as such, it has never been used in the analysis of banking. It is old because it draws on a well-established and time-honored strand of literature, that of input-output analysis.

Taking the balance sheet of the banking system as a point of departure, we derive expressions that closely resemble the traditional Leontief (1941) input-output

model. The input-output model makes evident the fact that the production of any sector has two distinct effects on the remaining sectors: on the one hand by increasing production it will demand more inputs from other sectors ("upstream"), on the other it will be able to provide more output to the sectors that depend its production as input to their own production process ("downstream"). The analogy with the problem of interconnected banks is quite obvious. If an individual bank is subject to a negative liquidity shock, it will transmit effects to the rest of the system via a contraction of lending to other banks, hence amplifying the effect on the entire system. The way the shock is transmitted depends on the matrix of interbank linkages

With this benchmark at hand and making use of the literature on linkages in input-output analysis and the transmission of risk in infrastructural systems, we present six measures of systemic importance, which rely heavily on the matrix of lending and borrowing positions in the interbank market. Each of these measures has an intuitive economic story behind, which is itself derived from the very structure of the model. The measures presented here aim at capturing different aspects of systemic importance, namely: *(i)* how does a shock to the funding side of one bank disperse through the rest of the system? *(ii)* how sensitive is a bank to a shock hitting simultaneously all other banks? *(iii)* what happens when the shock comes from interbank flows themselves? In particular, what happens if a bank sees its sources of interbank funding reduced? *(iv)* what if it is the bank itself who decides to cut financing to all other banks?, *(v)* what if the last two events happen simultaneously?, and finally, *(vi)* what if a bank is being completely cut off the interbank system?

The measures are illustrated by means of a simple numerical example which highlighted how the indicators operate and in which way they capture different aspects of risk stemming from the balance sheet of banks. We also draw some parallels to network centrality measures, both at a formal level as well as by means of a simulated network designed to resemble real-world characteristics.

Input-Output-based measures of systemic importance*

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Abstract

The analyses of intersectoral linkages of Leontief (1941) and Hirschman (1958) provide a natural way to study the transmission of risk among interconnected banks and to measure their systemic importance. In this paper we show how classic input-output analysis can be applied to banking and how to derive six indicators that capture different aspects of systemic importance, using a simple numerical example for illustration. We also discuss the relationship with other approaches, most notably network centrality measures, both formally and by means of a simulated network.

Keywords: *banks, input-output, systemic risk, too-interconnected-to-fail, networks, interbank markets*

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Contents

1 Introduction

One of the remarkable consequences of the recent financial crisis has been the shift in emphasis, in economic analysis as well as policy practice, from the *micro*-prudential to the *macro*-prudential approach to banking regulation and supervision (see Hanson et al. (2011) and references therein). The micro-prudential approach focuses on controlling the risk of individual banks, taking the rest of the system as given. Since banks are funded largely by deposits, an instrument protected in most countries by a collectively funded safety net, the micro-prudential regulator/supervisor needs to control that bank managers do not take advantage of such protection to assume an unduly high level of risk (*moral hazard*). However, in the presence of risk externalities, i.e. when risk-taking behavior of individual institutions affects other institutions and the system as a whole, another source of social inefficiency arises, requiring the prudential regulator/supervisor to take a macro-prudential approach. Banks need to be controlled not only in relation to their propensity to undertake high risk due to the protection they enjoy on their liability side, but also in relation to the risk they transmit to other banks with which they are connected by a reciprocal web of exposures. To guide policy it becomes necessary, in this context, to measure the systemic importance of individual banks, i.e. the degree to which they propagate systemic risk by exerting influence on the rest of the system.

Following the seminal work of Allen and Gale (2000) and especially in the aftermath of the crisis, there has been a growing interest in understanding, from a theoretical standpoint, which configuration of the interbank market might deliver more stability and render the system resilient to shocks (see also Freixas et al. (2000) or more recently Acemoglu et al. (2013) among others). While highly relevant and insightful, this type of inquiry need not concern us here, since our focus is on the assessment of systemic importance in order to pinpoint those banks that are more systemically relevant, for a given configuration of the banking system. In that respect, even when the measures proposed here are grounded on economic theory, the ultimate motivation is practical in nature.

A large body of literature has developed to define and measure systemic risk and to analyse its implication for the conduct of prudential regulation and supervision – we survey the most relevant part of this literature below. Our contribution here consists in proposing an alternative method for measuring the transmission of risk within banking systems. Our approach is new and old at the same time. It is new because, as such, it has never been used in the analysis of banking[1]. It is old because it draws on a well-established and time-honored strand of literature, that of input-

[1] Input-Output style algebra has been used for the analysis of cross-holdings in business groups in general. Brioschi et al. (1989), for instance, use matrix algebra quite close to ours in order to determine the value of member firms of a group (see also Fedenia et al. (1994)). More recently, Elliott et al. (2013) note that the critical matrix for their analysis of financial contagion (what they call "dependency matrix") has a Leontief flavour. As we will see below, we start with an explicit balance sheet representation of the banking sector and then perform transformations analogous to those done in traditional input-output analysis. Furthermore, we use this structure to propose measures of systemic importance with an economic interpretation derived from the construction itself.

output analysis. Since it was originally developed by Leontief in the interwar period to analyse the structure of the US economy (see Leontief (1941)), input-output analysis has enjoyed a broad popularity and has been applied to a variety of problems, ranging from energy and environmental analyses to computable general equilibrium models. Hirschman (1958) used this approach to identify key industrial sectors capable of sparking economy-wide growth in developing economies. In his analysis, key sectors are those characterised by strong linkages with others, because they use output of other sectors as intermediate inputs and/or their own output is used as intermediate input by other sectors. For this reason, their production can "activate" a large number of other sectors[2]. In recent years, input-output techniques have moved beyond economics, being increasingly used in the engineering literature to study the transmission of risk in interrelated infrastructural systems. This extension is close in spirit to our approach, because its focus shifts from the activation of sectoral production – the central focus of classic input-output analysis – to the transmission of risk.

The remainder of the paper is organized as follows: in Section 2 we briefly go over the concept of systemic risk and sketch the main research avenues on systemic risk measures which put our contribution into context; Section 3 presents the framework used to construct the measures, which are then presented and illustrated with a simple numerical example; Section 4 draws parallels to measures of systemic importance derived from network theory and illustrates the connection with simulated data; finally, Section 5 concludes.

2 Systemic risk: concepts and measures

The notion of systemic risk is closely linked to that of externalities; systemic is a risk whose consequences are not confined to an individual financial institution but extend beyond it: to other financial institutions, to the real domestic economy, or to the global economy, etc.. The transmission can take place through a variety of channels, balance sheet exposures, asset market bubbles and can include feedbacks as well as second (or multiple) round effects. The presence of systemic risk calls for enhanced regulatory and supervisory vigilance, on the system as a whole as well as on individual banks that are deemed to have systemic importance. A significant part of the effort to reform international financial architecture after the crisis, conducted by the Financial Stability

[2]While in classic input-output analysis the combination of inputs to achieve an output has a clear-cut (even *physical*) interpreation, when applied to banking the logic might not be so straightforward, since the inputs/outputs are homogenous (i.e. "money is money"), the heterogeneity being given by the identity of the sender/receiver of funds. Think for example of a car producer who suddenly suffers a drop (or even a complete cut) in its supply of tires; unless he is able to find another supplier, he might see his own production severely impaired. In a banking context there are obviously more potential sources for substitution, in theory actually as many possibilities as counterparties in the system. This calls for a degree of caution when translating one-to-one the insights from input-output analysis into a banking setting. Yet we believe that even when substitution might be perfect (something which in fact does not seem to be supported by the literature on relationship lending in the interbank market, see Footnote 9 below), a reduced supply of (or demand for) funds provided puts a strain in counterparties and the measures presented here provide a good way of capturing this.

Board and the Basel Committee on Banking Supervision under the aegis of the G20, has been aimed precisely at establishing a framework for controlling systemic risks in the financial sector (see Angeloni (2008)).

Recently, many indicators of systemic risk have been proposed. Among them, one variety that is of high importance in the context of the current debate on reforming the financial system is that of indicators of systemic influence, that is, indicators of the effects single financial institutions can have on the system as a whole. More systemically important institutions, so the argument goes, should be subject to stricter prudential requirements.

Doing full justice to the literature on systemic risk assessment would require an additional paper altogether. For this reason we refer to the contribution by Bisias et al. (2012), which to our knowledge is the most comprehensive review of systemic risk measures attempted thus far. We limit ourselves instead to a broad cathegorization that helps to put our contribution into context.

Generally speaking, there are two classes of indicators of systemic importance proposed in the recent literature. The first and more widely used class of indicators relies, directly or indirectly, on asset prices distributions and correlations across institutions, assets and time. Probability distributions of asset prices are used to calculate probabilities of default. Historical correlations are used, in various ways, to estimate contagion effects. This group of measures essentially tries to capture, in some way or the other, co-dependence at the tails of the distribution of returns, and is hence intimately related to the *Value at Risk (VaR)*. The main advantage of this approach is data availability; the main drawback is that using asset market data to quantify default probabilities assumes market efficiency, and estimating contagion effects with historical correlations assumes, at least, constancy of some structural parameters over time. It is well known that these assumptions cease to hold under market stress, hence invalidating previous inferences precisely at the time when accurate risk measures are most needed[3].

The second approach, with a rapidly rising literature, is that of network analysis (see ECB (2010))[4]. Network analysis estimates systemic importance from interlinkages maps among financial institutions, using techniques that were developed in other disciplines ranging from sociology to epidemiology. Links are typically estimated by aggregating reciprocal exposures, whose overall size determines the intensity of the links. This approach does not rely on strong hypotheses regarding the functioning of asset markets, but it comes at the cost of being more data demanding (bilateral exposure data do not normally exist or are not published)[5].

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[3]Danielsson et al. (2011) and Löffler and Raupach (2013) emphazise this line of criticism for the first group of systemic risk measures, in particular for $CoVaR$ and $Marginal\ Expected\ Shortfall$ (MES), the two most popular measures from this class of indicators proposed by Adrian and Brunnermeier (2011) and Acharya et al. (2012) respectively.

[4]In Section 4, when comparing our measures to network centrality measures, we shortly review the main contributions in this literature.

[5]Some recent contributions have tried to bridge the gap between the two broad categories just outlined, see for example Diebold and Yilmaz (2011), Dungey et al. (2012), Billio et al. (2012) or Barigozzi and Brownlees (2013), among others. Diebold and Yilmaz (2011) discuss briefly the connection between their measures and network

3 The Input-Output approach to measuring systemic importance

Our approach to measuring systemic influence logically belongs to the second class mentioned in the previous section, namely that based on balance sheet measures of interconnection. It improves upon traditional network analysis, however, because it avoids a major criticism raised against it, that of lumping together different types of exposures including borrowing and lending. Furthermore, it builds on a well-established tradition in economics and therefore provides more economic intuition than several network-based measures, which may seem of a rather mechanical nature.

At the heart of our analysis is the matrix of lending and borrowing positions in the interbank market. Though this is also the center of network analyses of interbank markets, the measures presented here go beyond this matrix through different transformations that allow for a richer economic interpretation. The measures obtained by this approach therefore exclude other potentially important channels of transmission, and as such they should be seen as complementary to other measures incorporating alternative sources of information.

3.1 General set-up

Consider a banking system composed of n banks[6]. Each bank collects deposits and equity, lends to non bank customers and lends to and borrows from other banks. The balance sheet of bank j is given by[7]:

$$e_j + d_j + a_{1j} + ... + a_{nj} = a_{j1} + ... + a_{jn} + l_j \tag{1}$$

where e_j , d_j , l_j are, respectively, equity, deposits and total non-interbank lending (composed of loans, net securities holdings and lending to (reserves at) the central bank) and a_{ij} is net interbank lending from bank i to bank j. All magnitudes are expressed in monetary terms, say euros. By construction, $a_{jj} = 0$ for all j. Reserves at the central bank are assumed riskless, whereas all other forms of assets (including interbank) normally carry some degree of default risk.

We can aggregate the balance sheets of the n banks and write it in matrix form as follows:

$$\mathbf{e} + \mathbf{d} + \mathbf{A}'_M\mathbf{i} = \mathbf{A}_M\mathbf{i} + \mathbf{l} \tag{2}$$

centrality measures, resembling at times the comparison we present in Section 4.

[6]We assume a closed economy, so that there is neither lending abroad nor borrowing from abroad.

[7]Unless otherwise specified, throughout the paper we use standard notation from matrix algebra. Hence, the identity matrix is indicated by $\mathbf{I}$, the unit (column) vector is indicated by $\mathbf{i}$, $\mathbf{i}_j$ denotes a vector with a 1 in its j^{th} position and zeros elsewhere (or alternatively the j^{th} column of $\mathbf{I}$), and a matrix that is full of ones is indicated by $\mathbf{J}$. By capital bold fonts (e.g. $\mathbf{X}$) we denote an $n \times n$ matrix with generic elements x_{ij}, whereas lower case bold fonts (e.g. $\mathbf{x}$) represent $n \times 1$ column vectors with generic elements x_i. $\mathbf{x}_j$ denotes de j^{th} column of matrix $\mathbf{X}$. The transpose of a matrix or vector is indicated with a prime. Finally, a lower case bold letter with a "hat" on it (e.g. $\hat{\mathbf{x}}$) denotes an $n \times n$ diagonal matrix with the vector $\mathbf{x}$ on its main diagonal, such that $\hat{\mathbf{x}}\mathbf{i} = \mathbf{x}$.

where $\mathbf{e}$, $\mathbf{d}$, $\mathbf{l}$ are appropriate column vectors, $\mathbf{i}$ is a unit vector of appropriate size, and $\mathbf{A}_M$ is the matrix of interbank bilateral positions, still expressed in money terms. Let $\mathbf{q}$ be a vector with total bank assets, $\mathbf{q} = \mathbf{e} + \mathbf{d} + \mathbf{A}'_M\mathbf{i} = \mathbf{A}_M\mathbf{i} + \mathbf{l}$, and $\hat{\mathbf{q}}$ be a corresponding diagonal matrix, such that $\hat{\mathbf{q}}\mathbf{i} = \mathbf{q}$.

The right hand side of Equation 2 can be written in the following form:

$$\mathbf{q} = \mathbf{A}_M\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}\mathbf{i} + \mathbf{l} = \mathbf{A}\mathbf{q} + \mathbf{l} \tag{3}$$

where $\mathbf{A} = \mathbf{A}_M\hat{\mathbf{q}}^{-1}$ is the matrix of interbank positions in which each *column* is divided by the total assets of the *borrowing* bank. Hence, the columns of $\mathbf{A}$ are fractions of unity and express, for each bank, the share of funding from other banks as a ratio to total funding. Hence we have $\mathbf{A}'\mathbf{i} = \mathbf{a}$, where $\mathbf{a}$ is the vector of total interbank borrowings divided by total assets.

Equation 3 is similar in form and interpretation to the familiar input-output model[8]. In the Leontief system, each firm or productive sector j uses a_{ij} units of output of sector i ($i = 1, ..., n$), plus labor and other primary inputs, to produce a unit of final output. In our case, bank j "uses" (borrows) funds from other banks, in an amount equal to a fraction a_{ij} of its assets, as well as funding from other non-bank sources (deposits and equity), to lend l_j to its final (non-interbank) borrowers. If all a_{ij} are constant (though interbank borrowing and lending relations tend to be rather stable through time[9], these ratios are of course not literally constant; we will discuss below the consequences of changes in these parameters), then the relation between loans and total assets is fixed and given by the well-known Leontief inverse $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$:

$$\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{l} = \mathbf{B}\mathbf{l} \tag{4}$$

Since it was originally proposed by Leontief in the 1930s, input-output analysis has enjoyed a broad popularity and has been applied to a variety of problems, ranging from energy and environmental analyses to computable general equilibrium models. Hirschman (1958) aimed at identifying key industrial sectors capable of sparking economy-wide growth in developing economies. In his analysis, key sectors are those characterised by many linkages with others. These sectors use the output of a large number of other industries as intermediate inputs, hence their production can "activate" a large number of other sectors.

In recent years, input-output techniques have trespassed the boundary of economics and have been increasingly used in the engineering literature to study the transmission of risk in interrelated infrastructural systems. The basic idea (see Haimes (2009), chapter 18) is that of inoperability; a given sector or infrastructure may be partially inoperable, i.e. operate below full capacity, for

[8] For a complete treatment of input-output analysis including several different applications of the model see Miller and Blair (2009).

[9] See for example Furfine (1999) for the case of the U.S., Cocco et al. (2009) for Portugal, Affinito (2012) for Italy and Bräuning and Fecht (2012) for Germany.

example due to a natural accident, a technical failure, a terrorist attack or other reasons. An interdependence matrix expresses the way in which the inoperability of each sector or infrastructure affects others. Through an analogue of the Leontief inverse one can calculate the total knock-down effect on each sector, and on the system as a whole, of a given initial shock.

The analogy with the problem of interconnected banks is quite obvious. If an individual bank is subject to a negative liquidity shock, it will transmit effects to the rest of the system via a contraction of lending to other banks, hence amplifying the effect on the entire system[10]. The way the shock is transmitted depends on the matrix of interbank linkages. To see how this can happen, write Equation 2 as

$$\mathbf{l} = (\mathbf{A}'_M - \mathbf{A}_M)\mathbf{i} + \mathbf{e} + \mathbf{d} \tag{5}$$

Note that $(\mathbf{A}'_M - \mathbf{A}_M)\mathbf{i}$ is the vector of net interbank borrowing (for each bank, the difference between the total amount it borrows and the total amount it lends in the interbank market). Hence, the right hand side of Equation 5 shows the sum of the three funding sources used by banks to finance their loans, i.e. net interbank borrowing, equity and deposits (net of liquidity). Let this sum be $\mathbf{s}$. Then we can write Equation 4 as $\mathbf{q} = \mathbf{B}\mathbf{s}$; any shock to the bank sources of funding is transmitted to the total size of their balance sheets $\mathbf{q}$ in a way that depends on the matrix of bank interconnections $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$. In addition, the shock can originate from the matrix $\mathbf{A}$ itself, that is, from a credit squeeze within the interbank market. This happens if one or several elements of $\mathbf{A}$ change.

3.1.1 Reformulation as a supply-side model

The traditional Leontief input-output model is usually considered a demand-driven model, since it describes the relationship between final demand and output (in our setting total non-interbank lending and total assets respectively), by means of the Leontief inverse $\mathbf{B}$. For reasons that will become clear in the next section, we note that with the same set of data it is possible to construct a supply-side model.[11] As opposed to the demand-driven standard input-output model, the Ghosh model is supply-driven and focuses on the relationship between the gross assets of the different banks and the primary inputs entering the system, in our setting represented by equity and deposits $(\mathbf{e} + \mathbf{d})$. The starting point is again the balance sheet of the different banks expressed in matrix form, namely Equation 2, reproduced here for convenience:

[10] This assumes that the funding shortfall cannot be absorbed by equity. If it can, the shock will be mitigated in relation to the size of the bank's equity buffer.

[11] This version is usually referred to also as "Ghosh" model after its proponent (see Ghosh (1958) and Miller and Blair (2009)). This implicit duality inherent to the Input-Output model is also exploited in the analysis of Brioschi et al. (1989) mentioned in Footnote 1.

$$\mathbf{e} + \mathbf{d} + \mathbf{A}'_M\mathbf{i} = \mathbf{A}_M\mathbf{i} + \mathbf{l} \tag{6}$$

Using $\mathbf{q}$ as before to indicate the vector of assets/liabilities, we can write:

$$\mathbf{q} = \underbrace{\mathbf{e} + \mathbf{d}}_{\equiv\mathbf{v}} + \mathbf{A}'_M\mathbf{i} = \mathbf{v} + \mathbf{A}'_M\mathbf{i} \tag{7}$$

By transposing and operating we get:

$$\mathbf{q}' = \mathbf{v}' + \mathbf{i}'\hat{\mathbf{q}}\hat{\mathbf{q}}^{-1}\mathbf{A}_M \tag{8}$$

Define $\hat{\mathbf{q}}^{-1}\mathbf{A}_M = \mathbf{O}$ as the matrix of output coefficients. This is the matrix of interbank positions in which each *row* is divided by the total assets of the *lending* bank (recall that our "input" matrix $\mathbf{A}$ represented the matrix of interbank positions in which each *column* is divided by the total assets of the *borrowing* bank). Hence the *rows* of $\mathbf{O}$ are fractions of unity that express, for each bank, the share of funding provided to other banks as a share of total funding provided.

It is then straightforward to see that Equation 8 yields the following:

$$\mathbf{q}' = \mathbf{v}'\mathbf{G} \tag{9}$$

where $\mathbf{G} = (\mathbf{I} - \mathbf{O})^{-1}$

Equation 9 represents the supply-side input-output model.

Note that $\mathbf{A}$ and $\mathbf{O}$ are *similar* matrices (i.e. $\mathbf{O} = \hat{\mathbf{q}}^{-1}\mathbf{A}\hat{\mathbf{q}}$) and therefore they share the dominant eigenvalue. Furthermore, if we denote by $\mathbf{x}$ and $\mathbf{y}$ the right Perron eigenvectors of $\mathbf{A}$ and $\mathbf{O}$ respectively, then it holds that $\mathbf{x} = \hat{\mathbf{q}}\mathbf{y}$. This has a bearing on the discussion at the end of Section 4, since the right eigenvector of $\mathbf{A}$ is a scaled version of the right eigenvector of $\mathbf{O}$. Further interpretations based on this alternative model are possible, though we do not pursue them here.

3.2 Input-Output measures of systemic importance

The input-output model makes evident the fact that the production of any sector j has two distinct effects on the remaining sectors: on the one hand by increasing production it will demand more inputs from other sectors ("upstream"), on the other it will be able to provide more output to the sectors that depend on j's production as input to their own production process ("downstream").

Early on in the history of input-output analysis, this inherent structure of interdependence was used as the basis for the identification of key sectors in the economy, which are taken to be those that by increasing their production activate several other sectors both as purchaser of inputs and seller of outputs to be further used as inputs. The pioneering analyses of Rasmussen (1958) and

Hirschman (1958) provide a framework that can be readily adapted and interpreted in terms of the set-up described above and used as a starting point in the study of systemic importance in banking systems.

It is useful, at this stage, to discuss a few specific cases with intuitive appeal.

Case A. Suppose first for example that the banking system undergoes a shock originating from the deposit side of one bank, say bank 1. The shock will result, at first impact, in a balance sheet loss for bank 1 itself. Subsequently, however, other banks may be affected: bank 1 may curtail credit to other banks, spreading the effect through the system[12]. In our model, $\mathbf{q} = \mathbf{Aq} + \mathbf{s}$, the first impact would be on the first element of $\mathbf{s}$ and of $\mathbf{q}$. The second round effects to each bank in the system would occur depending on the elements of the first column of $\mathbf{A}$. And so on, until the overall effects are expressed by the Leontief inverse (Equation 4). In the end, a unitary deposit drawdown in bank 1 would affect the system to an extent given by the first column of $\mathbf{B}$. The effects on each bank are given by $\mathbf{Bi}_1$, and the total system effect by $\mathbf{i}'\mathbf{Bi}_1$, the sum of all elements in the first column of $\mathbf{B}$ ($\mathbf{i}_1$ denotes a column vector with 1 in the first element and 0 elsewhere, see footnote 7). In the literature, the so-called Rasmussen-Hirschman "backward" index has been used (see Sonis et al. (1995), Sonis and Hewings (2009), Miller and Blair (2009)), which we denote by $\mathbf{h}_{(b)}$[13] and can be defined for each bank j as follows:

$$h_{b_j} = \mathbf{i}'\mathbf{Bi}_j \tag{10}$$

For ease of interpretation and comparison across j, one can normalize $\mathbf{h}_{(b)}$. Here we follow the standard in the literature and perform such normalization using the "intensity" of $\mathbf{B}$ (the sum of all elements of $\mathbf{B}$) and multiplied by n, so that the sum of h_{b_j} for all j is equal to n. We indicate the normalized version of indices by an overbar:

$$\bar{h}_{b_j} = n\frac{\mathbf{i}'\mathbf{Bi}_j}{\mathbf{i}'\mathbf{Bi}} \tag{11}$$

This is what Rasmussen (1958) labeled a "power of dispersion" index, because it measures the strength with which the initial shock is dispersed through the system, measured relative to the balance sheet size of the bank in which the shock originates (recall that matrix $\mathbf{A}$ expresses, by column, interbank *borrowing* as a share of total assets of the *borrowing* bank). A value of $\bar{h}_{b_j}$ greater than one means that a funding shock in bank j affects the system more than the same shock in the average of all banks.

A similar "forward" index is also used in the literature, but an important remark is in order.

[12]A further set of second round effects would occur if the credit squeeze from bank 1 is large enough to affect the real sector of the economy, hence affecting other banks also via the real sector, rather than only via interbank linkages. In this paper we do not consider this case, which would require combining the interbank linkage matrix with a macroeconomic model, but leave it to future developments of this line of research.

[13]We use the parenthesis notation to distinguish the vector $\mathbf{x}_{(j)}$ from $\mathbf{x}_j$, which as noted in footnote 7 would be used for the j^{th} column of matrix $\mathbf{X}$.

For the computation of these Rasmussen-Hirschman forward indices, it has been argued in the input-output literature[14] that the output matrix of the so-called "Ghosh" model introduced in Section 3.1.1 is more appropriate. The reason for this is that contrary to backward measures, which imply summations along the column dimension of the Leontief inverse, forward measures are based on summations along the row dimension. If one were to sum along the row dimension of the Leontief inverse **B**, which is based on the input matrix **A**, then one would be summing elements with different denominators, which is not ideal. With this in mind, the forward Rasmussen-Hirschman index is constructed in its un-normalized and normalized versions respectively as follows:

$$h_{f_j} = \mathbf{i}'_j \mathbf{G} \mathbf{i} \tag{12}$$

$$\bar{h}_{f_j} = n \frac{\mathbf{i}'_j \mathbf{G} \mathbf{i}}{\mathbf{i}' \mathbf{G} \mathbf{i}} \tag{13}$$

This index can be interpreted as the effect of a systemic shock on bank j. In other words, if a funding shock hits simultaneously all banks, $\bar{h}_{f_j}$ measures how sensitive bank j is; for this reason it is usually referred to as a "sensitivity of dispersion" index. Since this indicator builds on matrix **O** instead of **A**, the sensitivity of bank j to a shock to all other banks simultaneously is measured relative to the balance sheet size of the same bank[15].

Putting together the information provided by the backward and forward linkage indicators it is possible to classify banks into four different categories in order to identify key players as shown in Table 1:

Table 1: Classification based on backward and forward linkages

		$\bar{h}_{f_j}$	
		< 1	> 1
$\bar{h}_{b_j}$	< 1	Generally independent	Important provider of funds
	> 1	Dependent on funds from others	Key bank

This taxonomy tries to combine the different dimensions implied by the Rasmussen-Hirchsman indices. A systemically important bank according to such a classification would be a bank that is both an important provider of funds to other banks in the system ($\bar{h}_{f_j} > 1$) and that is also a relatively heavy user of funds from other banks ($\bar{h}_{b_j} > 1$). This could correspond for instance to a core bank in a standard "core-periphery" structure; such banks typically have significant cross borrowing and lending with other core banks and they also intermediate between periphery banks

[14] See for example Beyers (1976), Jones (1976) or Dietzenbacher (1992).

[15] Recall that matrix **O** expresses, by row, interbank *lending* as a share of total assets of the *lending* bank.

that are not directly tied through borrowing/lending.

Case B. A second case that is intuitively interesting occurs when the initial shock originates not in the bank deposit funding, but in the interbank flows themselves. For example, the shock could derive from a crisis of confidence by the market in a particular bank, that induces some or all other banks to cut credit to it. Or alternatively, a bank can curtail funding to all other banks, due to an idiosyncratic increase in (perceived) counterparty risk. Several combinations can be examined. For simplicity, we restrict ourselves here to three cases that seem of particular interest: in the first, all banks cut financing to a given bank; in the second, a given bank cuts financing to all others; in the third, both events happen, so the bank in question both sees its sources of interbank funds restricted, and (perhaps as a consequence) cuts its own supply to other banks.

The notion of "fields of influence" used in the input-output and engineering literature is of help here (see Sonis et al. (1995) and the references therein[16]). What is of interest is how the matrix $\mathbf{B}$ ($\mathbf{G}$) changes when one or more elements of the bank interconnection matrix $\mathbf{A}$ ($\mathbf{O}$) change. The "field of influence" measures the change of each element of $\mathbf{B}$ ($\mathbf{G}$) in response to a small change in $\mathbf{A}$ ($\mathbf{O}$). Suppose only one element of matrix $\mathbf{A}$ changes, say a_{ij}. Then it can be shown that the change in $\mathbf{B}$ is given by the following $n \times n$ matrix:

$$\mathbf{F}(i,j) = (\mathbf{B}\mathbf{i}_i)\left(\mathbf{i}'_j\mathbf{B}\right) = \mathbf{b}_i\mathbf{b}'_j \tag{14}$$

where $\mathbf{b}_i = \mathbf{B}\mathbf{i}_i$ and $\mathbf{b}'_j = \mathbf{i}'_j\mathbf{B}$ are respectively the i^{th} column and the j^{th} row of $\mathbf{B}$. This matrix is called the *first order field of influence* of a given element a_{ij} of $\mathbf{A}$, and is simply the first derivative of $\mathbf{B}$ with respect to a_{ij}. Higher order fields of influence can be calculated using appropriate formulas (see Sonis and Hewings (2009)). A similar logic and formulae applies for the effect on matrix $\mathbf{G}$ of a change in one element of matrix $\mathbf{O}$[17].

The first sub-case we want to examine is that of a bank, say bank j, which is cut financing by all others. This is represented by a unit decline in all elements of column j of $\mathbf{A}$ (except a_{jj}, which is equal to zero by definition). The effect on the system's total assets is given by

$$f_{c_j} = \mathbf{i}'\left(\sum_{i\neq j}\mathbf{F}(i,j)\right)\mathbf{i} \tag{15}$$

that is, the intensity of the matrix obtained by summing all first order fields of influence be-

[16]The notion of fields of influence was originally developed in the input-output framework by Sonis and Hewings (1989, 1991). For a comprehensive review with further references see Sonis and Hewings (2009). A nice summary is provided by Percoco (2006).

[17]Note that if one defines $\mathbf{B}\mathbf{i} = \tilde{\mathbf{h}}_{(\mathbf{f})}$, then the intensity of $\mathbf{F}(i,j)$ (i.e. the sum of all its elements) can be expressed as $\mathbf{i}'\mathbf{F}(i,j)\mathbf{i} = \mathbf{i}'\mathbf{B}\mathbf{i}_i\mathbf{i}'_j\mathbf{B}\mathbf{i} = \mathbf{h}'_{(b)}\mathbf{i}_i\mathbf{i}'_j\tilde{\mathbf{h}}_{(f)} = h_{b_i}\tilde{h}_{f_j}$. An analogous expression can be derived when the field of influence is defined over matrix $\mathbf{G}$.

longing to column j, with the exception of that of row j.

Since we are interested in comparing relative values across firms, the index can be normalized by the sum of all individual bank effects and multiplied by n, to yield

$$\bar{f}_{c_j} = n \frac{\mathbf{i}' \left(\sum_{i \neq j} \mathbf{F}(i,j) \right) \mathbf{i}}{\mathbf{i}' \left(\sum_{j=1}^{n} \sum_{i \neq j} \mathbf{F}(i,j) \right) \mathbf{i}} \tag{16}$$

where $\frac{1}{n} \sum_{j=1}^{n} \bar{f}_{c_j} = 1$. In short, this "column field of influence index" expresses the strength of the systemic effects of bank j being cut credit by all other banks, by a unit amount. The strength is measured relative to the mean of the system, set equal to 1. A value of the index higher than unity then means that the systemic effect of bank j being cut credit is higher than average, by an extent given by the distance of the index value from unity.

Analogously, and with a slight abuse of notation, we can construct a "row" field of influence index, defined over matrices **O** and **G** instead of **A** and **B**, following the discussion for the forward Rasmussen-Hirschman index. The row field of influence is given in un-normalized and normalized form respectively:

$$f_{r_j} = \mathbf{i}' \left(\sum_{j \neq i} \mathbf{F}(i,j) \right) \mathbf{i} \tag{17}$$

$$\bar{f}_{r_i} = n \frac{\mathbf{i}' \left(\sum_{j \neq i} \mathbf{F}(i,j) \right) \mathbf{i}}{\mathbf{i}' \left(\sum_{i=1}^{n} \sum_{j \neq i} \mathbf{F}(i,j)) \right) \mathbf{i}} \tag{18}$$

This index expresses the strength of the systemic effects of bank i cutting credit to all other banks, by a unit amount. The strength is again measured relative to the mean of the system, set equal to 1.

Finally we can define a "total" index, expressing the systemic effect of a joint cut of credit, from and to bank j:

$$\bar{f}_{t_j} = \frac{\bar{f}_{c_j} + \bar{f}_{r_j}}{2} \tag{19}$$

Case C. Note that all measures defined so far are *relative*, i.e. they measure differences across banks of effects stemming from unitary changes in one or more balance sheet items, without taking into account the size of the banks in question.Yet, size is obviously an important factor behind the systemic relevance of a bank. In order to account for this, we can define another index that measures the total systemic effect of a complete "cut-off" of bank j from the interbank system. We calculate, in other words, the systemic effect of bank j ceasing to borrow or lend from the rest of the system. Note that bank j does not disappear, but continues to operate on a more limited scale as an autarkic bank, financing its (non-bank) lending only by deposits and equity.

To express this idea, the notions of "total linkage effect" (Cella (1984)) can be used[18].The total systemic effect of bank j being cut out from interbank markets is:

$$\begin{aligned} t_j &= \mathbf{i'q} - \mathbf{i'q}^{-j} \\ &= \mathbf{i'Bl} - \mathbf{i'B}^{-j}\mathbf{l} \\ &= \mathbf{i'}(\mathbf{B} - \mathbf{B}^{-j})\mathbf{l} \end{aligned} \tag{20}$$

where $\mathbf{B}^{-j} = (\mathbf{I} - \mathbf{A}^{-j})^{-1}$ and $\mathbf{A}^{-j}$ is the matrix obtained from $\mathbf{A}$ by setting all elements of the j^{th} row and j^{th} column to zero[19]. Instead of normalization by means of the average, a more instructive normalization in this case is through division by total assets of the system:

$$\bar{t}_j = \frac{\mathbf{i'q} - \mathbf{i'q}^{-j}}{\mathbf{i'q}} \tag{21}$$

$\bar{t}_j$ measures how much the banking system will suffer, in terms of total assets lost, if bank j is completely cut-off from the interbank market[20].

Table 2 summarizes the six indicators we have defined and their interpretation.

Table 2: Indicators of systemic influence

Index	Description	Interpretation
h_{b_j}	Backward Ras.-Hirschman	System effect of a unitary liquidity shock in bank j
h_{f_j}	Forward Ras.-Hirschman	Effect on bank j of a unitary system-wide liquidity shock
f_{c_j}	Column field of influence	System effect of a unitary cut of interbank lending by bank j
f_{r_j}	Row field of influence	Systemic effect of a unitary cut of interbank lending to bank j
f_{t_j}	Total field of influence	System effect of a unitary cut of all int. transactions by bank j
t_j	Total linkage effect	System effect of a cut of total interbank lending by and to bank j

[18] This is part of an approach labelled the "hypothetical extraction method", which tries to assess how much the total output of an economy would be reduced if a sector is eliminated.

[19] It is implicitly assumed that bank j can substitute its interbank liabilities by either equity or deposits (or both). It is possible to construct an alternative total linkage index for which this is not the case. Such index would be given by $t_j = \mathbf{i'Bl} - \mathbf{i'B}^{-j}\mathbf{l}^{\star j}$, where $\mathbf{l}^{\star j}$ denotes the vector $\mathbf{l}$ with element j consisting only of $e_j + d_j$ (i.e. no interbank borrowing for bank j).

[20] Recently, Denbee et al. (2013) proposed to identify what they refer to as the "level key player" as that which causes the maximum expected reduction in the overall level of bilateral liquidity in an interbank payment system. This is very close in spirit to the total linkage effect. One may also argue that their "risk key player" measure resembles the idea behind the Rasmussen-Hirschman backward measure presented above.

3.3 A simple numerical example

As an illustration of the measures proposed, we construct a simple hypothetical banking system composed of five banks. Their balance sheets reflect certain characteristics observed in real life banks. More precisely (see numerical values in Table 3[21]):

Bank 1. This bank is a large retailer, its assets accounting for over 30 percent of the entire banking system. It has a large client base for both deposits and loans. Its interbank activities are limited in size, but broadly distributed across all other banks.

Bank 2. Bank 2 is as large as Bank 1 but its business is more concentrated on the interbank market. Its direct customer base is small: deposits and loans relative to assets are limited. It borrows more than 50 percent of its funds from other banks, and lends to other banks almost 60 percent of its assets.

Bank 3. This bank is smaller than the previous banks (20 percent of the market). It has virtually no deposit base and borrows most of its funds short term from other banks. Its funding is very concentrated: most of it comes from Bank 2. Its loan portfolio is large but not predominant (40 percent of assets).

Bank 4. Bank 4 is a small local bank specialized in deposit collection. Its assets account for 8 percent of the total banking system. Its loan portfolio is small (20 percent of assets), the rest being lent in the interbank market.

Bank 5. Finally, Bank 5 is another small bank but with a different business profile: its deposits are still relative large (64 percent of total liabilities), but loans are also large (80 percent of assets). Its interbank business is small and concentrated in a limited number of counterparties. In essence, Bank 5 has the features of a small, local retail bank.

Table 3: Numerical values

Bank	ε_j	δ_j	a_{j1}	a_{j2}	a_{j3}	a_{j4}	a_{j5}	λ_j	q_j
Bank 1	0.10	0.5125	0	0.1500	0.1600	0.1200	0.0800	0.70	8
Bank 2	0.10	0.3875	0.2000	0	0.5600	0.0400	0.0800	0.42	8
Bank 3	0.10	0.1000	0.1250	0.2500	0	0	0	0.40	5
Bank 4	0.10	0.7400	0.0375	0.0875	0.0800	0	0.1000	0.20	2
Bank 5	0.10	0.6400	0.0250	0.0250	0	0	0	0.80	2
$\sum$	-	-	0.3875	0.5125	0.8000	0.1600	0.2600	-	25

where ε_j stands for the equity to asset ratio and for simplicity is set to 0.10 (i.e. leverage is assumed to be equal to 10), δ_j denotes deposits as percentage of liabilities, a_{jk} are the entries of

[21] Table 3 presents matrix **A**. For space considerations we do not present matrix **O**, though it is straightforward to construct it from the data given in Table 3.

the matrix $\mathbf{A}$, λ_j is the loan portfolio as a percentage of assets, and q_j indicates the value of assets for bank j (i.e. $\frac{q_j}{\sum_{j=1}^{n} q_j}$ is the market share of bank j).

The values of the six indicators for these banks are reported in Table 4. Some remarks are in order. First, based on the criteria of using the Rasmussen-Hirschman backward and forward indices jointly (see Table 1), only Banks 2 and 3 are identified as key players, even when the Rasmussen-Hirschman forward index ranks bank 2 in the third place. With the exception of the forward Rasmussen-Hirschman indicator, all indices of systemic importance rank the first three banks as more systemically important. Interestingly, Bank 3, a midsize institution, is more systemically important than Bank 1, the large retailer, based on three indicators ($\bar{\mathbf{h}}_{(b)}$, $\bar{\mathbf{h}}_{(f)}$, $\bar{\mathbf{f}}_{(t)}$) out of six. Bank 3 is the most active on the interbank market in terms of its own balance sheet (60% and 80% of its assets and liabilities respectively are destined to or come from the interbank market). Third, once size is taken into account (indicator $\bar{\mathbf{t}}$), interconnection still matters: Banks 1, 2, 3, the most interconnected ones, still have the highest values. Note that no ranking is repeated for the six indicators proposed. It is clear that the indicators complement each other and help reveal different aspects of the balance sheet structure and different types of underlying risk.

By comparing the last indicator with the others one may conclude that size is the key factor in determining the systemic importance of a financial institution[22]. However, the degree and the pattern of interconnections, as measured by the bilateral exposures matrix $\mathbf{A}$ (and $\mathbf{O}$), also contributes to determine the systemic importance of financial institutions in a relevant way. A clear example of this is the fact that Bank 1 is never ranked in the first position despite having a considerable market share: Bank 3 has a smaller balance sheet but it is a bigger player on the interbank market. Our measures suggest that size, type of business and nature of the links with the rest of the system interact with each other and should be taken into account jointly.

Table 4: Measures of systemic importance

	$\bar{\mathbf{h}}_{(b)}$		$\bar{\mathbf{h}}_{(f)}$		$\bar{\mathbf{f}}_{(c)}$		$\bar{\mathbf{f}}_{(r)}$		$\bar{\mathbf{f}}_{(t)}$		$\bar{\mathbf{t}}$	
Bank	Index	#	Index	#	Index	#	Index	#	Index	#	Index	#
Bank 1	0.9906	*3*	0.8282	*4*	1.0895	*2*	1.2302	*2*	1.1598	*2*	0.3023	*3*
Bank 2	1.1237	*2*	1.0811	*3*	1.3937	*1*	1.3990	*1*	1.3963	*1*	0.3899	*1*
Bank 3	1.3903	*1*	1.1012	*2*	0.9444	*3*	1.2007	*3*	1.0725	*3*	0.3329	*2*
Bank 4	0.7096	*5*	1.2954	*1*	0.9254	*4*	0.5123	*5*	0.7189	*4*	0.0862	*4*
Bank 5	0.7859	*4*	0.6941	*5*	0.6471	*5*	0.6577	*4*	0.6524	*5*	0.0504	*5*
Mean/Sum[a]	1	-	1	-	1	-	1	-	1	-	1.1617	-

[a] Mean for all indicators except t_j, for which the sum is displayed.

[22] This echoes the conclusions of a recent study by the BIS; see Drehmann and Tarashev (2011b). However, these authors proxy size as total liabilities vs. non banks, while here size is proxied by all liabilities (or assets), including *with* banks.

4 Relation to network centrality measures

More than 60 years ago, Robert Solow (see Solow (1952), page 29) wrote: "It is by no means an infrequent occurrence in economics that theories which are "about" different things turn out to be formally similar or even identical"[23]. As emphasized at the beginning of Section 3, the measures proposed here belong to that class of indicators that are based on balance sheet measures of interconnection, in which network/graph-theoretic indicators feature prominently[24]. The latter measures typically build on the adjacency matrix defining the links that connect the different nodes in a network. As noted by McNerney (2009), the input-output network is a special kind of directed and weighted network that must obey some boundary flow conditions. Indeed, the link between input-output and network measures is not surprising when one notes that the former uses a matrix representation of the economy that allows for a complete picture of inter-sectoral relationships. We leave the relation to approaches other than network theory to Appendix B, given that at this stage the connection to other types of measures is of a rather intuitive nature.

While we don't claim to perform a comparison as concise and authoritative as that of Solow (1952), it is clear that several parallels can be drawn between the input-output and network literatures[25]. In the particular case that concerns us, the comparison is to be made with respect to network centrality measures, which attempt to capture the importance of nodes in an interrelated system using the matrix of interconnections as primitive datum. For the sake of brevity, we focus on some key measures from network theory, largely following ECB (2012).

The centrality measures considered are indicated by the vector $\mathbf{c}_{(i)}$, $i = in, out, cl\text{-}in, cl\text{-}out, bw, lev, rev$:

- $\mathbf{c}_{(in)}$, $\mathbf{c}_{(out)}$: these first two measures are the most basic and intuitive centrality measures and attempt to capture network activity. In-strength[26] centrality ($\mathbf{c}_{(in)}$) measures for node i the number of ties directed to it, using the value of the links as weights. It is basically calculated as the column sum of the matrix representing the network. Out-strength centrality ($\mathbf{c}_{(out)}$) measures, for node i, the number of ties going from i to all other nodes, again using the value of the links as weights. From a computational perspective, it involves the row sum of the

[23] Interestingly enough, Solow was specifically referring to the relationship between linear economic models as represented by the input-output framework and the finite Markov chains studied in probability theory, which have been recently used to identify systemically important banks in payment systems (see Soramäki and Cook (2012)).

[24] For papers studying the interbank markets through the lens of network theory see Boss et al. (2004), Soramäki et al. (2007) or Iori et al. (2008) among others.

[25] This link has been under-researched in the literature since network theory applied to economic phenomena is a relatively recent development. An early attempt taking input-output as the starting point can be found in Olsen (1992). More recently McNerney (2009) studies the network properties of input-output models, while Blöchl et al. (2011) use centrality measures and clustering techniques to uncover salient structural features of economies based on an interpretation of input-output tables as weighted, directed graphs. Acemoglu et al. (2012) combine network theory with input-output data to study to what extent idiosyncratic sectoral shocks can affect aggregate business cycle fluctuations.

[26] *Strength* measures are the weighted graph counterpart to standard *degree* centrality indicators for unweighted graphs, which measure the number of incoming and outgoing connections of any given node.

matrix representing the network. One could also combine the two to form an in-out strength centrality ($\mathbf{c}_{(in\text{-}out)}$) measure[27].

- $\mathbf{c}_{(cl\text{-}in)}$, $\mathbf{c}_{(cl\text{-}out)}$: closeness centrality measures, for node i, the shortest path between i and all other nodes reachable from it, averaged across all other nodes. It is a centrality indicator that aims to assess independence of nodes and assigns a high score to those nodes that are "close" to all other nodes, i.e. a node is central according to this measure if it is relatively easy to go from it to many other nodes. For a directed network it possible to distinguish between $\mathbf{c}_{(cl\text{-}in)}$ and $\mathbf{c}_{(cl\text{-}out)}$, the distinction following a similar logic to that of strength centrality.

- $\mathbf{c}_{(bw)}$: betweenness centrality gauges how often a given node lies in the shortest path between all other pairs of nodes and therefore attempts to quantify the importance of the node in terms of its role in the flow of activity in the network.

- $\mathbf{c}_{(lev)}$, $\mathbf{c}_{(rev)}$: eigenvector centrality identifies the importance of nodes by the Perron eigenvector (the eigenvector associated with the highest eigenvalue) and it is based on the idea that a node is central to the extent that it is connected to other nodes which are themselves central[28]. In particular, we have considered both right eigenvector centrality ($\mathbf{c}_{(rev)}$) and left eigenvector centrality ($\mathbf{c}_{(lev)}$)[29].

The basic degree/strength centrality measures are derived from the links coming to and from each node, something that can be inspected by direct observation of the matrix defining the network, which can be taken to be $\mathbf{A}$, $\mathbf{O}$ or $\mathbf{A}_M$[30], just as the basic linkage measures are derived from the same procedure but looking at the information contained in either the Leontief inverse $\mathbf{B}$ or the Ghosh inverse $\mathbf{G}$. The following simple propositions summarize their relationship.

Proposition 1. *Consider the $n \times n$ matrix $\mathbf{A}$, which when read by column expresses for each bank the share of funding from other banks as a ratio to total funding. By definition, $\mathbf{A}$ is a non-negative matrix, i.e. $\mathbf{A} \geq 0$. If $\sum_{i=1}^{n} a_{ij} < 1$ for $j = 1, ..., n$, or in matrix form, $\mathbf{i}'\mathbf{A} < \mathbf{i}$[31]; then there is a direct relationship between the Rasmussen-Hirschman backward index, $\mathbf{h}_{(b)}$, and the in-strength centrality measure, $\mathbf{c}_{(in)}$ (computed from matrix $\mathbf{A}$)*

[27] Due to space considerations we do not present results on this measure, though they are available upon request.

[28] Several other measures are in fact variations on eigenvector centrality and we therefore restrict our inquiry to this measure as representative. Some popular examples of eigenvector-related centrality measures are Bonacich centrality (Bonacich (1987)) and Google's *PageRank*.

[29] Left eigenvector centrality is essentially the same as right eigenvector centrality but using the original matrix transposed.

[30] In the context of input-output analysis, the basic strength centrality measures are equivalent to the direct linkage measures of Chenery and Watanabe (1958).

[31] The stated condition is commonly found in the input-output literature and in our context is equivalent to requiring that banks do not obtain their funding entirely from the interbank market.

Proof. Consider the Leontief inverse as defined before: $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$. Given that $\mathbf{A}$ is nonnegative ($\mathbf{A} \geq \mathbf{0}$), the condition that $\mathbf{i}'\mathbf{A} < \mathbf{i}$ guarantees that the Leontief inverse fulfills $\mathbf{B} \geq \mathbf{0}$.Then we can write the following:

$$\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{AB}$$

Then, premultiplying by $\mathbf{i}'$ and noting that $\mathbf{h}'_{(b)} = \mathbf{i}'\mathbf{B}$ and $\mathbf{c}'_{(in)} = \mathbf{i}'\mathbf{A}$, we have that

$$\mathbf{h}'_{(b)} = \mathbf{i}' + \mathbf{c}'_{(in)}\mathbf{B} \tag{22}$$

As $\mathbf{i} > \mathbf{0}$ and $\mathbf{B} \geq \mathbf{0}$, from Equation 22 the claim of the proposition follows[32]. □

Proposition 2. *Consider the $n \times n$ matrix $\mathbf{O}$, which by row expresses for each bank the share of lending to other banks as a ratio to total lending. By definition, $\mathbf{O}$ is a non-negative matrix. If $\sum_{j=1}^{n} o_{ij} < 1$ for $i = 1, ..., n$, or in matrix form, $\mathbf{Oi} < \mathbf{i}$[33], then there is a direct relationship between the Rasmussen-Hirschman forward index, $\mathbf{h}_{(f)}$, and the out-strength centrality measure, $\mathbf{c}_{(out)}$ (computed from matrix $\mathbf{O}$)*

Proof. Starting from the Ghosh inverse: $\mathbf{G} = (\mathbf{I} - \mathbf{O})^{-1}$ and given that $\mathbf{O}$ is nonnegative, the condition that $\mathbf{Oi} < \mathbf{i}$ guarantees that the Ghosh inverse fulfills $\mathbf{G} \geq \mathbf{0}$. Then we can write the following:

$$\mathbf{G} = (\mathbf{I} - \mathbf{O})^{-1} = \mathbf{I} + (\mathbf{I} - \mathbf{O})^{-1}\mathbf{O} = \mathbf{I} + \mathbf{GO}$$

Post-multiplying by $\mathbf{i}$ and using $\mathbf{h}_{(f)} = \mathbf{Gi}$ and $\mathbf{c}_{(out)} = \mathbf{Oi}$

$$\mathbf{h}_{(f)} = \mathbf{i} + \mathbf{Bc}_{(out)} \tag{23}$$

Hence, as $\mathbf{i} > \mathbf{0}$ and $\mathbf{G} \geq \mathbf{0}$, from Equation 23 the claim of the proposition follows[34][35]. □

Note that an element j of vectors $h_{(b)}$ and $h_{(f)}$ can be written respectively as $h_{b_j} = 1 + \sum_{i=1}^{n} b_{ij}c_{in,i}$ and $h_{f_j} = 1 + \sum_{i=1}^{n} g_{ji}c_{out,i}$. Then, given that as a general rule for Leontief (Ghosh) inverses the elements b_{jj} (g_{jj}) are bigger than one and the b_{ij} (g_{ij}), $i \neq j$, are smaller than one, the backward (forward) indicator for bank j will be relatively strongly linked to the in-strength (out-strength) centrality measure of the same bank.

[32]Note that if the in-strength centrality were to be computed based on the matrix $\mathbf{A}_M$ instead (say, $\tilde{\mathbf{c}}'_{(in)} = \mathbf{i}'\mathbf{A}_M$), the positive relationship would still hold, though it would be slightly tempered: $\mathbf{h}'_{(b)} = \mathbf{i}' + \tilde{\mathbf{c}}'_{(in)}\hat{\mathbf{q}}^{-1}\mathbf{B}$.

[33]Similar to the condition for matrix $\mathbf{A}$, this requires that banks do not concentrate their asset side entirely on interbank lending. This condition comes from Equation (9), by transposing it we get: $\mathbf{q} = \mathbf{G}'\mathbf{v} = (\mathbf{I} - \mathbf{O}')^{-1}\mathbf{v}$.

[34]Again, if we were to define the out-strength centrality based on matrix $\mathbf{A}_M$ instead ($\tilde{\mathbf{c}}_{(out)} = \mathbf{A}_M\mathbf{i}$), the positive relationship would still hold as: $\mathbf{h}_{(f)} = \mathbf{i} + \mathbf{G}\hat{\mathbf{q}}^{-1}\tilde{\mathbf{c}}_{(out)}$.

[35]Note that both proofs are done for the un-normalized version of the $\mathbf{h}_{(b)}$ and $\mathbf{h}_{(f)}$ indices, but they go through also for the normalized versions since the normalization implies dividing Equation 22 and Equation 23 by $\frac{\mathbf{i}'\mathbf{Bi}}{n}$ and $\frac{\mathbf{i}'\mathbf{Gi}}{n}$ respectively, which are positive scalars.

Eigenvector centrality and some of its variants are widely used in several applications of the networks literature. Though such literature is rarely specific about it, to the best of our knowledge normally right eigenvector centrality is computed. It seems that the remark from Solow quoted above applies here in full force, since Dietzenbacher (1992) proposed linkage measures in the input-output framework that are based on left and right Perron eigenvectors.

In fact, it can be shown that a generalized version of the Rasmussen-Hirschman backward and forward measures outlined above converge to the left and right eigenvectors of the input and output matrices $\mathbf{A}$ and $\mathbf{O}$ respectively. Following Dietzenbacher (1992, 1993), consider first the case of weighted backward linkages of the Rasmussen-Hirschman type, where $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse and $\mathbf{A}$ is the input matrix:

$$\mathbf{m}_1' = n\frac{\mathbf{r}'\mathbf{B}}{(\mathbf{r}'\mathbf{B}\mathbf{i})} \tag{24}$$

where $\mathbf{r}' > 0$ denotes the vector of row weights (note that if $\mathbf{r}'$ is set equal to $\mathbf{i}'$ we get the standard definition of the backward Rasmussen-Hirschman index presented in Equation 11). It makes sense to assume that the inputs from a sector with high backward linkages receive a larger weight than those from a low backward linkages sector[36]. As a logical consequence of this reasoning, the vector $\mathbf{m}_1'$ can be used as a weighting vector to yield:

$$\mathbf{m}_2' = n\frac{\mathbf{m}_1'\mathbf{B}}{\mathbf{m}_1'\mathbf{B}\mathbf{i}} = n\frac{n\mathbf{r}'}{n\mathbf{r}'}\frac{\mathbf{B}^2/\left(\mathbf{r}'\mathbf{B}\mathbf{i}\right)}{\mathbf{B}^2\mathbf{i}/\left(\mathbf{r}'\mathbf{B}\mathbf{i}\right)} = n\frac{\mathbf{r}'\mathbf{B}^2}{(\mathbf{r}'\mathbf{B}^2\mathbf{i})} \tag{25}$$

Following this train of thought, we could further use $\mathbf{m}_2'$ as weights and so on to get, after k iterations:

$$\mathbf{m}_k' = n\frac{\mathbf{m}_{k-1}'\mathbf{B}}{\mathbf{m}_{k-1}'\mathbf{B}\mathbf{i}} = n\frac{\mathbf{r}'\mathbf{B}^k}{(\mathbf{r}'\mathbf{B}^k\mathbf{i})} \tag{26}$$

Now assume that the input-output model is solvable, that is, $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ exists. This is equivalent to having the Perron eigenvalue of $\mathbf{A}$, which we denote by λ, smaller than 1, and also equivalent to having $\mathbf{B} \gg 0$. Hence, if $\lambda < 1$, the Leontief inverse $\mathbf{B}$ is a primitive matrix with Perron eigenvalue given by $\frac{1}{1-\lambda}$. Denoting by $\mathbf{x}'$ and $\mathbf{y}$ its left and right Perron eigenvectors respectively[37], we can use a well known property of primitive matrices (see Meyer (2000), p.674):

$$\lim_{k\to\infty}\left(\frac{\mathbf{B}}{1/(1-\lambda)}\right)^k = \frac{\mathbf{y}\mathbf{x}'}{\mathbf{x}'\mathbf{y}} \tag{27}$$

Now we can use Equation 27 to compute the limit of Equation 26 as k approaches infinity:

[36]Note that this is precisely the logic behind the eigenvector centrality (and related measures) in the network literature.

[37]Note that the Perron eigenvectors of $\mathbf{A}$ and $\mathbf{B}$ are the same.

$$
\begin{aligned}
\lim_{k\to\infty} \mathbf{m}'_k &= \lim_{k\to\infty} n\frac{\mathbf{r}'\mathbf{B}^k}{(\mathbf{r}'\mathbf{B}^k\mathbf{i})} \\
&= \lim_{k\to\infty} n\frac{\mathbf{r}'\left(\mathbf{B}(1-\lambda)\right)^k}{\left(\mathbf{r}'\left(\mathbf{B}(1-\lambda)\right)^k\mathbf{i}\right)} \\
&= \frac{\lim_{k\to\infty} n\mathbf{r}'\left(\mathbf{B}(1-\lambda)\right)^k}{\lim_{k\to\infty}\left(\mathbf{r}'\left(\mathbf{B}(1-\lambda)\right)^k\mathbf{i}\right)} \\
&= \frac{n\mathbf{r}'\lim_{k\to\infty}\left(\mathbf{B}(1-\lambda)\right)^k}{\mathbf{r}'\lim_{k\to\infty}\left(\mathbf{B}(1-\lambda)\right)^k\mathbf{i}} \\
&= \frac{n\mathbf{r}'\frac{\mathbf{y}\mathbf{x}'}{\mathbf{x}'\mathbf{y}}}{\mathbf{r}'\frac{\mathbf{y}\mathbf{x}'}{\mathbf{x}'\mathbf{y}}\mathbf{i}} \\
&= \frac{n\left(\mathbf{r}'\mathbf{y}\right)\mathbf{x}'\frac{1}{\mathbf{x}'\mathbf{y}}}{\left(\mathbf{r}'\mathbf{y}\right)\mathbf{x}'\mathbf{i}\frac{1}{\mathbf{x}'\mathbf{y}}} \\
&= \frac{n\mathbf{x}'}{\mathbf{x}'\mathbf{i}}
\end{aligned}
$$

We see then that the backward Rasmussen-Hirschman index converges to the normalized left Perron eigenvector of matrix $\mathbf{B}$[38], in a final expression that is independent from the original weighting vector. Furthermore, if we choose the weighting vector as $\mathbf{r}' = \frac{n\mathbf{x}'}{\mathbf{x}'\mathbf{i}}$, the Rasmussen-Hirschman backward indicator is the same as the left Perron eigenvector and as noted by Dietzenbacher (1992) is also the same as the weighted direct backward linkage indicator of Chenery and Watanabe (1958)[39].

Similarly, we can start from the weighted forward linkage as $\mathbf{p}_1 = n\frac{\mathbf{Gc}}{(\mathbf{i}'\mathbf{Gc})}$, where $\mathbf{c}$ is a vector of column weights. Following the same procedure as above, it can be shown that this indicator converges to the normalized right Perron eigenvector (denoted by $\mathbf{z}$) of the output matrix $\mathbf{O}$, that is:

$$
\lim_{k\to\infty} \mathbf{p}_k = \frac{n\mathbf{z}}{\mathbf{i}'\mathbf{z}} \tag{28}
$$

Network centrality measures are typically computed using the matrix of positions in the interbank market in monetary terms (what we have labelled $\mathbf{A}_M$). Input-output measures, on the other hand, provide an economic rationale for using different matrices depending on the issue that one aims to tackle (for example, whether one tries to assess the dispersion of a shock to a given bank to

[38] We have assumed that the limit in the denominator exists.

[39] And we should add, also the same as the in-strength centrality defined over the input matrix $\mathbf{A}$.

the rest of the system or whether one is trying instead to see how sensible a bank is to a shock in the rest of the system). Furthermore, by the way in which the model is built, these matrices actually link balance sheet characteristics of the different banks to the vector of total assets of the system, providing further economic intuition for the measures. That said, in practice one can expect some relation between the rankings given by the different measures.

Table 5 presents the ranking suggested by the network centrality measures for the simple example of Section 3.3, computed over the matrix $\mathbf{A}_M$[40]. The only input-output indicators which establish the same ranking as any network measure are the total linkage effect $\bar{\mathbf{t}}$, which ranks banks in the same order as the out-strength ($\mathbf{c}_{(out)}$) and right eigenvector ($\mathbf{c}_{(rev)}$) centrality indicators, and our backward indicator ($\bar{\mathbf{h}}_{(b)}$) which gives the same ranking as left eigenvector ($\mathbf{c}_{(lev)}$) centrality. Given the discussion above, this link is not particularly surprising.

Table 5: Ranking by network measures (matrix $\mathbf{A}_M$) - Five bank example

	$\mathbf{c}_{(in)}$	$\mathbf{c}_{(out)}$	$\mathbf{c}_{(cl\text{-}in)}$	$\mathbf{c}_{(cl\text{-}out)}$	$\mathbf{c}_{(bw)}$	$\mathbf{c}_{(lev)}$	$\mathbf{c}_{(rev)}$
Bank 1	3	3	3	3	2	3	3
Bank 2	1	1	4	1	3	2	1
Bank 3	2	2	5	5	4	1	2
Bank 4	5	4	1	4	2	5	4
Bank 5	4	5	2	2	1	4	5

4.1 A second numerical example

To give evidence of the link between the input-output-based measures of systemic importance and those stemming from the network literature, a graphical analysis based on a more complex example seems more illustrative. As is well known, a significant hurdle when it comes to analyzing systemic risk on the basis of balance sheet interconnectedness is the availability of data, since detailed information on cross exposures is rarely available. Previous studies that could not use detailed data on bilateral exposures via lending/borrowing typically went around this obstacle by using maximum entropy methods, where the total exposure to the interbank market (both on the lending and borrowing side, which are publicly available) is used to reconstruct the whole matrix of interbank exposures[41]. A problem with the maximum entropy method is that it tends, by

[40]The same can of course be computed for the matrices $\mathbf{A}$ and $\mathbf{O}$, though network theory does not provide any economic rationale for computing centrality measures based on these matrices, as is the case for input-output measures. For this reason, plus space considerations we do not report this results here, though as can be expected from the propositions discussed above, there are some similarities in ranking. For example $\bar{\mathbf{h}}_{(b)}$ ($\bar{\mathbf{h}}_{(f)}$) will typically give the same ranking as $\mathbf{c}_{(in)}$ ($\mathbf{c}_{(out)}$), when the latter is computed based on matrix $\mathbf{A}$ ($\mathbf{O}$) instead of $\mathbf{A_M}$.

[41]Examples of this approach include among others the above cited papers from the BIS, Upper and Worms (2004), van Lelyveld and Liedorp (2006), Degryse and Nguyen (2007) and Wells (2004). Castrén and Kavonius (2009) apply the same methodology to flow of funds data in order to build a network based on institutional sectors

construction, to generate matrices that are less sparse than real-world interbank matrices[42].

Here we follow Soramäki and Cook (2012) and simulate a network using the Barabási and Albert (1999) model for generating random scale-free networks with preferential attachment. The network is generated so as to resemble the topological properties of the Fedwire payment network analyzed in Soramäki et al. (2007)[43]. It is composed of a hundred banks and it features a few large and highly connected banks acting as a hub. Appendix A presents charts comparing the measures introduced here with the set of network measures considered. Figure 1a shows a circle representation of the network. The size of the nodes and the density in the lower part of this chart points to the crucial role played by a relatively small subset of banks, which transact a considerable amount of funds between themselves and are highly connected to smaller nodes in the rest of the system. Figure 1b provides a close-up picture of this group of core banks, showing how densely connected they are.

Table 6 and Table 7 present a comparison between input-output measures and the network measures presented in the previous subsection. In particular, Table 6 compares our measures (x-axis) to network measures (y-axis) where the latter are computed based on matrix $\mathbf{A}_M$. One can see positive relationships almost everywhere, though the intensity of this relationship varies. The total linkage effect, in particular, seems to be strongly related to in and out strength centrality and to the two eigenvector measures. Something similar applies to the fields of influence measures, though the relationship is not as strong. The Rasmussen-Hirschman indices do not seem to present a strong relationship with any of the network measures, and a simple inspection of the charts seems to suggest that the identification of the most systemically important banks will differ between these measures. This points to the importance of considering the assets of the borrowing or lending bank when analysing the effects of shocks going form one bank to the system or coming from the system to a given bank, respectively. The fact that the field of influence measures rank many banks together (see the vertical cloud of points in several charts) is a special feature of the network considered; it is to be expected that the less the network resembles a core-periphery structure, the more these measures will generate a dispersed cloud of points[44]. In this respect it is interesting to see that the closeness measures will give a high ranking of systemic importance to many banks that are actually not in the core.

for the Euro Area, while Castrén and Rancan (2013) extend this approach by considering Euro Area country-specific networks based on institutional sectors that are linked via cross-border linkages from the *Monetary and Financial Institutions* sector. Furthermore, the latter paper shows how the imposition of additional constraints helps improving the reliability of maximum entropy methods.

[42]Mistrulli (2011) explicitly compares an interbank matrix based on real data for Italy with matrices obtained by the maximum entropy method and concludes that the latter approach underestimates contagion risk.

[43]The network was generated using the software FNA. The payment data for a whole day was aggregated to obtain the interbank matrix. A snapshot of total assets/liabilities was generated for the hundred banks by randomly drawing from a uniform distribution such that total interbank lending/borrowing represents between 10% and 45% of total assets/liabilities respectively. For details on the topological characteristics of the generated network, see Soramäki and Cook (2012).

[44]This is confirmed for ongoing research on a real interbank network for large european banks, which doesn't feature a core-periphery structure as prominent as the network generated here.

Table 7 presents another type of comparison. Input-output measures are calculated as usual, while network measures are calculated either on matrix **A** or **O**, depending on which is the input-output measure they are being compared to. For instance, in the first column, corresponding to the backward Rasmussen-Hirschman index ($\bar{\mathbf{h}}_{(b)}$), which is based on matrix **A**, all network centrality measures are computed based also on matrix **A**. The relationships from the propositions become visually obvious in the corresponding charts.

5 Concluding remarks

Systemic risk, though it is a concept that pre-dated the recent financial crisis, seems to be intimately associated to the latter. A likely reason behind this is that the financial turmoil that emerged in 2007 brought to the forefront the importance of interconnectedness and risk externalities. As a consequence, we have witnessed a shift in emphasis from a *micro*- to a *macro*- prudential approach to banking regulation and supervision, both in economic analysis as well as in policy practice. It is now acknowledged that banks need to be controlled not only because of misincentives arising from the protection they enjoy on their liability side (e.g. deposit insurance), but also due to the risk they transmit to other banks through the web of exposures that ties the system together. With this transformed paradigm for policy a crucial practical issue arises, since it becomes necessary to measure the systemic importance of individual banks.

Taking the balance sheet of the banking system as a point of departure, we have derived expressions that closely resemble the traditional Leontief (1941) input-output model. With this benchmark at hand and making use of the literature on linkages in input-output analysis and the transmission of risk in infrastructural systems, we have presented six measures of systemic importance. Each of these measures has an intuitive economic story behind, story which is itself derived from the very structure of the model. The measures presented here aim at capturing differents aspects of systemic importance, namely: *(i)* how does a shock to the funding side of one bank disperse through the rest of the system?, *(ii)* how sensitive is a bank to a shock hitting simultaneously all other banks?, *(iii)* what happens when the shock comes from interbank flows themselves? in particular, what happens if a bank sees its sources of interbank funding reduced?, *(iv)* what if it is the bank itself who decides to cut financing to all other banks?, *(v)* what if the last two events happen simultaneously?, and finally, *(vi)* what if a bank is being completely cut off the interbank system?

The measures were illustrated by means of a simple numerical example which highlighted how the measures operate and in which way they capture different aspects of the balance sheet. Some parallels to network centrality measures were drawn, both at a formal level as well as by means of a simulated network designed to resemble real-world characteristics.

The present paper sets up a benchmark and there is much that can be done along these lines. For example, more connections could be established at the formal level with existing measures and with other branches of theory like for example finite Markov chains. The framework could also be readily adapted to compute rankings of systemic importance in models which generate the balance sheet items needed to compute our measures (endogenous banking network models, agent-based models, etc.). In ongoing research, we aim to apply these measures to a dataset of exposures between large european banks, with the idea of establishing a ranking of systemically important banks and with the goal of establishing an extended comparison with existing measures.

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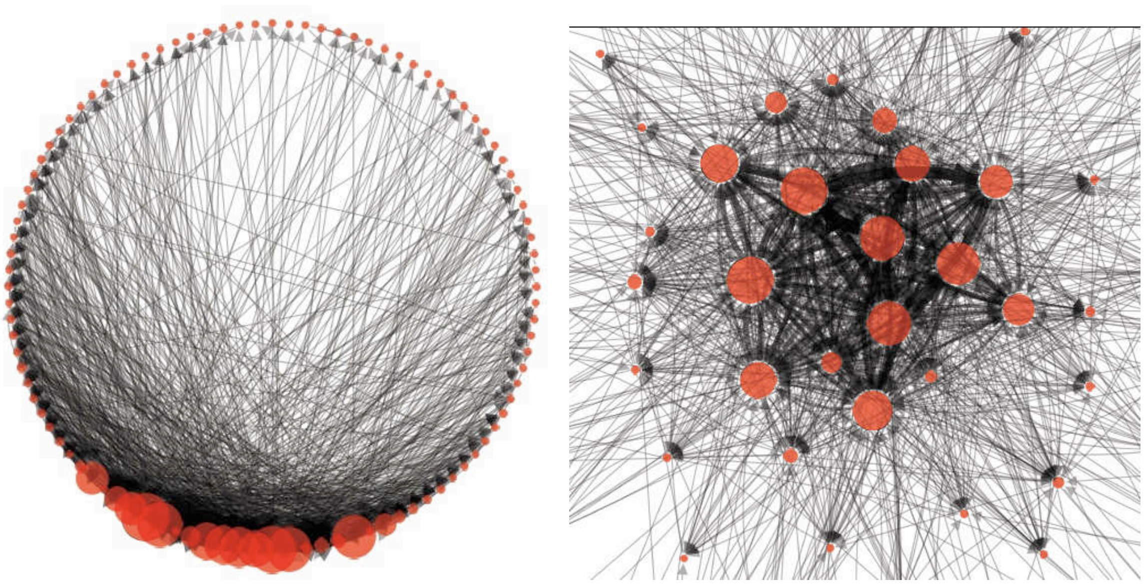
A Figures and tables

All figures are based on the second example presented in Section 4.1. Figure 1 presents a visualization of the network where the size of nodes is given by total assets of the corresponding banks and the width of the arrows connecting the different nodes represent the lending/borrowing between the banks (i.e. "thicker" arrows represent bigger positions).



(a) Circle representation **(b)** Core

Figure 1: Simulated network visualization

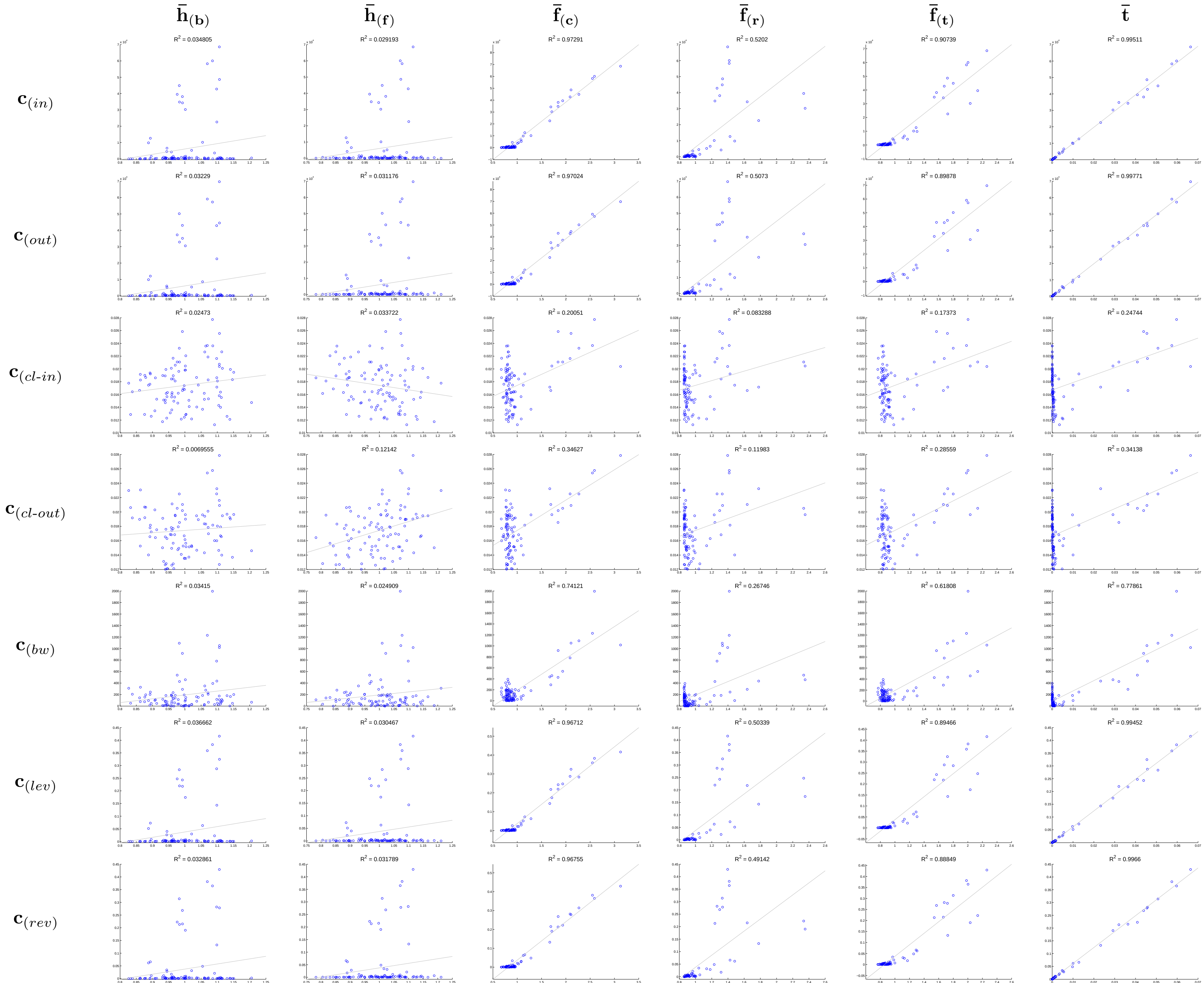


Table 6: Comparison of IO measures and network centrality measures based on matrix $\mathbf{A}_M$

Note: IO measures calculated as usual, network measures based on matrix $\mathbf{A}_M$.

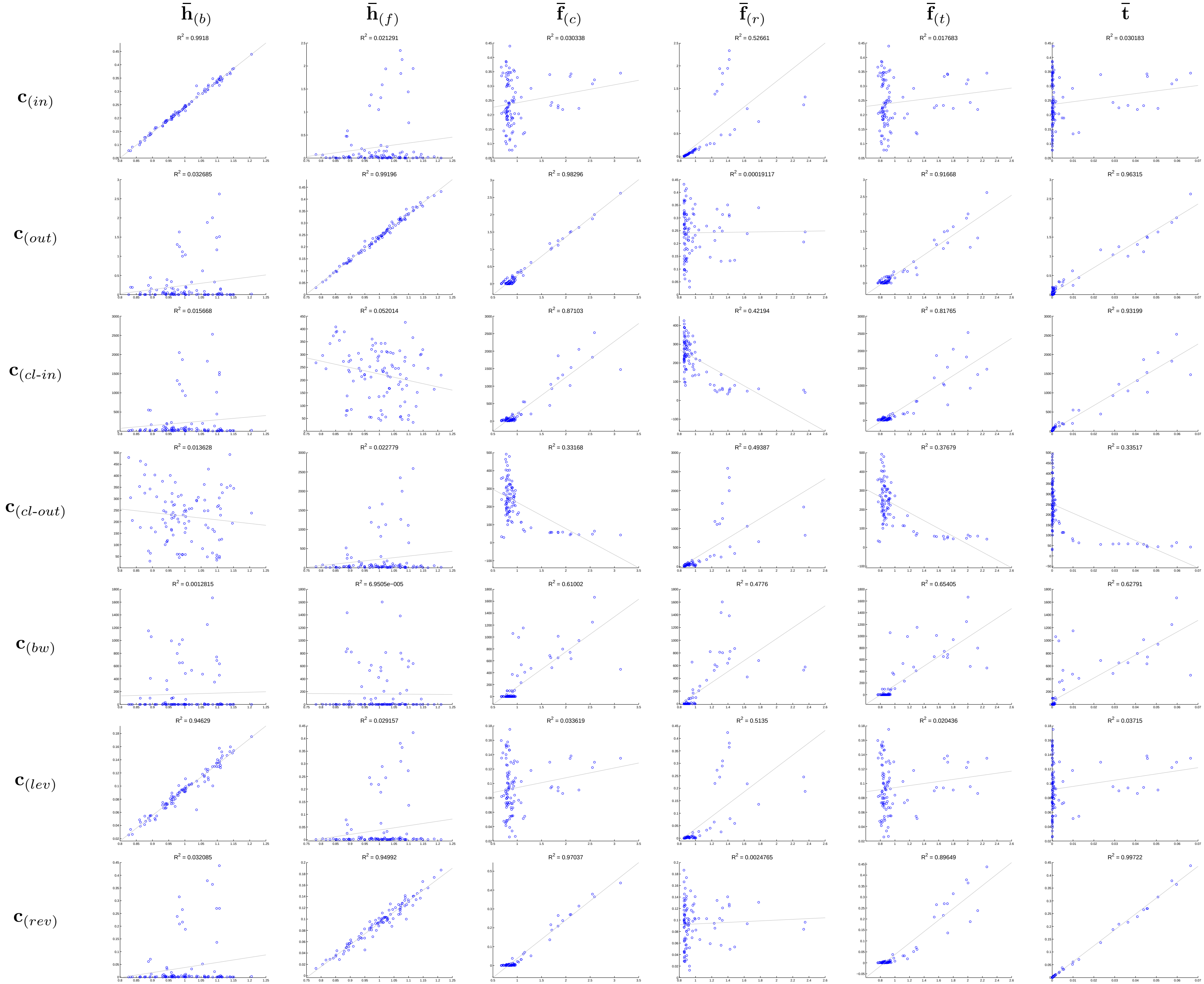


Table 7: Comparison of IO and network centrality measures

Note: When compared to $\bar{\mathbf{h}}_{(f)}$ and $\bar{\mathbf{f}}_{(r)}$, the centrality measures were computed based on the output matrix $\mathbf{O}$, otherwise they were computed based on the input matrix $\mathbf{A}$.

B ANNEX: Relation to other approaches

B.1 Asset price distributions and correlations

Within that class of indicators that relies on asset prices distributions and correlations, the two most prominent measures are $CoVaR$ proposed by Adrian and Brunnermeier (2011) and the *Marginal Expected Shortfall* (MES) by Acharya et al. (2012)[45]. Both are essentially measures of codependence at the tails of the distribution of returns and are hence intimately related to the *Value-at-Risk* (VaR).

Adrian and Brunnermeier (2011) have proposed $CoVaR$ as a measure of the contribution of each individual financial institution to systemic risk. $CoVaR_{j^\star,S}$ of financial insitution j is the value-at-risk (VaR) of the entire financial system S conditional on j being in distress. $\Delta CoVaR_{j,S} = CoVaR_{j^\star,S} - CoVaR_{j,S}$, the difference between the system's VaR when j is distressed (denoted by $CoVaR_{j^\star,S}$) and the same VaR when j is in a normal state (denoted by $CoVaR_{j,S}$), is the contribution of j to systemic risk.

It is possible to establish some links at the intuitive level between $CoVaR$ and the measures of systemic influence discussed in this paper, though the precise relation depends on the origin of the shock and the measure considered. Consider first the simple case already examined, where bank 1 is subject to a unitary deposit drawdown. To fix ideas, assume this is followed, on impact, by a squeeze of that bank's liquidity held at the central bank, $d_1 = l_1 = -1$. On impact, bank 1's VaR increases by the same amount[46]. In a second round of effects, bank 1 starts withdrawing deposits held with other banks, spreading the effect through the system; one can think of this as bank 1 trying to mitigate the increase in its own VaR by scaling down interbank loans (risky) and replacing them with central bank liquidity (riskless). The squeeze to other banks reduces interbank lending further. The final result is measured by $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots$. The total system effect is $\mathbf{i}'\mathbf{B}\mathbf{i}_1$, the numerator of $\bar{h}_{b_1}$. If the covariance of cross-bank asset returns does not change, as we have assumed in this simple example, the system's VaR increases exactly by the same amount, so $\mathbf{i}'\mathbf{B}\mathbf{i}_1$ is in effect identical to the $CoVaR$ of bank 1. Our Rasmussen-Hirschman "backward" index simply scales all individual $CoVaR$ changes by the total change, so that the measures sum up to unity. In more complex examples, the distribution of asset returns may change through the process.

Adrian and Brunnermeier (2011) also introduce the "exposure $CoVaR$" (i.e. $\Delta CoVaR_q^{i|system}$), which basically reverses the conditioning and hence measures the extent to which an individual institution is affected by systemic financial events. The MES of Acharya et al. (2012) uses essentially the same conditioning, as do others like Huang et al. (2012). The "sensitivity of dispersion" index

[45] Several such measures can be found in the literature, see Bisias et al. (2012).

[46] We have assumed that bank reserves are riskless while other assets, including interbank loans, are risky, so the reduction of bank reserves amounts to a leftward shift of the distribution of asset returns. For a given confidence level, the VaR shifts by the same amount.

$\bar{h}_{f_j}$ is in spirit analogue to these measures, since it is to be interpreted as the effect of a systemic shock on bank j (i.e. how sensitive bank j is to a funding shock hitting all banks simultaneously). The measures proposed here are more leaned towards the measurement of interconnectedness and are based on the matrix of interbank linkages, whereas indicators such as $CoVaR$ and MES are measures of codependence at the tails of the distribution of asset returns and are hence of a different nature altogether. We hope that future research will shed light on this connection (or lack thereof).

B.2 The BIS approach

Research economists at the Bank of International Settlements (Drehmann and Tarashev (2011a,b), Borio et al. (2010)) have taken another approach, focusing on how total systemic risk in a given financial sector may be decomposed into contributions by individual banks. They distinguish between "top-down" measures, which start with the risk of the system and allocate it to individual institutions, and "bottom-up" measures, which make distress at a particular institution the point of departure in order to then compute the associated level of system-wide distress[47]. The former group is further subdivided into two approaches. The first consists in measuring how much each institutions's non-bank creditors would suffer (i.e., incur losses) as a result of a systemic event and thereby measures systemic importance as the participation of each institution in such events (the so called "participation approach"[48]). But, as they note, *participation* of individual institutions in systemic events is not necessarily the same as their *contribution* to such events. Hence, the second top-down measure consists in measuring how much each institution contributes to the risk of a systemic event ocurring (the so called "contribution approach"). Drehmann and Tarashev (2011a) modify the contribution approach in order to account for the fact that a bank not only contributes to systemic risk through its exposure to exogenous shocks but also by being a channel of propagation of such shocks through the system and by being itself vulnerable to propagated shocks. This "generalized contribution approach" basically aims to include in the assessment of systemic risk the role that the interbank network plays. As noted above, the network of interbank connections is at the heart of our analysis.

There is some analogy between this reasoning and our measures. The "participation approach" is analogous to what we have called "row" measure, that gauges the effect on a given bank of a simultaneous unitary squeeze of interbank lending by all banks. By attributing the risk associated with an interbank transaction entirely to the lending counterparty, this approach emphasizes what happens "downstream" in the network of interconnections, i.e. what happens to the borrowers of

[47] As noted by Drehmann and Tarashev (2011a), $CoVaR$ can be subsumed into the bottom-up approach: instead of the VaR just use the "expected shortfall" (ES) and they become in essence identical.

[48] The participation approach is closely linked to other contributions by Huang et al. (2012), Acharya et al. (2012) and Brownlees and Engle (2011). As emphasized by Drehmann and Tarashev (2011a), these measures do not consider explicitly the interbank network as a driver of systemic risk. There is a growing literature on the influence of linkages in interbank networks on systemic risk assessment (see Upper (2011) and Allen and Babus (2009) for an overview).

the funds lent by this counterparty to which the risk is attributed. The "contribution approach" is instead akin to our "backward" indices, that measure the system effects of a unitary shock in one bank. Note that in all cases, it is possible to construct measures that sum to unity, thereby decomposing any given degree of systemic distress into individual contributions.

These series of papers by Drehmann and co-authors implement the contribution approach by a method borrowed from cooperative game theory called "Shapley value" after Shapley (1953).In the context of a coalitional game in which different players cooperate to obtain a certain aggregate value, Shapley (1953) provided a way for decomposing this value into the contributions of the different players[49]. Individual contributions to systemic risk are obtained by averaging all contributions that institutions make to all possible subsets (coalitions) of other institutions. The resulting measures satisfy, note the authors, four desirable axioms[50]:

1. *Efficiency*: the grand total of value to be distributed is exactly distributed among all players.
2. *Symmetry*: if two institutions i and j make the same marginal contribution to any coalition that contains neither i nor j, then their Shapley values are equal.
3. *Dummy (zero player)*: an institution with zero marginal contribution to every coalition is assigned a Shapley value of zero.
4. *Additivity*: for any two games g_1 and g_2, the Shapley value of the game composed of g_1 and g_2 together equals the sum of the Shapley values of g_1 and g_2 taken separately (i.e. the value is an additive operator in the space of all games, see Temurshoev (2009))

Coming back to linkage measures in input-output analysis, note that the problem of identifying a key player under the hypothetical extraction method can be formulated as finding the bank j that solves the following problem:

$$max\{\mathbf{i}'\mathbf{q} - \mathbf{i}'\mathbf{q}^{-j} \equiv \mathbf{i}'(\mathbf{B} - \mathbf{B}^{-j})\mathbf{l} | i = 1, ..., n\} \tag{29}$$

One can define the vector of factor (asset) multipliers as: $\mathbf{m}' = \mathbf{i}'\mathbf{B}$[51]. Given this one can further define the asset worth and adjusted asset worth of bank j respectively as $\omega_j(\mathbf{A}, \mathbf{l}) = \frac{m_j q_j}{b_{jj}}$ and $\tilde{\omega}_j(\mathbf{A}, \mathbf{l}) = \omega_j - l_j$.

After some matrix manipulations it is possible to see that the following holds: $\mathbf{B} - \mathbf{B}^{-j} = \frac{1}{b_{jj}}\mathbf{B}\mathbf{i}_j\mathbf{i}_j'\mathbf{B} - \mathbf{i}_j\mathbf{i}_j'$. With this at hand, problem 29 can be re-written in two ways. First, recall from Equation 14 that $\mathbf{B}\mathbf{i}_j\mathbf{i}_j'\mathbf{B} = \mathbf{F}(j, j)$, hence the objective function can be stated as: $\frac{1}{b_{jj}}\mathbf{i}'\mathbf{F}(j, j)\mathbf{l} -$

[49] The "value" generated by the game could be quantified by means of measures such as a system-wide VaR or ES; then the Shapley value indicates how much of this VaR or ES is generated by each player/institution.

[50] Indeed, the unique value that satisfies these four axioms is the Shapley value.

[51] In this part we follow Temurshoev (2009), who performs this analysis at a more general level in order to provide a more compact and efficient way of identifying key sectors and key groups of sectors in an input-output framework.

l_j; this representation highlights the fact that the fields of influence and the hypothetical extraction method are closely related, which should not come as a big surprise since the two are variants of the problem of coefficient change. Second, using the definitions form the previous paragraph, the objective function of problem 29 can shown to be equivalent to $\tilde{\omega}_j$; then the key bank $j^\star$ that solves problem 29 will be the one with the highest adjusted asset worth $\tilde{\omega}_j$.

Temurshoev (2009) shows that the asset worth defined in above satisfies all except the first of these axioms, and the same applies to the adjusted asset worth just defined. Hence, even when the Shapley value and input-output analysis operate under quite different frameworks, it is nonetheless possible to establish some points of contact. Again, we hope to tackle this relationship in further research.

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