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Declining time preference rates have a large effect on optimal climate policy, but efforts to surmount time consistency concerns have forced modelers either to employ very simple models or to adopt quasi-hyperbolic rates. Using the integrated assessment model from Golosov et al. (2013), we derive an explicit formula for the optimal carbon tax when time preference rates are non-constant in an arbitrary way. Concerns about time inconsistency, concerns about multiple equilibria, and concerns about the sensitivity of results to assumptions about future time preferences are all resolved in a straightforward way. Quantitative results show a large effect on optimal policy.

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[Declining discount rates] "solve one problem by creating another. Unless the discount rate is constant, the policy path is subject to 'time inconsistency'. Suppose an intelligent decision-maker plans a strategy for the long future, beginning today. Five years from now, she reconsiders the strategy, having followed it so far. She will want to change to a different strategy for no other reason than the passage of time.
. . . This sounds like a poor way to run a railroad."

Robert Solow in forward to Portney and Weyant (1999)

Climate change is the quintessential long-term problem. Carbon dioxide decays slowly, with a nontrivial fraction of emissions remaining in the atmosphere for thousands of years. Meanwhile, projected damages include irreversible effects, such as species extinctions and the potential collapse of the Greenland and West Antarctic ice sheets (IPCC 2007). When valuing future damages with a constant discount rate, the importance of the chosen rate grows exponentially in the time horizon:[1] for example, over 10 years, lowering the rate from 5.5% (as in Nordhaus 2008) to 1.4% (as in Stern 2007) increases the present value by a factor of 1.5; over 100 years, the present value increases by a factor of 60 (1.5^{10}); and over 200 years, it increases by a factor of 3600 (60×60). For events in the distant future—hundreds or even thousands of years ahead—the choice of discount rate is overwhelmingly important.

A key determinant of the discount rate in dynamic models is the rate of time preference. While typically assumed constant, there is no deep reason for this (Frederick et al. 2002). A constant rate ensures time consistency with conventional methods of dynamic optimization, but time consistency is possible with non-constant time preference also—this just requires a more sophisticated solution strategy that ensures

[1]To see this in general, consider two discount rates: $r_1 > r_2$. The relative valuation of a payoff C in t periods is $\frac{C \cdot e^{-r_2 t}}{C \cdot e^{-r_1 t}} = e^{(r_1 - r_2)t}$, which grows exponentially in t.

subgame perfection (Strotz 1955). Nevertheless, solving for subgame perfect equilibria can be difficult, especially for quantitatively plausible climate policy models, which typically have several state variables and nonstationary forcing variables.

To make progress, all time consistent applications of non-constant time preference in the climate policy literature either assume that discounting is quasi-hyperbolic[2] (Karp 2005, Gerlagh and Liski 2012) or consider very simple models that abstract from the process of economic growth (Karp 2007, Fujii and Karp 2008). The quasi-hyperbolic assumption is useful for generating qualitative insights, but unsatisfactory for quantitative policy analysis. Most justifications for non-constant time preference suggest rates that decline continuously in time; approximating a continuous path with a simple step function introduces approximation error that becomes large when the long run path of discount rates matters a lot.

The current paper is the first to solve for time consistent optimal carbon taxes in a quantitatively plausible climate-economy model while allowing for arbitrary non-constant time preference rates. This is done in an analytically tractable setting in which the subgame perfect equilibrium to the planner's problem without commitment is decentralized using a tax on fossil energy firms and an explicit formula is provided for the optimal tax.[3] We further show that the most important obstacles to adopting non-constant time preference rates in practice are overcome in a straightforward way.

The most immediate obstacle is time inconsistency. This arises in typical models because the optimal policy changes—often substantially—depending on whether or not a commitment device is assumed. But for the considered model, this concern

[2]This applies a higher rate of time preference between the first and second periods, followed by a constant lower rate thereafter (Phelps and Pollak 1968, Laibson 1997). The assumption makes it possible to recover a highly tractable recursive structure when solving for symmetric Markov perfect equilibria (Harris and Laibson 2001).

[3]To our knowledge, this is the first application in the literature to explicitly decentralize the optimal carbon tax under non-constant time preference.

falls away in a rather stark way. Starting from a given initial period, the optimal first period tax derived by assuming a commitment device is exactly the same as the optimal tax in the subgame perfect equilibrium without commitment. It follows that the optimal tax formula can be applied to evaluate current climate investments without having to worry about time inconsistency.

A further obstacle is due to the way in which time consistency concerns are typically resolved in the recent literature on non-constant time preference: mainly, by solving for Markov perfect equilibria in stationary, infinite-horizon models (Harris and Laibson 2001, Krusell and Smith 2003, Karp 2005, Karp 2007, Fujii and Karp 2008, Gerlagh and Liski 2012). Multiple equilibria arise generically in this case because the derivative of the equilibrium response function is itself an unknown and there are not enough equations to solve uniquely for all the unknowns (Tsutsui and Mino 1990). In contrast, the subgame perfect equilibrium is unique for the model in this paper when we consider a finite horizon version of the model. Because an *arbitrarily long* finite horizon provides a reasonable description of the intended policy problem, we argue that this provides a reasonable resolution of this important concern.[4]

Finally, in typical models with non-constant time preference, equilibrium policies are highly sensitive to assumptions about future time preference rates. This is unsettling because assumptions about time preferences five or ten generations removed are essentially arbitrary. A specific example of the concern arises when applying the preference aggregation argument for DRTP.[5] Most applications of this approach assume that the current distribution of time preference views remains fixed over time.

[4]Uniqueness does not obtain generally when solving for subgame perfect equilibria in finite horizon models with non-constant time preference (Peleg and Yaari 1973, Goldman 1980). It should therefore be viewed as a special feature of the considered model.

[5]This argument is discussed below.

Thus, there is no opportunity for views in society to shift over time; an implausible assumption over long horizons. But for the considered model, the optimal tax in the initial period is invariant to assumptions about the path of time preference rates to be adopted by subsequent generations. Thus, the above concerns do not apply.

Over long horizons, declining rates of time preference (DRTP) are intuitive. The plausible requirement that the equilibrium return on capital in the model economy be historically consistent implies a rate of time preference of about 3% if utility is logarithmic. A constant 3% rate weights utility for agents in 200 years—independent of consumption levels—20 times more than for agents in 300 years[6]. But since it is already hard to imagine our relationship to people in 200 years, it is not clear why we would distinguish so severely across these horizons (Heal 2000, Rubinstein 2003, and Karp and Tsur 2011).[7] When the constant rate assumption is relaxed, observed returns in financial markets do not reveal information about time preference rates beyond the horizon for which savings instruments exist (40 years at most). It is then possible to calibrate the model so returns in general equilibrium are consistent with market interest rates over the short to medium term even as rates become low in the long run (Gerlagh and Liski 2012).

An alternative argument for DRTP applies when time preference rates differ in society. A social planner who aggregates preferences will act as if maximizing utility for a representative agent with DRTP. The argument is simplest when agents consume a public good—a constant fraction of an endogenous consumption sequence (Li and Löfgren 2000, Jackson and Yariv 2012). The rationale in this case is formally equivalent to Weitzman's uncertainty-based argument for declining consumption discount

[6] The difference is 7-fold when the rate of time preference is 2%.

[7] A variety of behavioral studies from economics and psychology also support the conclusion that time preference rates decline (Thaler 1981; Loewenstein 1987; Ainslee 1991; Cropper et al. 1994; Kirby and Herrenstein 1995).

rates (Weitzman 1998, Weitzman 2001).[8]

The model is a slightly simplified version of the model in Golosov, Hassler, Krusell, and Tsyvinski (2013) (hereafter, "GHKT").[9] The one simplification is to assume that fossil resource constraints are non-binding. Finite resource stocks are allowed in the full GHKT model, though they do not effect the optimal carbon price under constant discounting:[10] profit maximizing firms internalize the scarcity rent, so their is no need for policy to correct the market outcome. Intuition suggests that an analogous result might apply in the model with non-constant discounting, though we are not able to prove this. A possible justification for the assumption is that coal is the most important source of carbon emissions in the long run (van der Ploeg et al. 2012) and coal deposits are not used up along the optimal path under plausible parameter values in most integrated assessment models.

The GHKT model does a reasonable job replicating the essential quantitative features of DICE (Nordhaus 2008).[11] Unlike DICE, it allows for an arbitrary number of micro-founded energy sectors and so does a better job simulating the effect of a

[8]Gollier and Zeckhauser (2005) consider the preference aggregation problem in which consumption shares are endogenous while income is exogenous, and Heal and Millner (2013) consider the problem in which consumption shares and income are both endogenous. "As if" DRTP is also implied when Knightian Uncertainty applies across time preference rates and the decision maker adopts minimax regret (Iverson 2013).

[9]We have in mind the version of the GHKT model adopted in the quantitative section of their paper. This is the version for which their central result, a tractable formula for the optimal carbon tax, arises as a consequence of explicit optimizing behavior by all agents. The GHKT optimal carbon tax formula is also presented for a model that allows for more general functional form assumptions, but the result requires the accompanying assumption that savings rates are constant. This *ad hoc* assumption would be inappropriate in the current context since a primary goal is to ensure that time consistency concerns are adequately addressed. To do this, it is important that the decision problems for all agents are specified explicitly.

[10]As a result, the optimal tax formula derived here reduces to that in Golosov et al. (2013) when discounting is constant.

[11]One limitation of the GHKT model is that it abstracts from temperature inertia, so the dynamic link between emissions and damages is not fully realistic. This can be remedied using an alternative set of linear coefficients in the damage function as suggested in Gerlagh and Liski (2012). It would be straightforward to incorporate these coefficients in our analysis, though we maintain the GHKT specification to keep the presentation more closely in line with the results in Golosov et al. (2013).

carbon tax in a decentralized equilibrium. It also allows damages to be stochastic. Analytic tractability is attained by imposing judicious functional form assumptions: in particular, log utility, Cobb-Douglass production, full depreciation of physical capital each period, and "linear-exponential" climate damages. Strengths and weaknesses of these assumptions are discussed in section 3.

The paper also solves for the "imputed" Pigouvian tax—the marginal externality cost computed using discount rates derived from observations about savings rates in the capital market of the model economy. A key insight in Gerlagh and Liski (2012) is that non-constant time preference leads to distortions in the capital market that make observed returns an incorrect basis for social cost benefit analysis. As in their paper, the optimal carbon tax in our model exceeds the imputed Pigouvian tax in the equilibrium without commitment provided the delay between emissions and damages is sufficiently long. We also compute the Pigouvian tax for the equilibrium with commitment. In this case, the optimal and Pigouvian taxes are equal.

The quantitative section calibrates declining time preference rates using the preference aggregation argument of Li and Löfgren (2000). Preference heterogeneity is consistent with the discounting debate between Nordhaus (2008) and Stern (2007). We find that even a small weight on the Stern (2007) discounting model dramatically increases the optimal tax. For example, a 20% weight on Stern increases the optimal tax four-fold relative to the Nordhaus (2008) calibration. We also simulate the full dynamic trajectory of the model for the equilibria with and without commitment. As expected, the optimal taxes are the same in the initial period, though they differ substantially in later periods. It follows that a commitment device, were it available, would have considerable value for the initial generation.

The paper is closely related to the earlier work by Gerlagh and Liski (2012). They solve for the linear symmetric Markov Perfect Equilibrium in a simplified version of the GHKT model with quasi-hyperbolic discounting. In addition to accommodating arbitrary non-constant time preference, we allow for uncertainty about future damages and for multiple energy sectors. We also solve for the decentralized equilibrium, and we demonstrate several features of the GHKT model that make it a highly convenient setting in which to incorporate non-constant discounting into climate policy decision making.[12] Despite the differences across models, the optimal carbon tax formula derived here reduces to that in Gerlagh and Liski (2012) in the deterministic case with quasi-hyperbolic discounting.

Our finding that time consistency concerns are functionally "irrelevant" is closely related to a result in Phelps and Pollak (1968). They consider a finite horizon growth model with linear production and quasi-hyperbolic discounting. They refer to savings in the full commitment equilibrium as *first best* and savings in the no commitment equilibrium as *second best*. With log utility, first and second-best savings are the same. Indeed, no matter what savings rate is adopted by subsequent generations, optimal savings in the initial period are unchanged. They conclude, "This logarithmic case must be added to the curious list of examples in which first-best and second-best decisions do not differ."

The result does not carry over to the more general Ramsey model. Barro (1999) solves for the unique Cournot-Nash equilibrium in a Ramsey model with logarithmic preferences and continuously declining time preference rates. He also solves the model by assuming a commitment device for a planner in the initial period. In both cases, the marginal propensity to consume out of wealth is a constant, but the amount of savings in the initial period differs.

[12]The main emphasis of their paper is to demonstrate that hyperbolic preferences, embedded in a general equilibrium setting, provide a compelling resolution of the "puzzle" that evidence on historical savings behavior from financial markets would seem to rule out the possibility of putting nontrivial weight on climate damages that occur beyond a couple hundred years.

I. Model

The paper extends the GHKT model by allowing each generation to employ an arbitrary path of non-constant time preference rates. We simplify by assuming the finite resource constraint on fossil fuel reserves is non-binding in all fossil sectors.

A. Preferences, Technology, and Climate

Time runs from 0 to T. The horizon is initially finite, but we also consider the infinite horizon limit. A representative household derives utility from the time path of consumption. Utility is discounted with a sequence of potentially non-constant discount factors $\{\beta_j\}$. For each j, $0 < \beta_j < 1$. We are typically interested in the case in which the rate of time preference declines in time, thus where $\{\beta_j\}$ increases, though it is not necessary to assume this for our results. For each time horizon k periods ahead, we define a cumulative discount factor $R_k = \prod_{j=1}^{k} \beta_j$. By assumption, $R_0 = 1$.

The utility function of the representative household in generation t is

$$\mathbb{E}_t \sum_{\tau=t}^{T} R^{(t)}_{\tau-t} \ln(C_\tau).$$

Thus, utility is logarithmic. The superscript on the discount factor each period indicates that time preferences are those of generation t; this allows for the possibility that time preferences differ across generations.

The economy has $I+1$ sectors. Sector $i=0$ produces final goods; sectors $i = 1,\ldots,I_g-1$ are polluting (fossil) energy sectors; and sectors $i = I_g,\ldots,I$ are non-polluting energy sectors. Production in the energy sectors depends on labor only:

$$E_{i\tau} = F_{i,\tau}(N_{i,\tau}),\; i = 1,\ldots,I. \tag{1}$$

The time subscript allows for technological change. Labor input to each sector is indicated by $\mathbf{N}_\tau = (N_{0,\tau}, N_{1,\tau}, \ldots, N_{I,\tau})$, and energy input to final goods production from each energy sector is indicated by $\mathbf{E}_\tau = (E_{1,\tau}, \ldots, E_{I,\tau})$.

Carbon emissions arise as an externality from energy production. Units are chosen so $E_{i,\tau}$ for each fossil energy sector denotes energy output and CO2 emissions. Atmospheric carbon beyond its preindustrial level accumulates according to

$$S_\tau - \bar{S} = \sum_{j=0}^{\tau+t+H} (1 - d_j) E^f_{\tau-j}, \tag{2}$$

where

$$E^f_\tau = \sum_{i=1}^{I_g - 1} E_{i,\tau} \tag{3}$$

is total CO2 emissions (from fossil sectors) in period τ. $1-d_k$ is the fraction of a unit of emissions that remains in the atmosphere k periods after it is emitted. H is the number of periods between period zero in the model and the start of the industrial revolution.

Cumulative emissions lead to climate damages that impact economic output through a multiplicative damage function that takes the following "exponential–linear" form:

$$\omega(S_\tau) = \exp\left(-\gamma_\tau (S_\tau - \bar{S})\right). \tag{4}$$

γ_τ is an elasticity that denotes the percent output loss associated with an extra unit of atmospheric carbon in period τ. It is stochastic.

Net output in the final-goods sector is determined by

$$Y_\tau = K_\tau^\alpha A_\tau(\mathbf{E}_\tau, N_{0,\tau})\omega(S_\tau). \tag{5}$$

$A_t(\cdot,\cdot)$ is an unspecified energy-labor composite function. The time subscript allows for technological change. Labor is mobile across sectors with

$$\sum_{j=0}^{I} N_{j,\tau} = N_\tau. \tag{6}$$

Finally, the aggregate resource constraint is[13]

$$Y_\tau = C_\tau + K_{\tau+1}. \tag{7}$$

B. *Planning Problem Without Commitment*

For global climate policy, a commitment device that would enable policy-makers to force the hand of later generations is almost certainly infeasible.[14] This is important when time preference rates are non-constant because the optimal control solution for the initial generation is not time consistent. The (second-best) optimal allocation for the planning problem without commitment is therefore appropriately viewed as a subgame perfect equilibrium among a sequence of planners, each of whom controls the endogenous variables for one period only.

[13]This assumes 100% depreciation of physical capital each period. A strong assumption partly offset by the assumed period length of a decade in the calibrated model.

[14]It may be sensible to talk about a long-term commitment device for national tax policy, but the notion is far less compelling in a global public good provision problem. In this case, there are no viable institutions to overturn the actions of sovereign nations in the present—especially for large CO2 emitters—more less to commit their actions in the future. In our calibration, the period length is a decade, so the no commitment assumption means decision-makers in the model cannot commit action beyond a decade.

The planner at t anticipates how $\{C_\tau, K_{\tau+1}, \mathbf{N}_\tau, \mathbf{E}_\tau\}_{\tau=t+1}^{T}$ will be chosen by subsequent planners, then chooses C_t, K_{t+1}, $\mathbf{N}_t$, and $\mathbf{E}_t$ to maximize

$$\mathbb{E}_t \sum_{\tau=t}^{T} R^{(t)}_{\tau-t} \ln(C_\tau)$$

subject to (1) through (7) and non-negativity constraints on consumption and capital.

To solve for the subgame perfect equilibrium, we exploit two characteristics of optimal decisions. First, agents optimally save a stock-invariant fraction of income.[15] Second, optimal energy inputs each period ($\mathbf{E}_t$) are independent of the inherited stock variables (K_t and S_t). The proof is inductive.

The hypothesis is easy to confirm for the last period. Suppose, in arbitrary period t, it holds for all later periods. Then capital accumulates according to

$$K_{\tau+1} = s_\tau K_\tau^\alpha A_\tau(N_{0,\tau}, E_\tau) \exp\left(-\gamma_\tau S_\tau\right), \ \tau = t+1, \ldots, T, \tag{8}$$

where the savings rate s_τ is independent of the inherited state variables.

Taking logs gives a first-order linear difference equation in the log of capital.[16] Iterating, while ignoring variables that are exogenous from the perspective of the period t decision, the log of capital in $\tau > t+1$ can be written

$$\ln(K_\tau) = \alpha^{\tau-(t+1)} \ln(K_{t+1}) - \sum_{j=0}^{\tau-t-2} \alpha^{\tau-t-2-j} \gamma_{t+1+j}(1-d_{1+j}) E_t^f + \ldots \tag{9}$$

[15]As recognized at least since Brock and Mirman (1972), growth models with log utility, Cobb Douglass production and 100% depreciation imply a constant (thus stock-invariant) savings rate (Ljungqvist and Sargent 2004).

[16]I am grateful to Larry Karp for suggesting this method of proof. An earlier version of the paper solved the model using a modified version of finite horizon dynamic programming (see Iverson 2012). The approach here more cleanly exposes the intuition behind the paper's results.

The flow payoff in τ then becomes

$$\begin{aligned}\ln(C_\tau) &= \ln[(1-s_\tau)K_\tau^\alpha A_\tau(\mathbf{E}_\tau, N_{0,\tau})\exp(-\gamma_\tau S_\tau)] \\ &= \alpha\ln(K_\tau) - \gamma_\tau S_\tau + \ldots \\ &= \alpha[\alpha^{\tau-(t+1)}\ln(K_{t+1}) - \sum_{j=0}^{\tau-t-2}\alpha^{\tau-t-2-j}\gamma_{t+1+j}(1-d_{1+j})E_t^f + \ldots] - \gamma_\tau S_\tau + \ldots\end{aligned}$$

Combining terms gives

$$\ln(C_\tau) = \alpha^{\tau-t}\ln(K_{t+1}) - \sum_{j=0}^{\tau-t-1}\alpha^{\tau-t-1-j}\gamma_{t+1+j}(1-d_{1+j})E_t^f + \ldots \tag{10}$$

This is used to rewrite the planner's problem at date t. For reference, we refer to this problem as $\mathbf{PP(t)}$.

$$\max_{C_t, K_{t+1}, \mathbf{N}_t, \mathbf{E}_t, S_t} \ln(C_t) + \mathbb{E}_t \sum_{\tau=t+1}^{T} R_{\tau-t}^{(t)}\left[\alpha^{\tau-t}\ln(K_{t+1}) - \sum_{j=0}^{\tau-t-1}\alpha^{\tau-t-1-j}\gamma_{t+1+j}(1-d_{1+j})E_t^f + \ldots\right]$$

s.t.

$$C_t + K_{t+1} = K_t^\alpha \tilde{E}_t(N_{0,t}, E_t)\exp\left(-\gamma_t(S_t - \bar{S})\right) \tag{11}$$

$$E_{it} = F_{i,t}(N_{i,t}),\ i = 1, \ldots, I, \tag{12}$$

$$\sum_{j=0}^{I} N_{j,t} = N_t, \tag{13}$$

$$S_t - \bar{S} = \sum_{j=0}^{t+T} (1 - d_j) E^f_{t-j}. \tag{14}$$

Let $\lambda_{0,t}$, $\lambda_{i,t}$, for $i = 1, \ldots, I$, ζ_t, and ψ_t be the corresponding Lagrange multipliers on the period t constraints. Using the first-order conditions for C_t and K_{t+1}, it is easy to verify that agents optimally save a stock-invariant fraction of income. For each dirty energy sector i, the first-order condition with respect to $E_{i,t}$ implies

$$\frac{\partial F_{0t}}{\partial E_{i,t}} = \frac{\lambda_{i,t}}{\lambda_{0,t}} + \frac{\tilde{\Lambda}^s_{i,t}}{\lambda_{0,t}}. \tag{15}$$

The left-hand side gives the marginal value of an extra unit of energy from sector i denominated in units of the final good. The right-hand side gives the marginal cost of producing an extra unit of energy in sector i plus the associated marginal externality cost, all denominated in units of the final good. $\tilde{\Lambda}^s_{i,t}$ is the marginal externality cost denominated in period t utility units, while

$$\Lambda^s_{i,t} = \frac{\tilde{\Lambda}^s_{i,t}}{\lambda_{0,t}}$$

is the marginal externality cost in units of the period t consumption good. An expression for $\Lambda^s_{i,t}$ is given in the following proposition.[17]

PROPOSITION 1: *In the infinite horizon limit, the marginal externality cost associated with an extra unit of carbon emissions in t, as viewed by an agent in t who*

[17]It remains to show that $E_{i,t}$ is stock invariant. This is shown in the appendix. The remaining first-order conditions from $\mathbf{PP(t)}$ are employed in subsequent analysis.

takes the optimal decision rules of subsequent generations as outside its control, is[18]

$$\Lambda^s_{i,t} = \frac{\mathbb{E}_t\left[\sum_{\tau=t}^{\infty} R^{(t)}_{\tau-t}\sum_{j=0}^{\tau-t}\alpha^{\tau-t-j}\gamma_{t+j}(1-d_j)\right]}{\sum_{j=0}^{\infty}\alpha^j R^{(t)}_j/Y_t}, \quad \text{for } i=1,\ldots,I_g-1. \tag{16}$$

It is the same for all fossil energy firms and it is zero for all clean energy firms.

The numerator gives the marginal externality cost in utility units, while the denominator gives the shadow value of an extra unit of consumption in t. The quotient translates the marginal externality cost into period t consumption units.

The expression in the numerator reflects two mechanisms at work in the model. Emissions affect future utility directly by raising the pollution stock in subsequent periods; this lowers net output and thus consumption. In addition, lower net output from climate damages lowers the base from which subsequent savings are drawn. This has a cumulative effect on capital accumulation that lowers consumption in later periods.[19]

In principle, two other mechanisms are possible: emissions could affect the emission decisions of later agents, and they could affect the savings rule of later agents. The former channel does not arise because flow payoffs are linear in prior emissions. The second channel does not arise because the dependence on future output cancels as future damages are multiplied by future marginal utility when evaluating policy.

[18] A simpler expression for the marginal externality cost is given in proposition 3.

[19] $(1-d_j)\gamma_{t+j}$ is the utility cost of damages in $t+j$ attributable to emissions in t (damages times marginal utility), while $\alpha^k(1-d_j)\gamma_{t+j}$ is the period $t+j+k$ utility cost associated with the lower capital stock in $t+j+k$ that results from the portion of climate damages in $t+j$ that are attributable to emissions in t. Summing all such effects or each future τ gives the expression in the numerator of 16

C. Conditions for a Unique Equilibrium

In the existing climate policy literature with non-constant time preference (Karp 2005, Karp 2007, Fujii and Karp 2008, Gerlagh and Liski 2012) multiplicity is a significant concern. Multiple equilibria arise generically when solving for symmetric Markov Perfect Equilibria (MPE) in stationary models because there are not enough equations to determine the derivative of the equilibrium response function in the steady state; this derivative is itself an unknown when solving for the MPE (Tsutsui and Mino 1990).

For the problem considered above, the subgame perfect equilibrium is unique—as seen by construction. Uniqueness is pinned down, in part, by the assumption that the time horizon is finite, though a unique equilibrium remains in the infinite horizon limit of a sequence of such finite-horizon equilibria. The latter equilibrium concept has been referred to as the Strotz-Pollak equilibrium (Peleg and Yaari 1973). It was employed in the original literature on non-constant time preference (Strotz 1955, Pollak 1968, Phelps and Pollak 1968). In our example, the limiting Strotz-Pollak equilibrium is a symmetric MPE but a continuum of additional symmetric MPE exist for the stationary, infinite horizon model. It follows that a great deal hinges on the choice between a finite horizon model and an infinite horizon model.

Because an arbitrarily long finite horizon includes one that surpasses the time at which physicists expect the sun to burn out, it is hard to argue on *a priori* grounds that the finite horizon game is a less appropriate description of the relevant policy problem.[20] A more compelling argument for multiplicity is that the additional equilibria can alternatively be motivated as ϵ-equilibria to a finite horizon game

[20]Note, moreover, that the difference between a finite horizon game and an infinite horizon one hinges on the specification of preferences and the specification of the climate-economy system that lie beyond the designated horizon. If this is sufficiently long, the difference between the two specifications hinges on assumptions that are completely arbitrary.

(Fudenberg and Levine 1983).[21] But, if one needs to consider perturbed games in order to generate multiplicity, then it is natural to regard the equilibrium to the unperturbed game, when it is unique, as the obvious focal point for discussion.

Uniqueness of the finite horizon equilibrium does not obtain in general models with non-constant time preference and should be viewed as a special feature of the current model—a counterexample is presented in Peleg and Yaari (1973) and Goldman (1980). Another instance in the literature in which the backward induction solution is shown to be unique is the linear-quadratic climate policy game considered at the end of Karp (2005).

An alternative motivation for a unique equilibrium can also be made. For the subgame perfect equilibrium in our model to be unique, it is enough to ensure that the last generation saves a stock invariant fraction of income and chooses an emission level that is independent of the inherited stock variables. If the problem is posed as an infinite horizon game, but technology evolves exogenously in such a way that emissions eventually go to zero—eventually a clean backstop technology is invented that is cheaper than remaining coal deposits—then a unique equilibrium will obtain. In effect, a non-stationarity in the model forces the policy response to a boundary of the feasible set within finite time; once this happens, the emission decision of subsequent generations is independent of the inherited state variables for all subsequent periods.

II. Decentralized Equilibrium

This section describes the corresponding private ownership economy. We show that the optimal allocation can be decentralized in a competitive equilibrium using a carbon tax on fossil energy firms.

[21] This argument is noted in Karp (2013).

A. Households

Households are atomistic: they take prices as given and ignore the external effects of their actions. Households earn income renting the factors of production, collecting firm profits,[22] and collecting government transfers. Without commitment, they choose consumption and savings for the current period only, while correctly anticipating the decisions of future generations. The period t household problem is

$$\max_{C_t, K_{t+1}} \mathbb{E}_t \sum_{\tau=t}^{T} R^{(t)}_{\tau-t} \ln(C_\tau),$$

s.t.

$$\mathbb{E}_t \sum_{\tau=t}^{T} q_\tau (C_\tau + K_{\tau+1}) = \mathbb{E}_t \sum_{\tau=t}^{T} q_\tau (r_\tau K_\tau + w_\tau N_\tau + T_\tau). \tag{17}$$

Here, T_τ is a government transfer, and the $\{q_\tau\}$ denote state-contingent Arrow-Debreu prices.

B. Firms

Firms are also atomistic. Final-goods firms solve the following profit maximization problem at each date t:

$$\max_{K_t, N_{0,t}, \mathbf{E}_t} K_t^\alpha A_t(\mathbf{E}_t, N_{0,t}) \omega(S_t) - r_t K_t - w_t N_{0,t} - \sum_{i=1}^{I} p_{i,t} E_{i,t}.$$

Here, r_t is the rental rate of capital, w_t is the wage rate, and $p_{i,t}$ is the price per unit of output from energy sector i. All prices are denominated in units of the final output good.

[22]Though profits for all firms are zero in equilibrium.

In addition, firms in energy sector i solve[23]

$$\max_{N_{it}} (p_{i,t} - \tau_{i,t}) F_{i,t}(N_{i,t}) - w_t N_{i,t}.$$

$\tau_{i,t}$ is a sector-specific tax. Energy units are chosen so a unit of energy for fossil sectors equals a unit of emissions. The tax is therefore applied to the carbon content of emissions. The efficiency of energy production per unit of emissions is embedded in the production function for each sector.

C. Optimal Carbon Taxes

For a given set of taxes $\{\tau_{i,t}\}_{i=1}^{I}$, the decentralized allocation can be characterized by combining the optimality conditions from the firm and household problems with the market clearing conditions.[24] Comparing this with the optimal allocation, the solution to the planning problem without commitment, we show that the optimal allocation can be decentralized with an appropriate carbon tax. For the remainder of the paper, proposition proofs are in the appendix.

PROPOSITION 2: *The subgame perfect allocation from the planner's problem without commitment can be decentralized with a carbon tax on fossil-energy firms. The optimal tax on fossil energy firms ($i = 1, \ldots, I_g - 1$) is*

$$\tau_{i,t} = \Lambda_{i,t}^{s}. \tag{18}$$

The optimal tax on clean-energy firms ($i = I_g, \ldots, I$) is zero.

[23]The profit maximization problem for energy firms is static under the assumption that resource stocks are infinite. With finite resource stocks, the profit problem would be dynamic (see Golosov et al. 2012 for details).

[24]The definition of a competitive equilibrium is standard, and we omit it.

The formula for $\Lambda^s_{i,t}$ in (16) shows that the optimal carbon tax as a fraction of output is a constant that depends only on the path of time preference rates, the expected value of the future damage elasticities, the carbon cycle parameters, and the Cobb Douglass coefficient on capital in final-goods production. Notably, the formula does not depend on the endogenous paths for carbon stocks, output, or consumption.[25] The formula also demonstrates a form of certainty equivalence that is noted also in Golosov et al. (2013). In particular, the only feature of uncertainty about future damages that affects the current decision is the expected value of the future elasticity parameter conditional on the information set today. It follows that fat-tailed damages affect the optimal carbon tax in this model only so far as they affect the expected value of future realizations of the damage parameter.

An alternative formula

To compare with the optimal tax under constant discounting—as in Golosov et al. (2013)—it is useful to write the expression in a different way. With non-constant discounting, the cumulative discount factor between future periods depends on how far out the periods occur. To account for this, define $R^{(t)}_{l,m}$ as the price at $t + l$ of a unit of utility received at $t + m$, viewed by an agent in t. For $m \geq l$,

$$R^{(t)}_{l,m} = \begin{cases} \prod_{j=l+1}^{m} \beta^{(t)}_j, & m > l \\ 1\ , & m = l \end{cases} . \tag{19}$$

To motivate the alternative expression, recall the explanation for $\tilde{\Lambda}^s_t$ that followed proposition 1. There, the formula was said to arise from two effects: the direct effect

[25] The same features hold for the optimal tax formula in Golosov et al. (2013) with the exception that their formula does not depend on the Cobb Douglass coefficient on capital. The source of this difference is explained in the next subsection.

of climate damages on future output, and the repercussions of each such damage event on subsequent capital accumulation. The formula in (16) adds up the effect of these two mechanisms by looking at the combined effect of all prior damage events on equilibrium utility in each future τ, but we could just as easily arrange terms by multiplying each direct damage event by a subsequent repercussion term. This gives[26]

$$\tilde{\Lambda}_t^s = \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} R_{\tau-t}^{(t)} \gamma_\tau (1 - d_{\tau-t}) \cdot \sum_{m=0}^{\infty} R_{\tau-t,\tau-t+m}^{(t)} \alpha^m \right]. \tag{20}$$

The leading "1" in the second sum (when $m = 0$) captures the direct effect; the remaining terms capture the (infinite horizon) repercussions of this initial damage event for future capital accumulation. Note that

$$R_{\tau-t}^{(t)} \cdot R_{\tau-t,\tau-t+m}^{(t)} = R_{\tau+m-t}^{(t)},$$

so all outcomes are discounted in a way that is consistent with the preference structure of an agent in t.

Combining (20) with the expression for the shadow value of capital at date t gives the following expression for the optimal tax.

PROPOSITION 3: *The optimal tax in fossil sector i can be written*

$$\tau_{i,t} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} R_k^{(t)} \gamma_{t+k} (1 - d_k) \cdot \Gamma_k^{(t)} \right] Y_t, \tag{21}$$

[26] A rigorous proof is in the appendix.

where

$$\Gamma_k^{(t)} = \frac{\sum_{m=0}^{\infty} \alpha^m R_{k,k+m}^{(t)}}{\sum_{n=0}^{\infty} \alpha^n R_n^{(t)}}.$$

Under constant discounting, the numerator and denominator in $\Gamma_k^{(t)}$ are equal, so the term drops out of the expression. The optimal tax then reduces to the formula in Golosov et al. (2013). The numerator and denominator of $\Gamma_k^{(t)}$ value the opportunity cost of a unit of the consumption good received in different periods. The numerator gives the discounted payoff to holding an extra unit of capital starting in period $t+k$, while the denominator gives the discounted value of holding an extra unit of capital starting in period t. With constant discounting, these values are the same, but they differ with non-constant discounting. This explains why α drops out of the tax formula with constant discounting, but not when discounting is non-constant. When time preference rates decline, the $\Gamma_k^{(t)}$ terms are strictly greater than one[27]. This makes the optimal tax bigger than if these terms were ignored.

The formula reduces to the optimal tax in Gerlagh and Liski (2012) when discounting is quasi-hyperbolic.[28] Because Gerlagh and Liski (2012) solve for the linear symmetric MPE, it follows that the linear MPE in this model is also the (unique) Strotz-Pollak equilibrium. This is interesting because there is not a compelling reason for choosing the linear MPE from the set of MPE. In contrast, the Strotz-Pollak equilibrium is the unique MPE that can be viewed as the equilibrium of an arbitrarily long finite horizon game.

[27]This can be seen by comparing terms in the numerator and denominator.

[28]This is true provided the damage elasticity parameter is fixed at the period t expected value, and the carbon cycle coefficients are set to coincide.

III. A Case of Time Inconsistency "Irrelevance"

Time consistency concerns arise under non-constant time preference when the problem is solved in a way that incorrectly assumes a commitment device. Without commitment, the decisions of future decision-makers in general differ from what earlier generations would have chosen if commitment were possible. Thus, moving from an equilibrium with commitment to one without typically changes the effective marginal payoffs associated with actions in the initial period, and so also the optimal initial period action. But not in the current model.

PROPOSITION 4: *Starting from a given period t with the same initial conditions, the optimal carbon tax in t is the same when the planner's problem assumes a commitment device for the period t planner as it is in the equilibrium of the planner's problem without commitment.*

The intuition for the result can be seen in equation (10). This shows that the equilibrium flow payoff is linear in prior emissions. As a result, future generations emitting more (or less) does not change the optimal emission decision of earlier generations. Because of this, the change in future emission decisions that occurs when moving from the problem with commitment to the equilibrium without commitment does not affect the optimal emission level.

The result suggests a sense in which time consistency concerns are "irrelevant" when applying the optimal carbon tax in any given period. Provided the possibility of commitment is off the table, the optimal tax can be applied without having to worry about the future trajectory of the system. Thus, whether a commitment device is assumed or not, the optimal tax today is unchanged. Moreover, without commitment, the current carbon price summarizes everything current decision-makers need to know to evaluate current climate investments.[29] The proposition does not imply that the full dynamic path of carbon taxes in the two equilibria are the same. Indeed, the application shows that they typically differ in subsequent periods.

The result can be strengthened. In addressing time consistency concerns, the most straightforward solution is to look for a subgame perfect equilibrium in which decision makers in different periods are identical. But as hard as it is for the current generation to "know" its own time preference structure, they presumably know significantly less about the preference structure of subsequent generations (Beltratti, Chichilnisky and Heal 1998). Given this, one could be led to entertain a wide range of possible assumptions about the path of time preference rates likely to be applied by future generations. In a typical model, this would make the initial carbon price highly sensitive to these assumptions. But for the model here, the optimal tax is unchanged no matter what path of time preference rates we assume for future generations.

PROPOSITION 5: *Starting from a given period t, the optimal period t carbon tax, derived as the subgame perfect equilibrium to the planner's problem without commitment, is unchanged no matter what one assumes for the path of time preference rates to be applied by subsequent generations.*

A convenient implication arises when applying the preference aggregation argument for DRTP. A criticism of most applications is that they require the accompanying assumption that the current distribution of time preferences is frozen in time. Thus, there is no room for the distribution of views in society to evolve. This is implausible over a long horizon. But the expectation that the distribution of time preferences will change over time merely implies that the path of time preference rates to be

[29]It clearly provides enough information to evaluate projects to be implemented within the period; meanwhile, plans that require coordination across multiple periods are infeasible without commitment.

used by subsequent decision-makers will change, but due to proposition 4 this does not change the optimal tax today.

The strong features of the model arise because the equilibrium flow payoff is linear in prior emissions. This is naturally a product of the assumed functional forms. One view, discussed in Karp (2013), is that this feature should be viewed as a weakness since it rules out meaningful strategic interactions across generations. But a model in which strategic interactions across generations are important suffers from the alternative problem that the current carbon tax will be highly sensitive to the maintained time preference assumptions of future generations. Since these assumptions are largely arbitrary—especially for agents ten or twenty generations removed—one ends up trading one form of arbitrariness for another. For out purpose, it is at least comforting that the maintained functional form assumptions are close to those in most of the prior literature on climate policy (Golosov et al. 2013).

IV. "Imputed" Pigouvian Taxes

The Pigouvian prescription taxes externalities at the full social cost of damages. To appropriately capture intertemporal trade-offs in the economy, the relevant discount rates are typically estimated using data from capital markets. But as Gerlagh and Liski (2012) demonstrate, when time preference rates are non-constant, equilibrium returns in financial markets do not reflect actual trade-offs over long horizons. To quantify the effect the effect of this distortion on computed carbon prices, they construct an imputed Pigouvian tax that uses data on savings behavior in the model equilibrium to infer the relevant trade-offs and thus the implied marginal externality cost.

We do the same thing, showing for a model with arbitrary non-constant time preference—as Gerlagh and Liski (2012) show with quasi-hyperbolic rates—that the optimal carbon tax exceeds the imputed Pigouvian tax in the equilibrium without commitment provided the delay between emissions and damages is sufficiently long. Gerlagh and Liski (2012) emphasize the key role of the delay between emissions and damages in generating a wedge between the optimal and imputed Pigouvian taxes. This wedge reflects the "commitment value" of climate policy, analogous to the value of commitment devices in self-control problems. The model with arbitrary DRTP makes it possible to see a further cause: mainly, the faster the rate of decline of time preference rates, the bigger the wedge.[30]

We also compute the imputed Pigouvian tax for the equilibrium with commitment. Because the path of endogenous variables differ over time under the two equilibria, the imputed Pigouvian tax in the initial period also differ across equilibria. For the optimal solution with commitment, the optimal and Pigouvian taxes are equal. It follows that the optimal initial period tax—for both equilibria—can be interpreted as the marginal externality cost with future damages discounted in a way that is consistent equilibrium savings behavior for a model in which all future savings decisions are chosen to maximize welfare for agents in the initial period.

To compute the relevant subjective discount factor for use in computing the imputed Pigouvian tax, we follow Gerlagh and Liski (2012) in noting that an agent who uses capital markets to transfer wealth across periods would discount future outcomes using the utility discount factor, ϕ_t, from the following consumption Euler equation:

$$u'(C_\tau) = \phi_t \mathbb{E}_\tau u'(C_{\tau+1}) r_\tau.$$

Here, r_t is the gross interest rate in the decentralized equilibrium—or the real return on capital. For the case with log utility and Cobb Douglass final-goods production,

[30]This is shown in the quantitative section below.

this implies

$$\phi_\tau = \frac{1}{\alpha} \cdot \mathbb{E}_\tau \left[\frac{C_{\tau+1}}{Y_{\tau+1}} \cdot \frac{K_{\tau+1}}{C_\tau} \right]. \tag{22}$$

This equation is combined with the equilibrium conditions from **PP(t)** to determine the subjective discount factor ϕ_τ for each future period τ. This is done separately for each equilibrium. The following results can then be shown.

PROPOSITION 6: *In the equilibrium without commitment, the optimal tax in the initial period is greater than the imputed Pigouvian tax provided the delay between emissions and damages is sufficiently long. With no delay, the imputed tax is bigger.*

In the application, we find that the optimal tax is always bigger than the imputed Pigouvian tax for the equilibrium without commitment. We also show that the wedge between the optimal and Pigouvian taxes increases with the rate of decline of the path of time preference rates.

PROPOSITION 7: *In the equilibrium with commitment, the optimal tax in the initial period equals the imputed Pigouvian tax.*

With commitment, the initial-period decision maker sets consumption–savings decisions to ensure the marginal rate of substitution between adjacent periods, viewed from the initial period, equals the marginal rate of transformation in equilibrium. Since this is the same subjective discount factor that is applied when solving the full commitment equilibrium, it follows that the period one Pigouvian tax and the period one optimal carbon tax in the equilibrium with commitment are the same.

V. Application

A. Calibration of Declining Time Preference Rates

Our calibration of time preference employs the preference aggregation argument of Li and Löfgren (2000). As noted, the GHKT model is a highly convenient setting in which to consider this approach, since one does not need to assume that the current distribution of views is static. Our calibration builds on the discounting debate between Nordhaus (2008) and Stern (2007). Nordhaus (2008) argues that time preference and the elasticity of marginal utility should be jointly calibrated to ensure that the savings decisions of households in the model are consistent with the real return on capital—about 5.5% in Nordhaus (2008). Under log utility, this implies a time preference rate of 3.0% in DICE. Stern (2007) argues that the intergenerational distributional consequences of this assumption are too extreme when applied to climate change. He then argues on ethical grounds that the time preference rate under log utility should be 0.1%. Our goal is not to resolve this debate. Rather, we consider a variety of scenarios for the relative weight that a global decision-maker might assign to the competing approaches.

Figure 1 compares the implied discounting paths for a representative household when respectively 1%, 5%, 10% and 20% of the relevant population maintains the Stern time preference rate, while the remaining portion agrees with Nordhaus. We do not offer a deep reason for viewing the Stern rate as less probable then the Nordhaus rate, though we suspect that a review of general-interest economics journals would reveal a strong preference for the Nordhaus view. More importantly, our scenarios show that the effect of putting even a small weight on those members of society who maintain the lower rate is large.

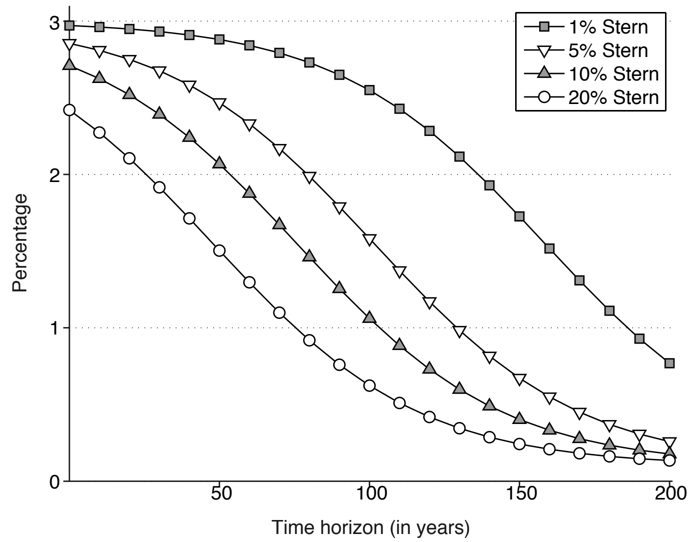

FIGURE 1. COMPARISON OF ALTERNATIVE HYPERBOLIC DISCOUNTING PATHS.

B. *Calibration of the Carbon Cycle and Damages*

Our calibration of the carbon cycle follows Golosov et al. (2013) with a small adjustment. Those authors calibrate the decay structure of atmospheric carbon dioxide to be consistent with recent evidence that the geometric decay structure in most climate policy models is incorrect. The revised scientific understanding of atmospheric carbon decay is described in the following quote from the IPCC (IPCC 2007): "About half of a CO2 pulse to the atmosphere is removed over a timescale of 30 years; a further 30% is removed within a few centuries; and the remaining 20% will typically stay in the atmosphere for many thousands of years". To replicate this, Golosov et al. (2013) assume that fraction ϕ_0 of emissions fall out of the atmosphere immediately. A further fraction ϕ_L remain forever. And the remaining carbon decays at a constant geometric rate. This implies the following formula for decay:

$$1 - d_s = \phi_L + (1 - \phi_L)\phi_0(1 - \phi)^s.$$

They calibrate this model by assuming $\phi_L = 0.2$, $\phi_0 = 0.393$, and $\phi = 0.0228$.

We adopt the same parameter values, but we alter the assumption that some fraction of the carbon stock remains in the atmosphere literally "forever". The assumption is innocuous when the discount rate is moderately high, since nothing beyond a couple hundred years matters in determining the carbon price. But the assumption becomes important when the subjective discount rate is very low (as in Stern 2007) or if it declines over time to near zero. Indeed, if the subjective discount rate declines to zero in finite time and a portion of emissions remains forever, the carbon price implied by our formula is infinite. To avoid this possibility, we take the IPCC description literally and assume instead that fraction ϕ_0 of emissions remains in the atmosphere for 2000 years.

Our calibration of the damage function follows Golosov et al. (2013). They assume that the expected value of the future damage elasticity parameter, conditional on information today, is the same for all future periods, and they calibrate this to match two data points from a meta analysis of damages in Nordhaus (2000). The first calibration point estimates that a 2.5 degree Celsius increase in mean global temperature would lead to a 0.48% loss of GWP. In addition, they allow for a 6.8% chance that damages from a 6 degree rise in temperatures would be catastrophic, leading to a 30% loss of GWP. These considerations imply an expected damage elasticity of 2.379×10^{-5}.

C. Optimal Taxes

In table 1, the optimal carbon tax and the imputed Pigouvian tax are shown for the discounting scenarios in figure 1. All values are computed for the equilibrium without commitment. Three characteristics stand out. First, even a small weight on the Stern model increases the optimal tax a lot. For example, a 20% weight on the Stern model increases the optimal tax 4-fold relative to what it is under the Nordhaus calibration. Second, the optimal tax and the Pigouvian tax are the same when discounting is constant. This is expected since the preferences of agents in different periods align. Third, the imputed Pigouvian tax is roughly constant across scenarios. Because of this, the wedge between the optimal tax and the Pigouvian tax increases with the rate of decline in the path of discount rates.

Table 1—Dollars per ton carbon. Hyperbolic paths from figure 1. Nordhaus (2008) assumes time preference rate of 3.0%; Stern (2007) assumes 0.1%.

	Pigouvian tax	Optimal tax
Nordhaus (2008)	31	31
1% Stern	32	36
5% Stern	33	54
10% Stern	35	77
20% Stern	39	122
Stern (2007)	450	450

The latter observation reflects the fact that without commitment, the equilibrium interest rate is determined predominantly by the subjective discount rates of agents in the short run. Because the Nordhaus (2008) calibration—zero weight on Stern—is calibrated to be consistent with historical returns in financial markets, it follows that savings in the model remains consistent with observed returns for the range of time preference rate paths considered in the table, even as the optimal carbon tax increases 8-fold over the range. Because the effects of climate change play out with long delay, climate policy provides a mechanism in the model for committing resources to future generations in a way that sidesteps the intermediate handling of subsequent generations who are more impatient then the initial generation would have wanted.

D. Equilibrium Trajectories

The optimal tax in the initial period can be computed without specifying functional forms for the energy composite or the energy sector production functions. But additional assumptions are needed to solve for the full dynamic path of the model. With a couple modifications indicated below, we draw the needed assumptions from Golosov et al. (2013).

First, output in each energy sector i is linear in labor:

$$E_{i,t} = A_{i,t} N_{i,t}.$$

Golosov et al. (2013) have three energy sectors: oil, coal, and wind. "Oil" in their model is viewed as a composite between oil and natural gas, and it differs from coal in that the resource stock is finite, while it is infinite for coal. In our model, the resource constraints are assumed to be non-binding in all fossil sectors. We therefore assume a single composite fossil energy sector that we call "coal". The second sector, "wind", is a composite of carbon-free energy sources, including solar, biomass, hydro, and nuclear.

Energy outputs are aggregated into a composite input to final production as follows[31]:

$$E_t = (\kappa_1 E_{1,t}^{\rho} + \kappa_2 E_{2,t}^{\rho})^{1/\rho}. \tag{23}$$

[31] Sector 1 is coal, and sector 2 is wind.

Finally, net output in the final-goods sector is given by

$$Y_t = e^{-\gamma_t(S_t-\bar{S})}A_{0,t}K_t^{\alpha}N_{0,t}^{1-\alpha-v}E_t^{\nu}.$$

The calibrated parameters are summarized in table 2.[32] Our calibration of the energy composite differs from Golosov et al. (2013) only because the number of energy sectors differs. We make the same assumption on the relative price between coal and wind, together with the assumption that $\kappa_1+\kappa_2=1$. This implies $\kappa_1=.2$ and $\kappa_2=.8$. In contrast, Golosov et al. (2013) have $\kappa_1=0.54$ (oil), $\kappa_2=0.10$ (coal), and $\kappa_3=0.36$ (wind). In the appendix, we repeat the analysis with $\kappa_1=\kappa_2=0.5$. This is consistent with a fossil sector more closely in line with oil. Carbon taxes are somewhat lower, but the paths are qualitatively similar. The damage elasticity is calibrated as before. We assume that current uncertainty persists over the time horizon of the simulation. The path of discount rates is fixed at the "10% Stern" scenario, and we assume that generations are identical.

Table 2—Calibration summary

ϕ	ϕ_L	ϕ_0	α	ν	κ_1	κ_2	ρ
0.0228	0.2	0.393	0.3	0.04	0.2	0.8	-0.058
$10^4\gamma_H$	$10^4\gamma_L$	p	$S_0(S_{1,0})$	K_0	$A_{1,0}$	$A_{2,0}$	$\frac{A_{1,t+1}}{A_{1,t}}=\frac{A_{2,t+1}}{A_{2,t}}$
2.046	0.106	0.068	802(684)	128,920	7,693	1,311	1.02^{10}

To simulate the equilibrium path without commitment, we derive the first-order conditions from problem $\mathbf{PP(t)}$. For each period t, there are nine first-order condi-

[32]The capital stock, K_0, is denominated in billions of dollars US. The calibration assumes that the initial capital stock is below the steady state capital stock. As in Barrage (2013), the initial level of TFP is set equal to 17,887 to ensure the endogenous energy choice replicates current GWP. TFP is assumed to be constant in the baseline calibration. See Golosov et al. (2013) and Barrage (2013) for further details.

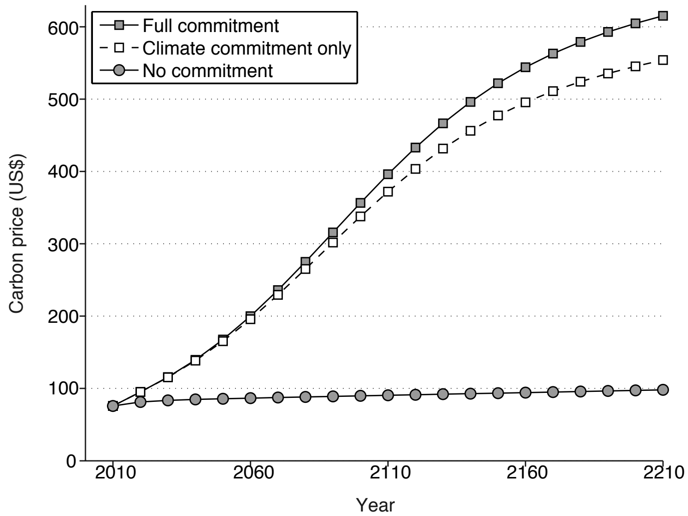

Figure 2. Carbon tax trajectories for alternative equilibria.

tions plus the original constraints. Simulating the model turns out to be relatively easy because the savings decision and the marginal externality cost do not depend on the response behavior of subsequent generations. As a result, the corresponding system of equations can be solved "forwards" even though the equilibrium conditions are derived backwards. This greatly simplifies the analysis relative to typical models with non-constant time preferences. The relevant equations are presented in the appendix.

The same steps can be used to simulate the equilibrium with commitment provided the cumulative discount factors in the period t planner's problem are suitably changed. In particular, replacing $\{R^{(t)}_{\tau-t}\}$ with $\{R^{(0)}_{t,\tau}\}$ imposes the initial generation's time preferences on each subsequent generation and thus solves for the equilibrium with commitment.

Figure 2 compares the time path of optimal taxes across scenarios. As expected, the optimal tax in the initial period is the same in the equilibria with and without commitment. But the figure also shows that a commitment device, were it feasible, would have enormous value for the initial generation. Without commitment, the path of carbon prices is roughly constant over time. This follows because the optimal tax is a constant fraction of GWP, and GWP is roughly constant under the calibration assumption of zero TFP growth.[33] In contrast, in the equilibrium with commitment, the optimal tax increases sharply. This is intuitive since the initial generation imposes lower and lower discount rates on later generations.

In addition, savings rates rise over time for the same reason in the equilibrium with "full commitment". Because our primary interest is in the effect of a commitment device for climate policy, it is useful to solve also for a third equilibrium in which the initial planner has a commitment device for climate policy only. Meanwhile, consumption-savings decisions are made each period by the contemporaneous generation—thus, in the same way as in the equilibrium without commitment. The results show that the effect of this further capital accumulation channel is small relative to the direct effect that lower discount rates have on the optimal tax.

VI. Conclusion

The paper solves for the optimal climate policy under non-constant time preference using a version of the integrated assessment model suggested in Golosov et al. (2013). The Golosov et al. (2013) model is modified by assuming that resource constraints are non-binding in all fossil energy sectors. The paper shows that the model provides a highly tractable and convenient setting in which to incorporate non-constant time preference rates. The full dynamic path of the subgame perfect equilibrium to the planner's problem without commitment and the corresponding

[33]Economic growth does arise in the model due to capital accumulation. This happens because the initial capital stock is assumed to start below the steady state level. Nevertheless, higher gross GWP is partly offset by rising climate damages over time.

decentralized equilibrium can be simulated in a straightforward way, and an intuitive formula can be derived for the optimal carbon tax. In addition, concerns about time inconsistency, concerns about multiplicity, and concerns about the sensitivity of results to assumptions about future time preferences—such as arise when applying the preference aggregation argument for declining rates—all fall away in a simple way.

These convenient features of the model come at a cost. While the assumed functional forms are close to those in most of the prior climate economics literature, they still comprise a subset of the plausible parameter space. When interpreting the results, it would be useful to understand how the results change as the functional form assumptions are relaxed. In particular, we would like to understand how the (time consistent) path of optimal taxes and the relationship between taxes across equilibrium scenarios (with and without commitment) change with the core assumptions—mainly, the elasticity of marginal utility, the depreciation rate, the final-goods production function, and the damage function. To accomplish this broader sensitivity analysis, it would be possible to proceed numerically using the algorithm suggested in the original version of this paper[34] (Iverson 2012). While feasible, this will be a nontrivial undertaking because the associated dynamic problem has four state variables[35] (three without uncertainty). We leave it for future research.

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[34]The algorithm there modifies the standard finite horizon dynamic programming algorithm to ensure that the continuation value functions appropriately account for the distinct time preferences of hyperbolic agents at different dates. It can be used to solve for the backward induction equilibrium in a finite horizon model with non-constant time preference.

[35]K, S_1, S_2, and γ.

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Mathematical Appendix For Online Publication

A1. Proposition 1: Proof that $E_{i,t}$ is stock invariant

For each dirty sector i, the first-order conditions for $E_{i,t}$ can be combined with the first-order conditions for $N_{i,t}$ to give

$$\begin{aligned}\frac{\partial F_{0t}}{\partial E_{i,t}} &= \frac{\lambda_{i,t}}{\lambda_{0,t}} + \frac{\Lambda_t^s}{\lambda_{0,t}} \\ &= \frac{\partial F_{0t}/\partial N_{0t}}{dF_{it}/dN_{it}} + \frac{\Lambda_t^s}{\lambda_{0,t}} \\ &= \frac{\partial F_{0t}/\partial N_{0t}}{dF_{it}/dN_{it}} + (1-s_t)\Lambda_t^s Y_t.\end{aligned}$$

Dividing through by Y_t and making use of the assumed form of the final goods production function gives

$$\frac{\partial A_t(N_{0t},\mathbf{E}_t)/\partial E_{i,t}}{A_t(N_{0t},E_t)} = \frac{\partial A_t(N_{0t},\mathbf{E}_t)/\partial N_{0t}}{A_t(N_{0t},E_t)} \cdot \frac{1}{dF_{it}(N_{it})/dN_{it}} + (1-s_t)\Lambda_t^s.$$

This is a stock-invariant equation in N_{0t}, N_{it}, and $\mathbf{E}_t$. We have one such equation for each dirty energy sector. We also have an analogous stock invariant equation for each clean energy sector (these just drop the externality term). These equations can be combined with

$$E_{it} = F_{i,t}(N_{i,t}), \quad \text{all } i$$

and

$$\sum_{j=0}^{I} N_{j,t} = N_t$$

to give $2I+1$ stock invariant equations in $2I+1$ variables. It follows that $E_{i,t}$ is stock invariant for each i.

A2. Proof of proposition 2

From the planner's problem, the first-order condition for $E_{i,t}$ implies

$$\frac{\partial F_{0t}}{\partial E_{i,t}} = \frac{\lambda_{i,t}}{\lambda_{0,t}} + \Lambda^s_{i,t}. \tag{A1}$$

$\Lambda^s_{i,t}$ is given by (16) for dirty energy firms, and it is zero for clean energy firms. In addition, the first-order conditions for labor imply

$$\frac{\lambda_{i,t}}{\lambda_{0,t}} = \frac{\partial F_{0t}/\partial N_{0,t}}{\partial F_{it}/\partial N_{i,t}}.$$

So,

$$\frac{\Lambda^s_t}{\lambda_{0,t}} = \frac{\partial F_{0t}}{\partial E_{i,t}} - \frac{\partial F_{0t}/\partial N_{0,t}}{\partial F_{it}/\partial N_{i,t}}. \tag{A2}$$

From the decentralized equilibrium, we have

$$\frac{\partial F_{0t}}{\partial N_{0,t}} = w_t = (p_{i,t} - \tau_{i,t})\frac{\partial F_{it}}{\partial N_{i,t}}$$

and

$$p_{i,t} = \frac{\partial F_{0t}}{\partial E_{i,t}}.$$

This implies

$$\tau_{i,t} = \frac{\partial F_{0t}}{\partial E_{i,t}} - \frac{\partial F_{0t}/\partial N_{0,t}}{\partial F_{it}/\partial N_{i,t}}.$$

It follows that the outcome in the decentralized competitive market allocation will equal the optimal allocation provided:

$$\tau_{i,t} = \Lambda^s_{i,t}, \quad i = 1, \ldots, I_g - 1.$$

It remains to show that the consumption-savings decisions in the two problems are the same.

The period t household problem is

$$\max_{C_t, K_{t+1}} \mathbb{E}_t \sum_{\tau=t}^{T} R^{(t)}_{\tau-t} \ln(C_\tau),$$

s.t.

$$C_\tau + K_{\tau+1} = (1 + r_\tau)K_\tau + w_\tau N_\tau + \Pi_\tau, \text{ for } \tau = t, \ldots, T. \tag{A3}$$

The last generation trivially saves a constant fraction of income. Suppose each remaining generation τ saves fraction $\hat{s}_\tau$ of income. Then the time t generation solves

$$\max_{K_{t+1}} \left\{ \ln[(1+r_t)K_t + w_t N_t + \Pi_t - K_{t+1}] + \mathbb{E}_t \sum_{\tau=t+1}^{T} R(\tau-t) \ln[(1-\hat{s}_\tau)((1+r_\tau)K_\tau + w_\tau N_\tau + \Pi_\tau)] \right\}$$

s.t.

$$K_{\tau+1} = \hat{s}_\tau[(1 + r_\tau)K_\tau + w_\tau N_\tau + \Pi_\tau] \quad \tau = t + 1, \ldots, T$$

The constraint is a linear difference equation in K_τ. Iterating on this, we can write

$$K_\tau = K_{t+1} \prod_{j=t+1}^{\tau-1} \hat{s}_j(1 + r_j) + \tilde{b}_\tau.$$

Throughout, let

$$I_\tau = (1 + r_\tau)K_\tau + w_\tau N_\tau + \Pi_\tau$$

Substituting back into the problem, we have

$$\max_{K_{t+1}} \Bigg\{ \ln[(1 + r_t)K_t + w_t N_t + \Pi_t - K_{t+1}]$$
$$+ \sum_{\tau=t+1}^{T} R(\tau - t) \ln \left[(1 + r_\tau)[K_{t+1} \prod_{j=t+1}^{\tau-1} \hat{s}_j (1 + r_j) + \tilde{b}_\tau] + w_\tau N_\tau + \Pi_\tau) \right] + \dots \Bigg\}$$

FOC(K_{t+1}):

$$\frac{-1}{I_t - K_{t+1}} + \sum_{\tau=t+1}^{T} R(\tau - t) \frac{(1 + r_\tau) \prod_{j=t+1}^{\tau-1} \hat{s}_j (1 + r_j)}{I_\tau} = 0$$

First, note that

$$\begin{aligned} s_j F_j &= \frac{K_j F_j}{F} \frac{K_{j+1}}{K_j} \\ &= \alpha \frac{K_{j+1}}{K_j} \end{aligned}$$

It follows that, for each τ,

$$\begin{aligned} \frac{(1 + r_\tau) \prod_{j=t+1}^{\tau-1} \hat{s}_j (1 + r_j)}{I_\tau} &= \frac{F_K(\tau)}{Y_\tau} \prod_{j=t+1}^{\tau-1} \alpha \frac{K_{j+1}}{K_j} \\ &= \frac{F_K(\tau)}{Y_\tau} \alpha^{\tau-t-1} \frac{K_\tau}{K_{t+1}} \\ &= \alpha^{\tau-t} K_{t+1}^{-1} \end{aligned}$$

Substituting back in the FOC, while noting that $I_t = Y_t$, gives

$$\frac{1}{Y_t - K_{t+1}} = \frac{1}{K_{t+1}} \sum_{\tau=t+1}^{T} R(\tau - t)\alpha^{\tau-t}$$

$$K_{t+1} = \frac{\sum_{\tau=t+1}^{T} R(\tau - t)\alpha^{\tau-t}}{1 + \sum_{\tau=t+1}^{T} R(\tau - t)\alpha^{\tau-t}} Y_t$$

It follows that

$$\begin{aligned} 1 - s_t &= \frac{1}{1 + \sum_{\tau=t+1}^{T} R(\tau - t)\alpha^{\tau-t}} \\ &= \frac{1}{\sum_{j=0}^{T} \alpha^j R(j)}, \end{aligned}$$

which is identical to the savings rater obtained along the optimal allocation.

A3. Proof of proposition 3

Consider the following double sum identity:

$$\sum_{p=0}^{\infty} \sum_{q=0}^{p} a_{q,p-q} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{n,m}.$$

The identify can be proved by listing the terms in a two by two grid, where the index of the first sum comprise the rows and the index of the second sum comprise the columns. The left-hand side is obtained by summing the rows of the grid, while the right-hand side is obtained by summing the same set of terms diagonally.

When $t = 0$ and $T = \infty$,

$$\begin{aligned}
&\sum_{\tau=t+1}^{T} R(\tau-1) \sum_{j=1}^{\tau-t} \alpha^{\tau-t-j} \gamma_{t+j}(1-d_j) \\
&= \sum_{p=0}^{\infty} R(p) \sum_{j=1}^{p+1} \alpha^{p+1-j} \gamma_j (1-d_j) \\
&= \sum_{p=0}^{\infty} R(p) \sum_{q=0}^{p} \alpha^{p-q} \gamma_{1+q}(1-d_{1+q}) \\
&= \sum_{p=0}^{\infty} \sum_{q=0}^{p} R(p-q+q) \alpha^{p-q} \gamma_{1+q}(1-d_{1+q})
\end{aligned}$$

Letting $n = q$ and $m = p - q$,

$$\begin{aligned}
&= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} R(m+n) \alpha^m \gamma_{1+n}(1-d_{1+n}) \\
&= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} R(n) R(n+1, n+m) \alpha^m \gamma_{1+n}(1-d_{1+n}) \\
&= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} R(n) R(n+1, n+m) \alpha^m \gamma_{1+n}(1-d_{1+n}) \\
&= \sum_{n=0}^{\infty} R(n) \gamma_{1+n}(1-d_{1+n}) \sum_{m=0}^{\infty} \alpha^m R(n+1, n+m) \\
&= \sum_{n=0}^{\infty} R(n) \gamma_{1+n}(1-d_{1+n}) \Gamma(n),
\end{aligned}$$

where

$$\Gamma(n) = \sum_{m=0}^{\infty} \alpha^m R(n+1, n+m).$$

A4. Proof of proposition 4

The proof follows from the proof of propostition 1. In solving for the subgame perfect allocation in period t, that proof employed backward induction in a way that allowed the time preference rate paths of each generation to be distinct (i.e., the cumulative discount factors were indexed by t). Nevertheless, the conditions for the subgame perfect allocation in t are independent of the time preference rates of future generations (thus, for example, the optimal carbon tax in t only depends on the discount factors for generation t). To compare the solutions with and without commitment, we can view the optimal allocation with commitment as the solution to the backward induction problem in which each future generation adopts the continuation path of time preference rates that the generation in t would have wanted them to. Because the period t allocation under backward induction does not depend on the time preference rates adopted by subsequent generations, the solution to this problem must be the same.

A5. Proof of proposition 5

The proof falls out from the proof of proposition 1. There, in solving for the optimal allocation in period t, the proof employed backward induction in a way that allowed the time preference rate paths of each generation to be distinct (i.e., the cumulative discount factors were indexed by t). Nevertheless, the conditions for the optimal allocation in t were independent of the time preference rates of future generations. It follows that the optimal carbon tax is unchanged when the backward induction problem is solved instead under alternative assumptions about the time preference rates to be adopted by subsequent generations.

A6. Proof of proposition 6

Again, consider the equilibrium path between t and T. The difference is that now the planner in each future period τ employs their own path of cumulative discount factors. From section I.B, the savings rule in $t \le \tau \le T-1$ is

$$K_{\tau+1} = s_\tau Y_\tau, \tag{A4}$$

where

$$s_\tau = \frac{\Gamma_\tau}{1+\Gamma_\tau}$$

and

$$\Gamma_\tau = \sum_{s=\tau+1}^{T} R_{s-\tau}\alpha^{s-\tau}.$$

By the same steps as above, it follows that

$$\phi_\tau = \frac{\Gamma_\tau}{\alpha(1+\Gamma_{\tau+1})}.$$

In the last period, this implies

$$\phi_{T-1} = \beta_1.$$

Moreover, for $\tau = t, \ldots, T-2$, we have

$$\phi_\tau = \beta_1 \cdot \frac{1+\alpha\beta_2+\alpha\beta_2\alpha\beta_3+\ldots+\alpha\beta_2\cdots\alpha\beta_{T-t}}{1+\alpha\beta_1+\alpha\beta_1\alpha\beta_2+\ldots+\alpha\beta_1\cdots\alpha\beta_{T-t-1}} > \beta_1. \tag{A5}$$

Without commitment, the agent in control in period τ discounts utility between τ and $\tau+1$ using the subjective discount factor β_1. Provided the preferences of subsequent agents aligned with their own, they would ensure that the expected marginal rate of transformation equaled $\frac{u'(C_\tau)}{\beta_1 u'(C_{\tau+1})}$. This is the case in $T-1$ since the preferences of the generation in the very last period align with those of agents in earlier periods (they consume everything, as prior generations would have wanted). But for $\tau < T-1$, it is not the case. The decision maker in τ saves more to account for the fact that subsequent decision-makers are going to save less then they would have if the hyperbolic agent in τ could force their hand with a commitment device.

The more interesting question is to compare the Pigouvian discount factors, $\{\phi_\tau\}$, with the discount factors applied in the full commitment equilibrium, $\{\beta_\tau\}$. Since the optimal carbon tax under commitment equals the optimal carbon tax without commitment (from Proposition 4) this comparison makes it possible to compare the optimal tax without commitment with the Pigouvian tax without commitment.

LEMMA 8: *In the equilibrium without commitment, the relevant subjective discount factor for use in constructing the Pigouvian tax starts out above the corresponding discount factor that would be applied under commitment, then declines monotonically. For large t, it is below the subjective discount factor under commitment. In particular,* $\frac{\phi_t}{\beta t} > 1$, $\frac{\phi_{T-1}}{\beta T-1} < 1$, *and* $\frac{\phi_{\tau+1}}{\beta \tau+1} \leq \frac{\phi_\tau}{\beta \tau}$ *for* $t \leq \tau \leq T-1$.

PROOF:

That $\frac{\phi_1}{\beta 1} > 1$ and $\frac{\phi_{T-1}}{\beta T-1} < 1$ follows from inspection. To prove that $\frac{\phi_{t+1}}{\beta t+1} \leq \frac{\phi_t}{\beta t}$, suppose instead that $\frac{\phi_{t+1}}{\beta t+1} > \frac{\phi_t}{\beta t}$. In particular, suppose

$$\text{(A6)} \qquad \frac{\Omega_{T-t-1}}{\alpha\beta_{t+1}(1+\Omega_{T-t-2})} > \frac{\Omega_{T-t}}{\alpha\beta_t(1+\Omega_{T-t-1})}.$$

Defining $\Psi_n = \prod_{m=1}^{n} \alpha\beta_m$, (A6) can be rewritten as

$$\frac{\Omega_{T-t-1}}{\alpha\beta_{t+1}(1+\Omega_{T-t-2})} > \frac{\Omega_{T-t-1} + \Psi_{T-t}}{\alpha\beta_t(1+\Omega_{T-t-2}+\Psi_{T-t-1})}.$$

Cross-multiplying, this is equivalent to

$$\alpha\beta_t\Omega_{T-t-1} + \alpha\beta_t\Omega_{T-t-1}\Omega_{T-t-2} + \alpha\beta_t\Psi_{T-t-1}\Omega_{T-t-1} >$$
$$\alpha\beta_{t+1}\Omega_{T-t-1} + \alpha\beta_{t+1}\Omega_{T-t-1}\Omega_{T-t-2} + \alpha\beta_{t+1}\Psi_{T-t}(1+\Omega_{T-t-2}).$$

But $\alpha\beta_{t+1}\Omega_{T-t-1} \geq \alpha\beta_t\Omega_{T-t-1}$, $\alpha\beta_{t+1}\Omega_{T-t-1}\Omega_{T-t-2} \geq \alpha\beta_t\Omega_{T-t-1}\Omega_{T-t-2}$ and $\alpha\beta_{t+1}\Psi_{T-t}(1+\Omega_{T-t-2}) > \alpha\beta_t\Psi_{T-t-1}\Omega_{T-t-1}$, which gives a contradiction.

It follows that the relationship between the optimal first-period carbon tax and the first-period Pigouvian tax in the equilibrium without commitment is in principle ambiguous. If climate damages were to all happen immediately, the Pigouvian tax would be bigger. But if climate damages occur with sufficient delay, the optimal tax is bigger.

A7. Proof of proposition 7

The equilibrium consumption path corresponding to the equilibrium with commitment can be determined by replacing $\{R^{(t)}_{\tau-t}\}$ with $\{R^{(0)}_{t,\tau}\}$ in problem $\mathbf{PP(t)}$ from section I.B. This imposes the initial generation's time preferences on subsequent generations, and therefore mimics the equilibrium with commitment. We assume that the preferences of all generations are identical.

To derive the period t Pigouvian tax, we combine equation (22) with the equilibrium savings rule for each subsequent period. For $t \leq \tau \leq T-1$, the savings rule is

$$K_{\tau+1} = s_\tau Y_\tau,$$

where

$$s_\tau = \frac{\Omega_\tau}{1+\Omega_{\tau+1}}$$

and

$$\Omega_\tau = \sum_{s=\tau+1}^{T} R^{(t)}_{\tau-t,s-t}\alpha^{s-\tau}.$$

It follows that

$$\frac{K_{\tau+1}}{C_\tau} = \frac{s_\tau}{1-s_\tau} = \Omega_\tau$$

and

$$\frac{C_{\tau+1}}{Y_{\tau+1}} = 1 - s_{\tau+1} = \frac{1}{1+\Omega_{\tau+1}}.$$

Substituting into equation (22) gives

$$\phi_\tau = \frac{\Omega_\tau}{\alpha(1+\Omega_{\tau+1})}.$$

Substituting and simplifying yields

$$\phi_t = \beta_t,$$

which holds for all t.

The result follows.

A8. System of equations to simulate the full dynamic path of the model

The following steps impose the additional functional form assumptions into problem $\mathbf{PP(t)}$ for each t.

To generate the modified savings rate, we combine the first-order conditions for C_t

and K_{t+1} with the aggregate resource constraint. This implies

$$K_{t+1} = \frac{\sum_{\tau=t+1}^{T} R_{\tau-t}\alpha^{\tau-t}}{1+\sum_{\tau=t+1}^{T} R_{\tau-t}\alpha^{\tau-t}} Y_t.$$

The fractional savings rate here reduces to $\alpha\beta$ when discounting is constant and the time horizon is infinite—a familiar expression from the Brock-Mirman growth model (and Golosov et al. 2013). The marginal externality cost falls out from the first-order condition for $E_{1,t}$:

$$\tilde{\Lambda}_t^s + \lambda_{1,t} = \phi_t(\kappa_1 E_{1,t}^{\rho} + \kappa_2 E_{2,t}^{\rho})^{1/\rho-1}\kappa_1 E_{1,t}^{\rho-1}$$

where

$$\tilde{\Lambda}_t^s = \mathbb{E}_t\left[\sum_{\tau=t}^{T} R_{\tau-t}^{(t)} \sum_{j=0}^{\tau-t} \alpha^{\tau-t-j}\gamma_{t+j}(1-d_j)\right].$$

The remaining equations can be combined as in the model with constant discounting. This gives

$$E_{1,t} = E_t^{-\rho/1-\rho}\epsilon_{1,t}, \tag{A7}$$

where

$$\epsilon_{1,t} = \left(\frac{A_{1t}N_{0t}\upsilon\kappa_1}{1-\alpha-\upsilon+A_{1t}N_{0t}\Lambda_t^s}\right)^{1/1-\rho},$$

and

$$E_{2,t} = E_t^{-\rho/1-\rho}\epsilon_{2,t}, \tag{A8}$$

where

$$\epsilon_{2,t} = \left(\frac{A_{2t}N_{0t}\upsilon\kappa_2}{1-\alpha-\upsilon}\right)^{1/1-\rho}.$$

These can be combined with

$$N_{0,t} + \frac{E_{1t}}{A_{1t}} + \frac{E_{2t}}{A_{2t}} = N_t$$

and

$$E_t = (\kappa_1 E_{1,t}^{\rho} + \kappa_2 E_{2,t}^{\rho})^{1/\rho}$$

to give two equations in two unknowns: $N_{0,t}$ and E_t. $E_{1,t}$ and $E_{2,t}$ are then derived from (A7) and (A8).

The steps here are shown for the equilibrium without commitment, but the substitutions indicated in the text can be used to simulate the equilibrium with commitment.

A9. Model simulation with $\kappa_1 = \kappa_2 = 0.5$