Macro Models: an APP for Macroeconomic Models. User Manual 2.0

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Macro Models
An App for Macroeconomic Models.
User Manual 2.0

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Abstract

Macro Models are a series of free Apps available in App Store and they works with the Ipad. Each App simulates a specific macroeconomic model and it shows both the static and the dynamic results. The first four Apps developed and published are: the Income-Expenditure model in two versions (I and II), the IS-LM model and the Taylor's rule (IS-MP model). In this paper we describe the economic model of each single App and we also show several examples on how it works.

Keywords: Macroeconomics, Income-Expenditure model, IS-LM, Taylor's rule, APP.
Jel Codes: A20; E20

Contents

Chapter 1

Introduction

This paper is a user manual of an APP that simulates the widely used Macroeconomic Models [1].The first two apps developed concern the Income- Expenditure Model and the IS-LM Model. Other Apps will be realized in the next months. The first question we have to answer is: why an App? There are two reasons.

Firstly, we think that tablets give another opportunity to learn. With the tablets you can play, write e-mail, and connect to Internet, everywhere. But you can also read articles and books and listen mp3. For these reasons an App is an opportunity to understand how the main macroeconomics models work. Secondly in the textbooks here are only few examples for each single model, and they concerns mainly the static aspects. With the app it is possible to simulate both the static and the dynamic results of the model. In fact with each app it is possible to input the parameters of the model in order to obtain both the static and the dynamic results.

But there is also another important issue. With this app and with this paper we would like to give another interpretation of the macroeconomic models. They are schemes useful to study the implication the instruments applied by the government in order to guarantee the social stability.

The logo of the app is a sphere over a picture of a water course. The sphere is un-

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[1]This paper is not a Macroeconomics text book. We suggest to use Dornbush et al. (2004) or Blanchard (2009).

real while the photo is real. The sphere is stable and it represents the "perfect" equilibrium: each of the infinite points of the sphere is an equilibrium and it is identical to all the others. The water is dynamic and it represents the unstable conditions of the reality. The sphere may represent the being, the metaphysic, while the water is the becoming, the nature. For Talete water was also the origin of all things. Parmenides says that two things, being and becoming, sphere and water, are conflicting. But the government has to rule the dynamic, thinking to the sphere.

The Apps can be downloaded by the Appl Store. They are free. Each App may contains one or more models.

The screen of the single app is divided into two parts that it can be scrolled. The first table of the upper side of the screen is the panel of inputs. It contains three columns, each of them represents a period. For example, the first one is the initial period, while in the second one there is a shock (i.e a decrease of the investment) and in the third period the Government reacts to that negative shock cutting the income tax rate.

The first panel of the lower part of the screen shows the results of the model in the equilibrium. You can obtain them by pressing the RESET button, while with SAVE you save them. In the lower part of the screen the are also some graphs. Some of them shows the model's static results while some others show the dynamics of the variables. It is possible to choice what variable to plot switching the cursors that are in "Graphic". The Legend, that is in the upper side of the screen, explains the meanings of the symbols, the results, and the graphs.

Chapter 2

The Income Expenditure model

1. Introduction

The Income Expenditure model is the first one that students find in the text-book of Macroeconomics. It is based on two assumptions. The first one is that prices are fixed. This implies that in the model the prices mechanism doesn't work. In order to reach the equilibrium it must change the quantity of goods and services offered. If the demand is greater than the supply, the production will fall, while it is the contrary, as to say the supply is greater than the demand, the production and the supply will rise.

The second assumption is that there are infinite equilibria but non all of them can ensure the social stability, because the economic system may reach an equilibrium were there isn't full employment. According the Keynesian theory, only with the Government's intervention, it will be possible to reach a full employment situation or a condition of social stability, even in the short run. In other words, the economic system is unstable. It is manly due to the Investments that are the unstable component of the aggregate demand. The Government can stabilize the business cycle through the fiscal policy. In order to make the production growth, the government can increase the economic expenditure, or reduce the taxes. In the first case it substitute the private sector, while in the

second case, it stimulates the private sector trough the disposable income and the consumption.

Anyway, in the recent years with the crises of the Eurozone, many European country, as Greece, Spain and Italy, were obliged to reduce their debt. In these cases the social stability was linked to the reduction of the public debts instead of the reduction of unemployment. For this reason in the software developed we give particular attention to the dynamic of the debt.

2. The Model I: Income-Expenditure Model

Be Y the income, t the Income tax rate, and TR the net Government Transfers. TR is positive if the amount of subsidies is greater than the lump-sum taxes, and it is negative otherwise. The after-tax income, or disposable income, YD, is equal to

$$YD = Y + TR - tY \tag{2.1}$$

The Aggregate Demand AD is equal to the sum of Consumption C, Investment I, Net Export NX, and the Government's expenditure G.

$$AD = C + I + G + NX + G \tag{2.2}$$

The Keynesian Consumption function is:

$$C = \overline{C} + cYD \tag{2.3}$$

where $\overline{C}$ is the autonomous consumption, and c is the marginal propensity to consume. $\overline{C} > 0$ and $0 < c < 1$. Substituting Yd into the equation (2.3), it obtains:

$$C = \overline{C} + cTR + c(1 - t)Y \tag{2.4}$$

$c(1-t)$ is the Net Marginal Propensity (NMP) to consume. The Investment function is:

$$I = \overline{I} + \theta Y \tag{2.5}$$

$\overline{I}$ is the autonomous investment and θ the marginal propensity to invest. $\theta \geq 0$.

The Net Exports NX are exogenous.

The Government can modify the expenditure G, the Net Transfer TR, and the income taxes rate t. They are the government's instruments.

The Income Expenditure Model is:

$$C = \overline{C} + cTR + c(1-t)Y$$

$$I = \overline{I} + \theta Y$$

$$G = \overline{G}$$

$$TR = \overline{TR}$$

$$NX = \overline{NX}$$

$$Y = AD \tag{2.6}$$

The equation (2.6) is the equilibrium where the supply Y is equal to the demand AD. It also possible to rewrite (2.6) as:

$$Y = \overline{C} + c\overline{TR} + \overline{I} + \overline{NX} + \overline{G} + C(1-t)Y + \theta Y \tag{2.7}$$

Solving for Y, we obtain the equilibrium income:

$$Y_e = \frac{1}{(1 - c(1 - t) - \theta)}[\overline{C} + c\overline{TR} + \overline{NX} + \overline{I} + \overline{G}] \tag{2.8}$$

$\frac{1}{(1-c(1-t)-\theta)}$ is the Keynesian Multiplier. It is possible to demonstrate that with $\theta = 0$ the Keynesian Multiplier is always greater than one. With $\theta > 0$ we assume that $0 < 1 - c(1 - t) - \theta < 1$. For this reason also in this case the Keynesian Multiplier is always positive and greater then 1.

Once obtained the equilibrium income, it is possible to calculate the equilibrium consumption and the equilibrium investment. They are respectively:

$$C_e = \overline{C} + c\overline{TR} + c(1 - t)Y_e \tag{2.9}$$

$$I_e = \overline{I} + \theta Y_e \tag{2.10}$$

The Balance Surplus BS of the government is equal to the difference between receipts and expenditure. The receipts are the amount of taxes tY and the expenditure are represented by the sum of the government's expenditure G and the Net Transfer TR [1] . In formula:

$$BS = tY - (\overline{G} + \overline{TR}) \tag{2.11}$$

The Balance Surplus in equilibrium is equal to:

$$BS_e = tY_e - (\overline{G} + \overline{TR}) \tag{2.12}$$

or

$$BS_e = t\frac{1}{(1 - c(1 - t) - \theta)}[[\overline{C} + c\overline{TR} + \overline{NX} + \overline{I} + \overline{G}] - (\overline{G} + \overline{TR}) \tag{2.13}$$

There is a deficit for $BS < 0$. The government's debt B at the time t is equal

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[1] if $TR < O$ the net transfer are receipts.

to the algebraic sum of the previous surplus and deficits[2]:

$$B = \sum_{t=0}^{T} BS_t \tag{2.14}$$

2.1. The Comparative Static

$\overline{C}, \overline{I}$ and NX are the exogenous variables of the model. A shock of one of these variables causes a variation of income equals to the Keynesian multiplier

$$\frac{dY}{d\overline{I}} = \frac{dY}{d\overline{C}} = \frac{dY}{d\overline{NX}} = \frac{1}{(1 - c(1-t) - \theta)} \tag{2.15}$$

The government can change the government expenditure G, the Net Transfers TR and the income tax rate t. The multiplier are respectively:

$$\frac{dY}{d\overline{G}} = \frac{1}{(1 - c(1-t) - \theta)} \tag{2.16}$$

$$\frac{dY}{d\overline{TR}} = \frac{c}{(1 - c(1-t) - \theta)} \tag{2.17}$$

$$\frac{dY}{dt} = \frac{-c}{(1 - c(1-t) - \theta)^2} [\overline{C} + c\overline{TR} + \overline{NX} + \overline{I} + \overline{G}] \tag{2.18}$$

The impact on government budget is:

$$\frac{dBS}{d\overline{I}} = \frac{dY}{d\overline{C}} = \frac{dY}{d\overline{NX}} = \frac{t}{(1 - c(1-t) - \theta)} \tag{2.19}$$

$$\frac{dBS}{d\overline{G}} = \frac{t}{(1 - c(1-t) - \theta)} - 1 \tag{2.20}$$

$$\frac{dBS}{d\overline{TR}} = \frac{ct}{(1 - c(1-t) - \theta)} - 1 \tag{2.21}$$

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[2]We are optimistic. We consider that the interest rate on the debt is equal to zero.

$$\frac{dBS}{dt} = \frac{(1-c-d)}{(1-c(1-t)-\theta)^2}[\overline{C} + c\overline{TR} + \overline{NX} + \overline{I} + \overline{G}] \tag{2.22}$$

The surplus is:

$$BS = tY - (\overline{G} + \overline{TR}) \tag{2.23}$$

The variation of surplus is equal to:

$$dBS = tdY - d\overline{G} - d\overline{TR} \tag{2.24}$$

For the sake of simplicity we suppose that $\theta = 0$. The variation of Equilibrium income is equal to:

$$dY_e = \left(\frac{1}{(1-c(1-t))}\right)[cd\overline{TR} + \overline{dG}] \tag{2.25}$$

and the change in the government Budget Surplus BS is:

$$dBS = t\left(\frac{1}{(1-c(1-t))}\right)[cd\overline{TR} + d\overline{G}] - [d\overline{G} + d\overline{TR}] \tag{2.26}$$

$$dBS = \left(t\frac{1}{1-c(1-t)} - 1\right)[\overline{dG}] + \left(t\frac{c}{1-c(1-t)} - 1\right)[d\overline{TR}] \tag{2.27}$$

$$dBS = \left(\frac{(1-c)(t-1)}{(1-c(1-t))}\right)[d\overline{G}] + \left(\frac{(c-1)}{(1-c(1-t))} - 1\right)[d\overline{TR}] \tag{2.28}$$

$$dBS = \left(t\frac{1}{(1-c(1-t))} - 1\right)[d\overline{G}] \tag{2.29}$$

The change in Government expenditure $d\overline{G}$ has an impact on Budget Surplus less than its amount, being

$$0 \le \left(t\frac{1}{(1-c(1-t))}\right) \le 1 \tag{2.30}$$

In other words an increase (or a decrease) in G causes also an increase (or decrease) in the Tax revenue tY equal to $t\frac{1}{(1-c(1-t))}$. The algebraic sum is less than dG.

2.2. Balanced Budget Multiplier

A government can increase spending and taxes keeping the budget in balance. In this case the Government expenditure multiplier has a different value, in another words, it is different the impact of a change in Government expenditure dG on income Y.

The variation of Budget Surplus BS is equal to:

$$dBS = tdY - d\overline{G} - d\overline{TR} \tag{2.31}$$

or

$$dBS = \left(\frac{(1-c)(t-1)}{(1-c(1-t))}\right)[d\overline{G}] + \left(\frac{(c-1)}{(1-c(1-t))} - 1\right)d\overline{TR}] \tag{2.32}$$

For $dBS = 0$

$$\left(\frac{(1-c)(t-1)}{(1-c(1-t))}\right)[d\overline{G}] + \left(\frac{(c-1)}{(1-c(1-t))} - 1\right)d\overline{TR} = 0 \tag{2.33}$$

$$d\overline{TR} = (t-1)d\overline{G} \tag{2.34}$$

Let's now consider the change in income

$$dY_e = \frac{1}{(1-c(1-t))}[cd\overline{TR} + d\overline{G}] \tag{2.35}$$

and substituting $\overline{TR}$ with $(t-1)d\overline{G}$, we obtain

$$dY_e = \frac{1}{(1-c(1-t))}[c(t-1)d\overline{G} + d\overline{G}] \tag{2.36}$$

$$dY_e = \frac{1}{(1 - c(1 - t))}[1 - c(1 - t)]d\overline{G} = 1 \qquad (2.37)$$

This result is known as Haavelmo Theorem (Haavelmo, 1945). When the Government increases spending and taxes keeping the budget in balance, the multiplier is equal to 1.

2.3. The reduction of Government Budget Deficit (keeping Income constant)

In this subsection we consider the case in which the Government reduces its budget deficit, keeping the income constant. We Know that the change in income is equal to:

$$dY = \frac{1}{(1 - c(1 - t) - \theta)}[d\overline{G} + cdTR] \qquad (2.38)$$

and the change in the Budget Surplus is:

$$dBS = tdY - (d\overline{G} + d\overline{TR}) \qquad (2.39)$$

For dY=0, we obtain:

$$d\overline{G} = -cd\overline{TR} \qquad (2.40)$$

or

$$dTR = -\frac{1}{c}d\overline{G} \qquad (2.41)$$

Substituting this result in the budget surplus equation, it becomes:

$$dBS = (d\overline{G} - \frac{1}{c}d\overline{G}) \qquad (2.42)$$

$$dBS = \frac{s}{c}d\overline{G} \qquad\qquad (2.43)$$

where $s = 1 - c$. This is the impact of a change in Government expenditure on the budget surplus when income is kept constant.

2.4.   The Fiscal Policy Options

In the next scheme, it is shown a list of feasible fiscal policy measures as consequence of a negative Investment shock. As it is well know, a decrease in investment ($I\downarrow$) causes a reduction of income ($Y\downarrow$) and of the Government Budget Surplus ($BS\downarrow$).

It is useful to distinguish two scenarios. In the first one, the budget surplus remains positive, while, in the second scenario, it becomes negative.

In the first case the government can decide to increase income ($Y\uparrow$) or to do nothing (0)[3]. In the second case the Government can pursue three aims (one more compared with the first one): 1) to increase income ($Y\uparrow$), to do nothing (0), and to reduce Deficit ($BS\uparrow$).

The first group includes the Keynesian fiscal Policies [1.], the second group is a "non-intervention" fiscal policy [2.], while the third one is direct to control the Government's Balance [3.].

The scheme 1 shows this possible list of fiscal policy,

Scheme 2. List of Fiscal Policies

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[3]to do nothing is always a political option.

$$I \downarrow \rightarrow (Y \downarrow; BS \downarrow) \begin{cases} Aim : Y \uparrow \begin{cases} single\ policy \begin{cases} [1.1.1](G \uparrow) \rightarrow (Y \uparrow; C \uparrow; BS \downarrow) \\ [1.1.2](TR \uparrow) \rightarrow (YD \uparrow; C \uparrow; Y \uparrow; BS \downarrow) \\ [1.1.3](t \downarrow) \rightarrow (YD \uparrow; C \uparrow; Y \uparrow; BS \downarrow) \end{cases} \\ policy - mix \begin{cases} [1.2.1](G \uparrow TR \downarrow) \rightarrow (Y \uparrow; C =; BS =) \\ [1.2.2](G \downarrow TR \uparrow) \rightarrow (Y \uparrow; C =; BS =) \end{cases} \end{cases} \\ 0 \\ Aim : BS \uparrow \begin{cases} single\ policy \begin{cases} [3.1.1](G \downarrow) \rightarrow (BS \uparrow; Y \downarrow; C \downarrow) \\ [3.1.2](TR \downarrow) \rightarrow (BS \uparrow; YD \downarrow; C \downarrow; Y \downarrow) \\ [3.1.3](t \uparrow) \rightarrow (BS \uparrow; YD \downarrow; C \downarrow; Y \downarrow) \end{cases} \\ policy - mix \begin{cases} [3.2.1](G \uparrow TR \downarrow) \rightarrow (BS \uparrow; YD \downarrow; C \downarrow; Y) \\ [3.2.2](G \downarrow TR \uparrow) \rightarrow (BS \uparrow; YD \downarrow; C \downarrow; Y) \end{cases} \end{cases} \end{cases}$$

2.5.   Some Examples (I): A reduction of the Investment (*with a Government budget still positive*)

In this section we show some examples. Each case is represented by a figure that includes 4 graphs: 1) Income - Expenditure Equilibrium, 2) The Government's Budget ($BS = f(Y)$), 3) The variables' dynamic, 4) the Government's budget's dynamic. There is a short comment for Each case. The Figure are token from the Macro Models' APP. The history begins from the Equilibrium: a negative shock of the investment causes a reduction of Income and of government budget.
In this first example

Table 2.1: Fiscal Policies

Symbol	t_0	t_1	t_2(I)	t_2 (II)	t_2 (III)	t_2 (IV)	t_2 (V)	t_2 (V)
I_0	900	**700**		700	700	700	700	700
	-	$I\downarrow$	-	$G\uparrow$	$G\uparrow; BS_{>0}$	$TR\uparrow$	$t\downarrow$	$G\uparrow; TR\downarrow$
$\overline{C}$	300	300		300	300	300	300	300
I_0	900	**700**		700	700	700	700	700
NX	-80	-80		-80	-80	-80	-80	-80
c	0.75	0.75		0.75	0.75	0.75	0.75	0.75
d	0	0		0	0	0	0	0
G	700	700		**900**	811.43	700	700	**1157.15**
TR	200	200		200	200	**466.6**	200	**-142.15**
t	0.25	0.25		0.25	0.25	0.25	**0.146**	0.25
NMP	0.56	0.56		0.56	0.56	0.56	**0.64**	0.56
K. M	2.29	2.29		2.29	2.29	2,29	**2.78**	**2.29 ; 1**
Y_e	4502.86	**4045.71**		4502.86	**4299.4**	4502.86	4502.91	4502.91
C_e	2982.86	**2725.71**		2982.86	2868.43	**3182.6**	**3182.91**	**2726.84**
I_e	900	700		700	700	700	700	700
BS_e	225.71	**111.43**		**25.71**	**63.86**	**-40.95**	**-40.97**	**111.43**
ΔY	-	**-457.14**		**457.14**	**253.71**	**457.15**	**457.15**	**457.15**
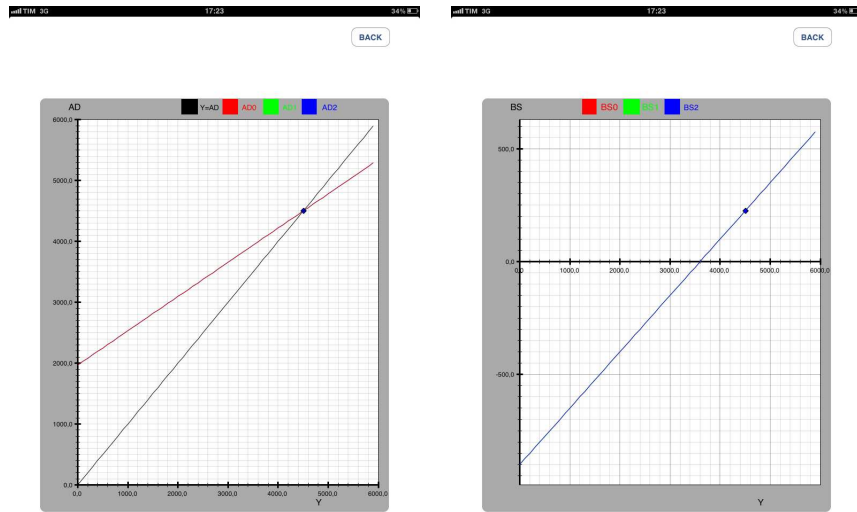
(a) Income Expenditure

(b) Government's Budget
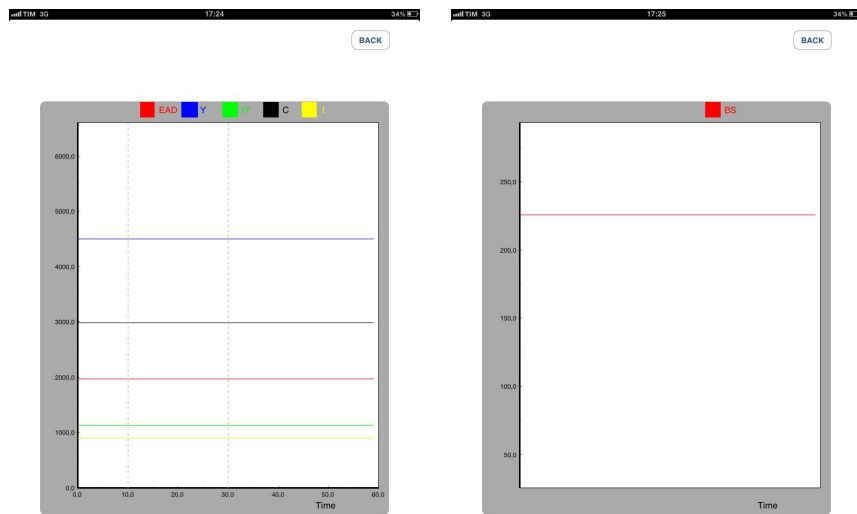
(c) Dynamic I

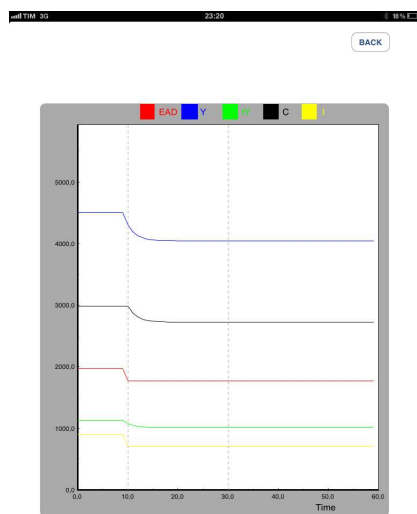(d) Dynamic II

Figure 2.1: At the Beginning of the History
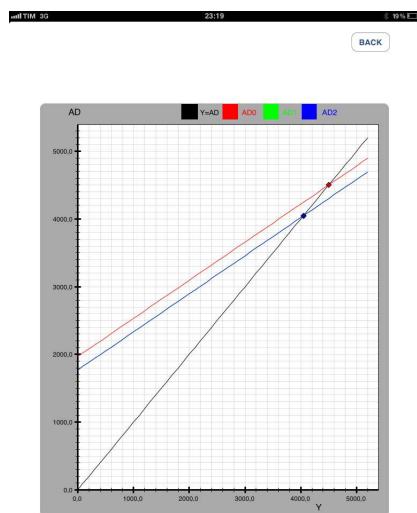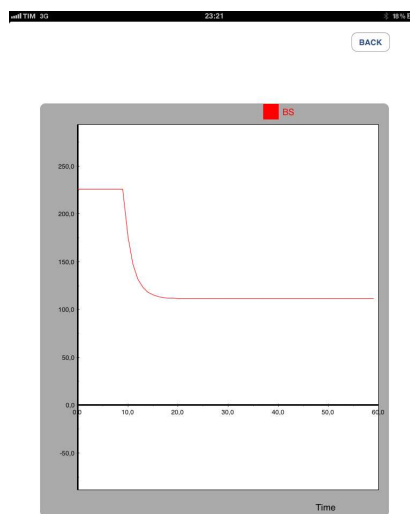
(a) Income Expenditure
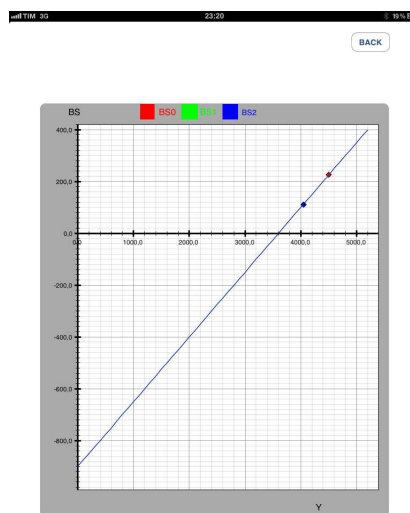
(b) Government's Budget

(c) Dynamic I

(d) Dynamic II

Figure 2.2: Case 1.I A negative shock: $I\downarrow$ and $BS > 0$
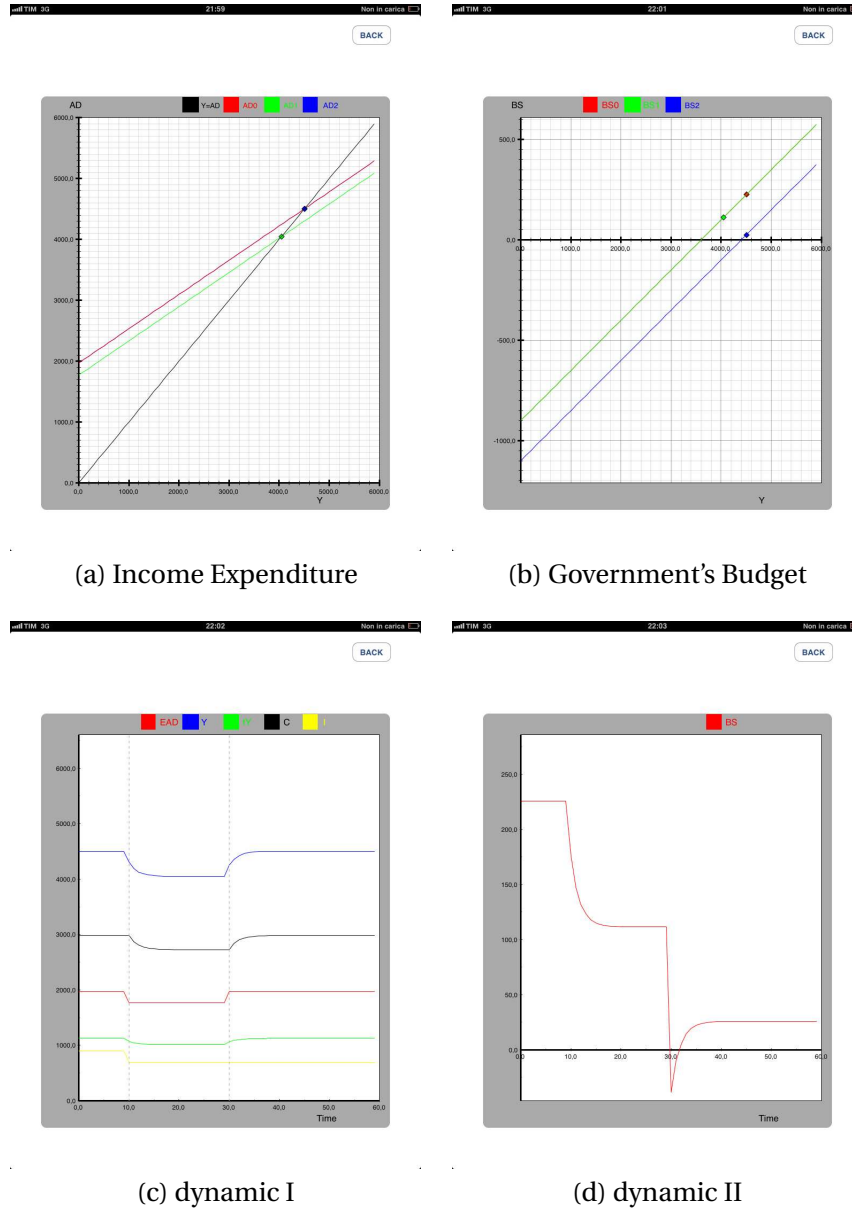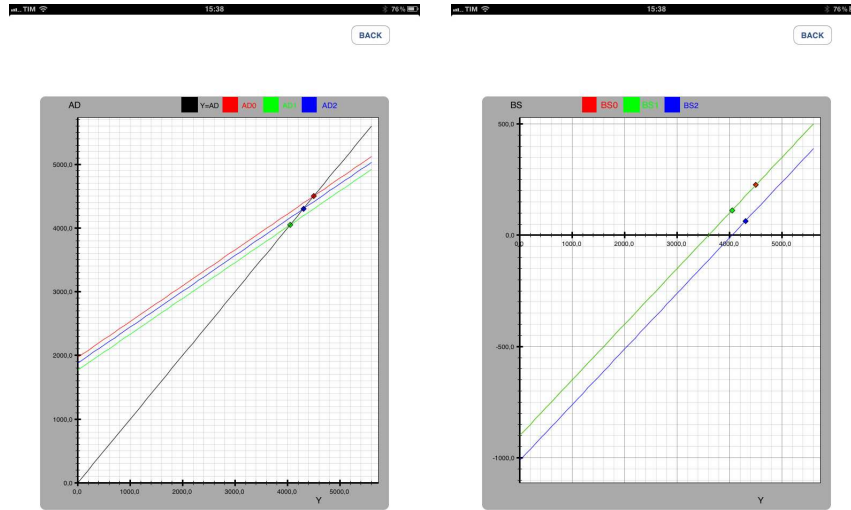
(a) Income Expenditure

(b) Government's Budget

(c) dynamic I

(d) dynamic II

Figure 2.3: Case 1.II. the Keynesian scenarios: I ↓ and G ↑
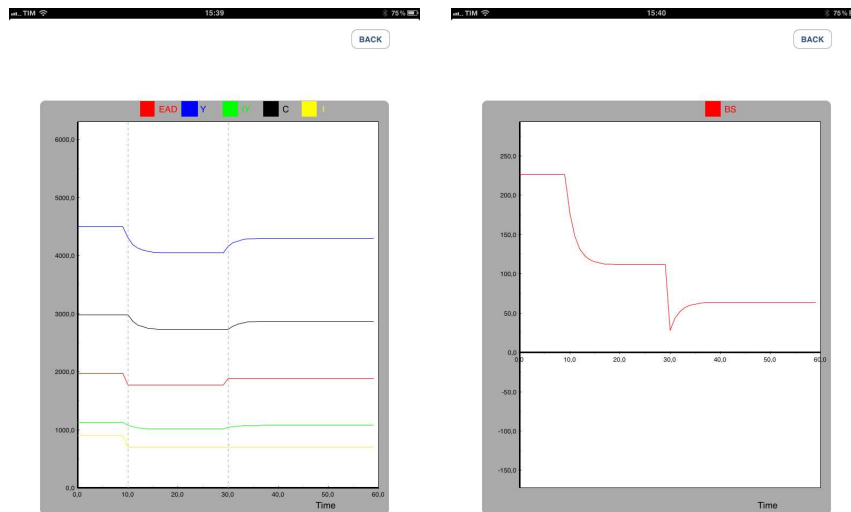BS becomes negative for "a while". Y returns at the initial level.

(a) Income Expenditure
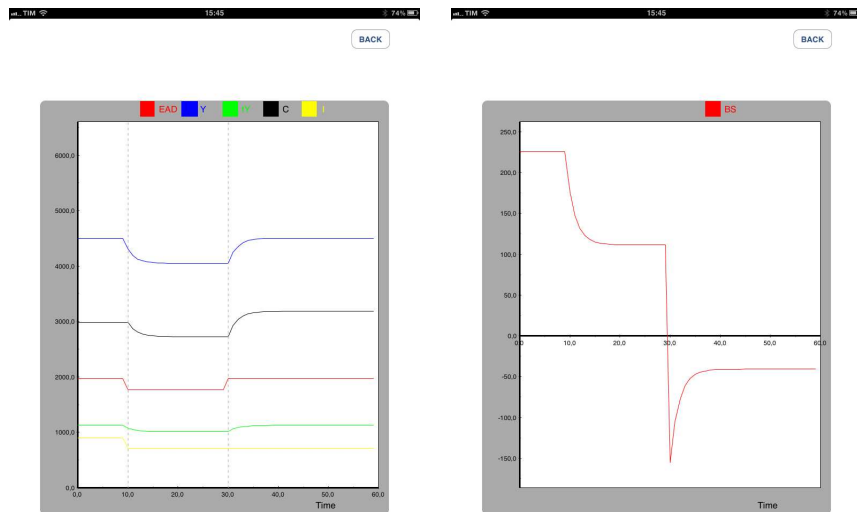

(b) Government's Budget

(c) dynamic I

(d) dynamic II

Figure 2.4: Case 1.III. I $\downarrow$ and G $\uparrow$
BS is always positive but Y does not return at the initial level.
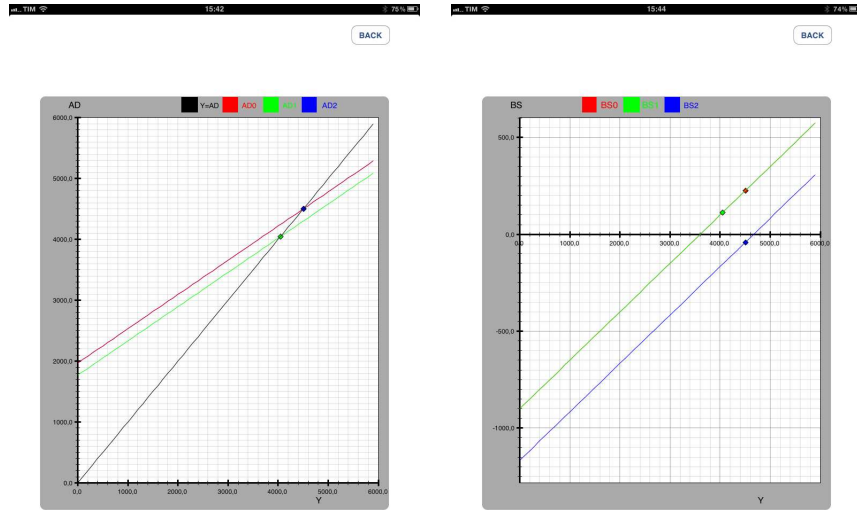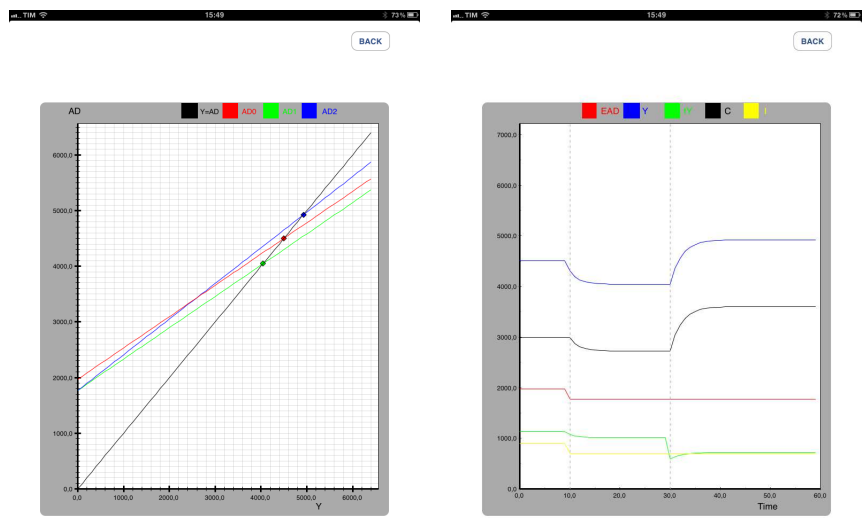
(a) Income Expenditure
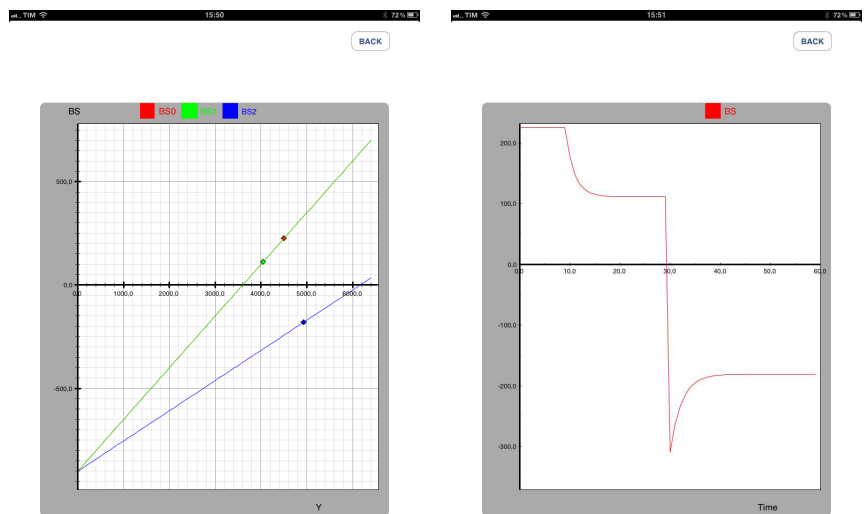

(b) Government's Budget

(c) dynamic I

(d) dynamic II

Figure 2.5: Case 1.IV $I \downarrow$ and $TR \uparrow$
$C \uparrow \uparrow$, BS is negative and higher.

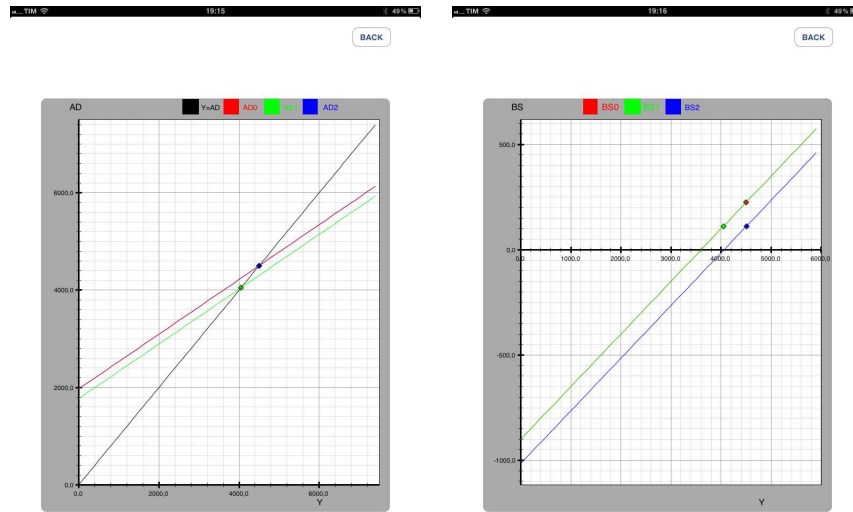(a) Income Expenditure

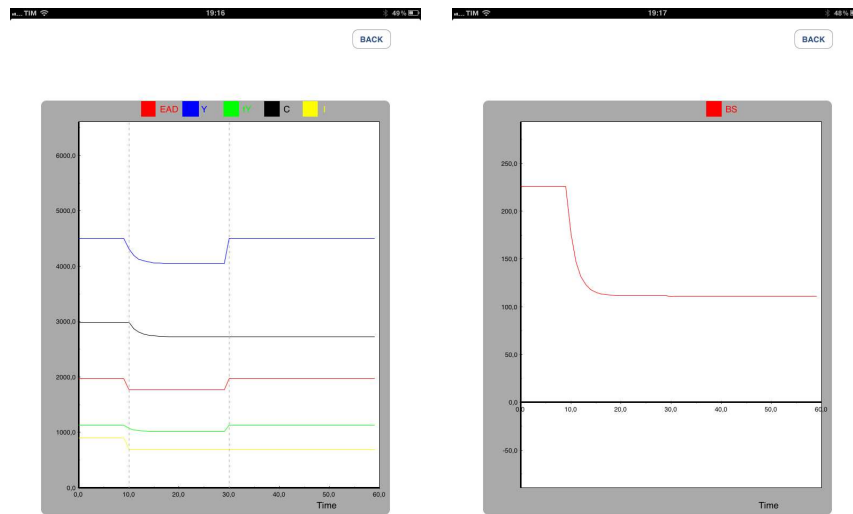(b) Government's Budget

(c) dynamic I

(d) dynamic II

Figure 2.6: Case 1.V. I - Haavelmo Theorem. $\downarrow$ - $G\uparrow, TR\downarrow$
BS is constant.

(a) Income Expenditure

(b) Government's Budget
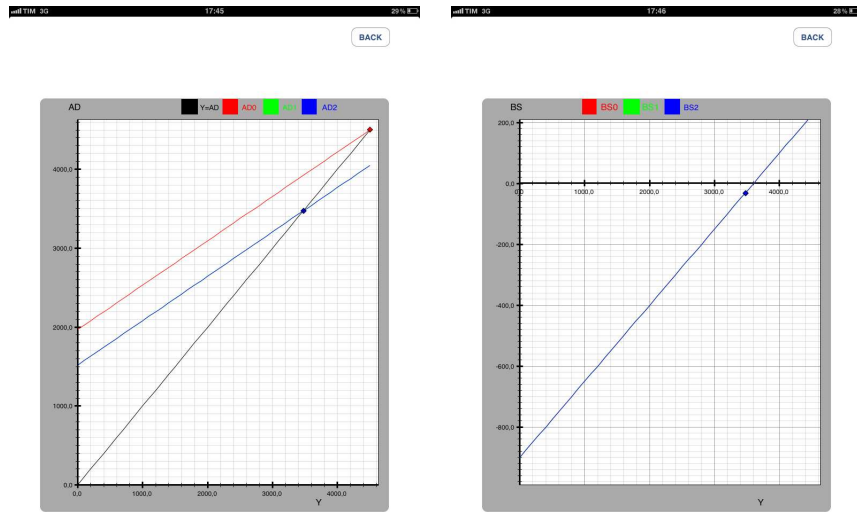

(c) dynamic I

(d) dynamic II

Figure 2.7: Case 1.VI I $\downarrow$ and $TR\uparrow$
$C\uparrow\uparrow$, BS is negative and higher (as Case 1.IV.)

2.6.   Some Examples (I): A reduction of the Investment (*with a Government budget that becomes negative*)

(a) Income Expenditure

(b) Government's Budget
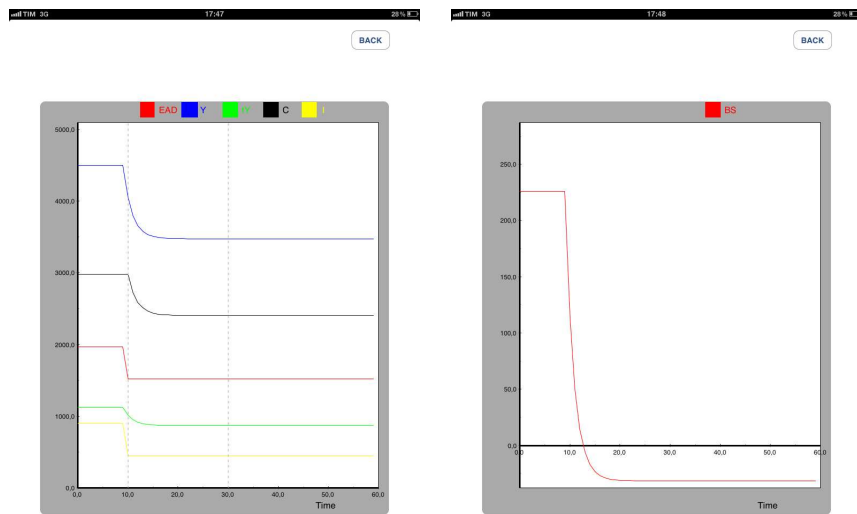
(c) dy

(d) BS

Figure 2.8: Case 2: A negative shock: $I\downarrow$ and $BS < 0$
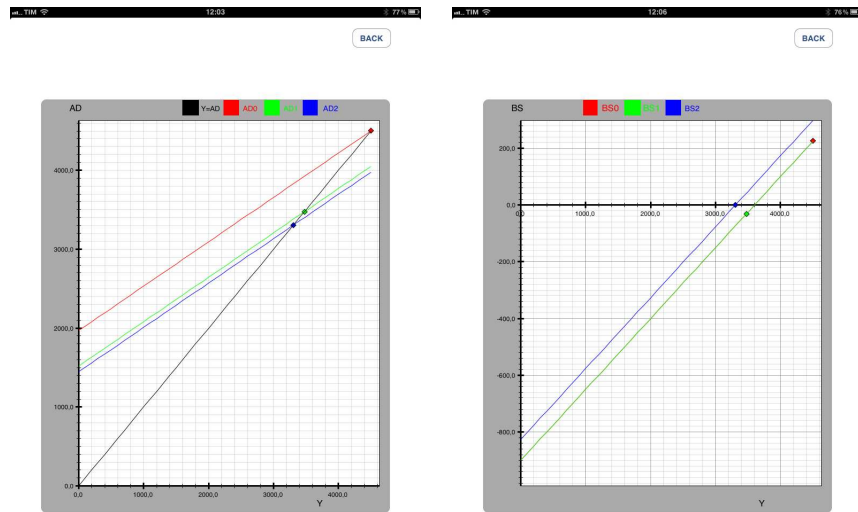
(a) Income Expenditure

(b) Government's Budget
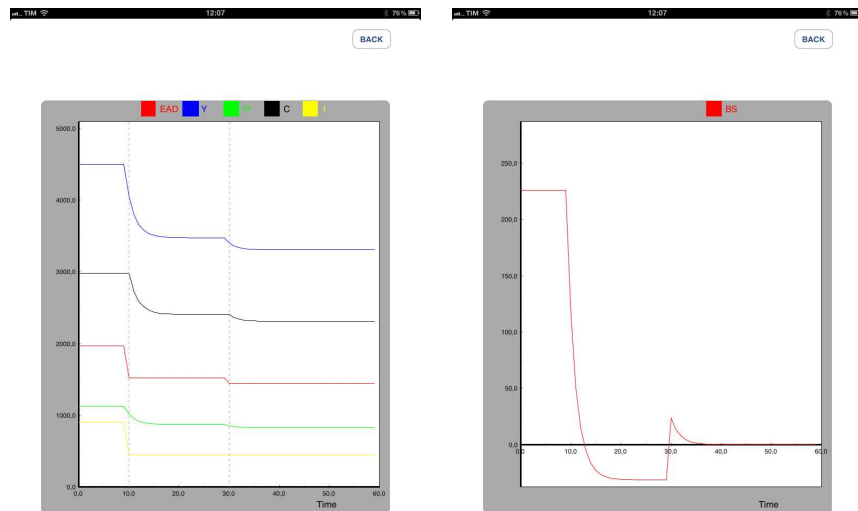
(c) dy

(d) BS

Figure 2.9: Case 2.1: A negative shock: $I\downarrow$ and $BS < 0$.
In order to reduce $BS \rightarrow 0$. $G \downarrow$, and also $C \downarrow$ and $Y \downarrow$

.
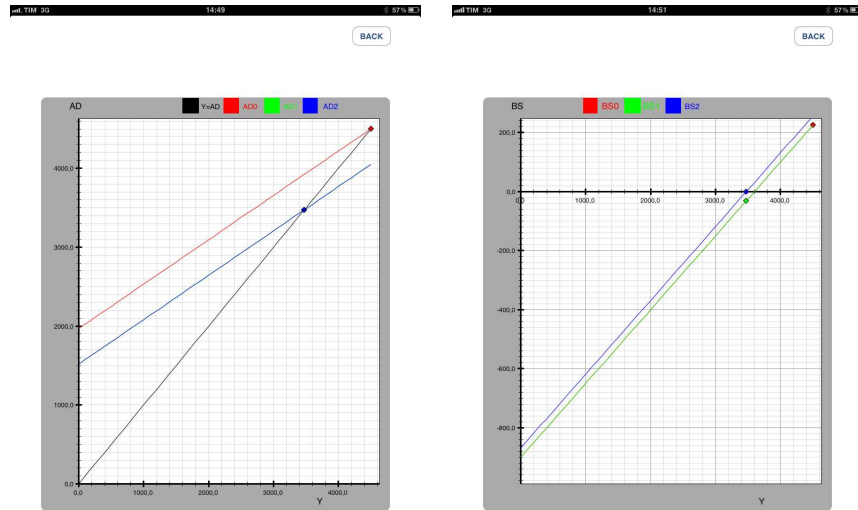
(a) Income Expenditure

(b) Government's Budget
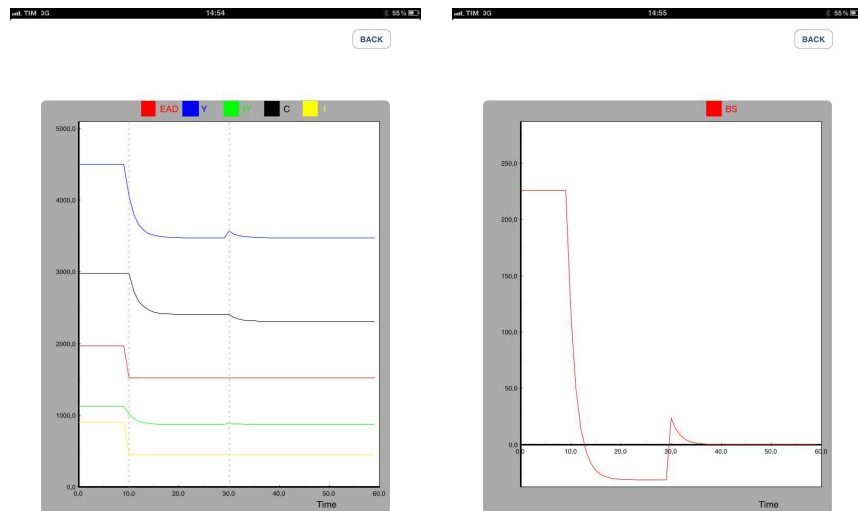
(c) dy

(d) BS

Figure 2.10: Case 2.2: A negative shock: $I\downarrow$ and $BS < 0$.
In order to reduce $BS \rightarrow 0$. $G \uparrow$, $TR \downarrow$, $C \downarrow$ but Y remains constant

.

(a) Income Expenditure

(b) Government's Budget

(c) dy

(d) BS
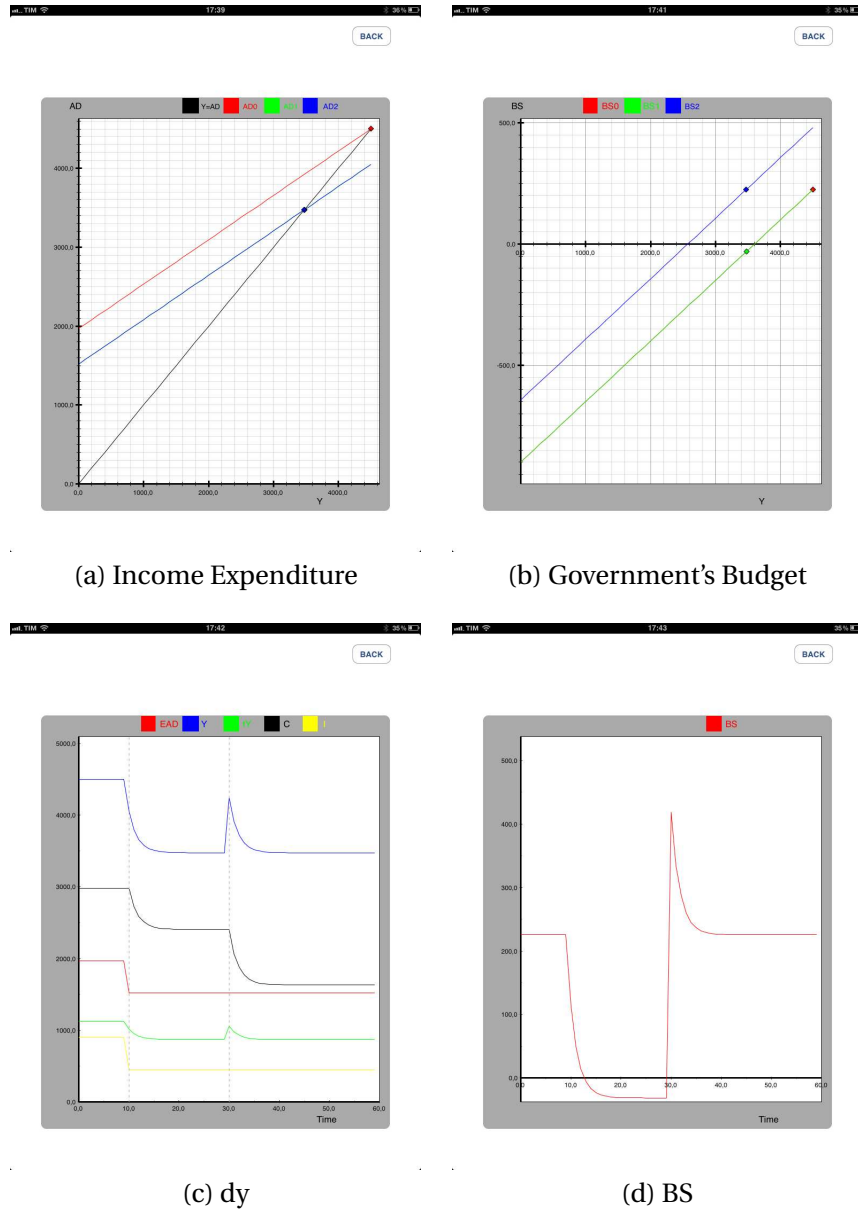
Figure 2.11: Case 2.2: A negative shock: $I\downarrow$ and $BS < 0$.
In order to reduce $BS \rightarrow$ "initial value"=225.71, $G \uparrow$, $TR \downarrow$, $C \downarrow$ but Y remains constant

2.7.   Legenda of the Income-Expenditure Model I

Legenda

Table 2.2: Input

Symbol	Variable / Parameter
$\overline{C}$	Autonomous (exogenous) Consumption
I_0	Net Investment
NX	Net Export
c	Marginal Propensity To Consume
d	Marginal Propensity to Invest
G	Government purchase of goods and services
TR	Net Government Transfers paryments
t	Income tax rate

Table 2.3: Output

acronymous	Parameter/Variable	formula
NMP	Net Marginal Propensity to consume	$c(1-t)$
Multiplier	Keynesian Multiplier	$\frac{1}{(1-c(1-t)-d)}$
Eq. Income	Equilibrium Income	Y_e
Eq. Consumption	Equilibrium Consumption	C_e
Balance	Government Surplus	$tY_e - (G + TR)$
ΔIncome	Income Variation	$Y_{e,t} - Y_{e,t-1}$

Table 2.4: Graph

EAD	Autonomous Aggregate Demand
Y	Income
tY	income tax
C	Consumption
I	Investment
D	Government Surplus
B	Government Debt

3. The Model II: the Samuelson's Multiplier Accelerator Model

In this version of the model we introduce the "Principle of Acceleration" as in Samuelson (1939). The model assumes that consumption depends on the previous income. In formulas, we have that

$$C_t = \overline{C} + c\overline{TR} + c(1 - t)Y_{t-1} \tag{2.44}$$

and the investment on the variation of consumption.

$$I_t = \overline{I} + n(C_t - C_{t-1}) \tag{2.45}$$

In this case it is possible to write:

$$I_t = \overline{I} + nc(1 - t)(Y_{t-1} - Y_{t-2}) \tag{2.46}$$

or

$$I_t = \overline{I} + \phi dY_{t-1} \tag{2.47}$$

where $\phi = nc(1 - t)$.

For $\phi > 0$ the APP shows only the dynamic results. Hereafter, we report some examples.

(a) c=0.5; n=0; ϕ=0, t=0

(b) c=0.5; n=0.98; ϕ=0.49, t=0

(c) c=0.8; n=1.25; ϕ=1, t=0
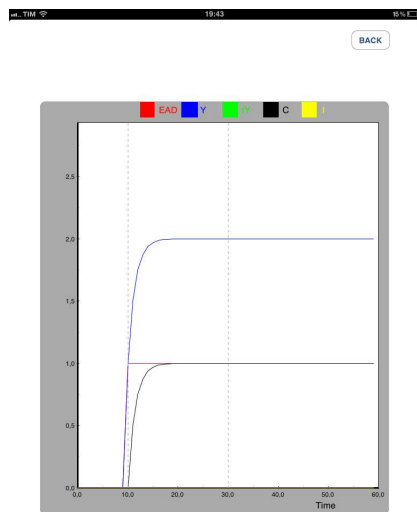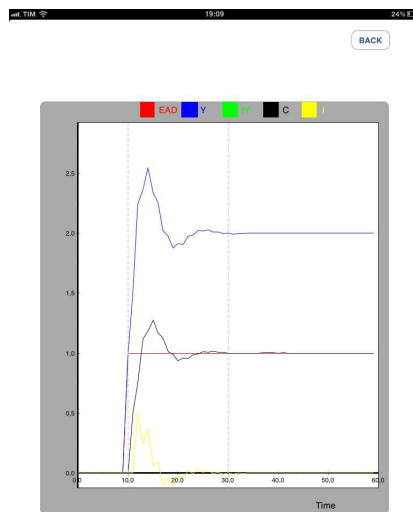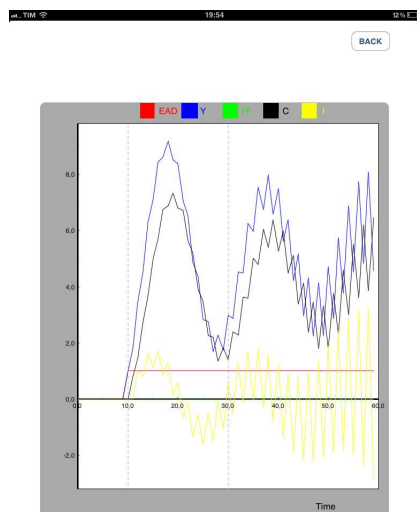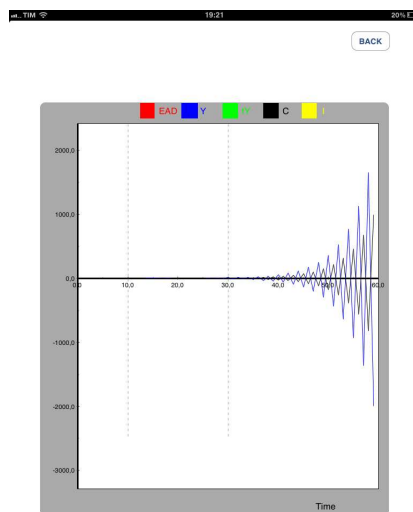
(d) c=0.6; n=2; ϕ=1.2, t=0

Figure 2.12: Case 3: Some Examples

3.1. Legenda of the Income-Expenditure Model II

Legenda

Table 2.5: Input

Symbol	Variable / Parameter
$\overline{C}$	Autonomous (exogenous) Consumption
I_0	Net Investment
NX	Net Export
c	Marginal Propensity To Consume
ϕ	Accelerator parameter
G	Government purchase of goods and services
TR	Net Government Transfers payments
t	Income tax rate

Table 2.6: Output

acronymous	Parameter/Variable	formula
NMP	Net Marginal Propensity to consume	$c(1-t)$
Multiplier	Keynesian Multiplier	$\frac{1}{(1-c(1-t)-d)}$
Eq. Income	Equilibrium Income	Y_e
Eq. Consumption	Equilibrium Consumption	C_e
Balance	Government Surplus	$tY_e - (G+TR)$
ΔIncome	Income Variation	$Y_{e,t} - Y_{e,t-1}$

Table 2.7: Graph

EAD	Autonomous Aggregate Demand
Y	Income
tY	income tax
C	Consumption
I	Investment
D	Government Surplus
B	Government Debt

Chapter 3

The IS-LM model

1.   The Real Sector (IS Curve)

We begin to consider the neoclassic Investment function in which the invest-ment depends on the interest rate r [1]. The function is:

$$I = \bar{I} - br + \theta Y \tag{3.1}$$

with $b > 0$

The model now becomes:

$$Y = AD$$

$$AD = C + I + G + NX$$

$$C = \overline{C} + c\overline{TR} + c(1 - t)Y$$

---

[1]In the IS-LM, the prices level $\overline{P}$ is constant, as to say, the inflation (π) is equal to zero. For the Fisher equation, the nominal interest rate (i) is equal to real rate (r) plus inflation $i = r + \pi$. In this model $\pi = 0$, so the nominal interest rate (i) is equal to real interest rate (r)

$$I = \bar{I} - br + \theta Y$$

$$TR = \overline{TR}$$

$$NX = \overline{NX}$$

The real sector is in equilibrium when the supply Y is equal to the demand AD

$$Y = AD$$

$$Y = \frac{1}{(1 - c(1 - t) - \theta)}[\overline{C} + \overline{G} + c\overline{TR} + \overline{NX} + \bar{I} - br] \tag{3.2}$$

This is the IS equation in which the income (Y) is function of real interest rate (r). It is possible to rewrite the 3.2 as:

$$Y = \frac{1}{(1 - c(1 - t) - \theta)}[\overline{A} - br] \tag{3.3}$$

where

$$\overline{A} = \overline{C} + \overline{G} + c\overline{TR} + \overline{NX} + \bar{I} \tag{3.4}$$

2. The Monetary Market (LM Curve)

The Money Supply of the Central Bank is $(\overline{M})$. The Prices Level $(\overline{P})$ is exogenous. So the real money supply is:

$$\frac{\overline{M}}{\overline{P}} \tag{3.5}$$

The demand of real money balance, or Liquidity (L), is increasing in income

(Y), and decreasing in interest rate r_{lt}.

$$L = \frac{M}{\overline{P}} = L(Y, r) \tag{3.6}$$

with $L_Y > 0$ and $L_r < 0$

It is possible to distinguish two cases. $L_r = 0$ is the case of classic school: interest rate is not a monetary variable and the demand of Money depends only on income (Y). On the contrary $L_r \to \infty$ is the case the Keynesian liquidity trap. the demand of money is infinite.

As functional form we use a linear equation.

$$L = \begin{cases} kY - hr & \textbf{if } r > r_{lt} \\ \infty & \textbf{if } r = r_{lt} \end{cases}$$

with $k > 0$, $h \geqslant 0$ and $r_{lt} \geqslant 0$

In order to encompass the "Liquidity Trap" hypothesis, we define r_{lt} as the "Liquidity Trap" interest. At this level of interest, the demand of real money is infinite.

The Equilibrium in the Monetary Market is given by:

$$\frac{\overline{M}}{\overline{P}} = L \tag{3.7}$$

$$\frac{\overline{M}}{\overline{P}} = kY - hr \tag{3.8}$$

It is also possible to write the last equation as:

$$Y = \frac{1}{k} \left(\frac{\overline{M}}{\overline{P}} + hr \right) \tag{3.9}$$

or

$$r = \frac{1}{h} \left(kY - \frac{\overline{M}}{\overline{P}} \right) \tag{3.10}$$

The 3.10 is the LM curve.

3. The IS-LM Equilibrium

The Equilibrium is given by the system of the two equation:

$$Y = \frac{1}{(1 - c(1-t) - \theta)}[\overline{C} + c\overline{TR} + \overline{NX} + \overline{I} - br + \overline{G}]$$

$$Y = \frac{1}{k}\left(\frac{\overline{M}}{\overline{P}} + hr\right)$$

The Equilibrium income is equal to:

$$Y = \frac{h}{(1 - c(1-t) - \theta)h + bk}(\overline{A}) + \frac{b}{(1 - c(1-t) - \theta)h + bk}\left(\frac{1}{\overline{P}}\right)\overline{M} \qquad (3.11)$$

or

$$Y = \frac{1}{(1 - c(1-t) - \theta) + \frac{b}{h}k}(\overline{A}) + \frac{b}{(1 - c(1-t) - \theta) + bk}\left(\frac{1}{\overline{P}}\right)\overline{M} \qquad (3.12)$$

while the Equilibrium interest rate is:

$$r = \frac{k}{h}\left(\frac{1}{(1 - c(1-t) - \theta) + \frac{b}{h}k}(\overline{A})\right) + \frac{1}{h}\left(\frac{kb}{(1 - c(1-t) - \theta)h + bk} - 1\right)\left(\frac{1}{\overline{P}}\right)\overline{M}$$
$$(3.13)$$

4. The impact of Fiscal Policy and of the Monetary Policy

We write again the equation 3.16 distinguishing which part affect the fiscal policy, and which one the monetary policy.

$$Y = \underbrace{\frac{h}{(1 - c(1 - t) - \theta)h + bk}(\overline{A})}_{\text{Fiscal Policy}} + \underbrace{\frac{b}{(1 - c(1 - t) - \theta)h + bk}\left(\frac{1}{\overline{P}}\right)\overline{M}}_{\text{Monetary Policy}} \qquad (3.14)$$

The impact of an increasing in $\overline{G}$ is equal to:

$$\frac{dY}{d\overline{G}} = \frac{h}{(1 - c(1 - t) - \theta)h + bk} \qquad (3.15)$$

The impact of an increasing in $\overline{M}$ is equal to:

$$\frac{dY}{d\overline{M}} = \frac{b}{(1 - c(1 - t) - \theta)h + bk}\left(\frac{1}{\overline{P}}\right) \qquad (3.16)$$

4.1. The Classic Hypothesis

a.1) $\underline{b \to \infty}$

The impact of an increasing in $\overline{G}$ is equal to:

$$\frac{dY}{d\overline{G}} = 0 \qquad (3.17)$$

The impact of an increasing in $\overline{M}$ is equal to:

$$\frac{dY}{d\overline{M}} = \frac{1}{k}\frac{1}{\overline{P}} \qquad (3.18)$$

a.2) $\underline{h = 0}$

The impact of an increasing in $\overline{G}$ is equal to:

$$\frac{dY}{d\overline{G}} = 0 \tag{3.19}$$

The impact of an increasing in $\overline{M}$ is equal to:

$$\frac{dY}{d\overline{M}} = \frac{1}{k}\frac{1}{\overline{P}} \tag{3.20}$$

4.2.   The Keynesian Hypothesis

b.1) $b = 0$

The impact of an increasing in $\overline{G}$ is equal to:

$$\frac{dY}{d\overline{G}} = \frac{1}{(1 - c(1 - t) - \theta)} \tag{3.21}$$

The impact of an increasing in $\overline{M}$ is equal to:

$$\frac{dY}{d\overline{M}} = 0 \tag{3.22}$$
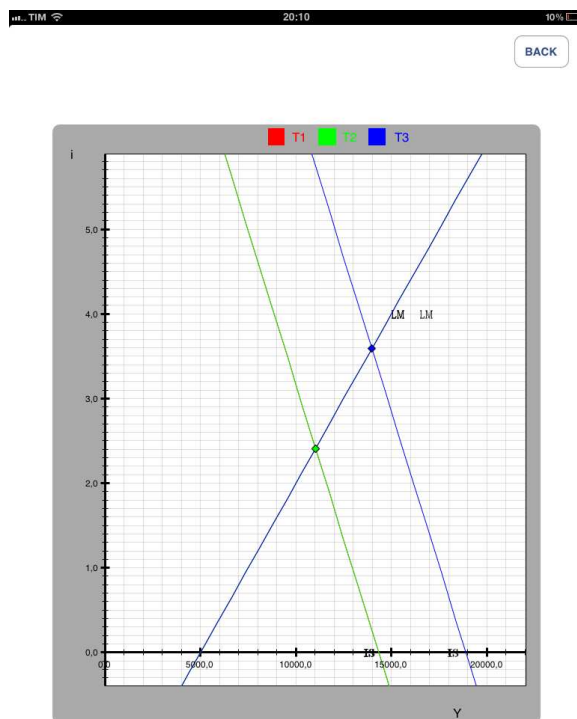
b.2) $h \to \infty$

The impact of an increasing in $\overline{G}$ is equal to:

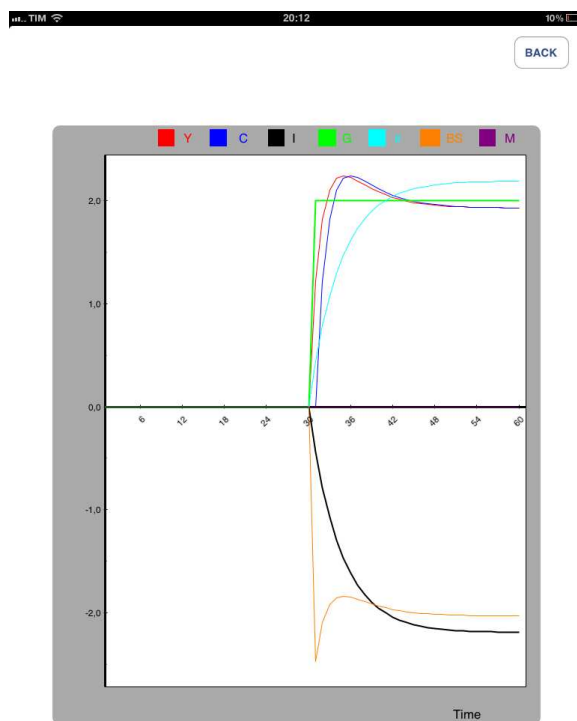$$\frac{dY}{d\overline{G}} = \frac{1}{(1 - c(1 - t) - \theta)} \tag{3.23}$$

The impact of an increasing in $\overline{M}$ is equal to:

$$\frac{dY}{d\overline{M}} = 0 \tag{3.24}$$

$$I \downarrow \rightarrow (Y \downarrow; BS \downarrow) \begin{cases} Aim : Y \uparrow \begin{cases} FP \begin{cases} \textbf{\textit{s. policy}} \begin{cases} [1.1.1](G \uparrow) \rightarrow (Y \uparrow; C \uparrow; r \uparrow; I \downarrow; BS \downarrow) \\ [1.1.2](TR \uparrow) \rightarrow (YD \uparrow; C \uparrow; Y \uparrow; r \uparrow; I \downarrow; BS \downarrow) \\ [1.1.3](t \downarrow) \rightarrow (YD \uparrow; C \uparrow; r \uparrow; I \downarrow; Y \uparrow; BS \downarrow) \end{cases} \\ policy - mix \begin{cases} [1.2.1](G \uparrow TR \downarrow) \rightarrow (Y \uparrow; C \uparrow; BS =) \\ [1.2.2](G \downarrow TR \uparrow) \rightarrow (Y \uparrow; C \uparrow; BS =) \end{cases} \end{cases} \\ \text{Monetary policy} \begin{cases} (M^s \uparrow) \rightarrow (r \downarrow; I \uparrow; Y \uparrow; C \uparrow; BS \uparrow) \end{cases} \\ FP + MP \begin{cases} (G \uparrow; M^s \uparrow) \rightarrow (Y \uparrow; C \uparrow; r =; I =; BS \uparrow\downarrow) \end{cases} \end{cases} \\ 0 \\ Aim : BS \uparrow \begin{cases} FP \begin{cases} \textbf{\textit{single policy}} \begin{cases} [3.1.1](G \downarrow) \rightarrow (BS \uparrow; Y \downarrow; C \downarrow) \\ [3.1.2](TR \downarrow) \rightarrow (BS \uparrow; YD \downarrow; C \downarrow; Y \downarrow) \\ [3.1.3](t \uparrow) \rightarrow (BS \uparrow; YD \downarrow; C \downarrow; Y \downarrow) \end{cases} \\ policy - mix \begin{cases} [3.2.1](G \uparrow TR \downarrow) \rightarrow (BS \uparrow; YD \downarrow; C \downarrow; Y) \\ [3.2.2](G \downarrow TR \uparrow) \rightarrow (BS \uparrow; YD \downarrow; C \downarrow; Y) \end{cases} \end{cases} \\ MP \begin{cases} (G \uparrow; M^s \uparrow) \rightarrow (Y \uparrow; C \uparrow; r =; I =; BS \uparrow\downarrow) \end{cases} \\ FP + MP \begin{cases} (G \downarrow; M^s \uparrow) \rightarrow (Y \uparrow\downarrow; C \uparrow\downarrow; r \downarrow; I \uparrow; BS \uparrow) \end{cases} \end{cases} \end{cases}$$
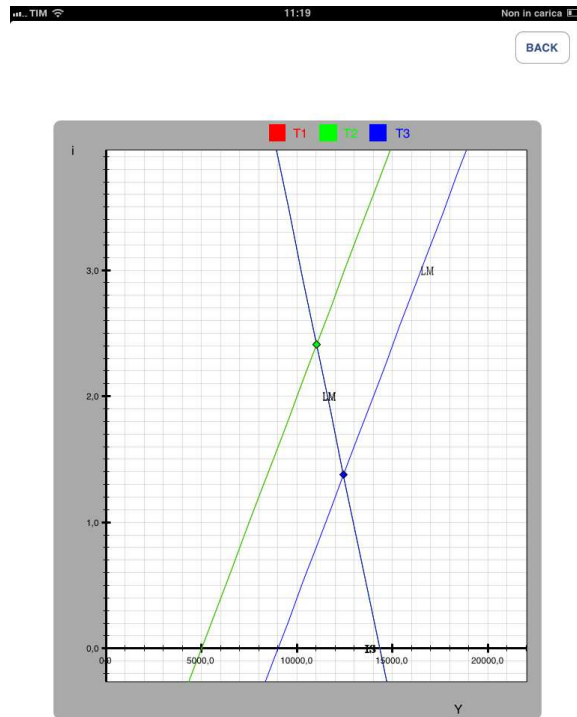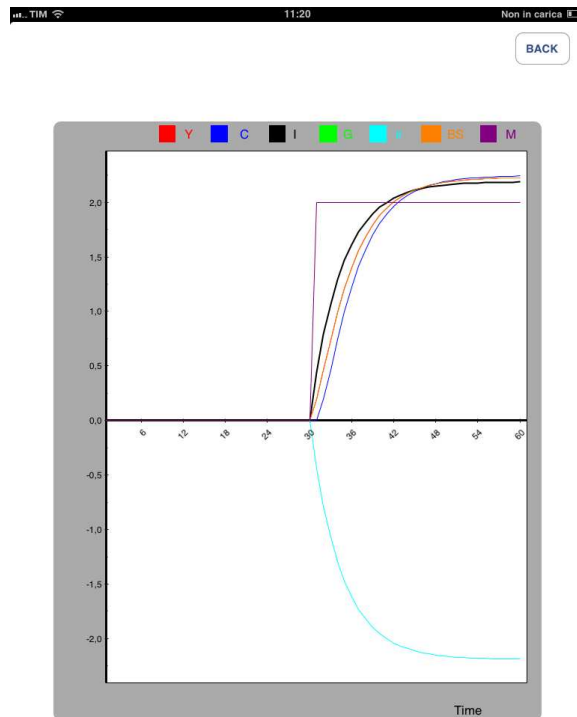
(a) IS-LM static

(b) IS-LM dynamics

Figure 3.1: Case 3.1 Fiscal Policy :$G \uparrow$;

(a) IS-LM static

(b) IS-LM dynamics

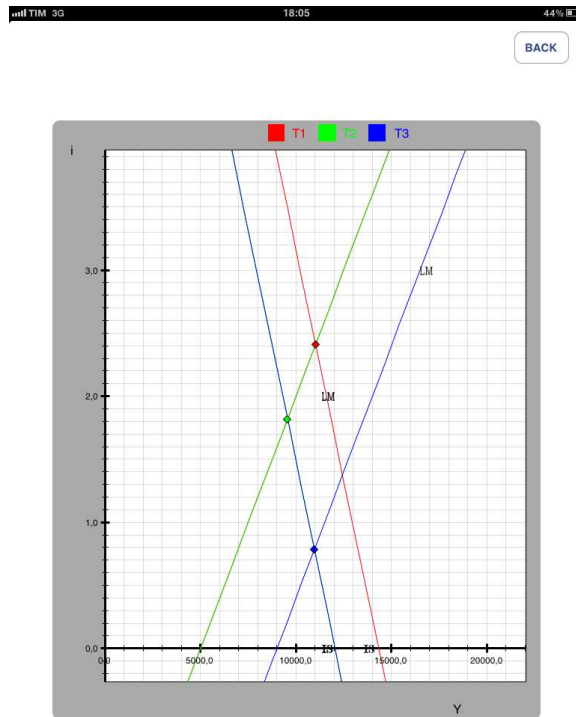Figure 3.2: Case 3.2 Monetary Policy: $M \uparrow$
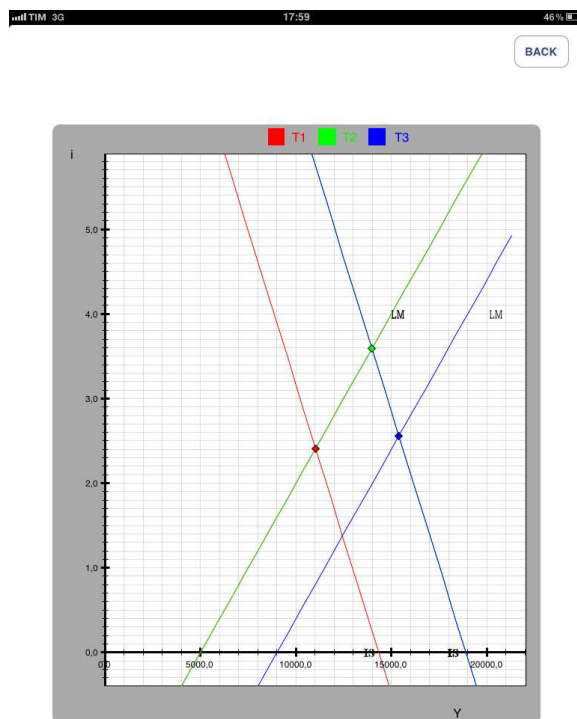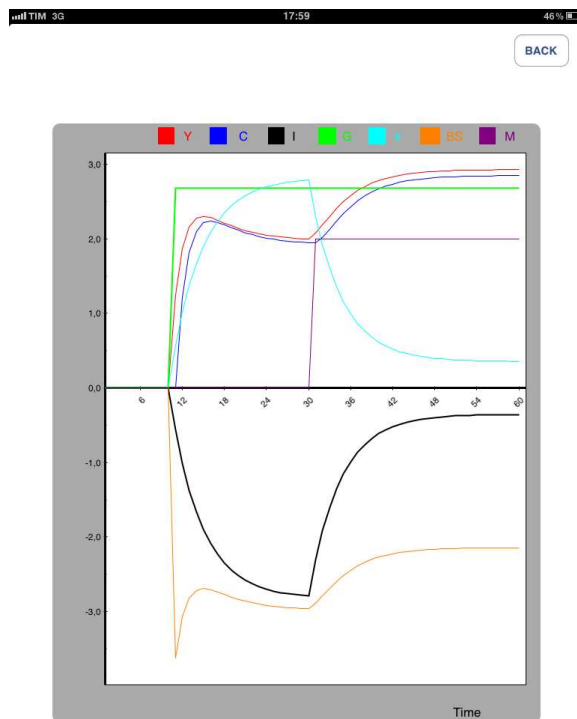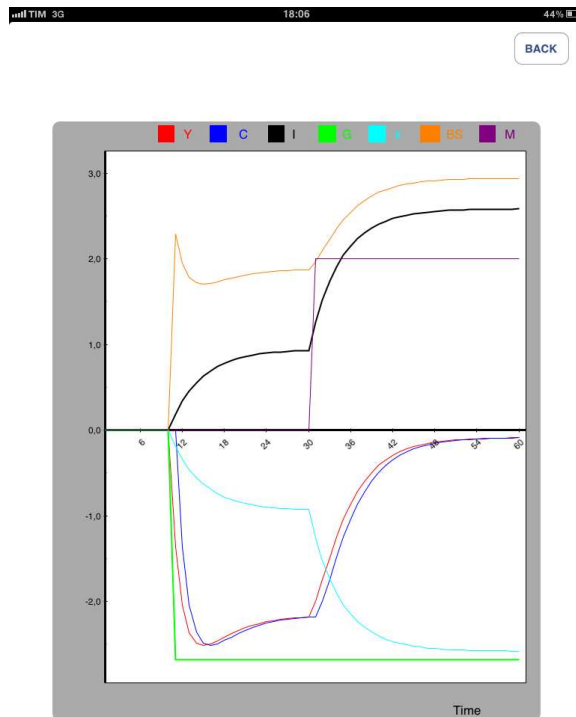
(a) IS-LM static

(b) IS-LM dynamics

Figure 3.3: Case 3.3 Fiscal Policy + Monetary Policy: $G \uparrow$ (at t_1); $M \uparrow$ (at t_2)

(a) IS-LM static

(b) IS-LM dynamics

Figure 3.4: Case 3.4 Fiscal Policy + Monetary Policy: $G \downarrow$ (at t_1); $M \uparrow$ (at t_2)

4.3.   Legenda of the IS-LM Model

Legenda

Table 3.1: Input Real Economy - Government

Symbol	Variable / Parameter
$\overline{C}$	Autonomous (exogenous) Consumption
I_0	Net Investment
NX	Net Export
c	Marginal Propensity To Consume
d	Marginal Propensity to Invest
G	Government purchase of goods and services
TR	Net Government Transfers paryments
t	Income tax rate

Table 3.2: Financial Market

Symbol	Variable / Parameter
k	Sensibility of money demand to income
h	Sensibility of money demand to interest rate
P	Prices level
lti	Liquidity trap interest rate
M	Money supply

Table 3.3: Output

Symbol	Variable / Parameter
Fisc Multiplier	Fiscal Multiplier
Mon Multiplier	Monetary Policy Multiplier
Eq. Income	Equilibrium Income
Eq. Consumption	Equilibrium Consumption
Eq. Investment	Equilibrium Investment
Balance	Government Surplus
Income	Income Variation

Table 3.4: Graph

Symbol	Variable / Parameter
Y	Income
C	Consumption
I	Net Investment
G	Government purchase of goods and services
i_r	Interest rate
BS	Government Surplus

Chapter 4

The IS-MP model

1.   The IS Curve

The real sector is given by the followinf system:

$$Y = AD$$

$$AD = C + I + G + NX$$

$$C = \overline{C} + cTR + c(1 - t)Y$$

$$I = \overline{I} - br + \theta Y$$

$$G = \overline{G}$$

$$TR = \overline{TR}$$

$$NX = \overline{NX}$$

In equilibrium the supply Y is equal to the demand AD, $Y = AD$:

$$Y = \overline{C} + cTR + C(1-t)Y + \overline{I} + \theta Y - br + \overline{G} + NX \tag{4.1}$$

or:

$$Y_e = \frac{1}{(1 - c(1-t) - \theta)}[\overline{C} + \overline{G} + cTR + NX + \overline{I} - br] \tag{4.2}$$

$$Y_e = \frac{1}{(1 - c(1-t) - \theta)}[A - br] \tag{4.3}$$

where

$$A = \overline{C} + \overline{G} + cTR + NX + \overline{I} \tag{4.4}$$

2.   The Monetary Market: the Taylor's rule

Here we introduce the simply Taylor's rule (Taylor, 1993) as in Romer (2006). It is based on two elements. Firstly, the nominal interest rate rise more than one-for-one with inflation. In other words, the real rate increases when inflation rises. Secondly, the interest rate rises (falls) when the output Y_t is above (below) the normal level Y^*. In formulas;

$$i = a + \pi_t + \alpha_\pi \pi_t + \beta_y(Y_t - Y^*) \tag{4.5}$$

with $\alpha_\pi > 0$ and $\beta_y > 0$.

$$i - \pi_t = a + \alpha_\pi \pi_t + \beta_y(Y_t - Y^*) \tag{4.6}$$

Taking into account the Fisher's equation

$$i = r + \pi_t \tag{4.7}$$

or

$$r = i - \pi_t \tag{4.8}$$

It is possibile to write the Curve MP (Monetary Policy):

$$r = a + \alpha_\pi \pi_t + \beta_y (Y_t - Y^*) \tag{4.9}$$

When $Y_t = Y^*$, we obtain:

$$r^* = a + \alpha_\pi \pi^* \tag{4.10}$$

$$\pi^* = \frac{r^* - a}{\alpha_\pi} \tag{4.11}$$

$$a = r^* - \alpha_\pi \pi^* \tag{4.12}$$

$$r = r^* - \alpha_\pi \pi^* + \alpha_\pi \pi_t + \beta_y (Y_t - Y^*) \tag{4.13}$$

and finally:

$$r = r^* + \alpha_\pi (\pi_t - \pi_t^*) + \beta_y (Y_t - Y^*) \tag{4.14}$$

that can be written as:

$$r = K + \beta_Y (Y_t - Y^*) \tag{4.15}$$

where:

$$K = r^* + \alpha_\pi (\pi_t - \pi^*) \tag{4.16}$$

The system to be solved is:

$$Y = A + [c(1 - t) + \theta]Y - br \tag{4.17}$$

$$r = K + \beta_Y(Y_t - Y^*) \tag{4.18}$$

The solution is:

$$Y = A + [c(1 - t) + \theta]Y - b[K + \beta_Y(Y_t - Y^*)] \tag{4.19}$$

$$Y = A + [c(1 - t) + \theta - b\beta_Y]Y - bK + b\beta_Y Y^* \tag{4.20}$$

$$Y - [c(1 - t) + d - b\beta_Y]Y = A - bK + b\beta_Y Y^* \tag{4.21}$$

The Equilibrium income is

$$Y_e = \frac{1}{1 - c(1 - t) - d + b\beta_Y}(A - bK + b\beta_Y Y^*) \tag{4.22}$$

The Equilibrium interest is:

$$r_e = K + \beta_Y(Y_e - Y^*) \tag{4.23}$$

or

$$r_e = K + \beta_Y\left(\frac{1}{1 - c(1 - t) - \theta + b\beta_Y}(A - bK + b\beta_Y Y^*) - Y^*\right) \tag{4.24}$$

The Fiscal Multiplier is:

$$\frac{1}{1 - c(1 - t) - \theta + b\beta_Y} \tag{4.25}$$

The Monetary Multiplier is:

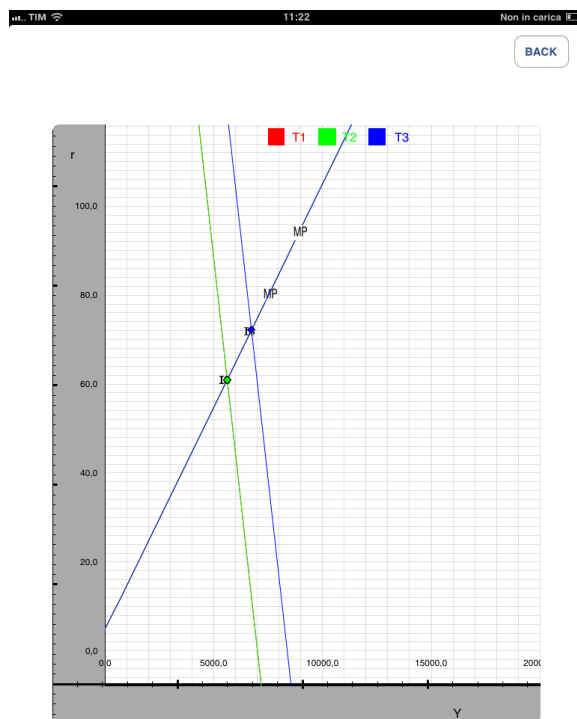$$\frac{-b}{1 - c(1 - t) - \theta + b\beta_Y} \tag{4.26}$$

Equilibrium Consumption:

$$C_e = \overline{C} + cY_e \tag{4.27}$$
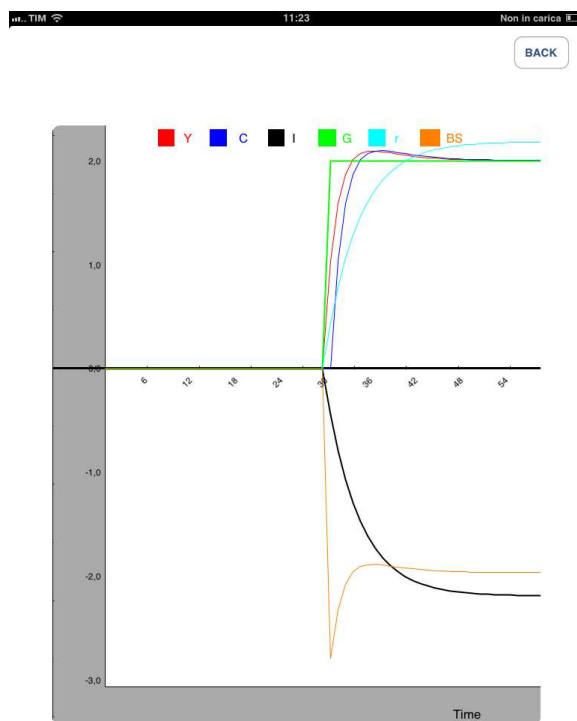
Equilibrium Investment:

$$I_e = \overline{I} + \theta Y_e - br_e \tag{4.28}$$

Here is a graphic example.

(a) IS-MP static

(b) IS-MP dynamics

Figure 4.1: Case 3.1 Fiscal Policy: $G \uparrow$

2.1.   Legenda of the IS-MP Model

Legenda

Table 4.1: Input Real Economy - Government

Symbol	Variable / Parameter
$\overline{C}$	Autonomous (exogenous) Consumption
I_0	Net Investment
NX	Net Export
c	Marginal Propensity To Consume
d	Marginal Propensity to Invest
G	Government purchase of goods and services
TR	Net Government Transfers paryments
t	Income tax rate

Table 4.2: Money Market - Central Bank

Symbol	Variable / Parameter
a	Nominal Interest Rate
$r*$	Long run or equilibrium real interest rate
$\pi*$	Central Bank's inflation objective
π	Current period inflation rate
α	Weight concerning income gap
β	Weight concerning inflation gap

Table 4.3: Output

Symbol	Variable / Parameter
Fisc Multiplier	Fiscal Multiplier
Mon Multiplier	Monetary Policy Multiplier
Eq. Income	Equilibrium Income
Eq. Consumption	Equilibrium Consumption
Eq. Investment	Equilibrium Investment
Balance	Government Surplus
Income	Income Variation

Table 4.4: Graph

Symbol	Variable / Parameter
Y	Income
C	Consumption
I	Net Investment
G	Government purchase of goods and services
R	Interest rate
BS	Government Surplus

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