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Cross-Hedging of Inflation Derivatives on Commodities:
The Informational Content of Futures Markets

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Abstract

According to the macro-econometric literature, the impact of exogenous oil price shocks on Inflation have greatly increased in the last two decades throughout OECD countries while the persistence of those shocks on long-term inflation, namely core inflation, has dramatically decreased. In the meantime, the market for inflation derivatives soared, spurred by a revival of the primary inflation-linked bond market. As the contribution of core inflation to the total headline inflation volatility bottomed, most of the volatility of headline inflation should thus be explained by changes in the spread between headline and core inflation indicators: a factor closely linked to commodity markets. This economic analysis should have important financial arbitrage implications in the futures market: are exogenous shocks on oil futures markets incorporated into zero coupon inflation indexed swap prices? To investigate this issue, we propose on the one hand a four-factor model for both inflation and nominal rates, and on the other hand a two-factor model for commodities. We proceed to an empirical estimation of the model using prices of oil futures contracts and inflation breakeven rates from which we can in particular extract a synthetic core inflation forward curve.

Keywords: Inflation, Core Inflation, Commodity Futures, Oil Futures, Breakeven Inflation Rates, Cross-Hedging, Inflation Pass-Through, Multi-dimensional Gaussian Model, Signal Processing, Kalman Filter, Equilibrium Pricing, Schwartz-Smith Model.

JEL classification: C51, C58, E31, E44, G13, Q43.

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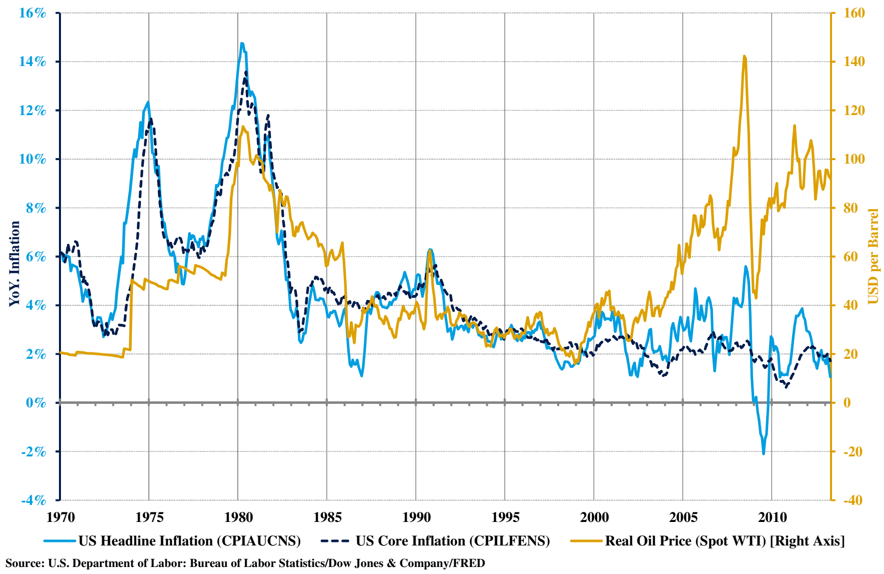
Contents

1. Introduction

Barely two decades ago, few papers would have endeavored to model the joint evolution of Core Inflation (CI) and Headline Inflation (HI) as the former was but a lagged indicator of the latter [17]. A basic Lag Operator would most probably have been an efficient modeling framework. Indeed, while the headline inflation measure derived from the evolution of the Consumer Price Index (CPI) became a household name for market practitioners and journalists, its Core Inflation counterpart derived from the CPI excluding Food and Energy prices remained mostly unknown except for a few central bankers and monetary economists. Moreover, and as a lasting legacy of the seventies' oil shocks, we have come to accept that exogenous oil price shocks (EOPS) are the driver of long term inflation. Thus, we used to firmly believe that any spread between the two was consistent with the lag induced by the pass-through of EOPS to the general level of prices in the economy. The effect of which should be persistent, thus leading CI to catch up with HI's level and thereby closing the spread between both inflation measures as illustrated by Figure 1. Yet, a structural macroeconomic shift that took place in all industrialized economies during the late eighties would challenge this status quo as [1] explained in their seminal paper.

Figure 1: US's headline vs. core inflation compared to the real price of oil over 40 years.

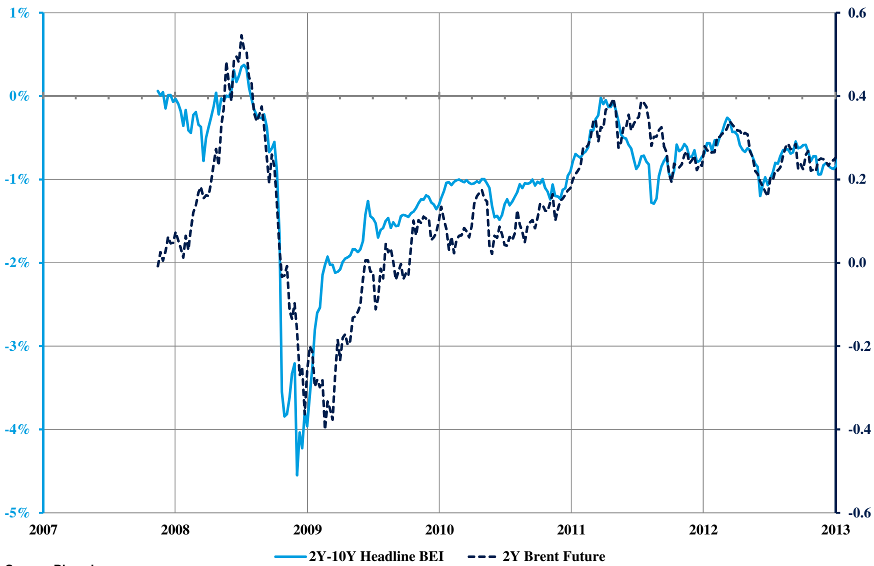


As advanced economies worldwide transformed themselves profoundly during the eighties, particularly through the rapid development of services to the detriment of energy hungry industrial activities, the energy intensity of output dropped. Moreover, during those years, new monetary policies in the form of inflation targeting were progressively put in place and seemed to have performed better than previous ones at reducing the macroeconomic impact of EOPS. Additional evidence of relaxation of nominal wage rigidity all feed into the macroeconomic regime change evidenced by [1]. The consequences of which being that during the last decade, EOPS have had virtually no impact on CI as is strikingly evidenced by both [15] and [3] even though their impact on HI had greatly increased according to [1]. As a result, we now have to deal with an increasingly more volatile HI which, however, mean reverts to the level of the CI [9], which has become increasingly stable in the meantime.

As evidenced by [8], the absence of EOPS pass-through into CI has several interesting implications for portfolio management: the first of which being that while the historic correlation between a proxy of traded commodities (GSCI-TR index) and CI is extremely low, conversely its correlation with the (HI-CI) spread has been extremely high (significantly above 80%). The second one is that since CI became very stable while HI became increasingly volatile, the share of the variance of HI explained by changes in CI is fairly low (slightly under 16%), thus highlighting the increasing role of commodities as the driver of inflation volatility even if its long-term level is determined by CI as a result of the mean reversion of [9]. Modeling the HI-CI spread, itself intrinsically linked with commodities, is clearly of the essence if we want to investigate cross-hedging possibilities between commodities and inflation derivatives.

Most inflation derivatives existing today were designed in the early eighties, at a time when choosing between core and headline inflation measures was not really an issue. The development of new products using CI as an underlier has been severely curtailed by the fact that it is currently an untraded quantity, thus also lacking a mark-to-market (MtM) reference valuation. HI-linked derivatives have been arbitrable because there is a primary market for HI, which is mostly made-up of inflation-linked government bonds from which can be extracted a forward curve for HI called the Breakeven Inflation rate curve (BEI), which enables the development of the inflation-linked derivative market. As appealing as CI-linked instruments might look for sophisticated investors, the lack of a primary market is a true hindrance. It should be noted that there exists a measure of expectations regarding future core inflation in the form of the quarterly Survey of Professional Forecasters (SPF) conducted by the Philadelphia's Federal Reserve. It is nonetheless a very low frequency indicator compared with BEIs which can be obtained instantaneously from market quotes, and its reliability is subject to the honesty of contributors, not on any indisputable transaction price. A first attempt at providing this service was done by Deutsche Bank in 2012 when they launched an investable proxy of the US's CI. The proxy forward curve for CI was obtained using a linear regression model applied to an energy futures index and an inflation futures index [12]. Yet, this still falls far short of an outright model which could extract a market-based price for core inflation: a proxy for a Core Breakeven curve.

Figure 2: Slope of the Breakeven inflation curve (2Y-10Y) vs. 2Y oil future.



Source: Bloomberg.

Figure 3: 10Y BEI vs. the 3Y CI forecast from the SPF.

Source: Bloomberg.

Modeling CI and HI, either together or independently is now common practice. But as we mentioned earlier, little attention has been given in the literature on the forward aspect of CI, and even less so on its interactions with the forward HI curve or the forward crude oil curve. Yet, hedging derivatives is precisely performed on valuations and sensitivities with respect to forward curves, not spot prices. Thus any cross-hedging strategy should be made according to a model of this joint future evolution. Building upon the macroeconomic arguments and their econometric consequences presented above, we aimed to design a parsimonious model for the joint evolution of CI and HI based on the following hypothesis:

One of the US Federal Reserve's mandates is to ensure price stability. Though many academics present CI as the most efficient monetary policy target as in [13] or [17], there is a carefully managed ambiguity regarding the precise nature of the targeted inflation measure. Yet, if we are to believe that the Federal Reserve monetary policy is successful, it seems reasonable to assume that CI mean reverts to the target of the Fed. We thus chose a simple Vasicek model [16] to model a core factor c which should thus be constrained to mean revert around the targeted inflation value θ_c with speed k_c.

- The mean reversion property of HI to CI evidenced by [9] led us to model HI as a time series reverting around the CI anchor at speed k_i. Since [8] showed that the HI-CI spread is strongly cointegrated to traded commodities, it opens a way to link our inflation model to the desired commodity market, thus also reflecting [1] findings on the major role of oil prices on inflation's volatility.
- As we want to extend [8] work into futures markets, we aim to link the HI-CI spread with commodity futures. To simplify our modeling, we chose to reduce the commodity universe to crude oil futures which are both very liquid and there is an easily available historic dataset. Yet, even if crude oil futures are heavily traded, they experience a very significant level of volatility, the magnitude of which greatly dwarfs that of inflation indices. To extract a much less noisy equilibrium price from the volatile market quotes, we used the model of [14], as well as their filtering procedure to reduce the information of oil futures prices to a set of underlying state variables.

We aim to calibrate our model to relevant market information which is made up of the forward HI curve and oil futures. We expect the calibrated model to exhibit the following properties:

- As a result of its mean-reversion property, HI should converge toward the forecasted CI long-term level. Thus the long BEI should be used to calibrate the CI proxy BEI as can partially be seen in Figure 3.
- Since oil price movements should impact the short end of the curve more than the long one, the slope of the curve should be a relatively good proxy to calibrate our head-core spread factor. This intuition is validated by
- Figure 2.

The paper is organized as follows, section 2 presents a factorial model for inflation and nominal market. Section 3 describes the estimation procedure for the model. We propose a pragmatic estimation procedure which requires a minimal set of assumptions and "tests" the model from a phenomenological point of view only relying on the functional representation of the HI forward curve and the nominal yield curve. We also apply Kalman filtering to the model. Section 4 briefly presents the commodities model of [14]. Section 5 presents the results of the estimation and links the signal extracted from the inflation market with the ones extracted from the commodities market.

2. A factorial model for Inflation

In this section we build a factorial model for Inflation. The pricing of Inflation derivatives cannot be made without a model for the nominal discount curve. Classic approaches such as [11] model inflation as the exchange rate between the nominal and the real economy, instead we model inflation directly as a traded asset. Though this assumption is inconsistent with market reality[2], it leads to very similar formulae for ZCIIS prices. We only apply our model to study the phenomenology of the market inflation forward curve and do not use it for accurate and consistent pricing of inflation derivatives instruments. This allows assumptions to be simplified; in particular it allows us to neglect the convexity adjustments coming from the lag between the date of observation of the inflation index and the payment date of the coupon in the derivative.

2.1. Dynamic assumptions

In all the following we consider a continuous trading economy with a trading interval $[0, T^*]$ for a fixed $T^* > 0$ The uncertainty in the economy is characterized by a probability space $(\Omega, \mathcal{F}, P)$. Information evolves according to the augmented filtration $\{\mathcal{F}_t\}_{t\in[0,T^*]}$ generated by a $d + 2$ - dimensional standard Brownian motion W. We assume that the market is arbitrage-free which implies the existence of an equivalent martingale measure Q under which discounted asset prices are martingales. We directly model the dynamics of the underlying state variables under this measure.

We consider a vector dynamics (i_t, c_t, X_t^T), where X is a vector process in $\mathbb{R}^d$. The dynamics under the risk-neutral measure is given by

$$d\begin{pmatrix} i \\ c \\ X \end{pmatrix} = \begin{pmatrix} k_i(c-i) \\ k_c(\theta_c - c) \\ K_X(\theta_X - X) \end{pmatrix} dt + \Sigma dW$$

Where $\Sigma \in \mathcal{M}_d(\mathbb{R})$. We then assume that the nominal spot rate is given by

[2] Note that assuming the existence of a real market with traded real assets is no better assumption in terms of realism.

$$r_t = \delta + \sum_{i=1}^{d} X_t^i$$

The underlying market inflation index follows the dynamics under the risk-neutral measure

$$\frac{dI_t}{I_t} = i_t dt$$

The model belongs to the class of multi-factor Gaussian models. Both the ZC bonds and the ZCIIS are explicit functionals of the underlying state variables of the model it , ct , Xt . Given our specification, the factors (it , ct) are the factors driving the Inflation market, while the factors Xt are driving the nominal bond market.

2.2. Model interpretation

The state variables of the model admit a clear interpretation. The nominal market model is a classic linear Gaussian model [7],[6], and it is well known that by choosing different scales for the mean reversion parameters K, the factors can be identified with the main drivers of the curve (the long term rate, the slope, the curvature). The interpretation of the inflation market state variables is similar. The factor i is the instantaneous growth rate of inflation and can thus be seen as the market equivalent of HI. Precisely as it will be endogenous to our model, it is the instantaneous HI extrapolated by our model from the market inflation forward curve. The factor c is the mean to which HI reverts, this mean is stochastic (probably less volatile than i). According to economic literature, HI shows significant mean reversion to CI. Then the factor c shall be viewed as a market indicator for CI, again to be more precise than the instantaneous CI extrapolated by our model from the market inflation forward curve.

2.3. Yield and breakeven curve reconstruction formulas

The expression of nominal ZC bonds is easily derived from arbitrage-free constraints. We refer to [2] for further details, we have

$$\begin{aligned} P_{t,T} &= \exp(A(T-t) + B(T-t)^T X_t) \qquad (4) \\ B(\tau) &= -(K_X)^{(-1)}(I_d - e^{-K_X\tau})1_{R^d} \\ A(\tau) &= -\tau\delta + \int_0^\tau du B(u)^T K_X \theta_X \sum_{ij=1}^{d} (\Sigma\Sigma^T)_{i+2,j+2} \int_0^\tau du B_i(u) B_j(u). \end{aligned}$$

We now turn into the pricing of ZCIIS, let us note by $BEI\ (t,\tau\ )$ the date t value of the Breakeven rate for maturity τ from t and by $YI_{t,\tau}$ the continuously compounded rate, we have

$$P_{t,T}(1+BEI(t,\tau))^{\tau} = P_{t,T}e^{YI_{t,\tau}\tau} = \mathbb{E}^{Q}\left[e^{-\int_t^{t+\tau} dsr_s}\frac{I_T}{I_t}|\mathcal{F}_t\right], \tag{5}$$

where Q stands for the risk-neutral measure and $\mathcal{F}_t$ stands for the information at date t. From standard change of numeraire techniques [5] we have

$$(1+BEI(t,\tau))^{\tau} = \mathbb{E}^{Q^T}\left[\frac{I_T}{I_t}|\mathcal{F}_t\right],$$

where Q^T stands for the T -forward neutral measure i.e. the equivalent martingale measure associated to the numeraire $P_{t,T}$. We verify that the above conditional expectation admits an exponential affine expression w.r.t. the Inflation market factors i_t , c_t , and furthermore that this expression is homogeneous in t i.e. it only depends on τ.

$$\begin{aligned}
\mathbb{E}^{Q^T}\left[\frac{I_T}{I_t}|\mathcal{F}_t\right] &= \exp(C(\tau)+D_1(\tau)i_t+D_2(\tau)c_t) \qquad (6)\\
D(\tau) &= \begin{pmatrix} k_i & 0 \\ -k_i & k_c \end{pmatrix}^{(-1)}\left(I_2-\exp\left(\begin{pmatrix} k_i & 0 \\ -k_i & k_c \end{pmatrix}\tau\right)\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\\
C(\tau) &= \int_0^{\tau} duD_2(u)k_c\theta_c + D(u)^T\Sigma^2(2,2)^T D(u) + \sum_{i=1}^{d}(D_1(u)\Sigma^2_{1,i+2}+D_2(u)\Sigma^2_{2,i+2})B_i(u)
\end{aligned}$$

Where $M\ (2,2)\ =\ (M_{i,j}\ )_{1\le i,j\le 2}$ of the matrix Σ and $\Sigma^2 = \Sigma\Sigma^T$. In all the following we assume that K_X is a diagonal matrix.

Remark 1: Let us note that while keeping a correlation structure between the inflation market and the nominal market factors, the model exhibits a certain orthogonality property. The nominal yield curve only depends on the factors X while the inflation forward curve only depends on the factors (i,c). This allows us to confirm the model interpretation we have given above and will allow an efficient estimation of the model on market data.

2.4. A market forward curve for core inflation

The model allows us to build a “pseudo” CI forward curve. Miming the construction of the HI BEI curve, we define the CI forward curve. Let us denote by $BEIC(t,\tau\ )$ the date t value of the breakeven rate for maturity τ for CI and by $YIC_{t,\tau}$, the continuously compounded version of this rate. Then we take the following definition

$$(1+BEIC(t,\tau))^{\tau}=e^{YIC_{t,\tau}\tau}=\mathbb{E}^{Q^T}\left[e^{\int_t^{t+\tau}dsc_s}|\mathcal{F}_t\right]. \qquad (7)$$

The dynamics of the core factor under the risk neutral measure is given by

$$dc=k_c(\theta_c^T(T-t)-c)dt+\Sigma(2)^T dW_t^{Q^T}.$$

Where $\theta^T(\tau)=\theta_c+\frac{1}{k_c}\sum_{i=1}^{d}\Sigma_{2,2+i}^2 B_i(\tau)$. Then the expression of the forward core inflation curve is given by

$$(1+BEIC(t,\tau))^{\tau}=e^{YIC_{t,\tau}\tau}=\exp\left(A^c(T-t)+\frac{1-e^{-K_c(T-t)}}{k_c}c_t\right),$$

Where

$$A^c(\tau)=\int_0^{\tau}duB(u)^T k_c\theta_c^T(u)+\frac{1}{2}\Sigma_{22}^2\frac{(1-e^{-K_c(T-t)})^2}{k_c^2}.$$

We interpret the curve YIC as the market anticipation on the future CI.

Remark 2: Let us note that while the forward HI curve depends on both the headline i and the core c factors, the market CI forward curve is entirely determined by the core factor c.

3. Empirical estimation of the model

Both the dynamics of the nominal and the inflation market are entirely explained by the underlying state variables (i,c) and X. These variables are not directly observable in the market and must be estimated using the market data on the nominal yield curve and the inflation forward curve. Rather than proceeding to a complete but necessarily unstable (given the high dimensional setting) estimation of both the model factors and parameters, we propose an intuitive and efficient estimation procedure; specifically, it is a signal processing procedure. Our aim is not necessarily to estimate the model parameters but rather to extract a "clean" signal from market data. We test the model from a phenomenological point of view, meaning we will not test the relevance of the dynamical assumptions on the underlying state variables (1) but only the functional representation of the nominal yield curve (4) and of the inflation forward curve (6). We propose two alternative signal processing methods: a "pragmatic" one based on the variations of the yield curve and the forward inflation curve, and a classic Kalman filtering procedure. The results obtained by these two methods are very close.

3.1. Dataset description

Our dataset consists in weekly observations of prices for US ZCIIS of 1, 2, 5 and 10-year maturities for the inflation market (4 contracts in total), and of the yield curve build from US swap rates (we only take 5 pillars corresponding to 3 months, 1, 2, 5 and 10 years). We use close prices on Wednesdays (to avoid noise coming from end/beginning of week market jumps) from November 2007 to January 2013, making a total of 273 sets of observations of the 4 ZCIIS and of the 5 pillars. These prices were obtained through Bloomberg, specifically using the following Tickets:

- Ticket USSWIT1, USSWIT2, USSWIT5 and USSWIT10 for HI BEI
- Ticket US0003M, US0001Y, US0002Y, US0005Y and US00010Y for the swap rates
-

3.2. Signal processing methodologies

Our signal processing methods are based on two key facts: the homogeneity of the model, and the fact the nominal yield curve only depends on the factors X, while the inflation breakeven curve only depends on the factors i and c.

3.2.1. Pragmatic signal processing: "variations"

In the model the movements of the yield curve are entirely captured by the movements of the underlying state variables X . For each market we have an historic series of the nominal yield curve $(Y_{t_1,\tau})_\tau, \dots, (Y_{t_N,\tau})_\tau$ and of the forward Inflation curve $(YI_{t_1,\tau})_\tau, \dots, (YI_{t_N,\tau})_\tau$. We build the series of variations

$$\begin{aligned} \Delta Y_{t_n,\tau} &= Y_{t_n,\tau} - Y_{t_{n-1},\tau} \\ \Delta YI_{t_n,\tau} &= YI_{t_n,\tau} - YI_{t_{n-1},\tau}. \end{aligned}$$

We estimate the optimal factors X variations vector $\widehat{\Delta X_{t1}}$ as the solution of the following optimization problem

$$\widehat{\Delta X}_{t_n} = \mathrm{argmin}_{\delta x \in \mathbb{R}^d} \sum_{s=1}^{M_1} \left(\Delta Y_{t_n,\tau_s} - \sum_{p=1}^{d} \frac{1 - e^{-k_p^X \tau_s}}{k_p^X \tau_s} \delta x_p \right)^2 .$$

Where $\tau_1, \dots, \tau_{M1}$ is a set of tenors. The above optimization problem admits an explicit solution given by

$$\widehat{\Delta Y}_{t_n} = \left(\sum_{s=1}^{M_1} \omega(\tau_s)\omega(\tau_s)^T\right)^{-1} \sum_{s=1}^{M_1} \Delta Y_{t_n,\tau_s}\omega(\tau_s).$$

where we have defined

$$\omega(\tau) = \left(\frac{1-e^{-k_1^X\tau}}{k_1^X\tau}, ..., \frac{1-e^{-k_d^X\tau}}{k_d^X\tau}\right).$$

Likewise, we estimate the inflation factors

$$(\widehat{\Delta i, \Delta c})_{t_n} = \mathrm{argmin}_{(\delta i,\delta c)\in\mathbb{R}^2} \sum_{s=1}^{M_2} \left(\Delta YI_{t_n,\tau_s} - D_1(\tau)\delta i - D_2(\tau)\delta c\right)^2.$$

Where $\tau_1, \dots, \tau_{M1}$ is a set of tenors. The above optimization problem admits an explicit solution given by

$$(\widehat{\Delta i, \Delta c})_{t_n} = \left(\sum_{s=1}^{M_2} \omega^I(\tau_s)\omega^I(\tau_s)^T\right)^{-1} \sum_{s=1}^{M_2} \Delta YI_{t_n,\tau_s}\omega^I(\tau_s).$$

where we have defined

$$\omega^I(\tau) = (D_1(\tau), D_2(\tau)).$$

Let us note that the optimal factors only depend on the values of the functions B and D at the tenors chosen for the estimation procedure. These functions are entirely determined by the mean reversion parameters k_i, k_c and K_X . We claim that these parameters can be arbitrarily set (to keep interpretability of the factors) and that their value does not have a major impact on the capacity of the model to reproduce the variations of the curves.

3.2.2. Kalman filtering

Kalman filtering can naturally apply to our context, for a detailed description of the method we refer to [10]. The SDEs (1) can easily be discretized in order to derive the transition equation of the filter

$$d\begin{pmatrix} i \\ c \\ X \end{pmatrix}_{n+1} = e^{-K\Delta t}X_n + (I - e^{-K\Delta})\begin{pmatrix} 0 \\ \theta_c \\ \theta_X \end{pmatrix} + \sqrt{\int_0^\Delta due^{-Ku}\Sigma^2 e^{-Ku}}G_n \tag{8}$$

where Δ is the time-step, and (G_{n}) is a sequence of $d + 2$-dimensional random vectors made of i.i.d. Gaussian variables, the square root is defined for positive semi defined matrices and

$$K = \begin{pmatrix} k_i & -k_i & 0 \\ 0 & k_c & 0 \\ 0 & 0 & K_X \end{pmatrix}.$$

Since the relationship between the observations (yield curve, continuously compounded inflation breakeven curve) and the state variables is linear, the modeling framework perfectly fits into the Kalman filter. The observations equation is the following

$$\begin{pmatrix} Y_{n,\tau_1} \\ \cdot \\ Y_{n,\tau_{M_1}} \\ YI_{n,\tau_1^I} \\ \cdot \\ YI_{n,\tau_{M_2}^I} \end{pmatrix} = \begin{pmatrix} A(\tau_1) \\ \cdot \\ A(\tau_{M_1}) \\ C(\tau_1^I) \\ \cdot \\ C(\tau_{M_2}^I) \end{pmatrix} + \begin{pmatrix} \omega((\tau_1)^T X_n \\ \cdot \\ \omega((\tau_{M_1})^T X_n \\ D_1((\tau_1^I)i_n + +D_2((\tau_1^I)c_n \\ \cdot \\ D_1(\tau_{M_2}^I)i_n + +D_2(\tau_{M_2}^I)c_n \end{pmatrix} + W_n \tag{9}$$

where $(W_n)_n$ is a sequence of independent Gaussian vectors of variance R. We do not go further into implementation details here, since it is not the purpose of the paper. As mentioned before we do not perform a proper estimation of the model parameters, classic estimation methods use maximum likelihood or mean square estimators for the diffusion parameters. These methods lack of robustness, especially in a multi-dimensional setting as ours, and robust estimation procedures require sophisticated techniques. Limiting ourselves to the estimation of the factors is sufficient for our purposes. We conjecture that the simple estimation procedure presented above gives results that do not significantly differ from the ones given by the Kalman filter. Yet, we can play on the observations noise parameter R to smooth the signal extracted from the Kalman filter.

3.3. Numerical results

The behavior of the nominal interest rate model is well known [4]. Let us just mention that by choosing different scales for K_X the factors X will naturally be identified with the long term rates, the slope, the curvature, etc, which is illustrated in Figure 4.

The factor i is the instantaneous market inflation rate. It can be viewed as a (continuously compounded) HI market quote. The factor c a stochastic long-term mean of instantaneous inflation. Inflation rates as Interest rates are known to be mean reverting, and it is natural to consider mean reverting dynamics for the instantaneous inflation i. These factors happen to be highly related to the short-term rates (2Y breakeven rate) and the long-term rate of the forward inflation curve, see Figure 5. Note that the very short inflation futures are not very liquid in the market trades and the short-term curve is usually determined by trading desk economists. We therefore only used the ZCIIS of maturity longer than 2Y for our estimation.

4. Filtering the oil futures signal

A rough analysis of the US market data suggests a strong link between the slope inflation forward curve and the oil futures (see

Figure 2). Our factorial inflation model allows us to map the slope of the forward curve by a stochastic factor $i - c$. In this section we use the Schwartz-Smith [14] model of commodities futures prices to filter the signal coming from the oil futures market. In the two-factor model, one of the factors represents the equilibrium price of the market, the second represents the short-term deviation in log prices and futures prices are mean reverting with respect to this factor.

4.1. Model definition

The spot price is assumed to be driven by two underlying factors. Let us denote by S_t the spot price of oil at time t and $s_t = log\ S_t$. We assume that s is the sum of two factors

$$s_t = \chi_t + \xi_t$$

The dynamics of the factors under the risk neutral measure are given by

$$d\chi_t = (-k\chi_t - \lambda_\chi)dt + \sigma_\chi dZ_\chi$$

$$d\xi_t = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dZ_\xi$$

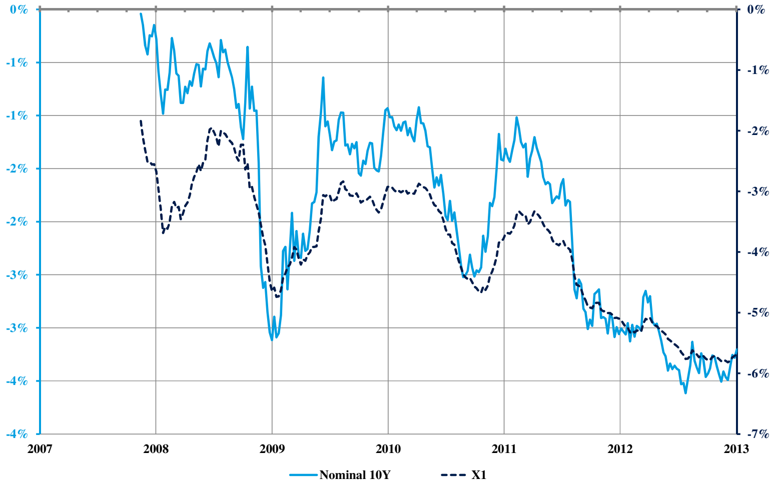
where Z_χ and Z_ξ are correlated standard Brownian motions with correlation parameter $\rho_{\chi\xi}$. The parameters λ_χ and λ_ξ are interpreted as the risk premiums for the factors χ and ξ. We can write the dynamics of the log prices in the model

$$ds_t = -k(s_t - \tilde{\xi}_t)dt + (\sigma_\xi\ ,\ \sigma_\chi)dB_t$$

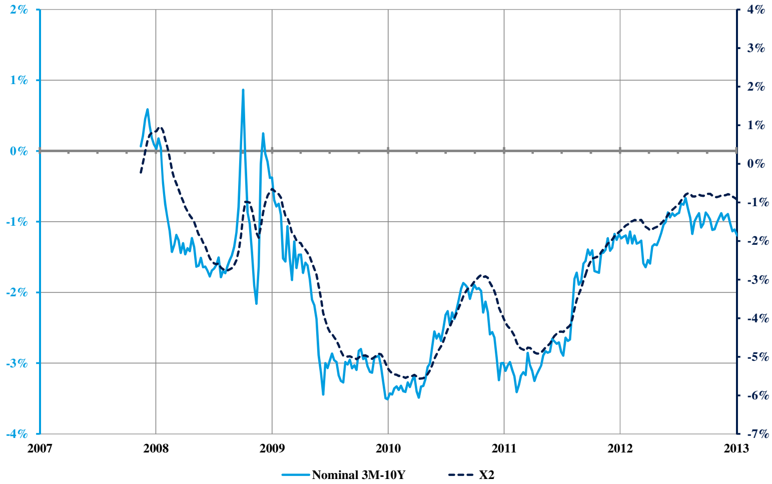
Where $B_t = (Z_\xi\ , Z_\chi)$ and $\tilde{\xi}_t = \xi_t + \frac{(\mu_\xi - \lambda_\xi - \lambda_\chi)}{k}$. The log price is thus mean reverting to the process $\tilde{\xi}$. From risk-neutral valuation theory we have that the time t price of a future of maturity τ is given by

$$F_{t,\tau} = \mathbb{E}^Q[S_{t+\tau} \mid \mathcal{F}_t]$$

Figure 4: Nominal yield curve drivers against risk factors estimated by Kalman filter. 10Y nominal rate against first nominal factor X_1. 3M-10Y nominal slope against rescaled second nominal factor X2.

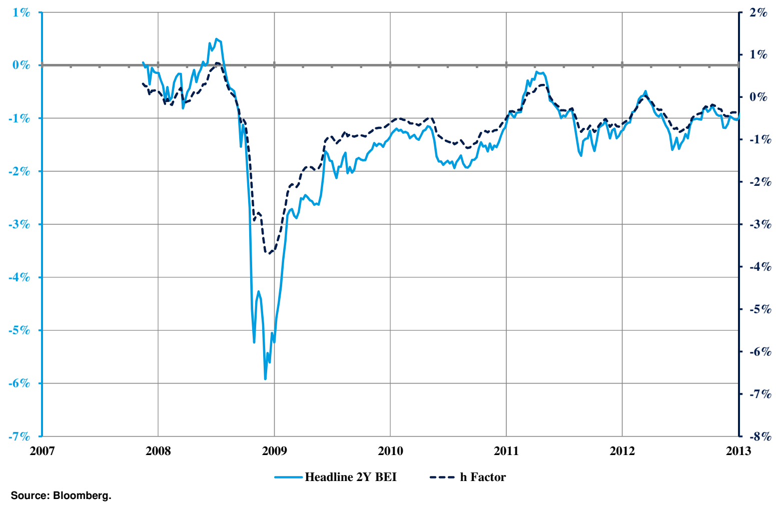


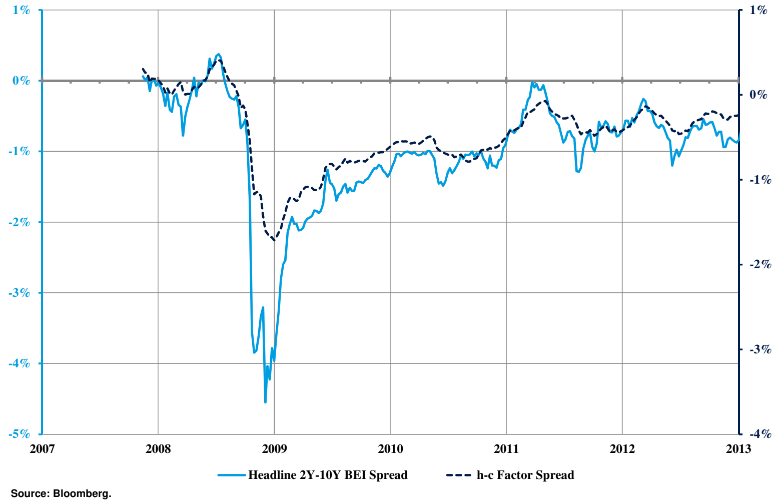
Source: Bloomberg.



Source: Bloomberg.

Figure 5: Inflation breakeven yield curve drivers against risk factors estimated by Kalman filter. Continuously compounded 2Y breakeven rate against rescaled factor i. 2Y-10Y breakeven slope against rescaled factor h-c.





We then deduce the following expression for the forward price

$$F_{t,\tau} = exp\left(e^{-k\tau}\chi_{\ t} + \xi_{\ t} + E(\tau)\right)$$

Where

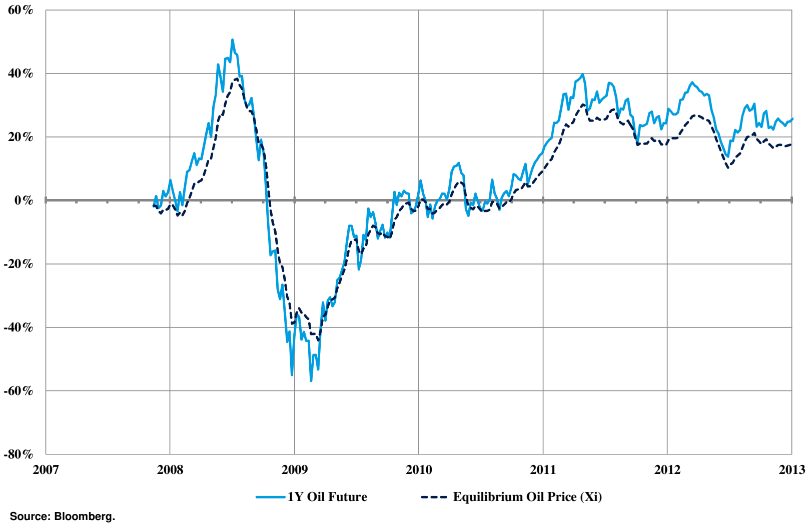
$$E(\tau) = \mu_{\xi}\tau - \frac{1 - e^{-k\tau}}{k}\lambda_{\chi} + \frac{1}{2}\left(\frac{1 - e^{-2k\tau}}{2k}{\sigma_{\chi}}^{2} + {\sigma_{\xi}}^{2}\tau + 2\frac{1 - e^{-k\tau}}{k}\rho_{\chi\xi}\sigma_{\xi}\sigma_{\chi}\right)$$

Since the model is homogenous, we can mimic the estimation procedure we have applied for the inflation model to estimate the state variables on Brent future prices.

4.2. Numerical results

The factor ξ is interpreted as the equilibrium price of the oil market. Empirical evidence shows that futures prices are mean reverting around this factor as illustrated by Figure 6.

Figure 6: Commodity futures prices and equilibrium price in Schwartz model estimated by Kalman filter.



The factor χ takes into account the short-term variations of the futures prices, this factor is more volatile than the equilibrium price factor price, it is also strongly related to the futures price (correlation between the series of 1Y futures prices and the factor χ is above 87%).

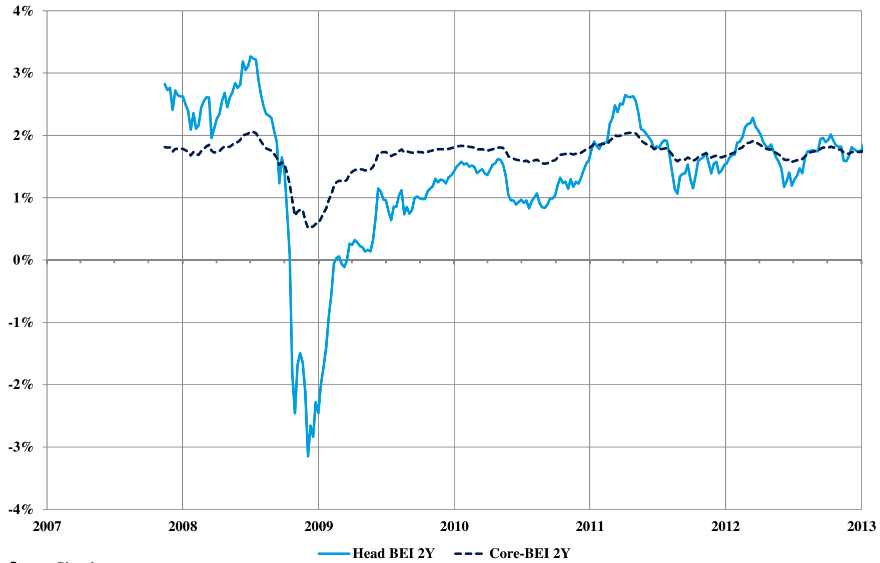
5. Linking commodities futures and inflation forwards

Our modeling frameworks for inflation and oil futures allowed us to filter the rough data and isolate the main drivers of the market materialized through underlying factors. Our inflation model allowed us to build a market indicator of the spread between headline and core inflation, which behaves similarly to the slope of the forward inflation curve. The commodities model of [14] allowed us to build a market indicator of the market equilibrium price of the oil market, as well as an indicator of the short term fluctuations of the futures prices.

5.1. Market estimation of core inflation

From (2.4) and the estimated core factor c, one can analyze the historic evolution of the market's CI anticipations. Figure 7 shows the evolution of the HI forward against the CI forward reconstructed from the estimated model. Consistently with the intuition coming from the macroeconomic analysis of the main inflation drivers, the HI BEI mean reverts to our CI BEI indicator. The model thus gives promising results for its capacity to isolate a market signal for CI enabling us to deduce a market signal for the spread between HI and CI.

Figure 7: 2Y BEI HI vs. 2Y BEI CI.



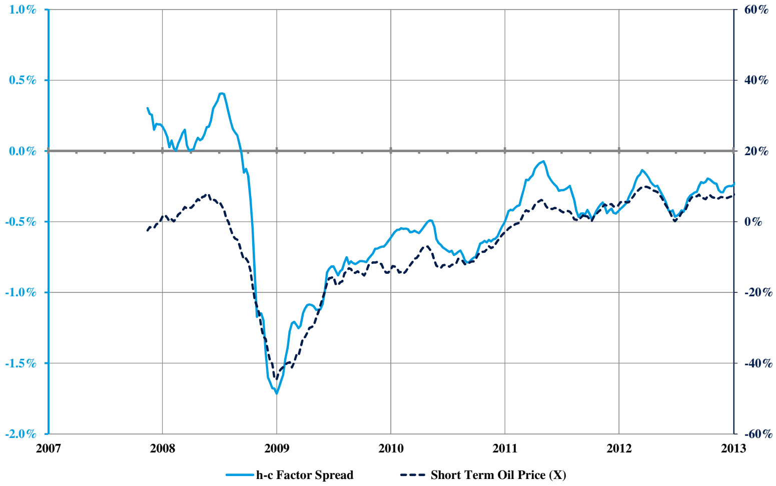
Source: Bloomberg.

5.2. A promising relationship

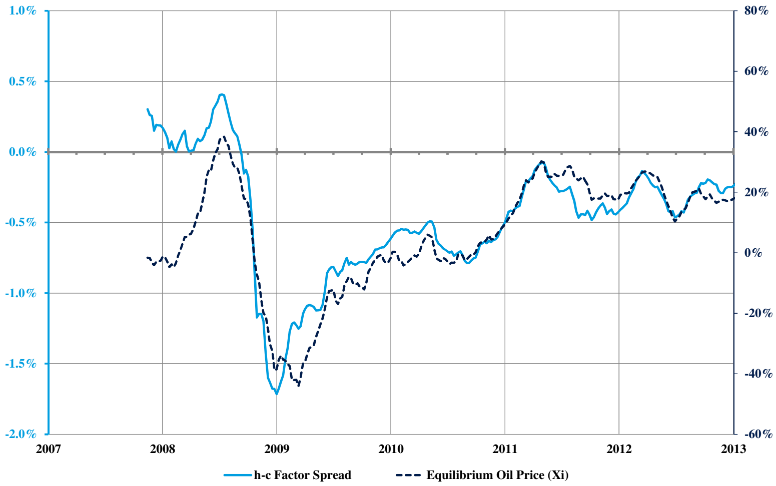
One of the aims of this work is to answer whether the strong link between the HI-CI spread and the commodities market is reflected in the corresponding derivatives markets. Empirical evidence shows that the estimated factors $i - c$ (which are the main drivers of the slope of the BEI curve) and the factors driving the commodities market are highly correlated and one seems to mean revert to the other as illustrated by Figure 8.

As mentioned before the Inflation-linked derivatives market's primary assets are the inflation-indexed bonds (these are mainly government bonds), the relationship with the commodities futures market seems to be so strong that it cannot be reduced to the relationship between HI and commodities prices. Though there is no primary market for CI based bonds, the model allows for an identification of a CI market indicator and a spread between a HI and CI market indicator which happen to be dramatically linked to the market indicators for oil prices, allowing the question to be answered positively.

Figure 8: Estimated spread $i_t - c_t$ against equilibrium price ξ_t and short term price fluctuations χ_t . In both cases the correlation between the series on the period is over 82%.



Source: Bloomberg.



Source: Bloomberg.

5.3. Potential trading strategies of commodities against inflation

As interest rates hardly reflect the economic reality of OECD countries, and monetary policies are failing to boost Inflation, the interest for Inflation protections has increased among investors, boosting the demand for Inflation-linked derivatives. This increase in demand has only partially been fulfilled by a slight increase in inflation-indexed bond issues. The primary Inflation market is often too small to hedge the demand of Inflation-linked derivatives. On the other hand the commodities derivatives market is large enough to fill the gap and some sections of the commodities market are very liquid. From a hedging perspective, cross-hedging of Inflation-linked derivatives with commodities seems a promising path. This would lead to the creation of CI based derivatives, which are potentially very attractive contracts for investors looking to hedge their long-term inflation position (such as pension funds for instance).

One can also look to the results of our estimation from a purely speculative point of view. The high correlation, and the mean reverting property of the signals $i - c$ and χ, ξ can be exploited to design an arbitrage trading strategy between oil futures and ZCIIS. However, the implementation of such a strategy needs to be thought through carefully. Suppose we want to exploit the relationship between $i - c$ and ξ. The 2Y − 10Y ZCIIS slope mean reverts towards $i - c$ and the 1Y oil future mean reverts towards ξ, then a natural strategy would be to go long when the market prices are lower than the estimated signal, and short when the market prices are higher than the estimated signals. Another strategy would be to build a long-short position depending on the position of one signal against the other. All these simple strategies require that we are able to roll a position on either the 2Y−10Y ZCIIS or the 1Y oil future, either directly on factors $i - c$ and ξ. While rolling a position on a commodity future is a fairly easy task, it is impossible to do so on a ZCIIS. Money management considerations are crucial to build a robust strategy. The appeal of the model is that in theory a position on the factor i − c (resp. ξ) can be replicated by different choices of ZCIIS (resp. oil futures). However our model is not designed for accurate consistency with market prices, it is rather a strategic tool to analyze the market main trends and it must be coupled with a more accurate pricing model if one aims to build a sophisticated cross trading strategy.

6. Conclusion

In this paper we have presented a detailed analysis of the links between the Inflation-linked derivatives market and the oil futures market. We proposed on the one hand a four-factor model for both inflation and nominal rates, and on the other hand a two-factor model for commodities. The inflation model allows us to break down the forward inflation curve into a part attributed to the market anticipations of CI and a part attributed to the spread between HI and CI. The CI forward curve is explicitly built from the model. The commodities model applied to the oil futures market allows to identify two factors which are associated to the equilibrium price and the short-term variations of the futures prices. The model is not designed for accurate pricing of both inflation-indexed and commodities derivatives, we are interested in its ability to capture the phenomenology of the market prices dynamics.

A signal processing procedure allowed an estimation on the US market data on the 2007-2013 period. We have not proceeded to an accurate estimation of the model parameter, but are rather interested in the estimation of the model factors, which we use as the “clean” signal coming from the market. Empirical results are consistent with macro-economic analysis. The actual BEI exhibits a mean reversion property towards the extracted market indicators for forward CI. The market factor of the spread between the HI and the CI appears to be highly correlated to the commodities market drivers.

The results suggest that the inflation market is mature enough to build CI based derivatives, that can be cross-hedged using oil futures. Though the study highlights an opportunity it does neglect some important aspects linked to the implementation of a cross-hedging strategy, and in particular the money-management dimension. Though the inflation-linked derivatives market has soared in recent years, it is still not very liquid compared to some sections of the commodities futures market. A strong link is evident from our study but how to exploit it needs some additional work.

References

[1] J. Blanchard, O. J. and Gali. The macroeconomic effects of oil shocks: Why are the 2000s so different from the 1970s? Working Paper 13368, National Bureau of Economic Research, September 2007.

[2] D. Brigo and F. Mercurio. Interest Rate Models - Theory and Practice: With Smile, Inflation and Credit (Springer Finance). Springer, 2nd edition, September 2006.

[3] T. E. Clark and S. J. Terry. Time variation in the inflation pass-through of energy prices. Journal of Money Credit and Banking, 42:1419–1433, 2010.

[4] Geman H. El Karoui, N. and A. Frachot. A note on the behavior of long zero coupon rates in a no arbitrage framework. Working paper, 1997.

[5] Geman H. El Karoui, N. and J. C. Rochet. Changes of numeraire, changes of probability measures and option pricing. Journal of Applied Probability, 32:443–458, 1995.

[6] Lacoste V. El Karoui, N. Multifactor models of the term structure of interest rate. AFFI Congress 1992, 1992.

[7] Myneni R. El Karoui, N. and R. Viswanathan. Arbitrage pricing and hedging of interest rate claims with state variables: I, ii. Working Paper, Paris VI and Stanford University, 1992.

[8] N. Fulli-Lemaire. An inflation hedging strategy with commodities. Journal of Investment Strategies, 2(4):1–28, 2013.

[9] G. Gelos and Y. Ustyugova. Inflation responses to commodity price shocks - how and why do countries differ? IMF Working Paper, 2(4):1–31, 2012.

[10] A. C. Harvey. Forecasting, Structural Time Series Models and the Kalman Filter. Cambridge University Press, March 1990.

[11] R. Jarrow and Y. Yildirim. Pricing Treasury Inflation Protected Securities and Related Derivatives Using an HJM Model. Journal of Financial and Quantitative Analysis, 38:337-358, 2003.

[12] A. Li and S. Zeng. DB core US CPI index. DB Fixed Income Special Report, pages 1–8, 2012.

[13] F. S. Mishkin and K. Schmidt-Hebbel. One decade of inflation targeting in the world: What do we know and what do we need to know? Working Paper 8397, National Bureau of Economic Research, July 2001.

[14] E.S. Schwartz and J. E. Smith. Short-term variations and long-term dynamics in commodity prices. Manage. Sci., 46(7):893–911, July 2000.

[15] Andrée C. Van den Noord, P. Why has core inflation remained so muted in the face of the oil shock? OECD Papers, 7(4):1–34, 2007.

[16] O. Vasicek. An equilibrium characterization of the term structure. Journal of Financial Economics, 5(2):177 – 188, 1977.

[17] M. A. Wynne. Core inflation: a review of some conceptual issues. Federal Reserve Bank of St. Louis Review, (May):205–228, 2008.

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