A Comparative Study for Estimation Parameters in Panel Data Model

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This paper examines the panel data models when the regression coefficients are fixed, random, and mixed, and proposed the different estimators for this model. We used the Mote Carlo simulation for making comparisons between the behavior of several estimation methods, such as Random Coefficient Regression (RCR), Classical Pooling (CP), and Mean Group (MG) estimators, in the three cases for regression coefficients. The Monte Carlo simulation results suggest that the RCR estimators perform well in small samples if the coefficients are random. While CP estimators perform well in the case of fixed model only. But the MG estimators perform well if the coefficients are random or fixed.

Key words: Panel Data Model, Random Coefficient Regression Model. Mixed RCR Model, Monte Carlo Simulation, Pooling Cross Section and Time Series Data. Mean Group Estimators. Classical Pooling Estimators.

1. Introduction

Econometrics commonly use "Time Series Data" describing a single entity. Another type of data called "Panel Data" which means any data base describing number of individuals across a sequence of time periods. To realize the potential value of the information contained in a panel data see Carlson (1978), and Hsiao (1985, 2003), and Baltagi (2008). `

When the performance of one individual form the panel data is interest, separate regression can be estimated for each individual unit. Each relationship, on our model studied, is written as follows:

$$y_{it} = \beta_{0i} + \beta_{1i} x_{it} + \varepsilon_{it}, \quad i = 1,2,3,\ldots,N \quad t = 1,2,3,\ldots,T, \tag{1}$$

where i denotes cross-sections and t denotes time-periods. The ordinary least squares (OLS) estimators of β_{0i} and β_{1i} will be best linear unbiased estimators (BLUE) under the following assumptions:

A1: $E(\varepsilon_i) = 0$

A2: $E(\varepsilon_i \varepsilon_i') = \sigma_i^2 I_T$

A3: $E(\varepsilon_i \varepsilon_j') = 0, \quad for\ all\ i \neq j$.

These conditions are sufficient but not necessary for the optimality of the OLS estimator, see Rao and Mitra (1971). If assumption 2 is violated and disturbances are either serially correlated or heteroskedastic, generalize least squares (GLS) will provide relatively more efficient estimator than OLS, see Gendreau and Humphrey (1980). If assumption 3 is violated and contemporaneous correlation is present, we have what Zellner (1962) termed seemingly unrelated regression (SUR) equations. There is gain in efficiency by using SUR estimator rather than OLS, equation by equation estimator, see Zellner (1962,1963).

Suppose that each regression coefficient in equation (1) is viewed as a random variable, that is, the coefficients β_{0i} and β_{1i} are viewed as invariant over time and varying from one unit to anther.

So, we are assuming that the individuals in our panel data are drown from a population with a common regression parameter,($\bar{\beta}_j,\ j=0,1$), which is fixed component, and a random component v_i which will allow the coefficients to differ from unit to unit, i.e.

A4: $\beta_{ji} = \bar{\beta}_j + v_{ji}$, *f*or i= 1,2,….,N, j=0,1.

Model (1) can be rewritten, under assumptions (1) to (4), as:

$$y_{it} = \bar{\beta}_{0i} + \vec{\beta}_{1i}\, x_{it} + e_{it}\,, \tag{2}$$

where

$$e_{it} = v_{0i} + x_{it}\, v_{1i} + \varepsilon_{it}\,, \quad i=1,2,\ldots,N,\ t=1,2,\ldots T,$$

model (2) is called “Random Coefficient Regression” model examined by Swamy (1970, 1971, 1973, 1974), Hsiao and Pesaran (2004), and Murtazashvili and Wooldridge (2008).

Equation (2) can be written in matrix form as

$$Y = X\bar{\beta} + e\,, \tag{3}$$

where

$$Y' = [Y_1\ Y_2 \cdots Y_N], \qquad Y_i' = [y_{1i}\ y_{2i} \cdots\ y_{Ti}], \qquad X' = [X_1\ X_2 \cdots X_N],$$

$$X_i = \begin{bmatrix} 1 & x_{i1} \\ 1 & x_{i2} \\ \vdots & \vdots \\ 1 & x_{iT} \end{bmatrix}, \quad \bar{\beta} = \begin{bmatrix} \bar{\beta}_0 \\ \bar{\beta}_1 \end{bmatrix}, \quad e = DV + \varepsilon,$$

$$D = \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_N \end{bmatrix}, \quad V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}, \quad v_i = \begin{bmatrix} v_{0i} \\ v_{1i} \end{bmatrix}.$$

The following assumptions are added to the previous assumptions:

A5: The vector V_i are independently and identically distributed with $E(v_i)=0$, and $E(v_i v_i')=\psi$, i=1,2,…,N.

A6: The ε_{it} and v_i are independent for every i and j, so the variance-covariance matrix of e is

$$E(ee')=\begin{bmatrix} X_1\psi X_1'+\sigma_1^2 I_T & 0 & \cdots & 0 \\ 0 & X_2\psi X_2'+\sigma_2^2 I_T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_N\psi X_N'+\sigma_N^2 I_T \end{bmatrix}=\Omega,$$

where zeros are $T\times T$ null matrices and ψ is the variance-covariance matrix of β_i as given in assumption (5). If assumptions (1) till (6) hold, then the GLS estimator of $\bar{\beta}$ is given by

$$\hat{\bar{\beta}}=(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y\,. \tag{4}$$

Swamy (1970) showed that

$$\hat{\bar{\beta}}=\left\{\sum_{i=1}^{N}\left[\psi+\sigma_i^2(X_i'X_i)^{-1}\right]^{-1}\right\}^{-1}\sum_{i=1}^{N}\left[\psi+\sigma_i^2(X_i'X_i)^{-1}\right]^{-1}\hat{\beta}_i, \tag{5}$$

where $\hat{\beta}_i$ is the OLS estimator of β_i. The GLS estimator cannot be used in practice, since ψ and σ_i^2 are unknowns. Swamy (1971) suggested the following unbiased and consistent estimators

$$\hat{\sigma}_i^2=\frac{1}{T-K}\hat{\varepsilon}_i'\hat{\varepsilon}_i\,, \tag{6}$$

and

$$\hat{\psi}=\frac{1}{N-1}S_{\hat{\beta}}-\frac{1}{N}\sum_{i=1}^{N}\hat{\sigma}_i^2(X_i'X_i)^{-1}, \tag{7}$$

where

$$S_{\hat{\beta}}=\sum_{i=1}^{N}\hat{\beta}_i\hat{\beta}_i'-\frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_i\sum_{i=1}^{N}\hat{\beta}_i'. \tag{8}$$

Note that $\hat{\sigma}_i^2$ is the mean square error from the OLS regression of Y_i on X_i, and $S_\beta/(N-1)$ is the sample variance-covariance matrix of β_i. Substitute (6), (7), and (8) in (5), we get the feasible generalized lest square (FGLS) estimator of $\hat{\bar{\beta}}$ as follows:

$$\hat{\hat{\beta}} = \left\{ \sum_{i=1}^{N} \left[\hat{\psi} + \hat{\sigma}_i^2 (X_i' X_i)^{-1} \right]^{-1} \right\}^{-1} \sum_{i=1}^{N} \left[\hat{\psi} + \hat{\sigma}_i^2 (X_i' X_i)^{-1} \right]^{-1} \hat{\beta}_i , \quad (9)$$

and the estimated variance-covariance matrix for the RCR model is

$$\begin{aligned} Var(\hat{\hat{\beta}}) &= (X' \Omega^{-1} X)^{-1}, \\ &= \left\{ \sum_{i=1}^{N} \left[\hat{\psi} + \hat{\sigma}_i^2 (X_i' X_i)^{-1} \right]^{-1} \right\}^{-1}, \end{aligned} \quad (10)$$

Swamy (1973, 1974) showed that the estimator $\hat{\hat{\beta}}_i$ is consistent as both *N* and $T \rightarrow \infty$ and is asymptotically efficient as $T \rightarrow \infty$.

Because v_i is fixed for given *i*, we can test for random variation indirectly by testing whether or not the fixed coefficient vectors β_i are all equal. That is, we form the null hypothesis

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_N = \overline{\beta} .$$

If different cross-sectional units have the same variance, $\sigma_i^2 = \sigma^2$, *i*=1,...,*N*, the conventional analysis of covariance test for homogeneity. If σ_i^2 are assumed different, as postulated by Swamy (1970, 1971), we can apply the modified test statistic

$$F = \sum_{i=1}^{N} \frac{(\hat{\beta}_i - \hat{\hat{\beta}}^*)' X_i' X_i (\hat{\beta}_i - \hat{\hat{\beta}}^*)}{\hat{\sigma}_i^2} , \quad (11)$$

where

$$\hat{\hat{\beta}}^* = \left[\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_i^2} X_i' X_i \right]^{-1} \left[\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_i^2} X_i' y_i \right].$$

Under H_0, (11) is asymptotically chi-square distributed, with *K* (*N* - 1) degrees of freedom, as *T* tends to infinity and *N* is fixed.

If the regression coefficients in model (3) contain both random and fixed coefficients, the model will be called “Mixed RCR” model. The Mixed RCR model is simply a special case of the RCR model where the variance of certain coefficients, which will be considered as fixed coefficients, are assumed to be equal to zero. Thus equation (9) still applies to estimation after certain elements of the ψ matrix are constrained to equal zero.

2. Mean Group Estimator

A consistent estimator of $\bar{\beta}$ can also be obtained under more general assumptions concerning β_i and the regressors. One such possible estimator is the Mean Group (MG) estimator proposed by Pesaran and Smith (1995) for estimation of dynamic random coefficient models. The MG estimator is defined as the simple average of the OLS estimators, $\hat{\beta}_i$:

$$\hat{\bar{\beta}}_{MG} = \frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_i \,. \tag{12}$$

When the regressors are strictly exogenous and the errors, ε_{it} are independently distributed, an unbiased estimator of the covariance matrix of $\hat{\bar{\beta}}_{MG}$ can be computed as

$$\hat{Cov}\left(\hat{\bar{\beta}}_{MG}\right) = \frac{1}{N}\hat{\psi}^{*} \,, \tag{13}$$

where $\hat{\psi}^{*} = \frac{1}{N-1}S_{\hat{\beta}}$. For a proof first note that under the random coefficient model we have

$$\beta_i = \bar{\beta} + v_i \,, \tag{14}$$

$$\beta_i + \hat{\beta}_i = \bar{\beta} + v_i + \hat{\beta}_i \,,$$

$$\hat{\beta}_i = \bar{\beta} + v_i + \hat{\beta}_i - \beta_i \,, \tag{15}$$

let $\hat{\beta}_i - \beta_i = \xi_i$ then we can rewrite the equation (15) as follows

$$\hat{\beta}_i = \bar{\beta} + v_i + \xi_i \,, \tag{16}$$

where

$$\xi_i = (X_i' X_i)^{-1} X_i' \varepsilon_i \,,$$

and

$$\hat{\bar{\beta}}_{GM} = \bar{\beta} + \bar{V} + \bar{\xi} \,, \tag{17}$$

where $\bar{V} = \frac{1}{N}\sum_{i=1}^{N} v_i$ and $\bar{\xi} = \frac{1}{N}\sum_{i=1}^{N}\xi_i$. Therefore

$$\hat{\beta}_i - \hat{\bar{\beta}}_{GM} = (v_i - \bar{V}) + (\xi_i - \bar{\xi}) \tag{18}$$

so

$$\left(\hat{\beta}_i-\hat{\bar{\beta}}_{GM}\right)\left(\hat{\beta}_i-\hat{\bar{\beta}}_{GM}\right)'=\left(v_i-\bar{V}\right)\left(v_i-\bar{V}\right)'+\left(\xi_i-\bar{\xi}\right)\left(\xi_i-\bar{\xi}\right)'$$
$$+\left(v_i-\bar{V}\right)\left(\xi_i-\bar{\xi}\right)'+\left(\xi_i-\bar{\xi}\right)\left(v_i-\bar{V}\right)',$$

and

$$\sum_{i=1}^{N}E\left[\left(\hat{\beta}_i-\hat{\bar{\beta}}_{GM}\right)\left(\hat{\beta}_i-\hat{\bar{\beta}}_{GM}\right)'\right]=(N-1)\psi+(1-\frac{1}{N})\sum_{i=1}^{N}\sigma_i^2(X_i'X_i)^{-1}. \tag{19}$$

But

$$Cov\left(\hat{\bar{\beta}}_{GM}\right)=Cov\left(\bar{V}\right)+Cov\left(\bar{\xi}\right)$$
$$=\frac{1}{N}\psi+\frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2\;E\left[\left(X_i'X_i\right)^{-1}\right], \tag{20}$$

from (7) we can get ψ as follows

$$\psi=\frac{1}{N-1}S_\beta-\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2(X_i'X_i)^{-1}, \tag{21}$$

and let

$$\psi^*=\frac{1}{N-1}S_\beta\,. \tag{22}$$

Substituting (22) into (21), we get

$$\psi=\psi^*-\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2(X_i'X_i)^{-1}, \tag{23}$$

and also substituting (23) into (20), we get

$$Cov\left(\hat{\bar{\beta}}_{GM}\right)=\frac{1}{N}\left[\psi^*-\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2(X_i'X_i)^{-1}\right]+\frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2\;E\left[\left(X_i'X_i\right)^{-1}\right]$$
$$=\frac{1}{N}\psi^*, \tag{24}$$

take the expectation for (13) then

$$E\left[\hat{Cov}\left(\hat{\bar{\beta}}_{MG}\right)\right]=\frac{1}{N}E\left[\hat{\psi}^*\right]=\frac{1}{N}\psi^*$$
$$=Cov\left(\hat{\bar{\beta}}_{GM}\right), \tag{25}$$

as required.

Finally, it is worth noting that the MG and the Swamy's estimators are in fact algebraically equivalent for T sufficiently large, namely

$$\lim_{T \to \infty} \left(\hat{\hat{\beta}} - \hat{\hat{\beta}}_{MG} \right) = 0. \tag{26}$$

To prove that, from (6) and when $T \to \infty$ we get

$$\lim_{T \to \infty} \sigma_i^2 = \lim_{T \to \infty} \frac{\varepsilon_i' \varepsilon_i}{T - K} = 0, \tag{27}$$

substituting (27) into (5), we get

$$\hat{\hat{\beta}} = \left\{ \sum_{i=1}^{N} \left[\psi + (0)(X_i' X_i)^{-1} \right]^{-1} \right\}^{-1} \sum_{i=1}^{N} \left[\psi + (0)(X_i' X_i)^{-1} \right]^{-1} \hat{\beta}_i,$$

$$\hat{\hat{\beta}} = \left[(\sum_{i=1}^{N} (\psi)^{-1})^{-1} (\psi)^{-1} \right] \hat{\beta}_i$$

$$\hat{\hat{\beta}} = \left[\frac{1}{N} \right] \hat{\beta}_i = \hat{\hat{\beta}}_{GM}$$

as required.

It is worth noting that $\psi = \psi^*$ when $T \to \infty$, to prove that, we substituting (27) into (23), we get

$$\psi = \psi^* - \frac{1}{N} \sum_{i=1}^{N} \sigma_i^2 (X_i' X_i)^{-1} = \psi^* - 0 = \psi^*, \tag{28}$$

as required.

3. Classical Pooling Estimator

When coefficients are equal for all individuals ($\beta_1 = \beta_2 = \cdots = \beta_N = \overline{\beta}$). We are assuming that the individuals in our database are drawn from a population with a common regression parameter vector $\overline{\beta}$. In this case the observations for each individual can be pooled and a single regression performed to obtain a more efficient estimator of $\overline{\beta}$. The equation system is now written as

$$Y = Z \; \overline{\beta} + \varepsilon \,, \tag{29}$$

where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}_{NT\times 1}, \qquad Z = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}_{NT\times K}, \qquad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix}_{NT\times 1},$$

and $\overline{\beta}$ is a $K \times 1$ vector of coefficients to be estimated.

If the error variance can be assumed equal for each individual ($E(\varepsilon_i \, \varepsilon_i') = \sigma^2 I_T$), then $\overline{\beta}$ is estimated efficiently and without bias by

$$\hat{\overline{\beta}}_{CP1} = (Z'Z)^{-1} \; Z'Y \,. \tag{30}$$

This estimator has been termed the Classical Pooling (CP) estimator. But if the error has different variances for each individual, then the CP estimator under this assumption would be

$$\hat{\overline{\beta}}_{CP2} = (Z'\Omega^{-1} Z)^{-1} \; Z'\Omega^{-1} Y \,, \tag{31}$$

where

$$\Omega = \begin{bmatrix} \sigma_1^2 I_T & & & 0 \\ & \sigma_2^2 I_T & & \\ & & \ddots & \\ 0 & & & \sigma_N^2 I_T \end{bmatrix}. \tag{32}$$

The unknown parameters σ_i^2 can be consistently estimated by

$$S_i^2 = \frac{1}{T-K} \sum_{t=1}^{T} \hat{\varepsilon}_{it}^2 \qquad \textit{for } \; i=1, \ldots, N \tag{33}$$

where $\hat{\varepsilon}_{it}$ are the residuals obtained from applying OLS to equation number *i*.

4. Design of the Simulation

We will use the Mote Carlo simulation for making comparisons between the behavior of RCR, CP, and MG estimators in three models (RCR, fixed, and Mixed RCR models). The settings of the model and results of the simulation study are discussed below.

The values of the independent variable x_{it}, were generated as independent normally distributed random variates with mean μ_X and standard deviation σ_X. The values of x_{it} were allowed to differ for each cross-sectional unit. However, once generated for all *N* cross-sectional units the values were held fixed over all Monte Carlo trials. The value of μ_X was set equal to zero and the value of σ_X was set equal to 10. The disturbances, ε_{it}, were generated as independent normally distributed random variates, independent of the x_{it} values, with mean zero and standard deviation σ_ε. The disturbances were allowed to differ for each cross-sectional unit on a given Monte Carlo trial and were allowed to differ between trials. The standard deviation of the disturbances was set equal to either 1 or 10 and held fixed for each cross-sectional unit. The values of *N* and *T* were chosen to be 10, 25, and 100 to represent small, medium and large samples for the number of individuals and the time dimension. The values 10 were chosen to represent small samples, and the values 25 were to represent medium samples, while the values 100 were to represent large samples.

The parameters, β_{0i} and β_{1i}, were set at several different values to allow study of the estimators under conditions where the model was both properly and improperly specified. The five different combinations of β_{0i} and β_{1i} used are detailed in Table (1) by giving the means and variances of the coefficients. Note that a variance of zero simply means that the coefficient is fixed and equal over all cross-sectional units.

Table (1) Values of Coefficient Means and Variances Used In the Simulation

Model	$\overline{\beta}_0$	$Var(\beta_0)$	$\overline{\beta}_1$	$Var(\beta_1)$
1	5	30	5	30
2	0	10	5	10
3	5	0	5	0
4	5	0	5	30
5	5	30	5	0

For each of the experimental settings 10,000 Monte Carlo trials were used and results were recorded in Tables (2) through (6), with each table consisting of two panels, numbered I and II, for the different samples size (10, 25, and 100). And each panel from this panels corresponding to two settings of the disturbance standard deviation (1 and 10). Each of the tables provides the results for a particular scheme of generation of the regression coefficients.

5. Monte Carlo Results

In tables results, several estimators and test statistics are of interest. Tables (2) through (6) are set up to show the following information:

The RCR estimators for the coefficient mean are computed as in equation (9). CP estimators for the coefficient mean are computed as in equation (31). While MG estimators for the coefficient mean are computed as in equation (12).

Table (2) Results of Different Estimation Methods When β_0 ~ N (5,30) and β_1 ~ N (5,30)

σ_ε	The Estimation Method	N=T	10		25		100	
			β_0	β_1	β_0	β_1	β_0	β_1
I. 1	RCR	Bias	-0.016	-0.019	0.008	0.005	-0.003	0.005
		MSE	3.025	3.007	1.202	1.202	0.300	0.300
		% Rejections $H_0:\beta_k=0$	72.8	73.3	99.2	99.1	100.0	100.0
		% Coefficients Contained in 95% CI	95.1	99.0	95.3	98.9	94.8	98.9
	CP	Bias	0.061	-0.017	0.013	0.004	-0.001	0.004
		MSE	0.012	0.000	0.002	0.000	0.000	0.000
		% Rejections $H_0:\beta_k=0$	86.8	99.1	97.4	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	3.8	1.0	2.6	0.9	1.8	0.3
	MG	Bias	-0.016	-0.019	0.008	0.005	-0.003	0.005
		MSE	3.025	3.007	1.202	1.202	0.300	0.300
		% Rejections $H_0:\beta_k=0$	72.8	73.3	99.2	99.1	100.0	100.0
		% Coefficients Contained in 95% CI	95.1	99.0	95.3	98.9	94.8	98.9
II. 10	RCR	Bias	-0.084	-0.014	-0.002	0.020	-0.002	-0.008
		MSE	4.133	3.017	1.368	1.198	0.310	0.300
		% Rejections $H_0:\beta_k=0$	60.6	73.5	98.3	99.2	100.0	100.0
		% Coefficients Contained in 95% CI	93.3	98.9	95.3	99.1	95.1	99.1
	CP	Bias	-0.024	-0.012	0.007	0.019	0.002	-0.008
		MSE	0.785	0.008	0.147	0.002	0.010	0.000
		% Rejections $H_0:\beta_k=0$	75.2	98.8	95.3	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	34.4	11.4	24.9	6.8	19.5	3.5
	MG	Bias	0.000	-0.012	-0.002	0.020	-0.002	-0.008
		MSE	4.014	3.018	1.370	1.198	0.310	0.300
		% Rejections $H_0:\beta_k=0$	60.2	73.5	98.4	99.2	100.0	100.0
		% Coefficients Contained in 95% CI	94.6	98.9	95.3	99.1	95.1	99.1

Table (3) Results of Different Estimation Methods When $\beta_0 \sim N(0,10)$ and $\beta_1 \sim N(5,10)$

σ_ε	The Estimation Method	N=T	10		25		100	
			β_0	β_1	β_0	β_1	β_0	β_1
I. 1	RCR	Bias	-0.009	-0.011	0.005	0.003	-0.002	0.003
		MSE	1.015	1.003	0.402	0.401	0.100	0.100
		% Rejections $H_0:\beta_k=0$	2.7	99.3	2.3	100.0	2.7	100.0
		% Coefficients Contained in 95% CI	95.1	99.0	95.2	98.9	94.8	98.9
	CP	Bias	0.035	-0.010	0.007	0.002	-0.001	0.002
		MSE	0.009	0.000	0.002	0.000	0.000	0.000
		% Rejections $H_0:\beta_k=0$	46.0	100.0	47.7	100.0	48.4	100.0
		% Coefficients Contained in 95% CI	6.9	1.7	4.4	1.4	3.1	0.5
	MG	Bias	-0.009	-0.011	0.005	0.003	-0.002	0.003
		MSE	1.015	1.003	0.402	0.401	0.100	0.100
		% Rejections $H_0:\beta_k=0$	2.7	99.3	2.3	100.0	2.7	100.0
		% Coefficients Contained in 95% CI	95.1	99.0	95.2	98.9	94.8	98.9
II. 10	RCR	Bias	0.157	-0.017	-0.005	0.011	-0.001	-0.004
		MSE	1.824	1.014	0.563	0.400	0.110	0.100
		% Rejections $H_0:\beta_k=0$	5.7	99.1	2.4	100.0	2.3	100.0
		% Coefficients Contained in 95% CI	88.5	98.9	95.2	99.1	95.2	99.1
	CP	Bias	-0.019	-0.006	-0.001	0.011	0.001	-0.005
		MSE	0.785	0.008	0.147	0.001	0.010	0.000
		% Rejections $H_0:\beta_k=0$	23.7	100.0	30.3	100.0	33.2	100.0
		% Coefficients Contained in 95% CI	53.2	19.8	40.0	11.9	33.4	6.4
	MG	Bias	0.000	-0.006	-0.005	0.011	-0.001	-0.004
		MSE	2.038	1.014	0.568	0.401	0.110	0.100
		% Rejections $H_0:\beta_k=0$	2.9	99.2	2.2	100.0	2.4	100.0
		% Coefficients Contained in 95% CI	94.4	98.9	95.4	99.1	95.1	99.1

The bias values of the coefficients mean estimators, $\hat{\bar{\beta}}_0$ and $\hat{\bar{\beta}}_1$, are computed as $bias(\hat{\bar{\beta}}) = \bar{\beta} - \hat{\bar{\beta}}$, where $\hat{\bar{\beta}}$ is a vector of coefficients mean estimators and $\bar{\beta}$ is a true vector of coefficients mean. The bias values shown in the first row of each panel (I and II).

Table (4) Results of Different Estimation Methods When $\beta_0 = 5$ and $\beta_1 = 5$

σ_ε	The Estimation Method	N=T	10		25		100	
			β_0	β_1	β_0	β_1	β_0	β_1
I. 1	RCR	Bias	0.022	0.003	-0.015	-0.004	0.000	0.000
		MSE	0.009	0.000	0.002	0.000	0.000	0.000
		% Rejections $H_0: \beta_k = 0$	99.5	99.9	100.0	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	67.5	75.4	82.7	85.1	94.3	98.9
	CP	Bias	0.000	0.000	0.000	0.000	0.000	0.000
		MSE	0.008	0.000	0.001	0.000	0.000	0.000
		% Rejections $H_0: \beta_k = 0$	100.0	100.0	100.0	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	90.5	97.9	93.9	99.1	94.7	99.1
	MG	Bias	0.001	0.000	-0.001	0.000	0.000	0.000
		MSE	0.011	0.000	0.002	0.000	0.000	0.000
		% Rejections $H_0: \beta_k = 0$	100.0	100.0	100.0	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	94.8	99.1	95.1	99.2	95.1	99.3
II. 10	RCR	Bias	-0.355	0.053	-1.323	0.698	-0.001	0.000
		MSE	0.916	0.010	1.840	0.487	0.010	0.000
		% Rejections $H_0: \beta_k = 0$	91.4	99.5	99.2	99.9	100.0	100.0
		% Coefficients Contained in 95% CI	68.1	75.2	82.1	85.4	94.4	98.7
	CP	Bias	0.010	0.001	-0.007	0.001	-0.001	0.000
		MSE	0.785	0.008	0.147	0.001	0.010	0.000
		% Rejections $H_0: \beta_k = 0$	99.5	100.0	100.0	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	91.0	98.0	94.1	98.9	94.5	99.0
	MG	Bias	0.014	0.000	-0.005	0.000	-0.001	0.000
		MSE	1.062	0.012	0.166	0.002	0.010	0.000
		% Rejections $H_0: \beta_k = 0$	99.0	100.0	100.0	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	95.2	99.0	95.2	99.1	95.0	99.0

The Mean Square Error (MSE) of coefficients mean estimators that are computed as $MSE(\hat{\bar{\beta}}_k) = \hat{Var}(\hat{\bar{\beta}}_k) + [bias(\hat{\bar{\beta}}_k)]^2$, where $\hat{Var}(\hat{\bar{\beta}}_k)$ is the estimated variance of the coefficient mean estimator and is computed as the *k*th diagonal element of the variance-covariance matrix. The estimated variances of RCR estimators are the diagonal elements in equation (10). The estimated variances of CP estimators are the diagonal elements in equation (32). While the estimated variances of MG estimators are the diagonal elements in equation (13). The MSE values shown in the row four of each panel.

Table (5) Results of Different Estimation Methods When β_0 = 5 and β_1 ~ N (5,30)

σ_ε	The Estimation Method	N=T	10		25		100	
			β_0	β_1	β_0	β_1	β_0	β_1
I. 1	RCR	Bias	-0.123	0.010	0.009	0.012	0.000	0.004
		MSE	0.021	2.995	0.002	1.198	0.000	0.299
		% Rejections $H_0:\beta_k=0$	99.7	72.5	100.0	99.2	100.0	100.0
		% Coefficients Contained in 95% CI	72.8	99.0	88.6	99.0	94.4	99.0
	CP	Bias	0.074	0.027	-0.030	0.010	-0.004	0.003
		MSE	0.013	0.001	0.002	0.000	0.000	0.000
		% Rejections $H_0:\beta_k=0$	89.1	99.0	98.7	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	4.0	1.2	2.6	0.7	2.6	0.4
	MG	Bias	0.000	0.016	0.000	0.012	0.000	0.004
		MSE	0.011	2.995	0.002	1.198	0.000	0.299
		% Rejections $H_0:\beta_k=0$	100.0	72.5	100.0	99.2	100.0	100.0
		% Coefficients Contained in 95% CI	94.5	99.0	95.3	99.0	95.1	99.0
II. 10	RCR	Bias	0.151	-0.180	-0.192	-0.015	-0.001	0.003
		MSE	0.723	3.040	0.174	1.195	0.010	0.300
		% Rejections $H_0:\beta_k=0$	93.6	72.7	99.7	99.3	100.0	100.0
		% Coefficients Contained in 95% CI	72.9	98.8	88.5	99.0	94.5	98.9
	CP	Bias	-0.082	0.007	-0.029	-0.012	0.000	0.003
		MSE	0.792	0.008	0.148	0.001	0.010	0.000
		% Rejections $H_0:\beta_k=0$	78.2	98.8	97.2	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	39.1	11.0	28.0	6.7	28.3	3.5
	MG	Bias	-0.013	0.013	-0.002	-0.010	-0.001	0.003
		MSE	1.043	3.010	0.166	1.195	0.010	0.300
		% Rejections $H_0:\beta_k=0$	99.2	72.7	100.0	99.3	100.0	100.0
		% Coefficients Contained in 95% CI	94.9	99.0	95.0	99.0	95.1	98.9

The third row shows the percentage of rejections of the null hypothesis $H_o:\overline{\beta}_k = 0$ for k = 0 and 1. The test uses the t-statistic computed as $t = \hat{\bar{\beta}}_k \,/\, se(\hat{\bar{\beta}}_k)$, where $se(\hat{\bar{\beta}}_k)$ is the square root of the kth diagonal element of the variance-covariance matrix. A nominal 5% level of significance was used so the expected percentage of rejections whenever the null hypothesis is true is 5%.

Table (6) Results of Different Estimation Methods When $\beta_0 \sim$ N (5,30) and $\beta_1 = 5$

σ_ε	The Estimation Method	N=T	10		25		100	
			β_0	β_1	β_0	β_1	β_0	β_1
I. 1	RCR	Bias	0.002	-0.001	0.011	-0.001	0.004	0.000
		MSE	3.007	0.000	1.200	0.000	0.300	0.000
		% Rejections $H_0:\beta_k=0$	72.3	100.0	99.2	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	94.7	78.3	95.0	89.4	94.9	98.7
	CP	Bias	0.016	0.001	0.011	0.000	0.003	0.000
		MSE	0.008	0.000	0.002	0.000	0.000	0.000
		% Rejections $H_0:\beta_k=0$	99.1	100.0	100.0	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	8.5	50.4	5.6	38.5	2.8	36.3
	MG	Bias	0.015	0.000	0.011	0.000	0.004	0.000
		MSE	3.008	0.000	1.200	0.000	0.300	0.000
		% Rejections $H_0:\beta_k=0$	72.4	100.0	99.2	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	95.0	98.9	95.0	99.0	94.9	99.0
II. 10	RCR	Bias	-0.523	-0.055	-0.178	0.043	0.003	0.000
		MSE	4.108	0.014	1.384	0.003	0.310	0.000
		% Rejections $H_0:\beta_k=0$	60.1	99.4	98.1	99.9	100.0	100.0
		% Coefficients Contained in 95% CI	89.6	77.3	93.7	88.6	95.0	98.8
	CP	Bias	-0.008	0.000	-0.015	0.000	0.002	0.000
		MSE	0.785	0.008	0.147	0.001	0.010	0.000
		% Rejections $H_0:\beta_k=0$	91.1	100.0	100.0	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	61.9	97.1	47.6	97.8	28.1	97.6
	MG	Bias	-0.002	0.002	-0.012	0.000	0.003	0.000
		MSE	4.033	0.012	1.360	0.002	0.310	0.000
		% Rejections $H_0:\beta_k=0$	60.0	100.0	98.6	100.0	100.0	100.0
		% Coefficients Contained in 95% CI	94.9	98.9	94.8	99.1	95.0	99.0

The percentage of time a 95% confidence interval estimate of $\bar{\beta}_k$ contained the true value of the coefficient is reported in row four. The confidence interval is computed as $\hat{\bar{\beta}}_k \pm t \; se(\hat{\bar{\beta}}_k)$.

As a guide to interpreting the tables, let us consider Table (2) as an example. The RCR estimators when $\sigma\varepsilon = 1$ and N=T=10 as follows: The values of bias and MSE for $\hat{\bar{\beta}}_0$ are -0.016 and 3.025 respectively. The percentages of rejections of the null hypothesis $H_0 : \beta_k = 0$ for β_0 and β_1 are 72.8 and 73.3. The percentages of time a 95% confidence interval estimate of $\bar{\beta}_0$ and $\bar{\beta}_1$ are 95.1 and 99.0. As the variation in the disturbances increase from $\sigma\varepsilon = 1$ to $\sigma\varepsilon = 10$ the estimators get worst. Increasing the sample size will make the estimators better.

6. Concluding Remarks

From Tables (2) through (6), several observations concerning the RCR, CP, and MG estimators for small, medium, and large samples can be made:

1- The CP estimators of the fixed coefficient perform well when the coefficient is fixed but this is not true for the fixed coefficient in the mixed models of Tables (5) and (6).

2- When coefficients are random, the CP estimators appear to be unbiased (for the proof see Dielman (1989)). The problem with using CP when coefficients are random is not bias in the estimates but in the performance of the hypothesis test for significance of the coefficients and in the performance of confidence interval estimators. For example, comparing the results for the three estimation methods in the Table (3), the RCR and MG hypotheses tests for significance are obviously superior to the CP test. The CP test has rejection rates much higher than the 5% level of significance set for the test. The RCR and MG rejection rates are much closer to the nominal 5% level. Also note that the CP enclosure rates for the 95% confidence interval are very low when the coefficients are random.

3- The RCR estimator performs well when the coefficients are random, even though the samples are small (T=10). From Tables (2) and (3), the bias and MSE are doing better in small and large variation of the parameters. In general, the RCR estimator performs best when both coefficients are random.

4- When one of the coefficients is fixed and the sample size is small, the RCR estimator will not perform as well as might be expected. But if the samples sizes are medium or large, the RCR estimator performs well.

5- The RCR and CP methods perform well when the respective required assumptions are met. However, both deteriorate rapidly when used improperly. This suggests the importance of being able to choose the assumptions which are appropriate in each particular situation. The RCR test for randomness should prove useful in this respect.

6- The MG estimators for the three models (fixed, RCR, and Mixed RCR) performs well even in small samples: When coefficients are fixed, Table (4), the MG estimators for the coefficients are better than the RCR estimators.

7- The MG estimators for the fixed coefficient in the mixed models, Tables (5) and (6), perform well and better than the RCR estimators. By using MG method, it is not possible to obtain negative estimates of the coefficients variances. So, we can say that the MG method is the general estimation method for fixed, RCR, and Mixed RCR models.

The Monte Carlo simulation results suggest that the RCR estimators perform well in small samples if the coefficients are random and but it does not in fixed or Mixed RCR models. But if the samples sizes are medium or large, the RCR estimators perform well for the three models. While CP estimators perform well in the fixed model only. But the MG estimators perform well if the coefficients are random or fixed. So, we can say that the MG method is the general estimation method for fixed, RCR, and Mixed RCR models. This simulation has been limited in scope, as all simulations must be. Hopefully it will shed some light on performance of several estimation methods for the panel data models when the regression coefficients are random.

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