On the cost of misperceived travel time variability

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Abstract

Recent studies show that traveler's scheduling preferences compose a willingness-to-pay function directly corresponding to aggregate measurement of travel time variability under some assumptions. This property makes valuation on travel time variability transferable from context to context, which is ideal for extensive policy evaluation. However, if respondents do not exactly maximizing expected utility as assumed, such transferability might not hold because two types of potential errors: (i) scheduling preference elicited from stated preference experiment involving risk might be biased due to misspecification and (ii) ignoring the cost of misperceiving travel time distribution might result in undervaluation. To find out to what extent these errors matter, we reformulate a general scheduling model under rank-dependent utility theory, and derive reduced-form expected cost functions of choosing suboptimal departure time under two special cases. We estimate these two models and calculate the empirical cost due to misperceived travel time variability. We find that (i) travelers are mostly pessimistic and thus tend to choose departure time too earlier to bring optimal cost, (ii) scheduling preference elicited from stated choice method could be quite biased if probability weighting is not considered and (iii) the extra cost of misperceiving travel time distribution contributes trivial amount to the discrepancy between scheduling model and its reduced form.

Keywords: travel time variability, scheduling delay, departure time choice, rank-dependent utility

JEL classification: D61, D81, R41

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1 Introduction

The concept of value of travel time has been well established in the long history of economics (Becker, 1965; DeSerpa, 1971), and it accounts a significant share of social benefit of infrastructure investment and social cost of traffic congestion. Meanwhile, travelers are confronting increasingly uncertain travel time because the pervasive congestion makes trip duration more sensitive to non-recurrent variations (e.g., weather, accident) and consequently less predictable. This leads to user's additional scheduling cost (i.e., commuter departs earlier than what is needed to buffer the potential loss) and psychological anxieties, making travel time variability (unreliability) equally costly as mean travel time. Therefore policy-makers have been gradually moving their eyeballs on how travel time variability should be valued and how to improve the travel time reliability in road network.

In the substantial body of research, mean-variance approach (Brownstone and Small, 2005; Small et al., 2005) and scheduling approach are two mainstream approaches of analyzing the value of travel time variability (VTTV). The former is the single viable option for cost-benefit analysis relevant to travel time reliability because its results directly associate with statistical measures of variability (e.g., standard deviation, inter-quantile range) from supply side. However, it is a "black box" model where the microeconomic mechanism of how travel time variability incurs scheduling cost is hidden. By contrast, scheduling model (Small, 1982; Noland and Small, 1995) is formulated with more realistic behavioral assumptions, where the stochastic travel time is considered to affect traveler's utility by making one arrive early or late relative to his/her preferred arrival time. The traveler is assumed to optimize the departure time (thus probability of being late) in accordance with the importance of activities before and after the trip as well as expected travel time distribution available from experience. Nonetheless, its formulation stands on individual's perspective, which makes it not suitable for appraisal purpose. A desirable solution is to depart from estimating individual's preferences and convert them to willingness-to-pay that links to statistical measures of variability, since people's trip timing preferences are relatively stable.

This solution requires establishing theoretical equivalent between scheduling model and mean-variance model, i.e., preference parameters in each can be transfered mutually. Noland and Small (1995) and Bates et al. (2001) showed that the scheduling model will give rise to a reduced form that is analogous to mean-variance model by assuming (i) travel time is exponential or uniform distributed, (ii) change of recurrent delay equals to 0, (iii) fixed penalty of arriving late is absent and (iv) travelers try to maximize expected utility under risky choice scenario. Fosgerau and Karlström (2010) further generalized aforementioned result to any distribution as long as its standardized travel time distribution is independent of departure time. Following papers (Fosgerau and Engelson, 2011; Engelson and Fosgerau, 2011) adopt other assumptions on scheduling preference and give rise to reduced-form expressions correspond to different variability measures, but the practical meaning remains the same. However, significant discrepancies between scheduling model and its reduced-form derived from aforementioned studies are found in some empirical studies (Börjesson et al., 2012), indicating scheduling model has not caught all disutilities of travel time variability as its reduced-form.

Since the theoretical equivalence is established under standard expected utility theory (EUT) formalized by von Neumann and Mogenstern, it is natural to suspect these discrepancies are incurred by the deficiency of this theory. Particularly, the experimental findings in Allais (1953) has shown simple re-framing of question can fail the independence axiom in standard EUT. Hence we argue that it is necessary to verify whether using standard EUT in scheduling context is viable before taking advantage of its mathematical convenience. Otherwise, two types of errors are about to occur.

First, misspecification. The estimation of scheduling model relies on data from stated preference experiment, where risk is generally taken as one of the design attributes (e.g., occurrence frequency of given travel time). Respondents, however, might perform less rationally as EUT in an unfamiliar risky experiment. Letting the expected value enters as a proxy for certainty equivalent is likely to be misspecification (De Palma et al., 2008), and will bias the model estimates. Second, undervaluation. Deriving a reduced-form model under EUT will ignore the cost of probability misperception, that is, the additional cost results from that traveler's subjectively optimized departure time cannot perfectly minimize the cost. Bates et al. (2001) mentioned the cost of misperception (see Figure 1) but no analytical result was given. Essentially its value depends on how much the subjective decision weight deviates from the objective probability. The mixed effect of aforementioned errors is supposed to exist in many relevant empirical studies so far. One generalization of expected utility theory for accommodating those behavioral anomalies and catching cost of probability perception will be using the concept of rank-dependence, i.e., individual processes objective probability to decision weight based on her favor to given outcome. From a normative analysis point of view, it is preferable to cumulative prospect theory because individual's reference point is hardly measurable and subjects to change. Koster and Verhoef (2012) formulated a rank-dependent scheduling model and showed the cost of misperception ranges 0-24% of total travel cost assuming a series of values for weighting parameters. Hensher and Li (2012) estimated a rank-dependent model but implicitly assumed marginal cost of time equals to that of scheduling delay. To the best of our knowledge, the empirical estimation of weighting parameters in scheduling context is still missing.

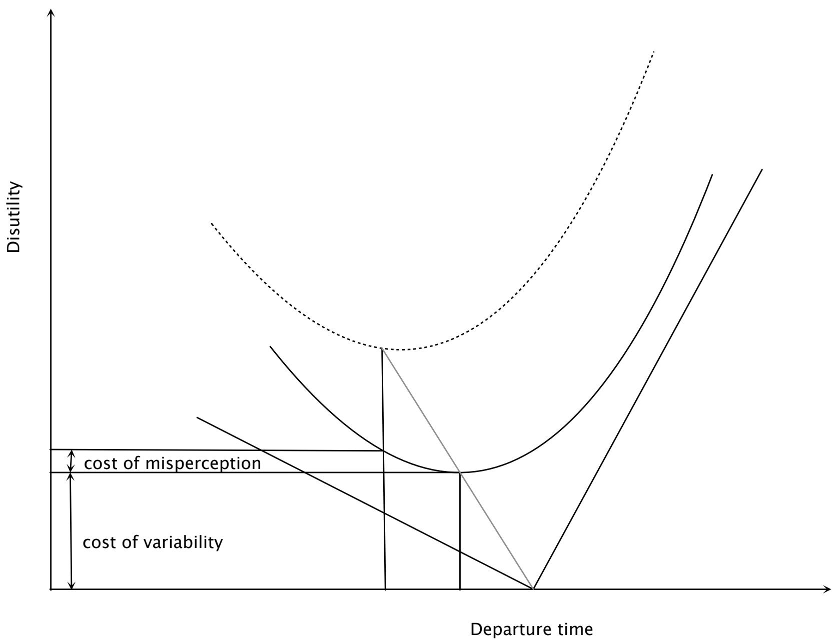

Figure 1: Additional cost due to misperception of variability (Bates et al., 2001)

It may be true that the misperception will gradually disappear as accumulating experience from recurrent choice, and thus less concerned in revealed preference setting. However, there is a lack of existing studies that prove such tendency. Hence it remains unclear how large an extra cost traveler's suboptimal departure time choice will result in, and whether such cost should be included into cost benefit analysis. Besides, scheduling preferences should be stable in a long run regardless of perception to risk. This implies that estimating a scheduling model considering probability weighting with data from risky situation will give the same estimates as estimating a standard one with data from certain situation. Therefore it is reasonable to estimate scheduling preferences with non-EUT formulation and apply its reduced-form equivalence to evaluate social cost from objective travel time distribution.

To sum up, the objective of this paper is to (i) empirically estimate scheduling model with rank-dependent utility, (ii) testify the existence of significant discrepancy between scheduling model and mean-variance model and (iii) if any, investigate to what extent traveler's violation of expected utility maximization contributes. If (iii) turns out to be significant, formulating the scheduling model under non-EU framework might bring more robust estimates of scheduling preferences and hence value of travel time variability.

This paper is organized as follows. In section 2, we reformulate a general scheduling model (Vickrey, 1973) under rank-dependent utility theory (Quiggin, 1982), analyze its properties and demonstrate two special cases: one with piece-wise constant marginal utility of time (Vickrey, 1969; Small, 1982) and another with linear marginal utility of time over time-of-day (Fosgerau and Engelson, 2011), and derive their reduced-form expressions. In section 3, we present details of SP experiment design and data description. In section 4, we specify the empirical model for estimating our data and define the measurement for two types of errors as mentioned. In section 5, we discuss about the model estimation result and its implications. Henceforth, our contribution is that we provide a general framework to analyze the cost of probability misperception, and demonstrate with empirical data that cost benefit analysis without considering such cost will still be good approximation.

2 Theoretical framework

2.1 Rank-dependent utility

Allais (1953) paradox showed that individual's behavior violates the independence axiom in EUT, and one possible explanation is that individual does not perceive probability linearly when multiple outcomes are possible from one choice. In rank-dependent utility theory (Quiggin, 1982), such non-linear probability perception is incorporated by using a probability weighting function, which transforms objective probability to subjective decision weight based on ranked position of given outcome. The shape of weighting function represents individual's attitude toward risky events as shown in Figure 2: in the case of worsening ranked outcomes, (i) convex W reflects pessimism, because the probability of good outcome is always underestimated and the bad one's probability is always overestimated, similarly (ii) concave W reflects optimism, (iii) inverse S-shaped W means individual overestimates the probability of both good and bad outcomes and insensitive to those intermediate and (iv) S-shaped W means the opposite.

2.2 General scheduling preference

Table 1: List of notation

Variable	Definition
T	Time-independent random travel time, $T = \mu + \sigma X$
d	Departure time from home
h	Marginal utility of time spending at home
w	Marginal utility of time spending at workplace
a^*	ideal arrival time, the intersection of h and w
W	Probability weighting function
π	Decision weight

The model formulation herein will start from the general scheduling preference in Vickrey (1973) and Tseng and Verhoef (2008). We consider a daily commuter who needs to travel from home to workplace, and she wants to maximize utility of conducting activities on both sides. The in-vehicle time is assumed

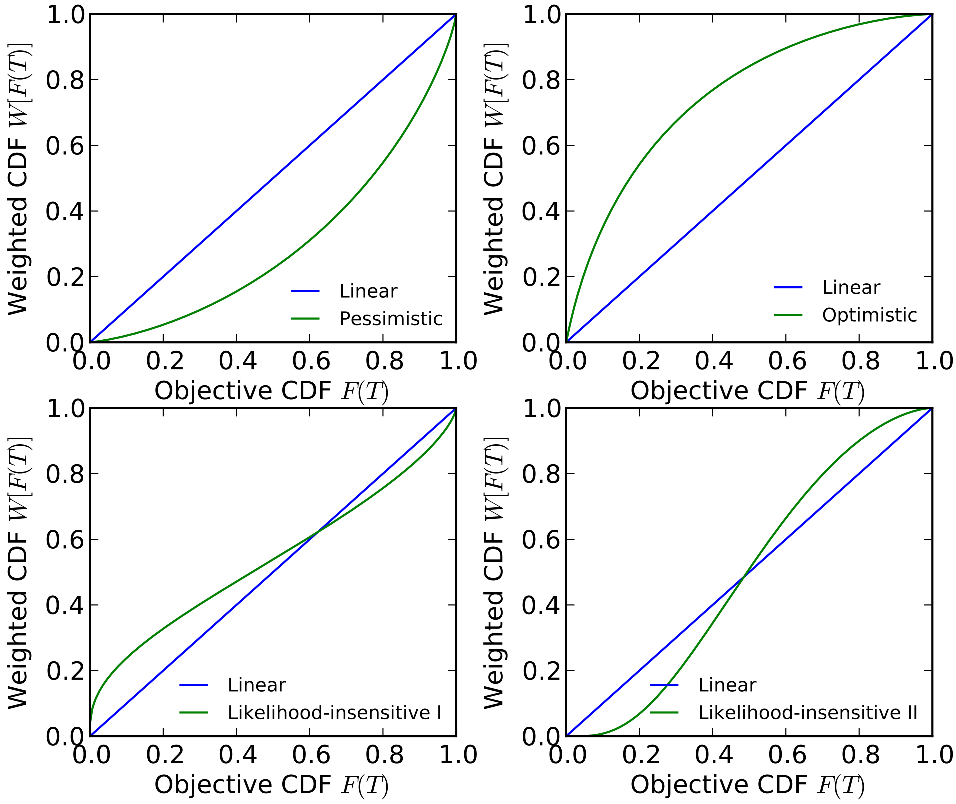

Figure 2: Behavioral interpretation of weighting function for worsening ranked positions

to be completely unproductive, so the marginal utility of spending time at home and at workplace relative to travel equal to $h(t)$ and $w(t)$ respectively. $h(t)$ and $w(t)$ are varying by time-of-day, and have an intersection a^*. It is interpreted as ideal arrival time as people will be best off arriving at a^* if the travel is instantaneous. Assumption 1 is needed to ensure the existence of optimal departure time and some properties shown afterwards. It is mild because otherwise it is behavioral implausible (e.g., no need to travel if $h(t)$ is increasing while $w(t)$ is decreasing).

Assumption 1. *$h(t)$ is non-increasing, $w(t)$ is non-decreasing, $h(t) > w(t)$ for all $t < 0$ and $h(t) < w(t)$ for all $t > 0$*

Assumption 2. *The travel time $T = \mu + \sigma X$ is stochastic, bounded at $[0, T_{max}]$ and its standardized distribution X is assumed to be independent of departure time and has cumulative distribution function G.*

The feasibility of Assumption 2 is empirically verified by Fosgerau and Fukuda (2012) where X is well-fitted by a stable distribution.

Normalizing $a^* = 0$ without loss of generality, the utility of time budget given random travel time T and departure time d is

$$U(d, T) = \int_0^d h(t)dt + \int_{d+T}^0 w(t)dt. \tag{1}$$

Traveler wants to maximize her utility by choosing an optimal departure time, which is equivalent to minimizing the opportunity cost of time budget

$$C(d, T) = \int_d^0 h(t)dt + \int_0^{d+T} w(t)dt. \tag{2}$$

As a sum of two convex function, $C(d, T)$ is necessarily convex in d for each realization of T. Therefore

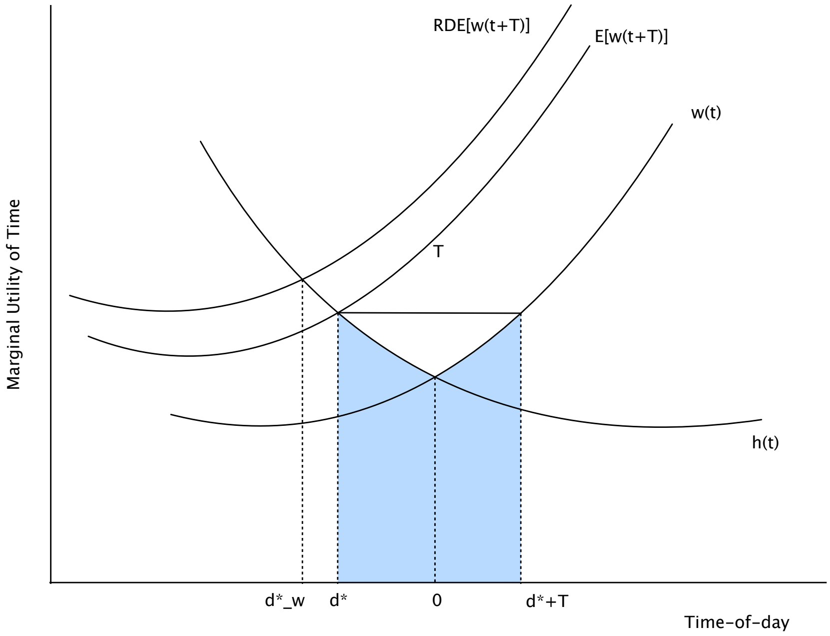

Figure 3: General scheduling preference

given the shape of marginal utility in Assumption 1, there exists a unique interior solution d^* that gives minimal travel time. The second derivative of $C(d, T)$ w.r.t. T is not less than 0, which means $C(d, T)$ is also convex and increasing in T given a chosen d. Under this setting, travelers have to be risk-averse on travel time, so a reduction of travel time variability is valuable. For continuous $h(t)$ and $w(t)$[1], the first-order optimality condition for minimizing expected cost is

$$h(d^*) = E[w(d^* + T)] = \int_0^{T_{max}} w(d^* + T)f(T)dT. \tag{3}$$

However, given the existence of probability weighting, the minimization problem individual processing is a subjective one, thus the optimality condition under rank-dependent utility is

$$h(d^*_w) = RDE[w(d^*_w + T)] = \int_0^{T_{max}} w(d^*_w + T)W'[F(T)]f(T)dT, \tag{4}$$

where W is a probability weighting function which will be explained later. Therefore, the probability misperception brings an extra cost of choosing a suboptimal departure time given by

$$\Delta = E[C(d^*_w)] - E[C(d^*)]. \tag{5}$$

Δ is necessarily not less than 0 because traveler can do no better than minimal expected cost. Equation (2) to (5) imply that the size of Δ is decided by $h(t), w(t), F$ and W. It will be insightful to know under what condition a traveler will depart earlier/later than optimal departure time, and which one of them

[1]Engelson and Fosgerau (2011) proved that an optimal solution still exists when $w(t)$ is a step function as assumed in Small (1982)

is more costly. We find that the order of stochastic dominance helps answering such questions.

Proposition 1. *For any $F(T)$ that first-order stochastically dominates $W[F(T)]$, i.e., $F(T) \geq W[F(T)]$ fot every T, $d^*_w \leq d^*$ and vince versa*

Proof. Since the optimal departure time d^* of $C(d,T)$ is given by the intersection of $h(t)$ and $E[w(t+T)]$ and $h(t)$ is non-increasing, whether $d^*_w \leq d^*$ depends on whether $RDE[w(t+T)] \geq E[w(t+T)]$ for every T;

$$\begin{aligned} RDE[w(t+T)] - E[w(t+T)] &= \int_0^\infty w(t+T)d(W[F(T)] - F(T)) \\ &= w(t+T)(W[F(T)] - F(T))|_0^\infty - \int_0^\infty w'(t+T)(W[F(T)] - F(T))dT \end{aligned}$$

since $W[F(0)] - F(0) = 0$ and $W[F(T)] - F(T) = 0$ for large T, the first term of this expression is 0. Furthermore, since $w'(t+T) \geq 0$ and $F(T) - W[F(T)] \geq 0$ for every T, the second term is not less than 0, consequently $d^* \leq d^*_w$ □

It is clear that a random travel time dominates its pessimistically weighted counterpart (convex W), so pessimism indicates early departure and similarly optimism indicates late departure. Particularly, d^*_w will equal to d^* if $w(t)$ is a constant, because in this case the departure time is always the intersection of $w(t)$ and $h(t)$ regardless individual's probability perception. A special case will be when $w(t)$ is linearly increasing, taking integration by parts once more will tell that the mean of $F(T)$ larger than the mean of $W[F(T)]$ implies early departure. However, the effects of S-shaped and inverse S-shaped weighting functions are uncertain if the weighting function is a mean-preserving transformation given non-decreasing $w(t)$.

Proposition 2. *If W is a mean-preserving transformation, $w(t)$ is convex, for any $F(T)$ that second-order stochastically dominates $W[F(T)]$, i.e., $\int_T^\infty F(s)ds \geq \int_T^\infty W[F(s)]ds$ for every T, $d^*_w \leq d^*$ and vince versa*

Proof. Follow the proof in Proposition 1 and repeat integration by parts twice, we have

$$\begin{aligned} RDE[w(t+T)] - E[w(t+T)] = w'(t+T)\int_0^T (W[F(T)] - F(T))dT|_0^\infty \\ - \int_0^\infty w''(t+T)\int_0^T (W[F(T)] - F(T))dT)dT \end{aligned}$$

taking integration by parts once more to the first term will yield $E[T] - RDE[T]$, which will equal to 0 given mean-preserving W. Since $w''(t+T) \geq 0$ and $\int_0^\infty (F(T) - W[F(T)])dT \geq 0$, the second term, and hence the whole expression is not less than 0, consequently $d^* \leq d^*$ □

This proposition provides a way to check effects of S-shaped and inverse S-shaped weighting function on departure time. Intuitively, if a probability weighting function preserve the mean but fatten the tails of given distribution, it is likely to causes a larger perceived loss if $w(t)$ is increasing faster than linearly. A mean-preserving S-shaped W will never make $W[F(T)]$ dominated by $F(T)$, so travlers will always choose early departure. Further, we try to investigate to which direction subjective optimal departure time shifts will be more costly.

Proposition 3. *If $h(t)$ and $w(t)$ are twice differentiable and $E[w''(d^*+T]) > h''(d^*)$, late departure is more costly, otherwise early departure is more costly*

Proof. $\Leftrightarrow \frac{\partial^3 E[C(d,T)]}{\partial d^3}|_{d=d^*} = E[w''(d^*+T)] - h''(d^*) \geq 0$ □

2.3 Special case: piece-wise constant marginal utility

Assuming time-invariant $h(t)$ and step function $w(t)$ with a jump at 0, namely

$$h(t) = \alpha, \quad w(t) = \begin{cases} \alpha - \beta, & d + T < 0 \\ \alpha + \gamma, & d + T \geq 0 \end{cases},$$

the travel cost becomes

$$C(d, T) = \alpha T + \beta \max(-(d + T), 0) + \gamma \max(d + T, 0) \tag{6}$$

Since $C(d, T)$ is a increasing function of T, the possible outcomes are already ranked from good to bad, which fits the setting in rank-dependent utility theory. So the rank-dependent expected travel cost is given by

$$RDE[C(d)] = \alpha\mu_w - \beta \int_0^{-d} (d + T) dW[F(T)] + \gamma \int_{-d}^{\infty} (d + T) dW[F(T)] \tag{7}$$

where $\mu_w = \int_0^\infty T \frac{\partial W[F(T)]}{\partial F(T)} f(T) dT$. First-order condition on departure time:

$$\frac{\partial RDE[C(d)]}{\partial d} = \gamma - (\beta + \gamma) W[F(-d)] = 0 \tag{8}$$

Standardizing the random travel time as $T = \mu + \sigma X$, where X has cumulative distribution function $G(x)$,

$$d_w^* = -\mu - \sigma G^{-1}[W^{-1}(\frac{\gamma}{\beta + \gamma})] \tag{9}$$

Therefore the expected cost of traveler who departs at a suboptimal departure time is

$$E[C(d_w^*)] = \alpha\mu + ([(\beta + \gamma) W^{-1}(\frac{\gamma}{\beta + \gamma}) - \gamma] G^{-1}[W^{-1}(\frac{\gamma}{\beta + \gamma})] + (\beta + \gamma) \int_{W^{-1}(\frac{\gamma}{\beta+\gamma})}^{1} G^{-1}(s) ds) \sigma \tag{10}$$

This formula tells that value of mean travel time remains the same, while VTTV has a increment depending on W.

$$E[C(d_w^*) - C(d^*)] = ([(\beta + \gamma) W^{-1}(\frac{\gamma}{\beta + \gamma}) - \gamma] G^{-1}[W^{-1}(\frac{\gamma}{\beta + \gamma})] - (\beta + \gamma) \int_{\frac{\gamma}{\beta+\gamma}}^{W^{-1}(\frac{\gamma}{\beta+\gamma})} G^{-1}(s) ds) \sigma \tag{11}$$

2.4 Special case: time-dependent linear marginal utility

Assuming marginal utility varying by time linearly, (2) can be rewritten as

$$C(d, T) = \int_d^0 (\beta_0 + \beta_1 t) dt + \int_0^{d+T} (\gamma_0 + \gamma_1 t) dt \tag{12}$$

First-order condition under RDEU,

$$\frac{\partial RDE[C(d)]}{\partial d} = \beta_1 \mu_w + (\gamma_0 - \beta_0) + (\gamma_1 - \beta_1)(d + \mu_w) = 0 \tag{13}$$

$$d_w^* = \frac{\beta_0 - \gamma_0 - \gamma_1 \mu_w}{\gamma_1 - \beta_1} \tag{14}$$

In this case, the optimal departure time will not depend on the variability of travel time. Without loss of generality we normalize $h(0) = w(0)$, namely $\gamma_0 = \beta_0$ and the suboptimal travel cost due to misperception will be

$$E[C(d_w^*)] = \frac{\gamma_1^2 \mu_w^2}{2(\gamma_1 - \beta_1)} + \frac{1}{2}\gamma_1 \mu^2 + \frac{1}{2}\gamma_1 \sigma^2 + \mu(\gamma_0 - \frac{\gamma_1^2 \mu_w}{\gamma_1 - \beta_1}) \tag{15}$$

The extra expected cost given a shifted d_w^* is symmetric, making early and late departure equally costly.

$$E[C(d_w^*) - C(d^*)] = \frac{\gamma_1^2}{2(\gamma_1 - \beta_1)}(\mu - \mu_w)^2 \geq 0 \tag{16}$$

The weighting function will make μ_w a function of μ and σ. Specifically, assuming W distorts X by a shift of location μ_Δ and a change of scale σ_Δ, then $T_w = \mu_w + \sigma_w X = \mu + \sigma(\mu_\Delta + \sigma_\Delta X)$ and thus $\mu_w = \mu + \sigma\mu_\Delta$ and $\sigma_w = \sigma\sigma_\Delta$. Inserting these into Equation 15 and taking differential with respect to μ and σ we can have the VMTT and VTTV. If X is time-invariant, then μ_Δ and σ_Δ are constants over time-of-day.

2.5 Valuation

Supposing that utility is money-metric, the value of mean travel time and value of travel time variability in these two special cases could be summarized as follow. It implies that VTTVs in both step and slope model are affected by probability weighting, while VMTTs are irrelevant.

	Step		Slope	
	EU	RDEU	EU	RDEU
μ	α	α	$\gamma_0 - \frac{\beta_1\gamma_1}{\gamma_1-\beta_1}\mu$	$\gamma_0 - \frac{\beta_1\gamma_1}{\gamma_1-\beta_1}\mu$
σ	$(\beta+\gamma)\int_{\frac{\gamma}{\beta+\gamma}}^{1} G^{-1}(s)ds$	$[(\beta+\gamma)W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma]G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]$ $+(\beta+\gamma)\int_{W^{-1}(\frac{\gamma}{\beta+\gamma})}^{1} G^{-1}(s)ds$	-	-
σ^2	-	-	$\frac{\gamma_1}{2}$	$\frac{\gamma_1}{2} + \frac{\gamma_1^2\mu_\Delta}{2(\gamma_1-\beta_1)}$

3 Data

The data used in this research originates from a tentative Internet-based stated preference study conducted in 2010, where the respondents were asked to put themselves into day-to-day car-commuting route choice scenario. The departure time is implicitly given and assumed to be unadjustable.

Regarding the experiment design, we first generated 1000 random draws $X = \{x_1, ..., x_{1000}\}$ based on a stable travel time distribution (see Figure 3) measured from "Tomei Express"—a toll road connecting Tokyo city and its south-west suburban. Its fat tail reflects the characteristics of congested time due to traffic incident. Subsequently, we specified a 5^3 fractional factorial design as shown in Table 2. Design factors are mean travel time μ, standard deviation σ, and optimal probability of lateness $\frac{\beta}{\beta+\gamma}$. Random travel time for choice profile is generated by $T = \mu + \sigma X$. Following Fosgerau and Karlström (2010), we calculated optimal departure time under linear probability weighting by $d^* = -\mu - \sigma\int_{\frac{\gamma}{\beta+\gamma}}^{1} G^{-1}(s)ds$ for each choice profile, and counted the frequencies of schedule delay within given intervals (e.g., $\#SDE_{0-10} = \#\{T_k : -10 \leq d^* + T_k \leq 0, k = 1, ..., 1000\}/10$). In this way travel time variability is converted and presented as a histogram-like choice situation as Table 3. Each possible schedule delay interval is corresponding to an occurrence frequency that respondent has experienced in the past 100 days. Such efforts were made to ensure even the respondent without knowledge of probability could understand the given information (Tseng et al., 2009).

Although some scheduling preference parameters are used as design attribute and the departure time is decided by interaction of design attributes rather than predefined levels, each alternative's departure

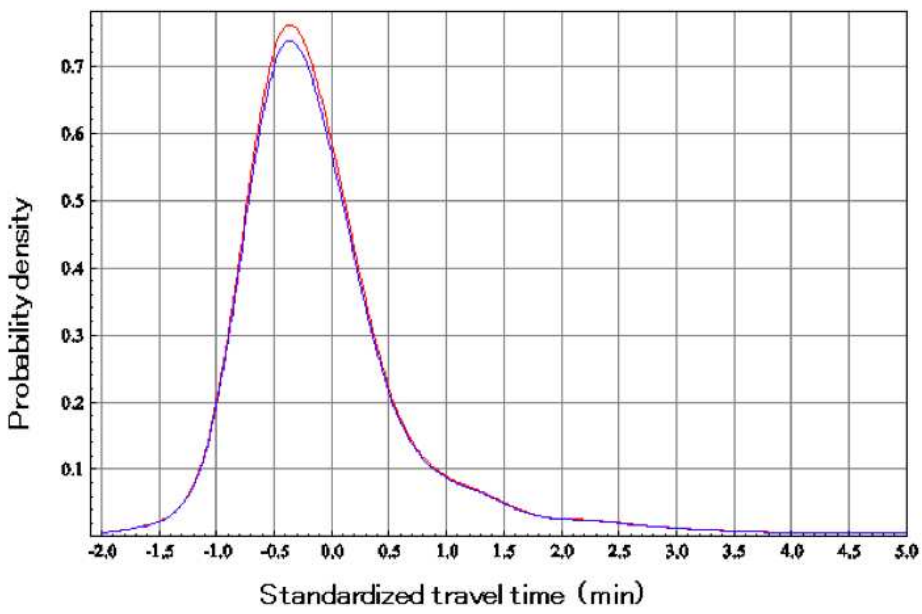

Figure 4: Stable distribution of travel time in Tomei Express

time remains exogenous from the respondent's point of view. The departure time for each profile is claimed to be unchangeable. Therefore the essence that respondent is choosing departure time given schedule constraint and stochastic travel time still holds. This could also be viewed as a bayesian approach where we first propose some prior belief of estimate values and the repeated choice updates the posterior distribution.

Table 2: Attribute level setting in SP experiment

Attributes	Level 1	Level 2	Level 3	Level 4	Level 5
Mean (min)	48	54	60	66	72
Standard Deviation (min)	8	10	12	14	16
Optimal probability of lateness ($\frac{\beta}{\beta+\gamma}$)	0.3	0.2	0.4	0.1	0.5

Table 3: Sample choice situation in our SP experiment

		Frequency of arriving early and late in past 100 days							
Route	Mean time in past 100 days	20^+ early	10-20 early	0-10 early	0-10 late	10-20 late	20-30 late	30-40 late	40^+ late
A	60 min	0	1	72	21	4	1	1	0
B	72 min	0	3	77	14	3	1	1	0

However, as a natural representation of travel time, ambiguity is inevitably existing in this experiments. Outcomes are not specific values but only known to be bounded within intervals in probability. The identification would be a problem if we do not impose further assumptions. Therefore, the travel time perceived by individual are assumed to be distributed uniformly within these intervals, such that the average of lower and upper bound of given interval could be regarded as a mass representing of specific outcome[2]. The travel time distribution is thus discretized. Despite a relatively strong distribu-

[2] 25 min and 45 min are arbitrarily chosen for representing schedule delay earlier than 20 min and later than 40 min because of the infinite upper bound. It is not expected affect the result much given low frequencies of these extreme outcomes.

tional assumption, it is still worth trying, since it could be the case that people use histogram-shaped approximation rather than perfectly forming a travel time distribution in their mind (Tseng et al., 2009).

All of the subjects are daily car commuter. After discarding the sample with (i) missing income, (ii) answering time shorter than 20 minutes or longer than 45 minutes, we have 4176 observations left, answered by 232 respondents each of whom facing 18 choice scenarios.

Table 4: Descriptive statistics

	Min	Median	Mean	Max
Age	20.00	45.00	43.44	60.00
Female dummy	0.00	0.00	0.39	1.00
Annual income (10^4 JPY)	50.00	600.00	587.10	1200.00
House ownership	0.00	1.00	0.68	1.00
Household size	1.00	3.00	2.90	8.00
Flexible workday	0.00	1.00	0.70	1.00
Commuting time (min)	1.00	20.00	26.08	110.00

4 Empirical specification

We specify econometric models for two aforementioned special cases of scheduling model, which are simply referred to as step model and slope model. The dichotomous choice data will be estimated by random utility discrete choice model with systematic utility function

$$V = \begin{cases} \mathit{Step}: & \sum_{i=1}^{n} \pi_i(\alpha T_i + \beta SDE_i + \gamma SDL_i) \\ \mathit{Slope}: & \sum_{i=1}^{n} \pi_i[(\gamma_0 - \beta_0)d + \gamma_0 T_i + (\gamma_1 - \beta_1)d^2/2 + \gamma_1(T_i^2/2 + T_i d)] \end{cases} \tag{17}$$

where $\{T_1, ..., T_i\}$ are i possible travel times from a decision with corresponding probabilities $\{p_1, ..., p_i\}$, such that $T_i < T_{i-1}$, i.e., travel time ranked from low to high, $\pi_i = w(p_i + p_{i-1} + \cdots + p_1) - w(p_{i-1} + p_{i-2} + \cdots + p_1)$ is the decision weight put on each T_i. Note that $T_i < T_{i-1}$ does not necessarily imply $T_i \prec T_{i-1}$, so the implicit assumption here is that travelers always prefer low travel time, namely $\beta < \alpha$ when departure time is given. Behaviorally it means travelers will prefer to terminate the trip as soon as they arrive rather than keep detouring until preferred arrival time comes. Such assumption is supported by the vast majority of empirical estimates. Two popular probability weighting function are

$$w[p] = \begin{cases} T-K: & \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \\ \mathit{Prelec}: & \exp(-\eta(-\ln p)^\gamma) \end{cases} \tag{18}$$

The criterion of selecting a probability weighting function is whether it is flexible enough to reflect four types of curves as mentioned. The Prelec (1998) weighting function fits this criterion as curvature (discriminality) is controlled by γ and elevation (attractiveness) is controlled by η. We keep Tversky-Kahneman (T-K) weighting function as a contrast though it does not match our criterion. The random utility for alternative j of individual n is

$$U_{nj} = V(\cdot) + \varepsilon_{nj}. \tag{19}$$

Alternative-specific constant is not considered because alternatives are unlabeled. Further, We assume the error term ε_{nj} is independent and identically Gumbel distributed so that a binary Logit framework can be applied. This model is estimated by maximum log-likelihood estimator with Pythonbiogeme 2.2. Type i error could be easily detected by comparing the preference estimates in EU-based model and

RDEU-based model. Because monetary cost is not included in our data, money-metric utility will be unavailable, and type i and type ii error will be measured by the ratio of parameters.

5 Estimation result

5.1 Piece-wise constant marginal utility

To make the results comparable, we first estimates a scheduling model with linear probability weighting, namely $\pi_i = p_i$ in Equation (17), as benchmark. According to the estimation result shown in Table 5, schedule delay early seems exerting no significant effect on the marginal utility while the schedule delay late ratio reaches as high as 11.3. Although the model fit is not bad, such a high SDL ratio is counter-intuitive and has not been found in any studies so far. This evidence casts a doubt on the validity of assuming individual's calculating expected value linearly when the experiment design is risk-involved. Given so, the non-linear scheduling model becomes a natural alternative. The model with Prelec weighting function outperforms the other w.r.t goodness-of-fit, while the one with T-K weighting function is not superior to the linear model. The SDE and SDL ratio in Prelec model are 0.92 and 6.0 respectively. This makes sense and we are not surprised that SDL ratio is relatively high since we did not consider lateness penalty.

Table 5: Step model

	Linear		T-K		Prelec	
Parameter	Value	t-stats	Value	t-stats	Value	t-stats
Mean time (α)	-0.0982	-22.47	-0.0985	-22.53	-0.101	-22.46
Schedule delay early (β)	-0.0178	-1.23*	-0.027	-1.65*	-0.0922	-3.41
Schedule delay late (γ)	-1.11	-24.04	-1.158	-20.22	-0.607	-18.89
Theta (θ)			1.03	1.05*	1.98	5.7
Eta (η)					10.5	3.43
No. of observation	4176		4176		4176	
Final log-likelihood	-1915.857		-1915.18		-1907.522	
Likelihood ratio ($\bar{\rho}^2$)	0.337		0.337		0.339	
Akaike information criteria (AIC)	3837.714		3838.36		3825.044	
SDE ratio	0.18		0.27		0.92	
SDL ratio	11.3		11.76		6.0	

Besides, two models indicate totally different pattern of probability weighting. θ in T-K model is not significantly different from 1, which means we cannot reject that people are perfectly following expected utility theory. If so, there is no need to consider probability weighting. An explanation for T-K weighting function reducing to a linear curve is that it gives best fit if the true probability weighting curve is some shape that cannot be reflected by T-K weighting function. In this case, the Prelec model is more reliable given its flexibility. By contrast Prelec model shows a s-shape weighting function that is convex within [0,0.92] and concave later on. Particularly probabilities smaller than 0.45 are under-weighted to 0. It implies that essentially people are very pessimistic and tend to choose a earlier departure time than they really need. However, they tend to undervalue the possibility of extremely bad outcomes, which means one requires very low risk of being late (smaller than 4%) tend to choose a departure time that makes him/her bear slightly higher risk. It also indicates that travelers are more focusing on intermediate outcomes rather than the extremely good or bad ones. One possible explanation is that the intermediate outcomes are what delineate the boundary of arriving early and late, and people do care much about lateness. This probability weighting curve should be robust in a sense that the utility function has captured the effect of loss aversion.

5.2 Time-dependent marginal utility

The Prelec model again outperforms other two models w.r.t. goodness-of-fit, and this superiority is larger compared with that in step model. In all models we find the slope of decreasing marginal utility at home $h(t)$ is very small compared with the slope of increasing marginal utility at workplace $w(t)$. Its implication is that traveler prefer to depart earlier rather than later in response to a change of travel time, which could be due to a tight workday of population[3]. An extreme case will be when $h(t)$ is flat, preferred arrival time is always 0 and travelers will assign all the increment of travel time to their headstart. The ideal arrival time calculated by $\frac{\beta_0-\gamma_0}{\gamma_1-\beta_1}$ from three models are negative and valued from -9 to -31. This meets the setting in slope model that ideal arrival time is supposed on the left of preferred arrival time given non-zero T.

The Prelec weighting curve displays similar pattern as its counterpart in step model, reflecting pessimistic attitude almost at the same magnitude. It is noted that the decision weights put on extremely bad outcomes are not quite different from linear, which means travelers can perceive probabilities of these outcomes fairly. Meanwhile, the T-K weighting function shows a completely convex curve, which also indicates pessimism. Two weighting functions are not mean-preserving transformations unless the travel time distribution has a very fat tail. So we argue that the probability weighting will result in early departure. Yet its cost is not expected to be large because of the relatively flat $h(t)$.

Table 6: Slope model

	Linear		T-K		Prelec	
Parameter	Value	t-stats	Value	t-stats	Value	t-stats
$\gamma_0 - \beta_0$	-0.52	-13.1	-0.496	-12.17	-0.308	-6.85
γ_0	-0.542	-27.6	-0.413	-15.57	-0.357	-26.52
$\gamma_1 - \beta_1$	-0.0324	-18.06	-0.0158	-3.18	-0.033	-11.3
γ_1	-0.0313	-16.3	-0.0129	-2.43	-0.0321	-9.84
Theta (θ)			2.71	8.75	2.29	21.14
Eta (η)					15.7	6.47
Final log-likelihood	-1957.869		-1944.492		-1914.151	
No. of Observations	4176		4176		4176	
Likelihood ratio($\bar{\rho}^2$)	0.322		0.327		0.337	

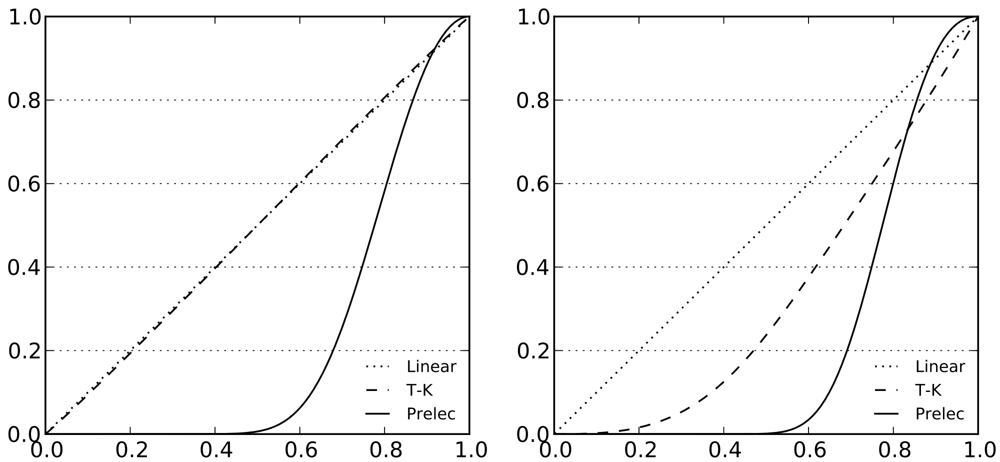

Figure 5: objective probability (ranked travel time) versus decision weight, step model on the left and slope model on the right

[3]we try segmenting population by flexibility of workdays and find steeper $w(t)$ for those who have flexible workdays

5.3 Type i and type ii error

Supposing the scheduling preferences estimated from models with Prelec weighting function are the true ones associated with its certainty equivalent, we can decompose the mixture of type i and type ii error into two term: (i) the difference between reliability ratios calculated by estimates from models with and without probability weighting function and (ii) the difference between reliability ratios derived with and without considering cost of misperception using same estimates from non-linear model. Because VMTTs are previously shown irrelevant to probability weighting in both model, they will serve well as the baseline and thus reliability ratios here actually reflect the level of expected cost. Although all estimates are supposed to be normal distributed in our econometric setting, the distribution of reliability ratio is unknown after the conversion. So we calculate confidence intervals of reliability ratio and two types of errors by generating 3000 multinomial random draws that subjects to the estimated variance-covariance matrix and convert them to reliability ratio. Subsequently, we use Wilcoxon rank-sum test to see if aforementioned two types of errors are of statistical significance.

	Step			Slope						
Valuation	EU1	EU2	RDEU	EU1	EU2	RDEU	δ_1	z-stats	δ_2	z-stats
$\frac{\sigma}{\mu}_{mean}$	1.79	2.39	2.42				-23.8%	-39.09	-0.9%	-2.76
$\frac{\sigma}{\mu}_{.025}$	0	1.77	1.77				-100%		-2.37%	
$\frac{\sigma}{\mu}_{.975}$	2.79	2.96	3.02				-30%		0.1%	
$\frac{\sigma^2}{\mu}_{mean}$				0.173	0.162	0.176	7.95%	25.26	-7.58%	-24.42
$\frac{\sigma^2}{\mu}_{.025}$				0.150	0.129	0.138	-17.8%		-9.59%	
$\frac{\sigma^2}{\mu}_{.975}$				0.197	0.202	0.220	39.6%		-5.85%	

Table 7: Type i and type ii error

The statistical test shows both types of errors are significant rather than being random variations. The type i error is sufficiently high, particularly -23.8% in step model and 7.95% in slope model. It has been a debate in reliability studies that in what way the variability should be incorporated into SP design. Probability is involved in the survey either explicitly or implicitly. Our findings imply that it is necessary to consider individual's non-linear probability weighting when modeling this kind of data, otherwise the estimates could be quite biased. Another choice is to not include risk as a design attribute, so that the elicited scheduling preferences are ensured to associate with certainty equivalent of random travel time. On the other hand, the effect of ignoring the cost of probability misperception on deriving reduced form, namely type ii error, will be as minor as -0.9% in step model, if weighting parameters are estimated correctly in our empirical application. This is a good news since it suggests we can keep taking advantage of analytical convenience of linearity without losing the accuracy. However, such effect weights -7.58% in slope model, which is not an trivial number. It is not surprising since the marginal utility of time is changing over time in slope model, making unit deviation from optimal departure time more costly. The conclusion depends pretty much on whether time-varying marginal utility is the case in reality. More empirical is apparently needed, yet we choose to hold a weak belief that the cost of misperception is minor given better model fit of step model.

6 Concluding remarks

The theoretical equivalence between mean-variance and scheduling model might not hold if individual is not expected utility maximizer. We argue the effect of this violation is twofold: the estimated scheduling preferences might be biased and the generalized travel cost might be underestimated. We reformulate a general scheduling model under rank-dependent theory and analyze its properties. It is found that the shape of weighting function and the trend marginal utility of time changes determine the way traveler's suboptimal departure time deviates from the optimal one, and thus the cost of probability perception.

With the data collected form a stated preference experiment, we estimated two special cases of proposed model. The estimation results indicates that (i) travelers are mostly pessimistic, (ii) the scheduling preference estimates are quite biased (around 20%) without considering probability weighting in a risky choice situation and (iii) the cost of probability misperception might be as minor as 1%. These imply that estimating scheduling preference from certain choice situation and converting it to generalized cost associating with aggregate measure from supply side is a viable approach for future practice. The probability misperception is not likely to be what causes the discrepancy between two models as found in Börjesson et al. (2012). However, more empirical studies are needed to confirm our argument.

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A Appendix.

$$
\begin{aligned}
E[C(d_w^*)] &= \alpha\mu + \beta\int_0^{F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]}(F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] - T)f(T)dT \\
&\quad + \gamma\int_{F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]}^{\infty}(T - (F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})])f(T)dT \\
&= (\alpha-\beta)\mu + [\beta W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma + \gamma W^{-1}(\frac{\gamma}{\beta+\gamma}))]F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] \\
&\quad + (\beta+\gamma)\int_{F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]}^{\infty} Tf(T)dt \\
&= (\alpha-\beta)\mu + [\beta W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma + \gamma W^{-1}(\frac{\gamma}{\beta+\gamma}))]F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] \\
&\quad + (\beta+\gamma)\int_{G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]}^{\infty}(\mu+\sigma x)g(x)dx \\
&= (\alpha-\beta)\mu + [\beta W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma + \gamma W^{-1}(\frac{\gamma}{\beta+\gamma}))](\mu + \sigma G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]) \\
&\quad + (\beta+\gamma)\mu(1 - W^{-1}(\frac{\gamma}{\beta+\gamma})) + (\beta+\gamma)\sigma\int_{W^{-1}(\frac{\gamma}{\beta+\gamma})}^{1} G^{-1}(s)ds \\
&= \alpha\mu + ([(\beta+\gamma)W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma]G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] + (\beta+\gamma)\int_{W^{-1}(\frac{\gamma}{\beta+\gamma})}^{1} G^{-1}(s)ds)\sigma
\end{aligned}
$$

$$
\begin{aligned}
E[C(d_w^*)] - E[C(d^*)] &= ([(\beta+\gamma)W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma]G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] - (\beta+\gamma)\int_{\frac{\gamma}{\beta+\gamma}}^{W^{-1}(\frac{\gamma}{\beta+\gamma})} G^{-1}(s)ds)\sigma \\
&= [(\beta+\gamma)W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma](G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] - G^{-1}(\Delta)), \quad \Delta \in (\frac{\gamma}{\beta+\gamma}, W^{-1}(\frac{\gamma}{\beta+\gamma})) \\
&> [(\beta+\gamma)W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma](G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] - G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]) = 0
\end{aligned}
$$