# A Neo-Kaldorian Approach to Structural Economic Dynamics

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**Abstract**

Although the Structural Economic Dynamic approach provides a simultaneous consideration of demand and supply sides of economic growth, it does not take into fully account the possible role played by demand in the generation of technical progress. From a neo-Kaldorian perspective, this paper seeks to establish the concepts of demand and productivity regimes in an open version of the pure labour Pasinettian model. In order to derive the demand regime, a disaggregated version of the Keynesian multiplier is derived for an open economy, while the productivity regime is built in terms of disaggregated Kaldor-Verdoorn's laws. The upshot is a multi-sector growth model of structural change and cumulative causation, in which an open version of the Pasinettian model may be obtained as a particular case. Besides, it is highlighted that the evolution of demand patterns, while being affected by differential rates of productivity growth in different sectors of the economy, also play an important role to establish the pace of technical progress.



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## 1. Introduction

While structural change and economic growth register as interrelated processes, the mainstream assigns to issues such as technical progress and capital accumulation the main role, relegating structural changes to a secondary position in explaining economic growth. The traditional Neoclassical approach with its emphasizes on the supply side, and originally built in terms of one or two sector models [see e.g. Solow (1956) and Uzawa (1961)] cannot take into account the possible links between growth and changes in the structure of an economy[1]. According to this view, structural changes are simply a by-product of the growth in per capita gross domestic product – GDP hereafter. [see McCombie (2006) and McMillan and Rodrik (2011)].

This reality is in sharp contrast with the post-Keynesian view, where success in structural change proves to be the key to economic development. Different approaches have taken into account the connections between growth and change in this tradition, with particular emphasis on the role played by demand even in the long run [see e.g. Pasinetti (1981, 1983), Setterfield (2010), Thirlwall (2013) and Ocampo et al. (2009)]. Within this tradition, the Structural Economic Dynamic view – SED hereafter – is distinguishable by its simultaneous considerations of supply and demand sides in a

[1] Authors such as Ngai and Pissarides (2007) and Echevaria (1997) among others have built multi-sectoral dynamic general equilibrium models that could in principle take into account structural changes. Although the new generation of multi-sectoral Neoclassical models acknowledges that demand plays an important role to explain structural change and, thus economic growth, it is still noted a passive role played by demand in such models. Arguably, being these analysis carried out in models of general equilibrium, some important issues related to structural change, such as structural unemployment and uneven development cannot be taken into account, calling these analysis into question. See the introductory chapter of Arena and Porta (2012) for a survey on the state of the art of the literature on structural change after the renewal interest of the mainstream by the theme.

multi-sectoral framework, being the interaction between the evolving patterns of demand and technical progress responsible for particular dynamics of output, prices and structural transformation of economies in different stages of the development process. Pasinetti's emphasis upon demand composition offers a significant qualitative improvement vis-a-vis traditional, aggregated models, which fail to adequately consider the composition of consumption demand thus concealing structural changes.

Although the SED approach provides a better treatment of structural changes, some authors have pointed to the necessity of a more inclusive treatment of the demand side in order to provide a fully characterization or even endogenisation of technical progress and structural changes[2]. Gualerzi (2012) for instance notes that the SED is an approach rooted in the theory of demand-led growth insofar as demand matters to shape how supply factors and technical change in particular will evolve not only in the short run but also in the long run. But elsewhere the author states that "[i]n Pasinetti's scheme, since the very source of income growth, technical change, is itself fully exogenous, potential demand is identified only with available disposable income; as such it is a passive notion". [Gualerzi (2001, p. 26)]

According to this view, demand still plays a somewhat passive role in the SED approach since increases in per capita income are motivated by technical change, which

[2] Pasinetti (1983, p.69) himself acknowledges the importance of considering a better treatment of the demand side when questing the origins of technological progress. According to him: "[t]his means that any investigation into technical progress, must necessarily imply some hypothesis on the evolution of consumers' preferences as income increases. Not to make such hypothesis and to pretend to discuss technical progress without considering the evolution of demand would make it impossible to evaluate the very relevance of technical progress and would render the investigation itself meaningless."

is wholly exogenous[3]. Admittedly, being the focus of Pasinetti's analysis the effect of productivity growth differentials on the sectoral dynamics, exogenous technical progress hinders a deeper understanding of the endogenous growth mechanisms. In this vein, if on one hand the Pasinettian model emphasizes the main channels of interdependence between economic growth and structural change in a multi-sectoral set up, on the other hand, it overlooks the emphasis of the demand-led-growth theory in which consumption and growth feedback in a cumulative process.

Hence, the SED approach in its original formulation is not able to take into account a *deeper* conception of endogenous technical change according to which the rate of technical progress is sensitive to the rate of output growth. That view of endogenous growth process is emphasized by the Neo-Kaldorian literature[4], which assigns to demand a central role in generating technical progress through the Kaldor-Verdoorn law. [Roberts and Setterfield (2007)].

In this article we intend to fill that gap by by building a bridge between the SED formulation and the Neo-Kaldorian theory, with its focus on cumulative causation and

[3] This view is also emphasized by Silva and Teixeira (2008, p.286) where they consider that: "Although Pasinetti relates both factors with the learning principle, learning itself is essentially unexplained and therefore the question of what moves the driving forces of the economy remains unanswered."

[4] Some developments of the neo-Kaldorian tradition within multisectoral set ups, mainly related to the balance of payments-constrained growth (BPCG), have yielded useful insights. Araujo and Lima (2007) and Araujo (2013) for instance have derived versions of the balance of payment constrained growth rates that explain the growth performance by considering that the evolution of patterns of consumption plays a crucial role in the performance of the external sector, and as a consequence of the overall economy.

endogenous growth[5]. To accomplish this task, we conceptualize the notion of a demand regime that departs from a multi-sectoral version of the Keynesian multiplier in an open version of Pasinetti's model. Trigg and Lee (2005) have shown within Pasinetti's pure production model that it is possible to derive a simple multiplier relationship from multisectoral foundations, meaning that a scalar multiplier can legitimately be applied to a closed multisector economy.

Here we extend the Trigg and Lee (2005) analysis to derive the Keynesian multiplier in an open version of Pasinettian model [see Araujo and Teixeira (2004)]. This derivation is a crucial step to establish the links with the Neo-Kaldorian literature since this literature assigns to exports a key role in autonomous aggregate demand. According to this view, the dynamism of the export sector may give rise to virtuous cycles of economic growth not only through its straight effect on aggregate demand but also due to dynamic economies of scale[6] that accrue from an increase in output.

Hence, the first contribution of this paper is the derivation of the multi-sectoral Keynesian multiplier in the Pasinettian model for an open economy. This derivation allows us to derive a proper demand regime for the model and, following the Neo-Kaldorian literature on growth regimes [see Blecker (2002)], we introduce concepts

[5] Pasinetti (2005, p. 839-40) assigns to a lack of theoretical cohesion amongst models in the Keynesian tradition a possible explanation for the difficulties faced by this theory as a successful alternative paradigm to mainstream economics.

[6] Cornwall and Cornwall (2002, p. 206) highlighted these mechanisms by considering that the contribution of the external sector to productivity growth is twofold: first it allows the larger scale production methods to improve productivity and second it encourages the adoption of the best available technologies spurring productivity.

such as demand and productivity regimes[7] in an open version of the Pasinettian model [Araujo and Teixeira (2004)]. With this analysis, we are able to endogenise technical progress in the SED model. Besides, it allows for connecting many of the arguments that underpin the importance of the endogenous concept of economic growth.

The second contribution of the paper rests in showing that the open version of the Pasinettian model derived by Araujo and Teixeira (2004) may be seen as a particular case of the approach derived here from the multi-sectoral Keynesian multiplier for an open economy. While the Pasinettian solution holds as a potential or equilibrium production, the solution derived here registers as effective production, being the latter equal to the former when the condition of full employment of the labour force is satisfied. This registers as a well-known result in the SED framework, and one of the main outcomes of the Pasinettian analysis is that in general it is not fulfilled, meaning that unemployment is the most probable outcome of structural chagnes. As a consequence, the SED solution of physical system is then shown to be a particular case of the solution derived here.

In order to emphasize this point, we carry out the formulation of a sectoral demand regime both in terms of effective and potential sectoral output. The first analysis is developed under the label of Sectoral Demand Regime – SDR hereafter – while the latter is referred as the Structural Economic Dynamic Regime – SEDR hereafter. Notwithstanding the Neo-Kaldorian emphasizes on the role of effective demand in interacting with productivity in a cumulative sense, the derivation of the SEDR allows us to take into account the role of potential demand plays in generating technical change, without denying the main role of effective demand.

[7] The sectoral productivy regime departs from Araujo (2013) who introduced sectoral Kaldor-Verdoorn's law to endogenize technical progress in Pasinetti's model.

Besides, it allows us to show that the Neo-Kaldorian analysis may also reap benefits from a disaggregated assessment of its basic framework. Even departing from a somewhat narrower view of cumulative causation based on Adam Smith dictum that "the division of labour is limited by the extent of the market" – which emphasizes the sectoral aspect of dynamic increasing returns of scale – we arrive at a Macroeconomic notion, in which technical change in one sector spurs productivity in other sectors through its effect on per capita income growth [see Young (1928)]. Central to this development is the concept of Engel's law, according to which an evolving pattern of consumption arises when per capita income grows.

This article is structured as follows: in the next section the demand and productivity regimes are modeled in the Pasinettian framerwork. In the third section we develop the SED regime, which may be compared to the previous regimes. Section 4 concludes.

## 2. The Sectoral Demand Regime [SDR]: The Derivation of the Multi-sectoral Multiplier for an open economy

The traditional Neo-Kaldorian growth schema [see McCombie and Thirlwall (1994) and Setterfield and Cornwall (2002)] is presented in terms of both demand – DR hereafter – and productivity regimes – PR hereafter. The latter is portrayed by a Kaldor-Verdoorn function while the former is depicted by the effects of growth rate of exports – and in some cases the growth rate of autonomous investment – on the growth rate of output via aggregate demand. [Setterfield and Cornwall (2002, p. 71)].

Following these lines, in order to develop a DR in the Pasinettian approach, we depart from Trigg and Lee (2005)[8], who derived a multisectoral version of the Keynesian multiplier. But due to the importance of foreign demand in the Neo-Kaldorian literature we go a step further by developing an extended version of the disaggregated Keynesian multiplier that takes into account international trade. Dealing with a closed version of the Pasinettian model, the authors had to assume that investment in the current period becomes new capital inputs in the next period and that the rate of depreciation is 100% (that is, all capital is circulating capital) in order to derive the Keynesian multiplier. By using matrix notation, they wrote the system of physical quantities as:

$$\begin{bmatrix} \mathrm{I} & -\mathrm{c} \\ -\mathrm{a} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ X_n \end{bmatrix} = \begin{bmatrix} \mathrm{M} \\ 0 \end{bmatrix} \tag{1}$$

Where I is an ($n$–1)x($n$–1) identity matrix, $\mathrm{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_{n-1} \end{bmatrix}$ is the ($n$–1) column vector of physical quantities, namely $X_i$, $\mathrm{c} = \begin{bmatrix} a_{1n} \\ \vdots \\ a_{n-1,n} \end{bmatrix}$ is the ($n$–1) column vector of consumption coefficients, $a_{in}$, and $\mathrm{a} = \begin{bmatrix} a_{n1} & \cdots & a_{n-1,n} \end{bmatrix}$ is the ($n$–1) line vector of labour coefficients,

[8]The idea of developing a multi-sectoral version of the Keynesian multiplier dates back to Goodwin (1949) and Miyazawa (1960) who accomplished to develop a disaggregated version of the income multiplier in Leontief's framework from the relatively simple Keynesian structure. Both authors emphasized that although there are important differences between the Keynes and Leontief approaches, a bridge between them, namely a disaggregated version of the multiplier, is an important development for both views.

$a_{ni}$, $i$ = 1,2,...,$n$ – 1. $X_n$ represents the quantity of labour in all internal production activities. M is assumed to be a ($n$ – 1) vector of physical quantities of investment goods produced in each sector. Taking into account the extension of the Pasinettian model to international flows [see Araujo and Teixeira (2004)], it is possible to introduce international flows in the model by writing the system of physical quantities as:

$$\begin{bmatrix} \mathrm{I} & -\mathrm{c} \\ -\mathrm{a} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ X_n \end{bmatrix} = \begin{bmatrix} \mathrm{M}+\mathrm{E} \\ 0 \end{bmatrix} \tag{2}$$

Where $\mathrm{E} = \xi X_n \hat{\mathrm{c}}$ denotes the exports. In this case, $\hat{\mathrm{c}} = \begin{bmatrix} a_{1\hat{n}} \\ \vdots \\ a_{n-1,\hat{n}} \end{bmatrix}$ refers to the ($n$–1) column vector of foreign demand coefficients, namely $a_{i\hat{n}}$, $i$ = 1,2,...,$n$ – 1. The population size in both countries is related by the coefficient of proportionality $\xi$. We may rewrite system (2) as:

$$\begin{cases} \mathrm{X} - \mathrm{c}X_n = \mathrm{M}+\mathrm{E} \\ -\mathrm{aX} + X_n = 0 \end{cases} \tag{3}$$

From the last line of system (3), it follows that:

$$X_n = \mathrm{aX} \tag{4}$$

By pre-multiplying throughout the first line of (3) by a , it follows that:

$$\mathrm{aX} - \mathrm{ac}X_n = \mathrm{a}(\mathrm{M}+\mathrm{E}) \tag{5}$$

By substituting (4) into expression (5), and isolating $X_n$, we obtain the employment multiplier relationship:

$$X_n = \frac{1}{1-\mathrm{ac}}\mathrm{a}(\mathrm{M}+\mathrm{E}) \tag{6}$$

where $1/1-\mathrm{ac}$ is a scalar employment multiplier [Trigg and Lee (2005)]. Through further decomposition [see Trigg (2006, Appendix 2)], (6) can be substituted into the first line of (3) to yield:

$$\mathrm{X} = \left(\mathrm{I} + \frac{\mathrm{ca}}{1-\mathrm{ac}}\right)(\mathrm{M}+\mathrm{E}) \tag{7}$$

This is a multiplier relationship between the vector of gross outputs $(\mathrm{X})$ and the vector representing final demand $(\mathrm{M}+\mathrm{E})$, where $\left(\mathrm{I} + \frac{\mathrm{ca}}{1-\mathrm{ac}}\right)$ is the output multiplier matrix. One of the main differences between this multi-sectoral multiplier for an open economy and the one derived by Trigg and Lee is that the latter is a scalar, and the former is a matrix. In order to highlight the working of the new version of the disaggregated multiplier, in what follows let us assume that $\mathrm{M}=\mathrm{O}$ for convenience only, which yields:

$$\begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} a_{1n} \\ \vdots \\ a_{n-1,n} \end{bmatrix} \begin{bmatrix} a_{n1} & \ldots & a_{n,n-1} \end{bmatrix} \begin{bmatrix} E_1/1-\mathrm{ac} \\ \vdots \\ E_n/1-\mathrm{ac} \end{bmatrix} + \begin{bmatrix} E_1 \\ \vdots \\ E_n \end{bmatrix} \tag{8}$$

The multiplier relationship for the *i*-th sector therefore takes the form:

$$X_i = E_i + \left(\frac{\mathrm{ca}}{1-\mathrm{ac}}\right)\mathrm{E} \tag{9}$$

In fact, the sectoral physical solution derived from the multi-sectoral Keynesian multiplier corresponds to the effective production, which contrasts with the potential or equilibrium production derived in the Pasinetti model. In order to fix the notation, let us follow the author convention of considering the existence of two countries. The advanced one is denoted by *A* and the underdeveloped one by *U*. Now it follows from (9), for the underdeveloped country $U$ :

$$X_i^U = \left( a_{in}^U \frac{\sum_{j=1}^{n-1} \xi a_{nj}^U a_{j\hat{n}}^U}{1 - \sum_{j=1}^{n-1} a_{jn}^U a_{nj}^U} + \xi \, a_{i\hat{n}}^U \right) X_n \tag{10}$$

where $a_{in}^U$ stands for the per capita demand coefficient of final commodity *i* for the *U* country, being the quantity demanded domestically multiplied by population of *U* country, $X_n$. The production coefficient $a_{ni}^U$ conveys the amount of labour per produced unit of final good *i* in countries *U*. Expression (10) plays a central role in our analysis. It shows that the effective demand for output of the *i*-th sector is due to two components: the domestic demand, conveyed by the domestic consumption coefficient $a_{in}^U$, and external demand, portrayed by the foreign demand coefficient $a_{in}^U$. Due to reasons that will become clearer latter, the domestic coefficient is affected by the structural economic dynamics of the economy as a whole, captured by the quotient: $\frac{\sum_{j=1}^{n-1} \xi a_{nj}^U a_{j\hat{n}}^U}{1 - \sum_{j=1}^{n-1} a_{jn}^U a_{nj}^U}$.

This quotient particularizes the solution obtained here, given by expression (10), from the solution derived by Araujo and Teixeira (2004) for an open version of the Pasinettian model. While the latter refers to sectoral potential output the former registers as effective sectoral production.

Following Araujo (2013) and considering that $p_i^U$ and $p_i^A$ stand for prices of the *i*-th consumption good in countries *U* and *A*, respectively, let us consider that per capita export coefficient $a_{i\hat{n}}^U$ is given according to:

i) On one hand, if $ep_i^A < p_i^U$, that is, if country *U* has no comparative cost advantage in the production of consumption good $i$, then the per capita foreign demand for good $i$ is assumed to be zero: $a_{i\hat{n}} = 0$. If $ep_i^A \geq p_i^U$, then let us consider that the foreign demand for the consumption good $i$ is given by an export function *à la* Thirlwall (1979) [see Araujo and Lima (2007)]:

$$a_{i\hat{n}}^U = \left(\frac{p_i^U}{ep_i^A}\right)^{\eta_i} y_A^{\beta_i} \; X_{\hat{n}}^{1-\beta_i} \tag{11}$$

where $y_A$ denotes the per capita income of country *A* and *e* stands for the the population of the *A* country, denoted by $X_{\hat{n}}$. $\eta_i$ designates a price elasticity of demand for exports of good $i$, with $\eta_i < 0$. While $\beta_i$ denotes an income elasticity of demand for exports, and with $\beta_i > 0$. According to this specification, it is not assumed *ex-ante* full specialization.

ii) On the other hand, if country *A* has no comparative cost advantage in the production of consumption good $i$, we assume country *U* does not import it, that is, $a_{\hat{i}n} = 0$, where $a_{\hat{i}n}$ stands for the per capita import coefficient for good $i$. But if $p_i^U \geq ep_i^A$, let us consider that the demand coefficients for imports are given by the following import function:

$$a_{\hat{i}n} = \left(\frac{ep_i^A}{p_i^U}\right)^{\varphi_i} y_U^{\phi_i} X_n^{\phi_i - 1} \tag{12}$$

where $\varphi_i$ is the price elasticity of the demand for imports of good *i*, with $\varphi_i < 0$, $\phi_i$ is the income elasticity of the demand for imports of good *i* and $y_U$ is the per capita income of country *U*. Following Pasinetti (1981), the coefficient of internal demand is assumed to vary according to:

$$a_{in}^{U}(t) = a_{in}^{U}(0)\exp(r_i^{U} t) \tag{13}$$

where $r_i^U$ stands for the growth rate of domestic demand of good *i* in the *U* country. In what follows let us assume that the evolution of consumption patterns is endogenous considering that the growth rate of sectoral demand is a function, not only of technical coefficients, $a_{ni}^{U}$, but also of their variations. From expressions (10) and (12) and by adopting the following convention: $\frac{\dot{p}_i^U}{p_i^U} = \sigma_i^U$, $\frac{\dot{p}_i^A}{p_i^A} = \sigma_i^U$, $\frac{\dot{e}}{e} = \varepsilon$ $\frac{\dot{y}_A}{y_A} = \sigma_y^A$, and $\frac{\dot{X}_{\hat{n}}}{X_{\hat{n}}} = \hat{g}$ we conclude that the growth rate of foreign and home demand for consumption good *i* are given respectively by:

$$\frac{\dot{a}_{i\hat{n}}^{U}}{a_{i\hat{n}}^{U}} = \hat{r}_i = \eta_i\left(\sigma_i^U - \sigma_i^A - \varepsilon\right) + \beta_i\sigma_y^A + (1-\beta_i)g \tag{14}$$

$$\frac{\dot{a}_{in}^{U}}{a_{in}^{U}} = r_i^U \tag{15}$$

In what follows, let us consider that the growth rate of foreign demand for the *i*-th consumption good is denoted by $\hat{r}_i = \eta_i\left(\sigma_i^U - \sigma_i^A - \varepsilon\right) + \beta_i\sigma_y^A + (1-\beta_i)g$. Following Pasinetti (1993) domestic and foreign prices are given by:

$$p_i^U(t) = a_{ni}^U(t)w^U \tag{16}$$

$$p_i^A(t) = a_{\hat{n}\hat{i}}^A(t)w^A \tag{17}$$

Where $w^U$ and $w^A$ stand for the wages in countries *U* and *A*, respectively, and $a_{\hat{n}\hat{i}}^A(t)$ stands for the labour coefficient of the *i*-th sector in country *A*. According to this formulation, prices are given by the costs of production. By taking logs, and differentiating these expressions in relation to time, we obtain the dynamics of prices as given by:

$$\sigma_i^U = \sigma_w^U - \rho_i^U \tag{18}$$

$$\sigma_i^A = \sigma_w^A - \rho_i^A \tag{19}$$

Where $\sigma_w^U$ and $\sigma_w^A$ stand for the growth rates of wages in countries *U* and *A* respectively, $\rho_i^U$ is the rate of technical progress in *i*-th sector of *U* country and $\rho_i^A$ represents technical progress in *i*-th sector of country *A*. The dynamics of technical coefficients, namely $a_{ni}^U$ and $a_{\hat{n}\hat{i}}^A$, in countries *U* and *A* are given respectively as:

$$a_{ni}^U(t) = a_{ni}^U(0)e^{-\rho_i^U t} \tag{20}$$

$$a_{\hat{n}\hat{i}}^A(t) = a_{\hat{n}\hat{i}}^A(0)e^{-\rho_i^A t} \tag{21}$$

By taking logs and differentiating expression (10) it is possible to obtain the growth rate of the production of the *i*-th sector as:

$$\frac{\dot{X}_i^U}{X_i^U} = \Pi_i^U \left\{ \left(\hat{r}_i^U + g\right) + \xi a_{i\hat{n}}^U X_n \left(1 - \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U\right) \left[\sum_{j=1}^{n-1} \left(\hat{r}_j^U + r_i^U + g - \rho_j^U\right) a_{nj}^U a_{j\hat{n}}^U a_{in}^U X_n\right]\right\} + \\ + \left(1 - \Pi_i^U\right) \frac{\sum_{j=1}^{n-1} \left(r_j - \rho_j^U\right) a_{jn}^U a_{nj}^U}{1 - \sum_{j=1}^{n-1} a_{jn}^U a_{nj}^U} \tag{22}$$

Where $\Pi_i^U = \dfrac{\xi\, a_{i\hat{n}}^U X_n}{\xi\, a_{i\hat{n}}^U X_n + \left(1-\sum_{i=1}^{n-1} a_{in}^U a_{ni}^U\right)^{-1}\left(\sum_{j=1}^{n-1} a_{nj}^U a_{j\hat{n}}^U a_{in}^U X_n\right)}$.

In order to make a parallel with the Neo-Kaldorian literature, in what follows let us rewrite expression (22), namely $\frac{\dot{X}_i^U}{X_i^U}$ as a linear function of technical progress of the $i$-th sector:

$$\frac{\dot{X}_i^U}{X_i^U} = \Gamma_{SDR}\rho_i^U + \Omega_{SDR} \tag{23}$$

Where: $\Gamma_{SDR} = -\Pi_i\eta_i + \dfrac{(1-\Pi_i)a_{ni}^U a_{in}^U}{\sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U} - \dfrac{(1+\eta_i)}{\left(1-\sum_{i=1}^{n-1} a_{nj}^U a_{jn}^U\right)\xi a_{i\hat{n}} X_n + \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U}$ and

$$\Omega_{SDR} = \Pi_i\left[\eta_i\left(\rho_i^A-\varepsilon\right)+\beta_i\sigma_y^A+g\right]+$$
$$+\frac{1}{\left(1-\sum_{i=1}^{n-1} a_{in}^U a_{ni}^U\right)}\left\{\frac{\left(1-\Pi_i\right)\left[\sum_{\substack{j=1\\ j\neq i}}^{n-1}\left(r_j-\rho_j^U\right)a_{jn}^U a_{nj}^U - r_i^U a_{ni}^U a_{in}^U\right]+\eta_i\left(\rho_i^A-\varepsilon\right)+\sum_{j=1}^{n-1}\eta_j(\rho_j^A-\rho_j^U-\varepsilon)+\beta_i\sigma_y^A+g}{\left(1-\sum_{i=1}^{n-1} a_{in}^U a_{ni}^U\right)\left(\sum_{j=1}^{n-1} a_{nj}^U a_{j\hat{n}}^U a_{in}^U X_n\right)+\xi a_{i\hat{n}} X_n}\right\}$$

(24)

Expression (23) is the sectoral counterpart of the DE, derived from a multi-sectoral Keynesian multiplier. We label this solution as the Sectoral Demand Regime (SDE), and it expresses the growth rate of the $i$-th sector as a function of technical progress. In order to fully determine the pace of technical progress and the growth rate of demand for the $i$-th sector, we also have to develop the notion of a productivity regime in a multi-sectoral set-up. We accomplish this task in the next subsection.

### 3.2. The Sectoral Productivity Regime

In order to establish the sectoral counterpart of the PR, namely a sectoral productivity regime – SPR hereafter – let us assume following Araujo (2013) that the sectoral growth rate of productivity is given by sectoral Kaldor-Verdoorn laws. According to this view, the dynamic economies of scale result from the increasing specialization of labor provided by sectoral market growth, and from the productivity gains that accrues from the learning by doing. Hence:

$$\rho_i^U = \frac{\dot{q}_i^U}{q_i^U} = \gamma_i^U + \alpha_i^U \frac{\dot{X}_i^U}{X_i^U} \tag{25}$$

Where $\rho_i^U$ is the rate of technical progress in *i*-th sector of *U* country, $\gamma_i^U$ is the intercept of the Verdoorn relation, and $\alpha_i^U$ poses itself as the Verdoorn coefficient. According to this view, it does not matter if the production increases occur at the firm level – that is, if they are restricted to one of the firms in a sector – or if they are widespread amongst firms. Both the individual firm and the aggregated sectoral production play an important role in the generation of sectoral productivity gains. Expression (25) may be rewritten as:

$$\frac{\dot{X}_i^U}{X_i^U} = -\frac{\gamma_i^U}{\alpha_i^U} + \frac{1}{\alpha_i^U}\rho_i^U \tag{25)'$$

Expression (25)’ plays the role of a DR in our formulation. By equalizing expression (25)’ to (23), namely the SDR to SPR, it is possible to obtain after some algebraic manipulation the rate of technical progress in the *i*-th sector as:

$$\left(\rho_i^U\right)_{SDR}^* = \frac{\gamma_i^U + \alpha_i^U \Omega_{SDR}}{1 - \alpha_i^U \Gamma_{SDR}} \tag{26}$$

Expression (26) conveys one of the important outcomes of this analysis, namely the endogenisation of technical progress in the SED model. This analysis has been suggested by Araujo (2013) who introduced sectoral Kaldor-Verdoorn's laws in the SED approach. But his analysis departs from sectoral equilibrium conditions in the Pasinettian model and not from a notion of aggregate demand. As a result the pace of technical progress is established but not in terms of the effective production as in the Neo-Kaldorian set up. Here, the pace of technical progress is determined according to the sectoral effective demand, which makes our analysis closer to the cumulative model. [See Dixon and Thirlwall (1975)].

One important and key property of expression (26) is that technical progress in the *i*-th sector, that is $\rho_i^U$, is affected by technical progress in other sectors, namely $\rho_j^U$. This raises an important property of the model: when demand is fully taken into account in Pasinetti's model, it highlights the role of productivity spillovers emphasized by the Neo-Kaldorian literature. The straight effect of an increase in $\rho_j^U$ is to increase $\rho_i^U$, meaning that positive effects of technical progress in the *j*-th sector will not be restricted to that sector, but will affect the generation of technical progress in other sectors[9].

The rationale behind this interaction may be grasped by considering that technical progress in the *j*-th sector has a negative effect on the price of good *j*. A smaller price for good *j* is translated in terms of higher purchasing power, which may be unevenly spent on consumption of other goods, let us say *i*. A higher level of consumption for good *i* means, through the Kaldor-Verdoorn relation, a higher level of

[9] This can be grasped from expression (24).

technical progress for the $i$-th sector[10]. By substituting expression (26) into expression (25)' we obtain the growth rate of production of the $i$-th sector in the $U$ country:

$$\left(\frac{\dot{X}_i^U}{X_i^U}\right)_{SDR}^* = \frac{\Gamma_{SDR}\gamma_i^U + \Omega_{SDR}}{1-\alpha_i^U\Gamma_{SDR}} \tag{27}$$

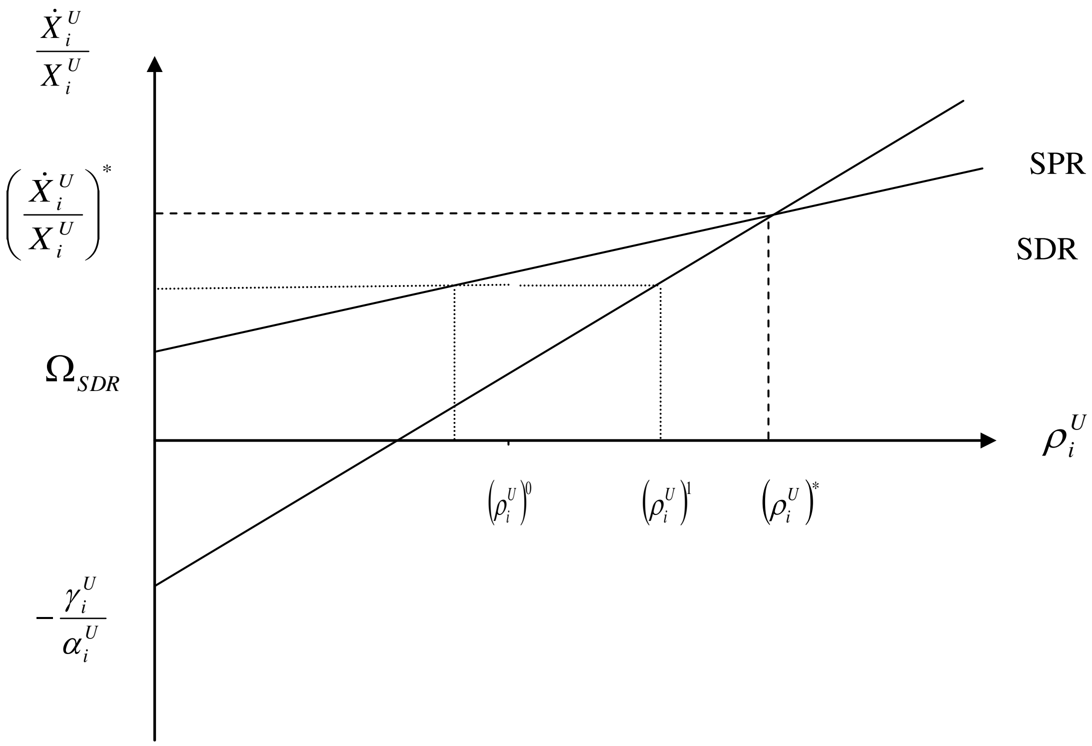
The analysis here is similar to the aggregated model. Since we are focusing on a sectoral aspect of the dynamics, let us consider as a device the case in which[11]: $r_j = \rho_j^U = 0, \forall j \neq i$. By following this approach we obtain $\Omega_{SDR}$ and $\Gamma_{SDR}$ as constants and a graphical device may be adopted. In this case, $\Omega_{SDR}$ may be rewritten as:

$$\Omega_{SDR} = \Pi_i\left[\eta_i\left(\rho_i^A - \varepsilon\right) + \beta_i\sigma_y^A + g\right] + \\ + \frac{1}{\left(1-\sum_{i=1}^{n-1} a_{in}^U a_{ni}^U\right)}\left\{\frac{\left(1-\Pi_i\right)\left[-r_i^U a_{ni}^U a_{in}^U\right] + \eta_i\left(\rho_i^A - \varepsilon\right) + \sum_{j=1}^{n-1}\eta_j(\rho_j^A - \varepsilon) + \beta_i\sigma_y^A + g}{\left(1-\sum_{i=1}^{n-1} a_{in}^U a_{ni}^U\right)\left(\sum_{j=1}^{n-1} a_{nj}^U a_{j\hat{n}}^U a_{in}^U X_n\right) + \xi a_{i\hat{n}} X_n}\right\}$$

Hence we plot the SDR and SPR in a graph as follows:

[10] Note that this property was not evinced in the SED version of the endogenised technical progress derived by Araujo (2013).

[11] Although this case is unrealistic it may evince the properties of our model. Note that Pasinetti (1993) considers in his structural economic dynamics as the first approximation the case in which $r_i = \rho_i^U, \forall i$.




The interpretation of this graph is similar to the traditional Neo-Kaldorian models. If we start with values of $\rho_i^U$ and $\frac{\dot{X}_i^U}{X_i^U}$ below their equilibrium values, then the *i*-th sector experience a rate of output growth that will induce the pace of technical progress, leading to higher price competitiveness that by its turn increase the exports. This will lead to a higher rate of output growth that will induce more productivity gains and further gains in terms of price competitiveness and export performance.

According to this view, structural changes are triggered by exogenous demand that induce technical progress through increasing returns of scale and learning-by-doing. The consequent increase in per capita income due to the raise in productivity will turn into an increase into per capita demand that may also induce more technical progress. In some moment of this virtuous cycle, structural changes are made endogenous. With this approach we overcome one of the shortcomings of the SED approach as pointed out by Gualerzi (1996, p. 157): “the integration of the demand side into the analysis of growth, which is potentially the most fruitful step forward, does not lead to an analysis of the endogenous growth mechanisms because of a fully inadequate theory of demand”.

## 3. A Structural Economic Dynamic Assessment of Macroeconomic Regimes

In order to derive a SED regime in the Pasinetti model let us depart from Araujo and Teixeira (2004) who derived a version of the full employment condition in an open economy. According to these authors the system of physical quantities may be expressed by[12]:

$$\begin{bmatrix} \mathrm{I} & -(\mathrm{c}+\xi\hat{\mathrm{c}}) \\ -\mathrm{a} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ X_n \end{bmatrix} = \begin{bmatrix} \mathrm{O} \\ 0 \end{bmatrix} \tag{28}$$

Note that system (28) is the same system expressed in (2) by considering that $\mathrm{E} = \xi X_n \hat{\mathrm{c}}$ and M = O. System (28) is a homogenous and linear system and, hence a necessary condition to ensure non-trivial solutions of the system for physical quantities is:

$$\det \begin{bmatrix} \mathrm{I} & -(\mathrm{c}+\xi\hat{\mathrm{c}}) \\ -\mathrm{a} & 1 \end{bmatrix} = 0 \tag{29}$$

Condition (29) may be equivalently written as:

$$\mathrm{a}(\mathrm{c}+\xi\hat{\mathrm{c}}) = 1 \tag{30}$$

By using summations it is possible to rewrite expression (30) as [see Araujo and Teixeira (2003)]:

$$\sum_{i=1}^{n-1} a_{ni}(a_{in} + \xi\, a_{i\hat{n}}) = 1 \qquad (30)'$$

If condition (30)' is fulfilled then there exists solution for the system of physical quantities in terms of a exogenous variable, namely $\overline{X}_n$. In this case, the solution of the system for physical quantities may be expressed as:

[12] The procedure adopted here is similar to the Pasinettian analysis.

$$\begin{bmatrix} \mathrm{X} \\ X_n \end{bmatrix} = \begin{bmatrix} (\mathrm{c} + \xi \hat{\mathrm{c}})\overline{X}_n \\ \overline{X}_n \end{bmatrix} \tag{31}$$

In order to particularize the production in one of the countries let us introduce the superscript $U$ do denote the components of vector X in the underdeveloped country, according to:

$$X_i^U = (a_{in}^U + \xi\, a_{i\hat{n}}^U)\overline{X}_n \tag{32}$$

From (32), we conclude that in equilibrium the physical quantity of each tradable commodity to be produced in country $U$, that is $X_i^U$, $i = 1,...,n-1$, will be determined by the sum of the internal, namely $a_{in}^U X_n$, and foreign demand, namely $\xi a_{in}^U X_n$.

Note then that we have two possibilities for the production of sector *i*. First, from what we call the DPR, we have obtained a production that is given by the multi-sectoral Keynesian multiplier, given by expression (10). This stands for the actual or effective production while expression (32) stands for the equilibrium or potential output for the *i*-th sector. We can prove that solution (32) is a particular case of solution (10) when condition (30)' holds. In other words, the potential solution is a particular case of the effective solution, given by multi-sectoral Keynesian multiplier, when the full employment condition is satisfied. Then we have the following:

**Proposition 1**

Expression (32) is a particular case of expression (10) when expression (30)' holds.

**Proof.**

The proof is straight. If condition (30)’ holds then rearranging it we obtain: $\sum_{j=1}^{n-1}\xi a_{nj}a_{j\hat{n}} = 1-\sum_{j=1}^{n-1}a_{nj}a_{jn}$. By replacing this result into numerator of the first term of the high rand side of expression (10), namely $X_i^U = \left( a_{in}^U \frac{\sum_{j=1}^{n-1}\xi a_{nj}^U a_{j\hat{n}}^U}{1-\sum_{j=1}^{n-1}a_{jn}^U a_{nj}^U} + \xi\ a_{i\hat{n}}^U \right) X_n$, it yields: $X_i^U = (a_{in}^U + \xi\ a_{i\hat{n}}^U)\overline{X}_n$, which is expression (32). □

Proposition 1 shows that the solution put forward by Araujo and Teixeira (2004) for an open version of the Pasinetti model is in fact a particular case of the solution obtained here. That result is of key importance. One of the central results of the SED analysis [See Pasinetti (1981, 1993)] is that even departing from an equilibrium position, where full employment prevails, condition (30)’ will not hold in the long run due to the particular dynamics of technical progress and evolution of demand for each sector. It means that, in general, we should expect that: $\sum_{i=1}^{n-1}a_{ni}(a_{in} + \xi\ a_{i\hat{n}}) < 1$. We may consider a symmetrical case, namely $\sum_{i=1}^{n-1}a_{ni}(a_{in} + \xi\ a_{i\hat{n}}) > 1$, which corresponds to the case of overemployment. Then we have the following proposition:

**Proposition 2**

If $\sum_{i=1}^{n-1}a_{ni}(a_{in} + \xi\ a_{i\hat{n}}) < 1$ then effective production is smaller than potential production. Otherwise, effective production is larger than potential production.

**Proof.**

If $\sum_{i=1}^{n-1} a_{ni}(a_{in} + \xi\, a_{i\hat{n}}) < 1$, then it is possible to show after some algebraic manipulation that: $1 - \sum_{j=1}^{n-1} a_{nj} a_{jn} > \sum_{j=1}^{n-1} \xi a_{nj} a_{j\hat{n}}$. As a consequence, $\dfrac{\sum_{j=1}^{n-1} \xi a_{nj}^U a_{j\hat{n}}^U}{1 - \sum_{j=1}^{n-1} a_{jn}^U a_{nj}^U} < 1$, and the sectoral output solution (10) obtained from the multi-sectoral Keynesian multiplier is smaller than the sectoral production from the SED approach (30)'. Now if $\sum_{i=1}^{n-1} a_{ni}(a_{in} + \xi\, a_{i\hat{n}}) > 1$ then $\dfrac{\sum_{j=1}^{n-1} \xi a_{nj}^U a_{j\hat{n}}^U}{1 - \sum_{j=1}^{n-1} a_{jn}^U a_{nj}^U} < 1$. In this case, solution (10), namely the sectoral effective production, is larger than the corresponding sectoral potential production. □

In sum, we should expect that the sectoral effective output will gravitate around the potential output. In the Pasinettian analysis the first case, namely $\sum_{i=1}^{n-1} a_{ni}(a_{in} + \xi\, a_{i\hat{n}}) < 1$, receives more attention since the one of the probable outcomes of structural change is structural unemployment[13]. That result is somewhat expected in the sense that if the full effective demand condition is not satisfied then effective output is smaller than the potential output. Here we show that this result is also valid within a multi-sectoral framework.

With the expression of the potential output in hands, it is possible to derive the growth rate of potential sectoral output – what we call here as our SEDR in contrast to

[13] The possibility of unemployment in a model of cumulative causation is also taken into account by Roberts (2002). But the author considers this possibility via endogenisation of the rate of nominal wage inflation, which may give rise to short run unemployment.

the SDR. By taking logs and differentiating expression (32) it is possible to obtain the growth rate of the production of the *i*-th sector as:

$$\frac{\dot{X}_i^U}{X_i^U} = \theta_i^U \frac{\dot{a}_{in}^U}{a_{in}^U} + (1-\theta_i^U)\frac{\dot{a}_{i\hat{n}}^U}{a_{i\hat{n}}^U} + \frac{\dot{X}_n}{X_n} \tag{33}$$

Where $\theta_i^U = \dfrac{a_{in}^U}{a_{in}^U + \xi a_{i\hat{n}}}$ stands for the share of internal demand in total demand of good *i*, $0 \le \theta_i^U \le 1$. By inserting (20) and (21) into expression (33), we obtain after some algebraic manipulation the growth rate of potential output for the *i*-th sector as:

$$\frac{\dot{X}_i^U}{X_i^U} = \theta_i^U r_i^U + (1-\theta_i^U)\left[\eta_i(\sigma_i^U - \sigma_i^A - \varepsilon) + \beta_i\sigma_y^A + (\beta_i - 1)g\right] + g \qquad (33)'$$

By adopting the same procedure of the previous section, from expression (33)', we can write the growth rate of output in the *i*-th sector as a function of technical progress in that sector. Hence expression (33)' may be rewritten as:

$$\frac{\dot{X}_i^U}{X_i^U} = \Gamma_{SED}\rho_i^U + \Omega_{SED} \tag{34}$$

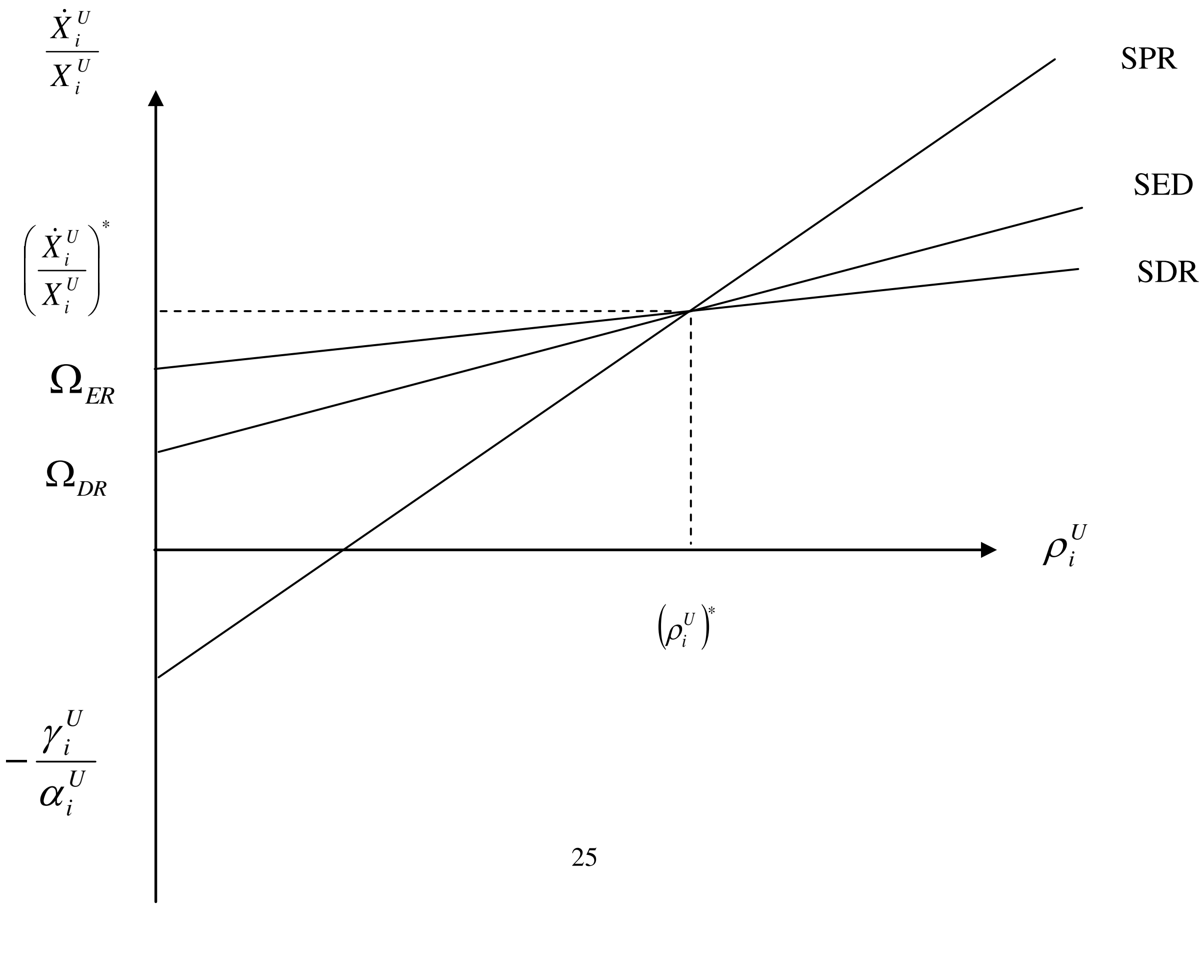
where: $\Gamma_{SED} = -(1-\theta_i^U)\eta_i$ and $\Omega_{SED} = \theta_i^U r_i^U + (1-\theta_i^U)\left[\eta_i(\gamma_i^A + \alpha_i^A\lambda_i^A\sigma_y^A - \varepsilon) + \beta_i\sigma_y^A\right]$. By replacing expression (25), which represents the SPR, into expression (34), we obtain after some algebraic manipulation the growth rate of productivity in the *i*-th sector:

$$\left(\rho_i^U\right)_{SED}^* = \frac{\gamma_i^U + \alpha_i^U\Omega_{SED}}{1-\alpha_i^U\Gamma_{SED}} \tag{35}$$

Expression (35) yields the pace of technical progress by considering the interaction between SPR with SEDR following Araujo (2013). By substituting expression (35) into expression (34) we obtain after some algebraic manipulation, the equilibrium growth rate of output under the SEDR regime:

$$\left(\frac{\dot{X}_i^U}{X_i^U}\right)_{SED}^{*} = \frac{\Gamma_{SED}\gamma_i^U + \Omega_{SED}}{1-\alpha_i^U\Gamma_{SED}} \qquad (36)$$

It is worth recalling that while the derivation of the SDR is based on actual production, the derivation of SEDR is based on potential production, we should expect at least gravitation of the production under SDR around production under SEDR in the short run. But in the long run, we should expect that the growth rate of production given by expressions (27) and (36) should be equal, that is $\left(\frac{\dot{X}_i^U}{X_i^U}\right)_{SDR}^{*} = \left(\frac{\dot{X}_i^U}{X_i^U}\right)_{SED}^{*}$. The graph below illustrates this point. Although the intercepts and slopes of the SPR and SEDR are different, there is a point in which they coincide and this corresponds to the long run solution.



Following this rationale, the pace of technical progress under SEDR and SDR should be equal in the long run. At this point it is important to consider an important difference between expressions (26) and (35). While the parameters that enter expression (35) are wholly exogenous, the technical progress of other sectors, namely $\rho_j^U$, that enter expression (26) are not exogenous. Hence, expression (26) generates a system of $n - 1$ variables and equations. If on one hand, this system is useful to evince the connections amongst technical progress in different sectors, on the other hand, the task of determining technical change for a specific sector from effective demand becomes cumbersome. In order to alleviate this difficulty we can use the pace of technical progress determined by SED regime, since its determination is straighter[14]. This point highlights the importance of the concept of potential output in the determination of the growth path in a multi-sectoral economy, thus emphasizing the importance of Pasinettian contribution.

But in any case it is possible to obtain a characterization of the equilibrium by following a sectoral approach that takes into account both the Neo-Kaldorian and the Pasinettian countributions. One strength of the approach presented here is its emphasis on the role played by demand in the process of economic growth. According to this view, demand cannot be limited to drive structural changes, but it should also be considered as one of the engines of economic growth via its effect on stimulating the creation and diffusion of technical progress.

[14] The value of $\left(\rho_i^U\right)_{SDR}^*$ may be endogenised if we consider that the rate of technical progress is given by $\left(\rho_j^U\right)_{SED}^*$.

When demand in a particular sector is fostered, the productivity in that sector is spurred due to the Kaldor-Verdoorn effect. But higher productivity is translated into higher real wages, which may give rise to further increases in demand, but not necessarily in demand for the good that kick started the process. Sectors producing goods with higher income elasticity of demand tend to increase their share in national income insofar as per capita income grows. Hence, those sectors will also enjoy higher rates of technical progress following the cumulative rationale.

Finally, the present approach stresses that the triggering point of this virtuous cycle is external demand, but once it is under way, indigenous demand may expand and may also be an important component to spur growth. In this vein a vigorous strategy of export led growth may play an important role to trigger the virtuous cycle motioned by cumulative causation.

## 4. Concluding Remarks

Notwithstanding Pasinetti's emphasis on the evolving patterns of demand within a multi-sectoral framework, demand still plays a somewhat passive role in his approach to the extent that its evolution registers as a function of technical progress, which is wholly exogenous. In this vein, although the original SED approach provides a simultaneous approach of demand and supply sides of economic growth, it does not take into account the role played by cumulative causation in the generation of technical progress. The present analysis aims to join these lines of research on structural factors in a more fully specified multi-sectoral framework and, in which demand interacts with technical progress.

With this inquiry we have introduced concepts such as demand and productivity regime in an open version of Pasientti's model, by showing that indeed it can be treated as a particular case of the multi-sectoral version of the Keynesian multiplier for an open economy. That was proven to be a required step to formulate a proper notion of demand regime in the SED framework. Besides, by considering the interaction between demand and productivity regimes, it was possible not only to endogenise technical progress in the Pasinettian approach but also to highlight the spillover connections between technical progress in different sectors.

If on one hand, endogenous technical progress is required to proper explain the evolving patterns of demand, on the other hand, the evolution of demand is seen as a function of the technical conditions. In this respect, a Neo-Kaldorian approach to the SED is convenient since it allows us to evince the connections between demand and technical change through the use of the cumulative causation concept. [See McCombie et al. (2002)].

If on the SED front, the gains from considering Neo-Kaldorian concepts are pervasive, also in the Neo-Kaldorian view we may reap some benefits from the cross-fertilization between these two strands. They accrue mostly from the use of a disaggregated model embedded with sectoral Kaldor-Verdoorn's law, thus emphasising the connections between demand and productivity growth not only in an aggregated but also in a disaggregated level. Following this view, once there is an exogenous increase of demand in a particular sector, the productivity increases gives rise to per capita income gains that will be translated into higher demand. This higher per capita income may be translated into higher demand for goods with higher income elasticity of demand.

## Appendix

In order to fix the ideas let us consider the case in which $n = \hat{n} = 3$. The system of physical quantities may be written as:

$$\begin{cases} X_1 - (a_{13} + \xi\, a_{1\hat{3}})X_3 = 0 \\ X_2 - (a_{23} + \xi\, a_{2\hat{3}})X_3 = 0 \\ X_3 - a_{31}X_1 - a_{32}X_2 = 0 \end{cases} \tag{A1}$$

The difference between this system and the closed Pasinettian formulation is confined to the foreign demand coefficients. If we assume that $a_{1\hat{3}} = a_{2\hat{3}} = 0$, then this system sums up to the original pure labour Pasinetti's model. The system above may be written in matricial notation as:

$$\begin{bmatrix} 1 & 0 & -a_{13} \\ 0 & 1 & -a_{23} \\ -a_{31} & -a_{32} & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} \xi\, a_{1\hat{3}}X_3 \\ \xi\, a_{2\hat{3}}X_3 \\ 0 \end{bmatrix} \tag{A2}$$

By using the following notation, $\mathrm{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ $\mathrm{c} = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$ $\hat{\mathrm{c}} = \begin{bmatrix} a_{1\hat{3}} \\ a_{2\hat{3}} \end{bmatrix}$ $\mathrm{a} = \begin{bmatrix} a_{31} & a_{32} \end{bmatrix}$

$\mathrm{E} = \begin{bmatrix} \xi a_{1\hat{3}}X_3 \\ \xi a_{2\hat{3}}X_3 \end{bmatrix}$, the system may be rewritten as:

$$\begin{cases} \mathrm{X} - \mathrm{c}X_3 = \mathrm{E} \\ -\mathrm{aX} + X_3 = 0 \end{cases} \tag{A3}$$

From the last line we conclude that: $X_3 = \mathrm{aX}$. By pre-multiplying throughout the first line of (A3) by $\mathrm{a}$, it follows that: $\mathrm{aX} - \mathrm{ac}X_3 = \mathrm{aE}$, which yields: $X_3 - \mathrm{ac}X_3 = \mathrm{aE}$. By isolating $X_3$ we obtain the employment multiplier relationship:

$$X_3 = \frac{1}{1-\mathrm{ac}}\mathrm{aE} \tag{A4}$$

where $1/1-\mathrm{ac}$ is a scalar employment multiplier. By considering three sectors the above expression yields:

$$X_3 = \frac{a_{1\hat{3}}a_{31}\xi + a_{2\hat{3}}a_{32}\xi}{1-a_{13}a_{31}-a_{23}a_{32}}X_3 \tag{A4)'$$

By substituting the value for $X_3$ in the first line of the system (A3) we obtain: $\mathrm{X}-\mathrm{c}\left[\frac{1}{1-\mathrm{ac}}\mathrm{aE}\right]=\mathrm{E}$, which yields :

$$\mathrm{X} = \left[\frac{1}{1-\mathrm{ac}}\mathrm{ca}+\mathrm{I}\right]\mathrm{E} \tag{A5}$$

This solution may also be expressed as:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}\begin{bmatrix} a_{31} & a_{32} \end{bmatrix}\begin{bmatrix} \xi a_{1\hat{3}}X_3/1-\mathrm{ac} \\ \xi a_{2\hat{3}}X_3/1-\mathrm{ac} \end{bmatrix} + \begin{bmatrix} \xi a_{1\hat{3}}X_3 \\ \xi a_{2\hat{3}}X_3 \end{bmatrix} \tag{A6}$$

Hence:

$$\begin{cases} X_1 = \xi a_{1\hat{3}}X_3 + \left(\dfrac{1}{1-a_{31}a_{13}-a_{32}a_{23}}\right)\left[a_{13}a_{31}\xi X_3 a_{1\hat{3}} + a_{13}a_{32}\xi X_3 a_{2\hat{3}}\right] \\ X_2 = \xi a_{2\hat{3}}X_3 + \left(\dfrac{1}{1-a_{31}a_{13}-a_{32}a_{23}}\right)\left[a_{23}a_{31}\xi X_3 a_{1\hat{3}} + a_{23}a_{32}\xi X_3 a_{2\hat{3}}\right] \end{cases} \tag{A6)'$$

By putting $a_{13}$ and $a_{23}$ in evidence in the second terms of the right rand side of $X_1$ and $X_2$ respectively, we can rewrite (A6)' as:

$$\begin{cases} X_1 = \xi a_{1\hat{3}} X_3 + \left( \dfrac{a_{31}\xi a_{1\hat{3}} + a_{32}\xi a_{2\hat{3}}}{1 - a_{31}a_{13} - a_{32}a_{23}} \right) a_{13} X_3 \\ X_2 = \xi a_{2\hat{3}} X_3 + \left( \dfrac{a_{31}\xi a_{1\hat{3}} + a_{32}\xi a_{2\hat{3}}}{1 - a_{31}a_{13} - a_{32}a_{23}} \right) a_{23} X_3 \end{cases} \quad \text{(A6)''}$$

which is the solution derived from the multi-sectoral open version of the Keynesian multiplier. In what follows let us derive the solution from the Pasinettian derivation as put forward by Araujo and Teixeira (2004). Alternatively the system may be rewritten as:

$$\begin{bmatrix} 1 & 0 & -(a_{13} + \xi\, a_{1\hat{3}}) \\ 0 & 1 & -(a_{23} + \xi\, a_{2\hat{3}}) \\ -a_{31} & -a_{32} & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(A7)}$$

Following the Pasinettian procedure, Araujo and Teixeira (2004) have shown that the necessary condition to the above system to yield solutions for physical quantities different from the trivial is:

$$\sum_{i=1}^{2} (a_{i3} + \xi\, a_{i\hat{3}}) a_{3i} = 1 \quad \text{(A8)}$$

This condition requires that the determinant of the coefficient matrix be equal to zero. If this condition holds then the solution of the physical system may be written as:

$$\begin{cases} X_1 = (a_{13} + \xi\, a_{1\hat{3}}) X_3 \\ X_2 = (a_{23} + \xi\, a_{2\hat{3}}) X_3 \end{cases} \quad \text{(A9)}$$

These solutions hold if $\sum_{i=1}^{2} (a_{i3} + \xi\, a_{i\hat{3}}) a_{3i} = 1$, otherwise the only solution of the system is the trivial one. Due to the dynamic path of coefficients that enter this expression, Pasinetti shows that even if it is satisfied when $t = 0$ , it is not possible to guarantee that it will hold for $t$>0. Hence, if $\sum_{i=1}^{2}(a_{i3} + \xi\, a_{i\hat{3}})a_{3i} \neq 1$, then the only solution of the above system is the trivial one. Of course this does not mean that the non-existence of meaningful solution. The solutions will be given by disaggregated version of the Keynesian multiplier (A6)''.

Note that if $\sum_{i=1}^{2}(a_{i3} + \xi\, a_{i\hat{3}})a_{3i} = 1$ then we can rewrite this equality as:

$1 - a_{31}a_{13} - a_{32}a_{23} = a_{31}\xi a_{1\hat{3}} + a_{32}\xi a_{2\hat{3}}$. By substituting these results into first and second lines of solutions (A6)'' they reduce to the first and second lines of (A9), respectively, which are exactly the solutions obtained under the hypothesis of equilibrium. Let us consider now the case in which $\sum_{i=1}^{2}(a_{i3} + \xi\, a_{i\hat{3}})a_{3i} < 1$. In this case the above solutions in (A6)'' are smaller than the potential solution (A9). But if $\sum_{i=1}^{2}(a_{i3} + \xi\, a_{i\hat{3}})a_{3i} > 1$, then the solutions in (A6)'' is larger than the potential solutions in (A9), as stated in Proposition 2.

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