# Adverse Selection and Search Frictions in Corporate Loan Contracts

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**Abstract**

We provide empirical evidence of both (1) price dispersion and (2) credit rationing in the corporate loan market. We argue that these properties are caused by two factors: an adverse selection resulting from the information asymmetry between lenders and borrowers, and search frictions in matching borrowers with lenders. We develop a model of loan markets in which lenders post an array of heterogeneous contracts, then borrowers tradeoff terms of loan contracts and matching probability between themselves. We show that a unique separating equilibrium exists where each type of borrower applies to a certain type of contract.

**Keywords:** loan contract, capital structure, debt heterogeneity, adverse selection, competitive search

**JEL classification:** G20, G21, G32, D86

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# 1 Introduction

In a friction-less credit market, firms that find it optimal to have bank debt in their capital structure should be able to issue bank debt at a correct market price. This correct price for debt (cost of debt) would depend on the incoming cash flow as well as the risk of the firm's future projects. In the real world, however, there is an excess demand for loanable funds; i.e., certain firms apply for loans but fail to receive them. Stiglitz and Weiss (1981) argue that potential borrowers who are denied loans would not be able to borrow even if they indicated a willingness to pay more than the market interest rate. Faulkender and Petersen (2006) and Colla et al. (2013) also show that bank credit is indeed rationed and certain types of borrowers end up with certain types of debt securities.

In the corporate finance literature, it is commonly assumed that information asymmetries between borrowers and lenders may be reflected by premiums in interest rates. This per se does not explain why some potential borrowers may fail to find a lender. Furthermore, there are other models suggesting that information frictions may be reflected in liquidity of funds. The literature, however, lacks models in which both liquidity distortions and price dispersions are allowed to operate simultaneously[1]. This paper attempts to bridge this gap by proposing a model of

[1] Stiglitz and Weiss (1981) provide the first theoretical framework that justifies credit rationing in equilibrium. They show that banks with imperfect information will formulate the terms of the contract, specifically the interest rate, to attract low-risk borrowers. Willingness to lend at a higher rate will attract riskier borrowers and the overall effect is value-destroying. In a similar environment with information frictions, Bester (1985) shows that when we allow banks to post different contracts to screen borrowers, credit rationing disappears. In an environment with only information frictions, one cannot explain both credit rationing and price dispersion. We add search friction to a standard environment with information frictions and show that a separating equilibrium with credit rationing always exists.

a credit market using the tools provided in the competitive search literature (i.e., Guerrieri et al. (2010)). We develop a model of a market with adverse selection and search frictions, and show that there exists a unique separating equilibrium in which each type of firm in need of bank credit applies to a different type of contract. In this model, banks post the terms and conditions of credit agreements and potential borrowers choose where to direct their search. Banks have imperfect information about borrowers, and through the posted terms of trade they can attract certain types of borrowers and screen out others. Contract terms and market tightness (i.e., ratio of borrowers to lenders in each submarket) is public information. In this framework, borrowers potentially face a trade-off between the terms of loans and market tightness. In a benchmark setup of the model with no information frictions, we show the properties of the offered loan contracts. In the second best version of the model with information frictions on the borrowers' side, we show how private information distorts the terms of loan contracts. Competitive search frictions allow us to investigate the effects of private information on two margins: 1- the extensive margin of loan contracts: the number of matched agents and offered loans is identified by the tightness in each submarket. 2- the intensive margin of the loan contracts: the amount of loans offered may be distorted downward when we add information problems to the model.

In the rest of the paper, we provide empirical evidence on the findings in the theoretical part of this paper. Using a novel dataset that records successful and unsuccessful applications for bank credit as well as the conditions of each credit agreement and firm characteristics, we verify the co-existence of unmatched credit

seekers and distribution in types of contracts. This finding has two immediate implications. First, there is a difference between a desired capital structure and a realized one. There are two types of firms whose capital structure does not have bank debt: firms that prefer to not have a bank loan and firms that prefer to have a bank loan but are unsuccessful in acquiring it. Even if a firm is successful in obtaining bank debt, due to restrictions on the supply side, the amount of debt obtained might be different from what is desired. Due to the lack of data on unsuccessful bank loan applications, empirical banking studies may ignore this difference and explain debt structure solely as a function of firm characteristics. Using data on a large cross-section of firms, we investigate the magnitude of this issue. We provide empirical evidence that potential borrowers exist who are unsuccessful in acquiring bank loans even if they are willing to pay higher interest rates, a point ignored or at best acknowledged based on anecdotal evidence in prior work.

Our paper also relates to the recent literature on corporate debt structure and debt heterogeneity. Rauh and Sufi (2010) and Colla et al. (2013) show that different borrowers specialize in certain types of debt. Furthermore, the degree and type of specialization varies widely across different firm types measured by size, degree of information asymmetry, maturity, profitability, etc., a finding that is also confirmed in our study. In analyzing debt specialization, Colla et al. (2013) extend the work of Rauh and Sufi (2010), who use a sample of 305 randomly selected rated public U.S firms for the period 1996 to 2006 by studying a sample of 3,296 U.S. public firms from 2002 to 2009. This number includes all the firms with available financial data on both Standard and Poor's Capital IQ database and the Compustat database. Our study

complements these two papers by taking advantage of the European Commission's EFIGE (European Firms in a Global Economy) database. This data is based on a survey of 15,000 manufacturing firms. This data helps us deepen our understanding of capital structure beyond prior findings in this area, especially since it includes rich data on the borrowing behaviour of private firms. Studying private firm-bank relationships is important because private firms have limited access to public funds, and as a result, acquiring bank loans is crucial for them. Furthermore, since the level of information asymmetry between lenders and private borrowers is higher, the role of search frictions in forming capital structure is more clearly demonstrated. Also, as mentioned earlier, this dataset provides additional insight by including data on unsuccessful bank loan applications.

The rest of the article proceeds as follows. The next section describes the data and empirical evidence. Section 3 provides the theoretical model. In this section, we develop the general environment, define equilibrium, and solve constrained optimization problems with and without borrower-specific information frictions. Section 4 concludes this article. Proofs for the solutions in theoretical models are relegated to the appendix.

# 2 Data and Evidence

We start with 14,759 European firms with available data on the EFIGE database[2]. EFIGE was primarily created to "examine the pattern of internationalization of

[2]EFIGE stands for "European Firms in Global Economy: internal policies for external competitiveness"; it is supported by the Directorate General Research of the European Commission through its 7th Framework Programme and coordinated by Bruegel, a European think tank.

European firms" by the European Commission. The database provides firm-level quantitative and qualitative information on about 150 items ranging from R&D and innovation, labour organisation, financing and organisational activities, and pricing behaviour. It is designed to be a representative sample of manufacturing firms in seven European economies (Germany, France, Italy, Spain, United Kingdom, Austria, Hungary). Data was collected in 2010 through survey questionnaires, covering the years from 2007 to 2009. Data gathered through surveys were validated by assessing the comparability of the survey data with official statistics[3]. We focus on the part of this database that is dedicated to firms' financing activities. Table 1 and Table 2 report descriptive statistics on the distribution of surveyed firms by country, industry, and size classes (extracted from Altomonte and Aquilante (2012)).

Table 1: Distribution of firms by country and size class

| Class Size | AUT | FRA | GER | HUN | ITA | SPA | UK | Total |
|---|---|---|---|---|---|---|---|---|
| Employees (10-19) | 132 | 1,001 | 701 | 149 | 1,040 | 1,036 | 635 | 4,694 |
| Employees (20-49) | 168 | 1,150 | 1,135 | 176 | 1,407 | 1,244 | 805 | 6,085 |
| Employees (50-249) | 97 | 608 | 793 | 118 | 429 | 406 | 519 | 2,970 |
| Employees (over 250) | 46 | 214 | 306 | 45 | 145 | 146 | 108 | 1,010 |
| Total | 443 | 2,973 | 2,935 | 488 | 3,021 | 2,832 | 2,067 | 14,759 |

The first relevant survey question for our research is "Did your firm recur to external financing in the period 2008-2009? By external financing we mean funds not generated internally (i.e., not [through] self-financing)". Possible responses are "Yes," "No," and "DK/DA" (Do not know/Did not answer)[4]. A follow-up question

[3]For more information see Altomonte and Aquilante (2012) or visit www.efige.org.

[4]In general respondent firms agreed to a minimum response rate of 70% of 15 important questions and an overall average of response rate not below 60% for the remaining part of the questionnaire (135 questions)

Table 2: Distribution of firms by country and NACE2 industries

| Industry | AUT | FRA | GER | HUN | ITA | SPA | UK | Total |
|---|---|---|---|---|---|---|---|---|
| 15 | 32 | 212 | 350 | 62 | 238 | 463 | 147 | 1,504 |
| 17 | 8 | 118 | 77 | 7 | 196 | 46 | 52 | 504 |
| 18 | 5 | 55 | 17 | 17 | 109 | 50 | 42 | 295 |
| 19 | 0 | 32 | 13 | 4 | 115 | 47 | 10 | 221 |
| 20 | 21 | 93 | 103 | 17 | 88 | 212 | 89 | 623 |
| 21 | 10 | 83 | 62 | 16 | 71 | 27 | 47 | 316 |
| 22 | 34 | 148 | 215 | 27 | 105 | 100 | 208 | 837 |
| 24 | 5 | 102 | 95 | 20 | 108 | 121 | 104 | 555 |
| 25 | 22 | 226 | 192 | 40 | 169 | 148 | 122 | 919 |
| 26 | 18 | 153 | 94 | 30 | 167 | 163 | 56 | 681 |
| 27 | 13 | 68 | 58 | 7 | 76 | 68 | 54 | 344 |
| 28 | 70 | 839 | 510 | 101 | 611 | 580 | 301 | 3,012 |
| 29 | 48 | 249 | 503 | 68 | 381 | 305 | 208 | 1,762 |
| 31 | 20 | 121 | 134 | 19 | 152 | 66 | 124 | 636 |
| 32 | 5 | 94 | 56 | 9 | 49 | 25 | 101 | 339 |
| 33 | 15 | 58 | 192 | 6 | 71 | 25 | 80 | 447 |
| 34 | 6 | 73 | 41 | 11 | 47 | 64 | 33 | 275 |
| 35 | 2 | 16 | 20 | 3 | 33 | 42 | 21 | 137 |
| 36 | 5 | 16 | 172 | 18 | 211 | 258 | 258 | 938 |
| Total | 339 | 2,756 | 2,904 | 482 | 2,997 | 2,810 | 2,057 | 14,345 |

is: "Have firms actually increased the total amount of external financing over that period?". In Table 3, we show that 43.0% of 14,759 firms (6,344 firms) sought for external financing during 2008-2009. Out of this number, 2,692 firms (42.4%) indeed increased the total amount of external financing and 3,636 firms (57.3%) were not successful in raising external financing.

Table 3: Distribution of firms in need of external financing

| Asked for External Financing | | | Raised External Financing | | |
|---|---|---|---|---|---|
| | Number | Percentage of Total | | Number | Percentage |
| Yes | 6,344 | 43.0% | Yes | 2,692 | 42.4% |
| | | | No | 3,636 | 57.3% |
| | | | DN/DK | 16 | 0.3% |
| | | | Total | 6,344 | 100.0% |
| No | 7,856 | 53.2% | | | |
| DN/DK | 556 | 3.8% | | | |
| Total | 14,759 | 100.0% | | | |

Table 4 shows the breakdown of the type of financial instrument used as a means of external financing. Of the 2,692 firms who succeeded in raising external financing, 16% (430 firms) used equity; only 3% (81 firms) relied on venture capital (VC) and private equity financing (PE); whereas 45.3% (1,220 firms) and 72.5% (1,952 firms) relied on short-term and medium- or long-term bank credit, respectively. This result confirms a heavy reliance of the European economy on bank financing as opposed to VC/PE financing. However, this result should be interpreted with caution as firms with less than 10 employees (which is more likely to include entrepreneurial firms and start-ups) are excluded from the data. Table 4 also shows that firms use other types of external financing, such as financial securities, public funds, tax incentives, leasing or factoring, and other (3.2%, 8.4%, 4.5%, 29.5% and 10.4% of all the firms

in the sample, respectively). In the rest of this section, we focus on the firm-bank relationship and bank credit rationing.

Table 4: Type of financial instrument used for external financing

| | Number | Percentage |
|---|---|---|
| Equity | 430 | 16.0% |
| Venture capital and private equity | 81 | 3.0% |
| Short-term bank credit | 1,220 | 45.3% |
| Medium or long term bank credit | 1,952 | 72.5% |
| Securities | 86 | 3.2% |
| Public funds | 226 | 8.4% |
| Tax incentives | 120 | 4.5% |
| Leasing or factoring | 794 | 29.5% |
| Other financing methods | 279 | 10.4% |

The most direct way of detecting credit rationing is through knowing who the unsuccessful credit seekers are and identifying the price they were willing to pay to obtain credit. Regulatory authorities around the world normally do not require firms to report unsuccessful bank loan applications. Also, there is no comprehensive data collected from the supply side (i.e., banks) that provide the details of denied corporate loans. We believe the EFIGE database can be used to provide new insights into the mechanism of credit rationing in the corporate loan market.

Surveyed firms were asked "During the last year, did the firm apply for more credit?" In table 5, we show that out of the 2,710 firms who responded to this question, 1,997 (73.7%) indicated they applied for more credit, however only 1,407 firms (70.4%) were successful in obtaining credit. Table 5 also shows that out of the 29.6% of unsuccessful applicants, almost everyone (98.5%) was willing to borrow at the interest rate that they currently pay or the rate they previously paid and 60.7%

(358 applicants) indicated they were prepared to borrow at a higher rate of interest if needed.

Table 5: Successful and unsuccessful credit applications

| Did the firm apply for more credit? | Number (% [a]) | Was the firm successful? | Number (% [b]) | Was the firm willing to increase borrowing at the same rate of interest? | Number (% [c]) | Was the firm prepared to increase borrowing at a higher rate of interest? | Number (% [d]) |
|---|---|---|---|---|---|---|---|
| Yes | 1,997 (73.7%) | Yes | 1,407 (70.4%) | Yes | 581 (98.5%) | Yes | 358 (60.7%) |
| | | No | 590 (29.6%) | No | 0 (0.0%) | No | 230 (39.0%) |
| | | | | DK/DY | 9 (1.5%) | DK/DY | 2 (0.3%) |
| | | | | Total | 590 (100.0%) | Total | 590 (100.0%) |
| | | Total | 1,997 (100.0%) | | | | |
| No | 713 (26.3%) | | | | | | |
| Total Number of Respondents | 2,710 (100.0%) | | | | | | |

[a] Percentage of applicants
[b] Percentage of applicants
[c] Percentage in success category
[d] Percentage in success category

We next focus on the screening process of credit applications. In the theoretical part of this paper, we show that banks screen borrowers by posting contracts with certain terms and conditions and by requiring certain types of information from potential borrowers. Their goal is to attract specific groups of borrowers by making it difficult for others to apply. Table 6 provides descriptive statistics on the distribution of the type of information and guarantees required in credit applications. We divide firms that apply for bank credit into three groups: 1. firms with successful applications; 2. firms with unsuccessful applications that were not willing to pay a higher interest rate; and 3. firms with unsuccessful applications that were willing to pay a higher interest rate. We exclude the two unsuccessful firms (Table 5) for which their willingness to pay a higher rate is not known. We also divide terms and conditions into two main groups: First, the information that banks required from potential borrowers in order to process their application, and second, the type of guarantee or collateral that borrowers were asked to provide. Information required includes these seven categories: collateral, balance sheet information, interviews with management on firm's policy and prospects, business plan and firms' targets, historical records of payments and debt service, brand recognition, and other. The types of guarantee/collateral required include five categories: personal guarantees from the person who manages or owns the firm, guarantees on assets belonging to the firm, guarantees on assets of the group the firm belongs to, third party collateral (i.e., by a consortium, etc.), and other collaterals. The number and percentage of loan contracts in each firm category that are subject to each term and condition are presented. For instance, Table 6 shows that under "Information Required," 61.5%,

77.0% and 79.3% of loans applied for by successful applicants, unsuccessful applicants not willing to pay a higher rate, and unsuccessful applicants willing to pay a higher rate, respectively, were required to provide information on some sort of collateral. Also, as an example under "Type of Guarantee/Collateral Required", it is shown that personal guarantees from the person who manages or owns the firm were required for 39.7%, 53.5% and 54.7% of loans applied for by successful applicants, unsuccessful applicants not willing to pay a higher rate and unsuccessful applicants willing to pay a higher rate, respectively. In addition to providing statistics on the conditions of acquiring loans, Table 6 also shows that denied loans were more difficult to acquire originally, as applicants have to provide more guarantees and collaterals at the time of loan applications.

Table 6: Distribution of conditions of granted and denied credit agreements

| | Successful credit application (N=1,407) | | Unsuccessful credit application, not willing to pay higher rates (N=230) | | Unsuccessful credit application, willing to pay higher rates (N=358) | |
|---|---|---|---|---|---|---|
| | Number | Percentage | Number | Percentage | Number | Percentage |
| *Information Required:* | | | | | | |
| Collateral | 865 | 61.5% | 177 | 77.0% | 284 | 79.3% |
| Balance sheet information | 1,253 | 89.1% | 212 | 92.2% | 322 | 89.9% |
| Interviews with management on firm's policy and prospects | 835 | 59.3% | 105 | 45.7% | 187 | 52.2% |
| Business plan and firms' targets | 746 | 53.0% | 117 | 50.9% | 187 | 52.2% |
| Historical records of payments and debt service | 613 | 43.6% | 104 | 45.2% | 179 | 50.0% |
| Brand recognition | 224 | 15.9% | 28 | 12.2% | 54 | 15.1% |
| Other | 147 | 10.4% | 21 | 9.1% | 38 | 10.6% |
| *Type of Guarantee/Collateral Required:* | | | | | | |
| Personal guarantees from the person who manages or owns the firm | 558 | 39.7% | 123 | 53.5% | 196 | 54.7% |
| Guarantees on assets belonging to the firm | 572 | 40.7% | 107 | 46.5% | 188 | 52.5% |
| Guarantees on assets of the group the firm belongs to | 114 | 8.1% | 22 | 9.6% | 32 | 8.9% |
| Third party collateral (i.e., by a consortium, etc.) | 63 | 4.5% | 22 | 9.6% | 40 | 11.2% |
| Other collaterals | 68 | 4.8% | 19 | 8.3% | 32 | 8.9% |

To sum up, we provide evidence on two issues: first, the existence of credit rationing in the corporate loan market, that is, the existence of excess demand for loanable funds. Our results demonstrate that there are indeed potential borrowers in need of bank credit who are willing to pay a higher interest rate but their applications are denied. This implies that firms are rationed by the lenders in the corporate loan market. Second, we provide evidence on the heterogeneity that exists in the corporate loan market in the terms and conditions of loans and also in the information required in the screening process of loan applications. In the next section we provide a theoretical model that includes these two observations.

# 3 Theoretical Model

There is a measure one of borrowers. Measure $0 < \pi_1 < 1$ of these borrowers are type 1, and the remaining ($\pi_2 = 1 - \pi_1$) are type 2 agents. Each period has three subperiods: matching, loan, and repayment. Borrowers have access to measure one of Lucas trees with stochastic return. Lucas trees produce fruits on the last subperiod (repayment). A type $i = 1, 2$ borrower receives $R$ units of fruit in the repayment subperiod with probability $p_i$, where $p_1 < p_2$, and with probability $1 - p_i$ this borrower receives 0 fruit. There is a large measure of ex ante homogeneous lenders who may decide to enter the market. If they decide to enter the market, they incur $k > 0$ in the fixed cost of entering. Lenders can produce in the loan subperiod and they incur $c(q)$ cost when they produce $q$ units. Borrowers cannot produce in the loan subperiod and their utility of consuming $q$ is $u(q)$. We assume $u'() > 0$,

$u''() < 0$, $c'() > 0$, and $c''() > 0$.

Lenders post contracts in the frictional loan market and each borrower directs his search to a single submarket. Contracts are observable by all participants in the market. Each contract is a loan amount ($q$) and a repayment level ($x$). The fraction of lenders to borrowers in each submarket is called tightness and is represented by $\theta$. In each submarket, borrowers and lenders match according to a matching function. Each borrower is matched with a lender with probability $\mu(\theta)$, where $\mu'() > 0$ and $\mu''() < 0$[5]. Each lender matches with a borrower with probability $\eta(\theta) = \frac{\mu(\theta)}{\theta}$, where $\eta(\theta)$ is nonincreasing. Timing of the events is shown in Figure 1

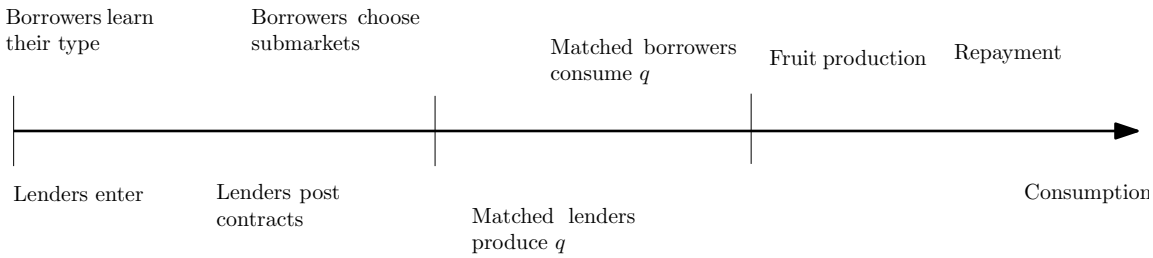


Figure 1: Timing of the events

## 3.1 First best

Here, we assume borrowers' types are common knowledge. Similar to Guerrieri et al. (2010), instead of solving the competitive search equilibrium directly, we solve a set of optimization problems for each type. Maximization problems in 1 show how

[5]The concavity assumption on the matching function is a standard assumption in many models with search frictions. For a survey on the properties of matching functions, see Petrongolo and Pissarides (2001).

the optimal contract is chosen.

$$\begin{aligned} \overline{U}_{i=1,2} = \quad & \max_{\theta,q,x} \ \mu(\theta)[u(q) + p_i(R - x)] \\ st. \quad & \eta(\theta)(-c(q) + p_i x) \geq k \end{aligned} \tag{1}$$

In the optimization problem for each type, market tightness ($\theta$), loan amount ($q$), and repayment level ($x$) are chosen to maximize the expected utility of each type subject to the bank making nonnegative profits when only type $i$ borrowers apply. A borrower's expected profit is the probability that he matches ($\mu(\theta)$) times his expected return given that he finds a match. If he finds a match, he will enjoy $u(q)$ in utility of consuming the loan. At the repayment period, a matched borrower pays back $x$ of his fruits, if his tree yields any.

A lender's expected profit conditional on entering is her matching probability ($\eta(\theta)$) times her expected profit. Her expected profit is her expected fruit collection in the repayment period, and she suffers ($c(q)$) when giving out loans. Here, we assume that $R$ is large enough such that the constraint $R > x$ is not binding. Later we will show the specific minimum value for $R$ that guarantees a nonbinding constraint.

The constraint in problem 1 is binding:

$$x_i = \frac{k}{p_i \eta(\theta)} + \frac{c(q)}{p_i} \qquad i = 1, 2 \tag{2}$$

Substitute in the objective function, and the problem becomes

$$\overline{U}_{i=1,2} = \max_{\theta,q} \mu(\theta)u(q) + \mu(\theta)p_i R - \theta k - \mu(\theta)c(q) \tag{3}$$

The first order condition for $q_i$ is:

$$u'(q_i^*) = c'(q_i^*) \tag{4}$$

$$q_1^* = q_2^* = q^*$$

Note that $q^*$ is the first best amount of lending in an environment with no frictions. This result shows that without information frictions, search frictions do not distort the optimal amount of loans. As seen in equation 2, search frictions may distort the repayment levels. The amount of loan is also independent of borrowers' types. The first order condition for $\theta_i$ gives the first best market tightness for each submarket:

$$\mu'(\theta_i^*) = \frac{k}{u(q^*) - c(q^*) + p_i R} \tag{5}$$

Using the concavity property of the matching function, we can see that $\theta_2^* < \theta_1^*$. Additionally, equation 2 shows that $x_2^* < x_1^*$. The market for high-type borrowers is tight comparing to the low-type borrowers, but high-type borrowers pay a lower interest rate. Without information frictions, search frictions only affect the number of successful matches through market tightness in different submarkets. There is no credit rationing along the intensive margin in the loan market: the amounts of offered loans are the same for high-type and low-type borrowers.

The lowest value for $R$ such that the constraint $R > x$ is nonbinding is $R^*_{min} = \max\{x^*_1, x^*_2\}$, where $x^*_i$ is the first best value for repayment level

$$x^*_i = \frac{k}{p_i \eta(\theta^*)} + \frac{c(q^*)}{p_i} \qquad i = 1, 2 \tag{6}$$

Therefore, the constraint in problem 1 is $R > R^*_{min}$.

## 3.2 Private information

Let us assume borrowers' types are their private information. Similar to Guerrieri et al. (2010), agents with the lowest type (type 1) do not face incentive compatibility constraints. Therefore, their problem is similar to the first best with complete information. We can find the market tightness that they face, their loan amount, and their repayment level by solving the following first best problem:

$$\begin{aligned} \overline{U}_1 = \quad & \max_{\theta,q,x} \ \mu(\theta)[u(q) + p_1(R - x)] \\ st. \quad & \eta(\theta)(-c(q) + p_1 x) \geq k \end{aligned} \tag{7}$$

In a type 2 optimization problem they face an incentive compatibility constraint: compared to type 2 contracts, type 1 contracts should be more attractive for a type

1 borrower.

$$\begin{aligned} \overline{U}_2 = \quad & \max_{\theta,q,x} \ \mu(\theta)[u(q) + p_2(R - x)] \\ st. \quad & \eta(\theta)(-c(q) + p_2 x) \geq k \\ & \mu(\theta)(u(q) + p_1(R - x)) \leq \overline{U}_1 \end{aligned} \tag{8}$$

In 8 we use the binding participation constraint to eliminate $x$ and the problem becomes

$$\begin{aligned} \overline{U}_2 = \quad & \max_{\theta,q} \ \mu(\theta)u(q) + p_2 R\mu(\theta) - \theta k - \mu(\theta)c(q) \\ & \mu(\theta)u(q) + \mu(\theta)p_1 R - \theta\frac{p_1}{p_2}k - \mu(\theta)\frac{p_1}{p_2}c(q) \leq \overline{U}_1 \end{aligned} \tag{9}$$

The algorithm to solve the type 2 problem is to first solve the first best problem without the incentive compatibility constraint. Then, we can check whether the solution for a type 2 borrower $(q^*, \theta_2^*, x_2^*)$ satisfies the incentive compatibility constraint. If the solution satisfies the incentive compatibility constraint then information problems do not distort the type 2 problem and we get the first best. The allocations are efficient.

If the solution to the first best problem for a type 2 borrower $(q^*, \theta_2^*, x_2^*)$ does not satisfy the incentive compatibility constraint in problem 9, we have to solve the above constrained optimization problem (8).

**Proposition 1.** *The amount of loan received by the good type (type 2) under binding*

*private information is less than the amount they receive in the first best case*

$$q_2 < q_2^* = q^*$$

The proof of the above proposition is in the appendix. The above proposition shows that the information problem intensifies credit rationing. Search friction generates credit rationing in the extensive margin of the loan contracts: the number of matched agents and offered loans is identified by the market tightness in each submarket, and market tightness may be distorted upward or downward[6]. Information problems affect both the extensive margin and the intensive margin of the loan contracts: the amount of loans offered may be distorted downward when we add the information problem to the model.

## 3.3 Existence and uniqueness

The existence and uniqueness of the equilibrium follow from the assumed preference and payoff structure. Guerrieri et al. (2010) show that under very mild assumptions on preferences and payoffs, this problem has a unique equilibrium. These assumptions are met here. Assumptions $A1$ and $A2$ hold because the assumed preferences are monotone, and also the contract allows transfers. We have the single crossing property here, therefore the sorting assumption $A3$ is met too. As a result a

[6]In problem 8, we can use the binding free entry condition to eliminate $x$. The first order condition for $\theta$ gives

$$\mu'(\theta_2) = \frac{k}{[\frac{1-\kappa}{1-\kappa\frac{p_1}{p_2}}]u(q_2) - c(q_2) + p_2 R}$$

where $\kappa$ is the multiplier on the incentive compatibility constraint. Then it is straightforward to show that compared to $\theta^*$, $\theta_2$ may be distorted upward or downward.

unique equilibrium always exists. Note that the nonexistence in the adverse selection problems is resolved here[7].

## 3.4 A model with multiple types

In previous sections, we developed a simple model with two types of borrowers to prove the main properties in the cases with and without private information. Here we generate the above model to include multiple types of borrowers. This version of the model will generate multiple submarkets, but the basic intuition of the model and the results are the same as before. Let us assume we have a measure 1 of agents and a fraction $\pi_i > 0$ of them are type $i$, where $0 \leq i \leq I$. The setup of the model is the same as in the previous section with $p_i > p_j$ for $i > j$. In the case without private information the setup of the model and the results are exactly the same as before.

In the case with private information the problem of the type 1 agent is the same as in 7. The problem of a type $i > 1$ agent is

$$\begin{aligned} \overline{U}_i = \quad & \max_{\theta,q,x} \ \mu(\theta)[u(q) + p_i(R-x)] \\ st. \quad & \eta(\theta)(-c(q) + p_i x) \geq k \\ & \mu(\theta)(u(q) + p_j(R-x)) \leq \overline{U}_j \quad i > j \end{aligned} \tag{10}$$

[7]Comparing to Rothschild and Stiglitz (1976) equilibrium always exists here. As Guerrieri et al. (2010) state :"...a key difference in our paper is that matching is bilateral and that each principal can serve at most one agent. This can create distortions along the extensive margin and implies that principals must form expectations about which agents are most attracted to a contract."

The solution algorithm for the above optimization problems is similar to that presented in the previous sections. First we solve the first best problem for the type 1 agents. Then we use the maximized value for a type 1 agent ($\overline{U}_1$) to solve the problem of a type 2 agent and continue the process until we solve the problem of all of the agents. Again, information problems do not distort the choices of the lowest types (type 1), however choices of higher type agents ($i > 1$) may be distorted in cases with private information.

# 4 Concluding remarks

If the price mechanism works then we should not observe credit rationing in corporate loan markets. Using a novel dataset on a large sample of European firms seeking external financing, we show that credit rationing indeed exists. Out of 1,997 firms that apply for more bank credit, 590 firms (29.6%) cannot obtain any. The application by 60.7% of these firms (358 firms) was denied in spite of them being prepared to borrow at a higher rate of interest. We also show that banks provide credit agreements with different terms and conditions. In addition, they screen potential borrowers by requiring various information and guarantees during the process of loan application.

The corporate finance literature clearly lacks models in which both credit rationing and distribution in the terms and conditions of loans are allowed to operate simultaneously. To fill this gap, we present a simple model of credit market with adverse selection and search frictions. In this model, banks post the terms and con-

ditions of credit agreements and potential borrowers with private information choose where to direct their search. In equilibrium, banks post separating contracts. The terms and conditions of each contract serve as a device by which lenders induce borrowers to self-select across credit sub-markets. This leads to a number of findings. First, in equilibrium, there exists certain firms in need of bank credit that remain unmatched. Second, the credit market becomes segmented into different "sub-markets," and each sub-market attracts certain types of borrowers.

Our work suggests several directions for further research. First, our finding that a significant fraction of firms fail to achieve their desired capital structure suggests that studies that link realized capital structure to firms' characteristics and even firms' unobserved heterogeneity are not complete unless they account for frictions in the credit market. In the empirical part of this paper, we focus on the cross-sectional heterogeneity in bank-firm relationships and variations in capital structure. It will be useful to study the impact of various credit supply shocks and regimes over time on the evolution of capital structure and heterogeneity in debt contracts. Another interesting research idea is how the deviation of realized capital structure from the desired capital structure of firms changes over time as a result of shocks in the supply of credit. Understanding how search frictions work in credit markets requires collecting a time-series of successful and unsuccessful bank loan applications as well as the terms and conditions of the granted and denied loan agreements.

Second, our treatment of credit agreement focuses on a simple contract that only includes interest rate. Another possible venue of future research is to extend the theoretical part of this paper and examine the joint determination of the amounts,

rates, and guarantees of credit agreements in each sub-market.

Appendix

# A Proof for proposition 1

The constraint problem is the following

$$\begin{aligned} \overline{U}_2 = \quad & \max_{\theta,q,x} \ \mu(\theta)[u(q) + p_2(R-x)] \\ st. \quad & \eta(\theta)(-c(q) + p_2 x) \geq k \\ & \mu(\theta)(u(q) + p_1(R-x)) \leq \overline{U}_1 \end{aligned} \tag{11}$$

Let us call the Lagrangian multipliers of the above constraints $\lambda$ and $\nu$. The solution for these multipliers are

$$\lambda = \theta \frac{\frac{p_2}{p_1} - 1}{\frac{p_2}{p_1} - \frac{c'(q_2)}{u'(q_2)}} \tag{12}$$

$$\nu = \mu(\theta) \frac{p_2(1 - \frac{c'(q_2)}{u'(q_2)})}{p_1(\frac{p_2}{p_1} - \frac{c'(q_2)}{u'(q_2)})} \tag{13}$$

We are interested in cases where multipliers are positive. Therefore, from 12

$$\frac{p_2}{p_1} > \frac{c'(q_2)}{u' q_2}$$

Moreover, from 13

$$\frac{c'(q_2)}{u'(q_2)} < 1$$

and using curvature properties of $u()$ and $c()$ we can see that

$$q_2 < q_2^*$$

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