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Abstract

This note formulates a dynamic two-country (developed and developing countries) Chamberlin-Heckscher-Ohlin model of trade with endogenous time preferences a la Uzawa (1968). We examine the relationship between initial factor endowment differences and trade patterns in the steady state. In particular, to highlight the integration of developing countries (e.g., China) into the world trading system, we concentrate on the case of *asymmetric size* of two countries (in terms of population). It will be shown that (i) given that the representative household in each country supplies

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an equal amount of labor, only intra-industry trade occurs in the steady state irrespective of differences in the number of representative households and that (ii) the number of households being equal, the country with less labor efficiency becomes the net exporter of the capital-intensive good.
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1 Introduction

In recent decades, many developing countries have opened their economies to international trade. As an example, China's integration into the world economy is one of the most important developments affecting the structure and evolution of the global trading system at the dawn of the 21st century. How does the integration of developing countries into the world economy affect world trading patterns?

It seems to be very important to consider this problem in a dynamic Heckscher-Ohlin trade model. However, while the static Heckscher-Ohlin theorem holds even if preferences and technologies are slightly different among countries, the dynamic Heckscher-Ohlin theorem under the assumption of exogenous time preference that was proved by Chen (1992) holds *only if* preferences and technologies are strictly identical among countries. In other words, under exogenous time preferences, at least one of the two countries should specialize in one of the two goods and it is very difficult to derive satisfactory results on trade patterns.[1] The state of the art in dynamic trade theory is apparently unsatisfactory. This seems to suggest that the traditional focus on exogenous time preferences should be accompanied by a focus on endogenous time preferences.[2]

Thus, we address the question of developing countries' integration in a dynamic Chamberlin-Heckscher-Ohlin (CHO) model with endogenous time preferences a la Uzawa (1968), in which there is a monopolistically competitive 'differentiated products' sector, and a perfectly competitive 'consumable capital' sector.[3] Consider the world economy as consisting of one developed country

[1] This was pointed out by Stiglitz (1970, p.463).

[2] A non-constant time preference rate has been empirically documented through panel data and cross-country data by Hong (1988), Lawrence (1991) and Ogawa (1993).

[3] The static Chamberlin-Heckscher-Ohlin model has been extensively investigated. Helpman's (1981) seminal integration of the monopolistic competition trade model into the two-

and one developing country. The *developed country* reached a steady state before the *developing country* (which corresponds to China) started the process of development (i.e., the removal of trade barriers). For simplicity, we call the former *Home* and the latter *Foreign*. Then China's decision to join the world trading system represents the opening of trade between Home and Foreign.

Kikuchi and Shimomura (2007) examine a similar problem using a dynamic two-country Chamberlin-Heckscher-Ohlin model.[4][5] They assume, however, that both countries are endowed with an equal number of households. Thus the role of size differences in factor endowment is downplayed in the analyses. In the real world, there is a significant size difference between developed and developing countries. For example, China's population is 20 percent of the world population. To our knowledge, little attention has been given to the relationship between timing of development and the size of developing countries. Thus, it is important to consider the case of the asymmetric size of countries.

In this note, we extend the analysis of Kikuchi and Shimomura (2007) to the case of *asymmetric size* of two countries (in terms of population). We demonstrate that, given that the representative household in each country supplies an equal amount of labor, only intra-industry trade occurs in the steady state irrespective of differences in the number of representative households. Even if there

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country by two-factor by two-good Heckscher-Ohlin (HO) framework, which was extended and made popular by Helpman and Krugman (1985), has led to the widely held belief that HO and Chamberlinian monopolistic competition are complementary in nature.

[4]Atkeson and Kehoe (2000) examine a similar problem using a dynamic Heckscher-Ohlin model composed of a larger number of small open economies.

[5]The literature on dynamic two-country models originated in Oniki and Uzawa (1965). While they assume exogenous saving rate in each trading country, most subsequent contributions, including Stiglitz (1970), Chen (1992), Shimomura (1992, 1993, 2004), Ventura (1997), Nishimura and Shimomura (2002, 2006), assume that households maximize their discounted sum of utility, i.e., saving rates are endogenously determined. Chen, Nishimura and Shimomura (2005) discuss other major problems within dynamic HO models.

is a larger amount of labor (in terms of population) in the developing country, due to catching-up by the developing country, sources of inter-industry trade based on differences in the capital-labor ratio vanish and only intra-industry trade occurs in the steady state.

This note is organized as follows. Section 2 sets up a dynamic CHO model and Section 3 discusses the existence, uniqueness and local stability of the steady state. Section 4 derives trade-pattern propositions. Section 5 provides concluding remarks.

2 The Model

Consider a world economy consisting of two countries, Home and Foreign, that differ in their factor endowments. There are two types of commodities, differentiated products (Good 1) and a consumable capital (Good 2), produced using reproducible capital, k, and a primary and time-invariant factor of production, l (labor). The consumable capital can be either consumed as a non-durable good or added to the existing capital stock. Labor is measured in efficiency units. Each Home (resp. Foreign) representative household supplies l (l^*) units of efficiency labor. The population of each country is assumed to be constant over time. The Home (resp. Foreign) population is m (resp. m^*). Thus, the Home (resp. Foreign) household is endowed with ml and mk (resp. m^*l^* and m^*k^*) units of factors of production. Note that Kikuchi and Shimomura (2007)'s case corresponds to $m = m^* = 1$.

Following the standard trade theory, we assume away international factor movements. Moreover, in order to focus on international trade, we assume that there is no international credit market, while there is a competitive domestic credit market in each country.

Each consumer maximizes the discounted sum of utility.

$$\int_0^\infty uXdt = \int_0^\infty f\left[U(V,C_2)\right]Xdt, \tag{1}$$

$$\dot{X} = -\rho(u)X, \tag{2}$$

where V is the quantity index for differentiated products, C_2 is the consumption of the consumable capital, and $X \equiv \exp\{-\int_0^t \rho(u)d\tau\}$ is the discount factor at time t which depends on the past and present level of utility through the function ρ.

Following Uzawa (1968), we assume that the variable discount rate $\rho(u)$ satisfies

$$\rho(0) > 0,\ \rho'(u) \equiv \frac{d\rho(u)}{du} > 0,\ \rho''(u) \equiv \frac{d^2\rho(u)}{du^2} > 0,$$
$$0 < \theta_\rho \equiv [u\rho'(u)/\rho(u)] < 1 \text{ for any positive } u < \infty. \tag{3}$$

It will be assumed that U is linearly homogeneous in its arguments and f satisfies

$$f(0) = 0,\ f'(U) > 0,\ f''(U) < 0. \tag{4}$$

Quantity index V takes the following Dixit-Stiglitz (1977) form:

$$V = \left[\int_0^N x(i)^{(\sigma-1)/\sigma} di\right]^{\sigma/(\sigma-1)},\quad \sigma > 1, \tag{5}$$

where N is the *total* number of differentiated products, $x(i)$ is the consumption of the i-th variety of differentiated products, and σ is the elasticity of substitution between varieties.

Solving the static expenditure minimizing problem, we can define the expenditure function as

$$e(P)\psi(u) \equiv \left(\min PV + C_2,\ \text{s.t.},\ u = f\left[U(V,C_2)\right]\right), \tag{6}$$

where the consumable capital serves as the numeraire, $P \equiv \left[\int_0^N p(i)^{1-\sigma} di\right]^{1/(1-\sigma)}$ is the price index for differentiated products, and $\psi(u)$ is the inverse function of f, which clearly satisfies

$$\psi(0) = 0,\ \psi'(u) > 0,\ \psi''(u) > 0. \tag{7}$$

Given that the equilibrium is symmetric, that is, $p(i) = p$ and $x(i) = x$ for $^{\forall}i \in [0, N]$, we can obtain the following condition from the envelope theorem, $\partial e(P)\psi(u)/\partial P = V$.

$$\begin{aligned} & e'[N^{1/(1-\sigma)}p]\psi(u) = N^{\sigma/(\sigma-1)}x \\ \text{or} \quad & N^{1/(1-\sigma)}e'[N^{1/(1-\sigma)}p]\psi(u) = Nx. \end{aligned}$$

Assume that differentiated products are more capital-intensive than the consumable capital.[6] Differentiated products are produced by monopolistically competitive firms under increasing returns technology, while the consumable capital is produced by competitive firms under constant returns technology. Assume that each firm in the differentiated products sector has the homothetic total cost function $c^1(w, r)\phi(y)$, where y is the output level of each firm. There are significant economies of scale: $\phi(y)/y$ is decreasing over the relevant range of output levels y. The marginal revenue will be equated to the marginal cost: $p\,[1 - (1/\sigma)] = c^1(w, r)\phi'(y)$.[7] Furthermore, free entry implies that price equals

[6]This assumption is just for simplification and this capital intensity ranking itself does not alter the results of this paper.

[7]We can obtain this relation as follows. Considering the subutility maximization problem: $\max V$, s.t., $\int_0^N p(i)x(i)di \leq I$, we obtain the inverse demand function of i-th variety as follows: $p(i) = [P^{(\sigma-1)}I/x(i)]^{1/\sigma}$. Therefore, the revenue of the i-th firm is given by

$$\begin{aligned} \pi_i &= p(i)x(i) - c^1(w, r)\phi[x(i)] \\ &= [P^{(\sigma-1)}Ix(i)^{(\sigma-1)}]^{1/\sigma} - c^1(w, r)\phi[x(i)]. \end{aligned}$$

and the first order condition, $d\pi_i/dx(i) = 0$, yields $p(i)\,[1 - (1/\sigma)] = c^1(w, r)\phi'[x(i)]$.

average cost: $p = [c^1(w,r)\phi(y)]/y$. By combining these conditions, one can easily see that all varieties will have the same output level $\bar{y}$, which is defined by[8]

$$1 - \frac{1}{\sigma} = \frac{\bar{y}\phi'(\bar{y})}{\phi(\bar{y})}.$$

The constraints on labor and capital within Home are[9]

$$c_w^1(w,r)\phi(\bar{y})n + c_w^2(w,r)y_2 = ml, \tag{8}$$

$$c_r^1(w,r)\phi(\bar{y})n + c_r^2(w,r)y_2 = mk, \tag{9}$$

where n is the number of differentiated products produced in Home and $c^2(w,r)$ and y_2 are the unit cost function and the output of the consumable capital, respectively.

Then, by defining $\xi \equiv \bar{y}/\phi(\bar{y})$, the zero-profit conditions can be written as

$$\xi p = c^1(w,r), \tag{10}$$

$$1 = c^2(w,r), \tag{11}$$

and we can obtain the factor price functions $w(\xi p)$ and $r(\xi p)$. Utilizing these factor price functions, the national income is shown as

$$r(\xi p)mk + w(\xi p)ml. \tag{12}$$

The partial derivative of the national income with respect to the price of differ-

[8]This result depends crucially on homotheticity in production. See Dixit and Norman (1980, pp. 284–5). To guarantee the existence and uniqueness of $\bar{y}$, we assume that ϕ satisfies

$$\phi'(0) < \infty,\ \phi''(0) > -\infty,\ \lim_{y\to\infty} \theta_\phi < 1 - \frac{1}{\sigma},\ \text{and}\ \frac{d\theta_\phi}{dy} < 0 \text{ for any positive } y < \infty,$$

where $\theta_\phi \equiv [y\phi'(y)/\phi(y)]$. An example of $\phi(y)$ is $\ln(y+1)$.

[9]As it is clear from these equations, any country's population size does *not* affect its relative factor abundance in the static sense. Our aim is to check whether population size affects long-run capital accumulation.

entiated products, p, is equal to the aggregate national output of those products:

$$n\bar{y} = \xi r'(\xi p)mk + \xi w'(\xi p)ml. \tag{13}$$

From (12), we can obtain another condition for each household:

$$\dot{k} = r(\xi p)k + w(\xi p)l - e[N^{1/(1-\sigma)}p]\psi(u). \tag{14}$$

Each household maximizes (1) subject to both (2) and (14). Associated with this problem is the Hamiltonian

$$H \equiv uX + \lambda\{r(\xi p)k + w(\xi p)l - e[N^{1/(1-\sigma)}p]\psi(u)\} - \delta\rho(u)X, \tag{15}$$

where λ and δ are the shadow prices of k and X. The necessary conditions for optimality are

$$0 = X - \lambda e[N^{1/(1-\sigma)}p]\psi'(u) - \delta\rho'(u)X, \tag{16}$$

$$\dot{\lambda} = -\lambda r, \tag{17}$$

$$\dot{\delta} = \rho(u)\delta - u. \tag{18}$$

Letting $Z \equiv \lambda/X$ and combining (2) and (17), we can obtain

$$\dot{Z} = Z[\rho(u) - r(\xi p)]. \tag{19}$$

Based on the foregoing argument, our dynamic general equilibrium two-country model is described as

$$\dot{k} = r(\xi p)k + w(\xi p)l - e[N^{1/(1-\sigma)}p]\psi(u), \tag{20}$$

$$\dot{k}^* = r(\xi p)k^* + w(\xi p)l^* - e[N^{1/(1-\sigma)}p]\psi(u^*), \tag{21}$$

$$\dot{Z} = Z[\rho(u) - r(\xi p)], \tag{22}$$

$$\dot{Z^*} = Z^*[\rho(u^*) - r(\xi p)], \tag{23}$$

$$\dot{\delta} = \rho(u)\delta - u, \tag{24}$$

$$\dot{\delta^*} = \rho(u^*)\delta^* - u^*, \tag{25}$$

$$0 = 1 - Ze[N^{1/(1-\sigma)}p]\psi'(u) - \delta\rho'(u), \tag{26}$$

$$0 = 1 - Z^*e[N^{1/(1-\sigma)}p]\psi'(u^*) - \delta^*\rho'(u^*), \tag{27}$$

$$0 = N\bar{y} - \xi[r'(\xi p)(mk + m^*k^*) + w'(\xi p)(ml + m^*l^*)], \tag{28}$$

$$0 = e'[N^{1/(1-\sigma)}p]N^{1/(1-\sigma)}[m\psi(u) + m^*\psi(u^*)] - N\bar{y}. \tag{29}$$

The system determines the equilibrium path of two state variables, k and k^*, and eight jump variables, Z, Z^*, δ, δ^*, u, u^*, p, and N.

3 The Steady State

The steady state is the solution for the system of equations

$$0 = r(\xi p)k + w(\xi p)l - e[N^{1/(1-\sigma)}p]\psi(u), \tag{30}$$

$$0 = r(\xi p)k^* + w(\xi p)l^* - e[N^{1/(1-\sigma)}p]\psi(u^*), \tag{31}$$

$$0 = \rho(u) - r(\xi p), \tag{32}$$

$$0 = \rho(u^*) - r(\xi p), \tag{33}$$

$$0 = \rho(u)\delta - u, \tag{34}$$

$$0 = \rho(u^*)\delta^* - u^*, \tag{35}$$

$$0 = 1 - Ze[N^{1/(1-\sigma)}p]\psi'(u) - \delta\rho'(u), \tag{36}$$

$$0 = 1 - Z^*e[N^{1/(1-\sigma)}p]\psi'(u^*) - \delta^*\rho'(u^*), \tag{37}$$

$$0 = N\bar{y} - \xi[r'(\xi p)(mk + m^*k^*) + w'(\xi p)(ml + m^*l^*)], \tag{38}$$

$$0 = e'[N^{1/(1-\sigma)}p]N^{1/(1-\sigma)}[m\psi(u) + m^*\psi(u^*)] - N\bar{y}. \tag{39}$$

For a given p, if

$$\rho(0) < r(\xi p),$$

then there exists a unique and positive u such that

$$\rho(u) = r(\xi p).$$

Let $u(\cdot)$ be the inverse function of $\rho(\cdot)$.[10] Since the shadow prices, Z, Z^*, δ, δ^*, are derived once the above system of equations determines p, k, k^*, N, we see

[10]As is clear from (32) and (33), $u = u^*$ holds at the steady state in which both countries are incompletely specialized.

that the main system consists of the four equations:

$$0 = r(\xi p)k + w(\xi p)l - e[N^{1/(1-\sigma)}p]\psi[u(r(\xi p))], \quad (40)$$

$$0 = r(\xi p)k^* + w(\xi p)l^* - e[N^{1/(1-\sigma)}p]\psi[u(r(\xi p))], \quad (41)$$

$$N\bar{y} = \xi[r'(\xi p)(mk + m^*k^*) + w'(\xi p)(ml + m^*l^*)], \quad (42)$$

$$N\bar{y} = e'[N^{1/(1-\sigma)}p]N^{1/(1-\sigma)}\{m\psi[u(r(\xi p))] + m^*\psi[u(r(\xi p))]\}. \quad (43)$$

Now, we can restate Kikuchi and Shimomura (2007)'s result.

Proposition 1: *Suppose that differences in initial factor endowments between Home and Foreign are not very large and that both the preference of each household and production technologies take the Cobb-Douglas form. Then there exists a unique steady state which is saddle-point stable. In the steady state both countries produce both goods.*

Proof: *See Appendix.*

4 Trade-Pattern Propositions

Let us focus on the Home (gross) excess demand for differentiated products in the steady state,[11]

$$ED_1 \equiv m\{e'[N^{1/(1-\sigma)}p]N^{1/(1-\sigma)}\psi(u) - \xi[r'(\xi p)k + w'(\xi p)l]\}.$$

Considering the steady-state Home budget constraint, (40), we obtain

$$k = \frac{e\psi - wl}{r}.$$

[11] As we will see later, even if $ED_1 = 0$ holds, there is an incentive for trade due to product differentiation. Thus, ED_1 (pED_1) can be interpreted as the Home *gross* excess demand for differentiated products (the Home excess supply of the consumable capital).

Substituting this into the Home excess demand and rearranging, we obtain the following condition:

$$ED_1 = (m/p)[e\psi(\theta_e - \theta_r) + wl(\theta_r - \theta_w)], \tag{44}$$

where $\theta_e \equiv [pe' N^{1/(1-\sigma)}/e]$, $\theta_r \equiv (\xi pr'/r)$, and $\theta_w \equiv (\xi pw'/w)$, respectively. Following the same procedure, we can obtain the Foreign excess demand for differentiated products in the steady state, ED_1^*:

$$ED_1^* = (m^*/p)[e\psi(\theta_e - \theta_r) + wl^*(\theta_r - \theta_w)]. \tag{45}$$

From these excess demand functions, we see that

$$ED_1 - ED_1^* = (1/p)[e\psi(\theta_e - \theta_r)(m - m^*) + w(\theta_r - \theta_w)(ml - m^* l^*)]. \tag{46}$$

Since differentiated products are assumed to be capital intensive,

$$\theta_r > 1 > \theta_e > 0 > \theta_w$$

holds. Let us examine the following two cases.

4.1 Case A: $m < m^*$ and $l = l^*$

If the representative household in each country supplies an equal amount of labor ($l = l^*$), the gross excess demands for differentiated products have the same sign in both countries (see (44) and (45)). Since demands have to add up to zero, this implies that both of them have to be zero and, therefore, there is no net trade (the value of imports equals the value of exports) in the differentiated products sector. This also implies that there is no incentive for inter-industry trade (i.e., the exchange of differentiated products for the consumable capital). Still, since each country specializes in a different range of differentiated products, an incentive for intra-industry trade remains. We obtain our main proposition on the patterns of intra-industry trade.

Proposition 2: Suppose that the representative household in each country supplies an equal amount of labor. Then, in the steady state, only intra-industry trade of differentiated products between countries occurs irrespective of differences in the number of households.

This case provides a complementary view for the existence of intra-industry trade between developed and developing countries. We implicitly assume that Foreign (the developing country) started the process of development late (i.e., its capital stock is relatively low initially). Then, Proposition 1 and Proposition 2 state that Foreign accumulates capital until its capital-labor ratio equals that of Home.[12] Therefore, due to catching-up by the developing country, sources of inter-industry trade based on differences in the capital-labor ratio vanish and only intra-industry trade occurs in the steady state. Furthermore, since Foreign has a larger amount of labor, that is, $m < m^*$, its share of differentiated products in the world market also becomes larger than Home. Note that, in the steady state, the share of Foreign varieties $[n^*/(n+n^*)]$ is equal to the share of Foreign households $[m^*/(m+m^*)]$. Our dynamic model reinforces the role of increasing returns and monopolistic competition as determinants of intra-industry trade: the importance of intra-industry trade remains in the dynamic setting while that of inter-industry trade is downplayed.

4.2 Case B: $m = m^*$ and $l > l^*$

In this case, each Foreign household is relatively less efficient in providing labor. And also, assume that capital-labor endowment ratio is lower in Foreign

[12] This point contrasts sharply with Atkeson and Kehoe (2000), in which the developing country accumulates capital until its capital-labor ratio equals the ratio used in the rest of the world to produce the *labor-intensive good*: the developing country never catches up in this setting.

initially. Since we assume that each household in both countries has the same instantaneous discount function, $u = u^*$ holds at the steady state (see (32) and (33)). Therefore, from (30) and (31), each Foreign household accumulates more capital (i.e., $k < k^*$). Then, $ED_1 - ED_1^* > 0$ holds and Foreign becomes a net exporter of differentiated products (i.e., capital intensive products) although it is a labor-rich country at the initial moment.[13]

Proposition 3: *If the number of households is equal, the country with lower labor efficiency becomes the net exporter of the capital-intensive good.*

This case highlights that the source of *inter*-industry trade crucially depends on the efficiency of *each* household, not on the number of households. It also highlights the importance of capital accumulation in dynamic trade patterns. Again, in this case, Foreign's share of differentiated products in the world market becomes larger than Home's.

5 Concluding Remarks

Based on the two-sector Chamberlin-Heckscher-Ohlin (CHO) framework, this note has formulated a dynamic model of international trade by introducing the Uzawa (1968) endogenous time preferences. Also, in contrast to Kikuchi and Shimomura (2007), the difference in the number of households has been emphasized. We have shown that there exists a unique and saddlepoint-stable steady state that is independent of the initial international distribution of capital. In that steady state production in both countries is incompletely specialized (Proposition 1). Making use of the new dynamic trade model, we have shown that, (i) given that the representative household in each country supplies an

[13]The case of $m < m^*$ and $ml > m^* l^*$ can be analyzed in a similar way.

equal amount of labor ($l = l^*$), only intra-industry trade occurs in the steady state irrespective of differences in the number of households (Proposition 2), (ii) if the number of households is equal, the country with higher labor efficiency becomes the net exporter of the labor-intensive good (Proposition 3). Propositions 2 and 3 highlight the dominance of the developing country in the world economy: although its capital-labor ratio is lower than that of the developed country, capital accumulation makes it a major exporter of differentiated products. Although our result depends critically on several restrictive assumptions (e.g., Uzawa's endogenous time preferences), it establishes a link between the workhorse model of monopolistic competition and the size of labor endowment. Hopefully this analysis provides a useful paradigm for considering how the labor endowment of developing countries (e.g., China) works as a determinant of world trade patterns.

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6 Appendix: Existence, Uniqueness and Stability of the Steady State with Incomplete Specialization in Both Countries

Here, we shall prove the existence, uniqueness and stability of the steady state with incomplete specialization in the present two-country dynamic general equilibrium model. We shall focus on the symmetric case where preferences, technologies, and initial factor endowments are common between Home and Foreign ($m = m^* = 1$, $l = l^*$). As we shall show later, the determinant of the Jacobian at a symmetrical steady state is not zero, which implies that as long as the international differences in those economic fundamentals are not very large, the existence, uniqueness and stability are guaranteed.

6.1 Existence

Let us consider the existence of the steady state. Since we assume $l = l^*$, it is clear from (30)-(33) that $k = k^*$ holds at the steady state. Therefore, the system of equations which describes the steady-state k, p, and N becomes

$$0 = r(\xi p)k + w(\xi p)l - e[N^{1/(1-\sigma)}p]\psi[u(r(\xi p))], \tag{47}$$

$$N\bar{y} = 2\xi[r'(\xi p)k + w'(\xi p)l], \tag{48}$$

$$N\bar{y} = 2e'[N^{1/(1-\sigma)}p]N^{1/(1-\sigma)}\psi[u(r(\xi p))]. \tag{49}$$

From (47),

$$k = \frac{e\psi - wl}{r} \tag{50}$$

holds. Combining (48) $-$ (49), one can obtain

$$0 = e'N^{1/(1-\sigma)}\psi - \xi(r'k + w'l).$$

Substituting (50) into this, one can obtain

$$0 = e' N^{1/(1-\sigma)}\psi - \xi \left[\frac{r'}{r}(e\psi - wl) + w'l \right].$$

Multiplying p and rewriting this in terms of elasticity,

$$0 = \theta_e e\psi - [\theta_r(e\psi - wl) + \theta_w wl],$$

where $\theta_e = [pe'N^{1/(1-\sigma)}/e]$, $\theta_r = (\xi pr'/r)$, and $\theta_w = (\xi pw'/w)$, respectively. Rearranging this, we obtain:

$$e\psi = \Theta wl, \tag{51}$$

where $\Theta \equiv [(\theta_r - \theta_w)/(\theta_r - \theta_e)]$, which is greater than 1.

Next, multiplying p to (49), one can obtain

$$\begin{aligned} pN\bar{y} &= 2e\psi \frac{pe'N^{1/(1-\sigma)}}{e} \\ \text{or} \quad N &= \frac{2\theta_e}{p\bar{y}} e\psi. \end{aligned} \tag{52}$$

Substituting (51) into (52), one can obtain

$$N = \frac{2\theta_e}{p\bar{y}} \Theta w(\xi p) l. \tag{53}$$

In terms of proportional change, we obtain the first relationship between N and p:[14]

$$\frac{\hat{N}}{\hat{p}} = \theta_w - 1. \tag{54}$$

Since the differentiated products are capital-intensive, that is, $\theta_w < 0$, (54) implies that N is decreasing in p: we can depict (53) as Curve AA in Figure 1.[15]

[14]Note that θ_e (θ_r and θ_w) is constant when the preference (the production technologies) takes the Cobb-Douglas form.

[15]It can be easily shown that the right-hand side of (53) goes to ∞ (0) when p goes to 0 (∞).

Now, let us turn to the other condition. From (51),

$$e[N^{1/(1-\sigma)}p] = \frac{\Theta w(\xi p)l}{\psi[u(r(\xi p))]}.$$

Let $e^{-1} \equiv \beta$, then we can obtain:

$$N^{1/(1-\sigma)}p = \beta\left(\frac{\Theta w(\xi p)l}{\psi[u(r(\xi p))]}\right).$$

Rearranging this, one can obtain

$$N = \left[\frac{p}{\beta(\Theta w(\xi p)l/\psi[u(r(\xi p))])}\right]^{\sigma-1}. \tag{55}$$

In terms of proportional change, we obtain the second relationship between N and p.

$$\frac{\hat{N}}{\hat{p}} = (\sigma-1)\left[1 - \frac{1}{\theta_e}\left(\theta_w - \frac{\theta_r\theta_\psi}{\theta_\rho}\right)\right], \tag{56}$$

where $\theta_\psi \equiv [u\psi'(u)/\psi(u)]$ and $\theta_\rho = [u\rho'(u)/\rho(u)]$. Since θ_ψ and θ_ρ are positive, (56) implies that N is increasing in p: we can depict (55) as Curve BB in Figure 1.[16] Based on the foregoing argument, one can conclude as follows.[17]

LEMMA A1: *There uniquely exists a steady state in which production is incompletely specialized.*

[16] Let us define $\underline{p}$ as the solution of $u(r(\xi\underline{p})) = 0 \iff r(\xi\underline{p}) = \rho(0)$. Then, $\lim_{p\to\underline{p}}\beta(\Theta w(\xi p)l/\psi[u(r(\xi p))]) = \infty$, which implies that the right-hand side of (55) goes to 0 when p goes to $\underline{p}$. On the other hand, the right-hand side of (55) goes to ∞ when p goes to ∞.

[17] It is apparent from (50) and (51) that the steady-state capital stock is positive.

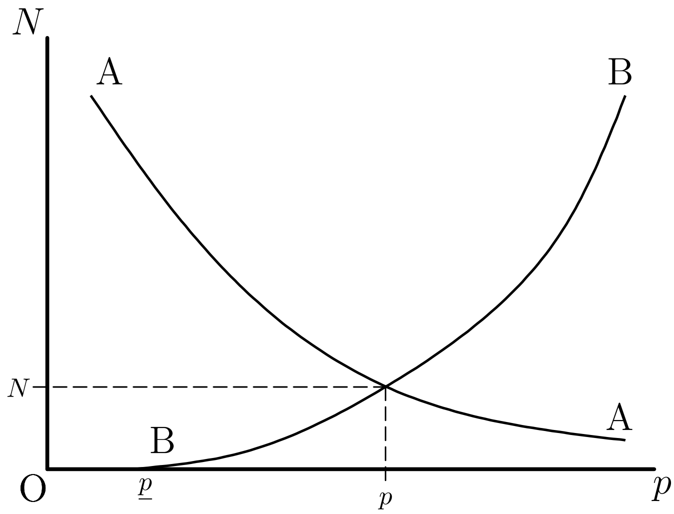

Figure 1

6.2 The Non-Existence of the Steady State with Complete Specialization in Home and/or Foreign

Now, what remains to be argued concerning uniqueness is to exclude a steady state where at least one country is completely specialized. For this purpose, let us consider the whole GDP function. In the case where the differentiated products are more capital-intensive than the homogeneous good, it can be expressed as follows.

$$F(k, \xi p) = \begin{cases} f^2(k, l), & 0 < k < k_2(\xi p), \\ r(\xi p)k + w(\xi p)l, & k_2(\xi p) < k < k_1(\xi p), \\ pf^1(k, l, \bar{y}), & k > k_1(\xi p), \end{cases}$$

where $k_i(\xi p) \equiv l\{c_r^i[w(\xi p), r(\xi p)]/c_w^i[w(\xi p), r(\xi p)]\}$, and $f^1(k, l, \bar{y}) \equiv \bar{y}n(k, l, \bar{y})$.[18] Making use of the above GDP function, we can express the steady-state Home

[18] For the derivation of the monopolistically competitive industry's implicit production function, f^1, see Helpman and Krugman (1985, p. 139).

and Foreign budget constraints as

$$
\begin{aligned}
0 &= F(k, \xi p) - e[N^{1/(1-\sigma)}p]\psi[u(F_k(k, \xi p))], && (57) \\
0 &= F(k^*, \xi p) - e[N^{1/(1-\sigma)}p]\psi[u(F_k(k^*, \xi p))]. && (58)
\end{aligned}
$$

If $k > k^*$, then both $F(k, \xi p) > F(k^*, \xi p)$ and $F_k(k, \xi p) \leq F_k(k^*, \xi p)$ hold from properties of the GDP function and vice versa. Thus, (57) and (58) together imply that there is no steady state such that $k \neq k^*$ holds. Therefore, we can conclude as follows.

LEMMA A2: *When the two countries are sufficiently close in terms of factor endowment ratio, no country can specialize in producing only one good in the steady state.*

6.3 Local Saddlepoint-Stability

Let us assume that the two countries are identical. Let us consider the Jacobian matrix of the steady state,

$$
\begin{bmatrix}
\rho & 0 & 0 & 0 & 0 & 0 & -e\psi' & 0 & \frac{p\bar{y}}{2(\sigma-1)} & 0 \\
0 & \rho & 0 & 0 & 0 & 0 & 0 & -e\psi' & \frac{p\bar{y}}{2(\sigma-1)} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Z\rho' & 0 & 0 & -Z\xi r' \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Z\rho' & 0 & -Z\xi r' \\
0 & 0 & 0 & 0 & \rho & 0 & -Ze\psi' & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho & 0 & -Ze\psi' & 0 & 0 \\
0 & 0 & -e\psi' & 0 & -\rho' & 0 & \begin{smallmatrix}-Ze\psi'' \\ -\delta\rho''\end{smallmatrix} & 0 & \frac{Zp\bar{y}\psi'}{2(\sigma-1)\psi} & -\frac{Z\bar{y}N\psi'}{2\psi} \\
0 & 0 & 0 & -e\psi' & 0 & -\rho' & 0 & \begin{smallmatrix}-Ze\psi'' \\ -\delta\rho''\end{smallmatrix} & \frac{Zp\bar{y}\psi'}{2(\sigma-1)\psi} & -\frac{Z\bar{y}N\psi'}{2\psi} \\
-\xi r' & -\xi r' & 0 & 0 & 0 & 0 & 0 & 0 & \bar{y} & -2\xi^2(r''k + w''l) \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\bar{y}N\psi'}{2\psi} & \frac{\bar{y}N\psi'}{2\psi} & -\left(\frac{\theta_e}{\sigma-1}+1\right)\bar{y} & 2\psi e'' N^{2/(1-\sigma)}
\end{bmatrix}.
$$

Denote the above matrix by J, and the corresponding eigenvalue as x. Then x is determined by the characteristic equation $\Omega(x) = |J - xI| = 0$, where $I \equiv \begin{bmatrix} I_6 & 0 \\ 0 & O_4 \end{bmatrix}$.

Let us make the following calculations to obtain the above determinant. First, let us add both the first row multiplied by $\xi r'/(\rho - x)$ and the second row multiplied by $\xi r'/(\rho - x)$ to the 9th row. Next, the 7th row minus the third row multiplied by $e\psi'/x$, and the 8th row minus the 4th row multiplied by $e\psi'/x$. Finally, we add the 5th row multiplied by $\rho'/(\rho - x)$ to the 7th row, and add the 6th row multiplied by $\rho'/(\rho - x)$ to the 8th row. Then, we see that

$$\begin{aligned}
\Omega(x) &= (\rho - x)^4 x^2 \begin{vmatrix} \frac{\Xi(x)}{x(\rho-x)} & 0 & \frac{pZ\bar{y}\psi'}{2(\sigma-1)\psi} & -\frac{Z\bar{y}N\psi'}{2\psi} + \frac{e\psi' Z\xi r'}{x} \\ 0 & \frac{\Xi(x)}{x(\rho-x)} & \frac{pZ\bar{y}\psi'}{2(\sigma-1)\psi} & -\frac{Z\bar{y}N\psi'}{2\psi} + \frac{e\psi' Z\xi r'}{x} \\ -\frac{\xi r' e\psi'}{\rho - x} & -\frac{\xi r' e\psi'}{\rho - x} & \bar{y} + \frac{p\xi r'\bar{y}}{(\rho-x)(\sigma-1)} & -2\xi^2(r''k + w''l) \\ \frac{\bar{y}N\psi'}{2\psi} & \frac{\bar{y}N\psi'}{2\psi} & -\left(\frac{\theta_e}{\sigma-1} + 1\right)\bar{y} & 2\psi e'' N^{2/(1-\sigma)} \end{vmatrix} \\
&= (\rho - x) \\
&\times \begin{vmatrix} \Xi(x) & 0 & \frac{(\rho-x)xpZ\bar{y}\psi'}{2(\sigma-1)\psi} & (\rho - x)x\left(-\frac{Z\bar{y}N\psi'}{2\psi} + \frac{e\psi' Z\xi r'}{x}\right) \\ 0 & \Xi(x) & \frac{(\rho-x)xpZ\bar{y}\psi'}{2(\sigma-1)\psi} & (\rho - x)x\left(-\frac{Z\bar{y}N\psi'}{2\psi} + \frac{e\psi' Z\xi r'}{x}\right) \\ -\xi r' e\psi' & -\xi r' e\psi' & (\rho - x)\bar{y} + \frac{p\xi r'\bar{y}}{\sigma-1} & -2(\rho - x)\xi^2(r''k + w''l) \\ \frac{\bar{y}N\psi'}{2\psi} & \frac{\bar{y}N\psi'}{2\psi} & -\left(\frac{\theta_e}{\sigma-1} + 1\right)\bar{y} & 2\psi e'' N^{2/(1-\sigma)} \end{vmatrix},
\end{aligned}$$

where $\Xi(x) \equiv (Ze\psi'' + \delta\rho'')x^2 - \rho(Ze\psi'' + \delta\rho'')x - Ze\psi'\rho'\rho$.

When the first column is subtracted from the second column, we obtain

$$\begin{aligned}
\Omega(x) &= (\rho - x)\Xi(x) \\
&\times \begin{vmatrix} 1 & 0 & \frac{(\rho-x)xpZ\bar{y}\psi'}{2(\sigma-1)\psi} & (\rho - x)x\left(-\frac{Z\bar{y}N\psi'}{2\psi} + \frac{e\psi' Z\xi r'}{x}\right) \\ -1 & \Xi(x) & \frac{(\rho-x)xpZ\bar{y}\psi'}{2(\sigma-1)\psi} & (\rho - x)x\left(-\frac{Z\bar{y}N\psi'}{2\psi} + \frac{e\psi' Z\xi r'}{x}\right) \\ 0 & -\xi r' e\psi' & (\rho - x)\bar{y} + \frac{p\xi r'\bar{y}}{\sigma-1} & -2(\rho - x)\xi^2(r''k + w''l) \\ 0 & \frac{\bar{y}N\psi'}{2\psi} & -\left(\frac{\theta_e}{\sigma-1} + 1\right)\bar{y} & 2\psi e'' N^{2/(1-\sigma)} \end{vmatrix}.
\end{aligned}$$

For $\Omega(x) = 0$, we have $x_1 = \rho > 0$. Furthermore, from $\Xi(x) = 0$, we have

$x_2 > 0 > x_3$ such that $\Omega(x_2) = \Omega(x_3) = 0$, since all of the first and second derivatives of ψ and ρ are positive.

Next, when the 1st row is added to the 2nd row, we obtain

$$\begin{aligned}
\Omega(x) &= (\rho - x)\Xi(x) \\
&\times \begin{vmatrix} 1 & 0 & \frac{(\rho-x)xpZ\bar{y}\psi'}{2(\sigma-1)\psi} & (\rho-x)x\left(-\frac{Z\bar{y}N\psi'}{2\psi} + \frac{e\psi' Z\xi r'}{x}\right) \\ 0 & \Xi(x) & \frac{(\rho-x)xpZ\bar{y}\psi'}{(\sigma-1)\psi} & (\rho-x)x\left(-\frac{Z\bar{y}N\psi'}{\psi} + \frac{2e\psi' Z\xi r'}{x}\right) \\ 0 & -\xi r' e\psi' & (\rho-x)\bar{y} + \frac{p\xi r'\bar{y}}{\sigma-1} & -2(\rho-x)\xi^2(r''k + w''l) \\ 0 & \frac{\bar{y}N\psi'}{2\psi} & -\left(\frac{\theta_e}{\sigma-1}+1\right)\bar{y} & 2\psi e'' N^{2/(1-\sigma)} \end{vmatrix} \\
&= (\rho - x)\Xi(x)\bar{y} \\
&\times \begin{vmatrix} \Xi(x) & \frac{(\rho-x)xpZ\psi'}{(\sigma-1)\psi} & (\rho-x)\left(-\frac{xZ\bar{y}N\psi'}{\psi} + 2e\psi' Z\xi r'\right) \\ -\xi r' e\psi' & \rho - x + \frac{p\xi r'}{\sigma-1} & -2(\rho-x)\xi^2(r''k + w''l) \\ \frac{\bar{y}N\psi'}{2\psi} & -\left(\frac{\theta_e}{\sigma-1}+1\right) & 2\psi e'' N^{2/(1-\sigma)} \end{vmatrix} \\
&\equiv (\rho - x)\Xi(x)\bar{y}A(x).
\end{aligned}$$

According to $A(x)$, the term on x^3 becomes

$$\begin{aligned}
&-(Ze\psi'' + \delta\rho'')2\psi e'' N^{2/(1-\sigma)} - \frac{\bar{y}N\psi'}{2\psi} \cdot \frac{pZ\psi'}{(\sigma-1)\psi} 2\xi^2(r''k + w''l) \\
&+\frac{\bar{y}N\psi'}{2\psi} \cdot \frac{\bar{y}NZ\psi'}{\psi} + (Ze\psi'' + \delta\rho'')2\xi^2(r''k + w''l)\left(\frac{\theta_e}{\sigma-1}+1\right),
\end{aligned}$$

where only the second term is negative, since the term, $r''k + w''l$, is positive due to the convexity of the GDP function with respect to p. Therefore, the term on x^3 becomes positive if σ is sufficiently large, which we assume.[19] Then, if

[19]Indeed, it can be shown that the sum of the second term and the third one becomes positive if $\sigma > [(\theta_r r'k + \theta_w w'l)/(r'k + w'l)]$.

$A(0) > 0$ holds, $A(x) = 0$ has one negative root x_4.

$$
\begin{aligned}
A(0) &= 2e\psi'\rho \\
&\times \begin{vmatrix} -Z\rho' & 0 & Z\xi r' \\ -\xi r' e\psi' & \rho + \frac{p\xi r'}{\sigma-1} & -\rho\xi^2(r''k + w''l) \\ \frac{\bar{y}N\psi'}{2\psi} & -\left(\frac{\theta_e}{\sigma-1}+1\right) & \psi e'' N^{2/(1-\sigma)} \end{vmatrix} \\
&= 2e\psi'\rho Z \\
&\times \left[\begin{array}{ccc} -\rho'\rho\left(1+\frac{\theta_r}{\sigma-1}\right)\psi e'' N^{2/(1-\sigma)} & + & (\xi r')^2 e\psi' \left(\frac{\theta_e}{\sigma-1}+1\right) \\ -\xi r'\rho\left(1+\frac{\theta_r}{\sigma-1}\right) e' N^{1/(1-\sigma)}\psi' & + & \rho'\rho\xi^2(r''k+w''l)\left(\frac{\theta_e}{\sigma-1}+1\right) \end{array} \right],
\end{aligned}
$$

which is positive, since the sum of the second term and the third one in the square brackets becomes

$$
\begin{aligned}
& \frac{\xi r' e\psi'\rho}{p}\left[\frac{p\xi r'}{r}\left(\frac{\theta_e}{\sigma-1}+1\right) - \frac{pe' N^{1/(1-\sigma)}}{e}\left(1+\frac{\theta_r}{\sigma-1}\right)\right] \\
= & \frac{\xi r' e\psi'\rho}{p}(\theta_r - \theta_e) \\
> & \ 0.
\end{aligned}
$$

Therefore, there are two negative characteristic roots, i.e., x_3 and x_4. Since there are two state variables, k and k^*, it follows that the steady state is a saddle point.

LEMMA A3: *When the two countries are sufficiently close, the steady state with both countries being incompletely specialized is locally saddlepoint-stable.*