Dual Banking and Financial Contagion

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Abstract
This paper builds a theoretical model based on Allen and Gale (2000) to analyse how unexpected shock affecting the banking assets in one region can generate bankruptcy in a second region. I also analyse the effect of the presence in a third region of an Islamic bank on the vulnerability of conventional banks to financial contagion. It is interestingly shown that the Islamic bank assets' diversification strategy across the regions reduces the likelihood of financial contagion among conventional banks.

Key Words: Islamic Banking, Conventional Banking, Financial Crisis

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1. Introduction

The Islamic banks (IBs) assets have continuously increased globally reaching 826 US$ billions in 2010 and projected to 1,130 US$ billions in 2012 (Ernst & Young, 2011). This tendency is also perceivable at the country levels where the share of the IBs assets is growing accounting for 14% in the Middle East and North Africa (MENA) region, 26% in Gulf Cooperation Council (GCC) and 17.3% in Malaysia. Therefore, in many countries the banking system is becoming dual with the simultaneous presence of conventional and Islamic banks. This transformation generated new challenges for the design of regulatory and supervisory frameworks by central banks. It continues also to stimulate studies trying to compare the behavior and performance of the two types of banks. The empirical studies revealed that the current practices of IBs deviate from their theoretical model as the majority of IBs mimic the conventional banks' (CBs) business model by creating assets through debt-like instruments with a predetermined fixed rate of return. In average almost 80% of the total assets of an IB are fixed income with short term maturity. While, only 20% are dedicated to long term and risk sharing investments.

El- Hawary el al (2007) and Greuning and Iqbal (2008) claim that the dominance of less risky, low return assets (e.g. Murabaha and Ijaras) deprives the IB of the benefits of the portfolio diversification, as Mudarabah and Mushrakah contracts are more profitable. According to Siddiqi (2006) this behavior could be explained by the low moral hazard and adverse selection problems associated with sale-based transactions compared to the profit and loss sharing (PLS) investments. Another salient divergence with the Islamic banking theory is also revealed by the income distribution. In many cases, IBs distribute profits to the investment depositors even when they accrue loss (El- Hawary et al, 2007; Greuning and Iqbal, 2008). This displaced commercial risk was confirmed by Zainol and Kassim (2010) and Cevik and Charap (2011) who found that the conventional banks' deposit rates Granger cause returns on PLS accounts in Malaysia and Turkey. An analogous result was established by Chong and Liu (2009) in the case of Malaysia showing that the retail Islamic deposit rates mimic the behavior of conventional interest rates. Beck et al (2010) carried an empirical investigation on a broad cross-country sample and reached the same conclusion since they identified few significant differences in business orientation, efficiency, asset quality between conventional and retail Islamic banks. The stability of IBs relatively to the CBs was analyzed by Cihak and Hesse (2010) who found that IBs are stronger only when they have small size. They also found no positive impact of the IBs' presence on the financial strength of conventional banks.

The analysis of the stability of IBs relatively to CBs becomes more relevant when the analysis's period include the recent global financial crisis. Indeed, the crisis induced a series of failure of many CBs and constitutes a good test of the stability of IBs. From a theoretical perspective, the Profit and Loss Sharing (PLS) principle enables the IBs to maintain its net worth under difficult economic situations. Indeed, any shock that could generate losses on their asset side will be absorbed on the liability side. (Ahmed, 2002; Cihak and Hesse, 2008). However, a growing number of studies show that the recent crisis has impacted not only the CBs but also IBs. Hasan and Dridi (2010) have reached the same conclusion using a sample of 120 Islamic and conventional banks in 8 countries. Beck et al (2010) showed that conventional banks operating in countries with high market share of Islamic banks are more cost-effective but less stable. Besides, their results confirmed the relatively better performance of IBs during the recent crisis. According to Syed Ali (2007) *Ihlas Finance House*, the Turkish Islamic financial institution was closed during the financial crisis of 2000-2001 due to liquidity problems and financial distress that originated from its strategic error to allow withdrawals from Investment Accounts. "On the contrary other SFHs (Islamic financial institutions) which survived the crisis did not en-cash the investment deposits and advised their clients to hold them to maturity."[1]

The objective of this paper is to shed light on another angle of the interaction between Islamic banks and conventional banks. More precisely I analyze the behavior of an IB when bankruptcy occurs in the conventional banking sector and delve if the presence of the IB reduces the likelihood of financial contagion within CBs[2].

One of the justifications of the existence of banks in the conventional "fractional reserve system" is their role as "pools of liquidity" providing depositors with insurance against idiosyncratic shocks that affect their consumption needs (Freixas and Rochet, 2008). In this system, banks hold a fraction of their deposits as cash reserve and the remainder fraction to finance profitable but illiquid investments. When offering demand deposit contracts, a bank become inherently vulnerable to bank run which takes place when all depositors panic and withdraw their deposits immediately irrespective to their consumption (liquidity) needs. This liquidity shock (excess of the immediate demand for liquidity which obliges the bank to liquidate its long assets since its short assets are insufficient) has been considered as the triggering event in the theory of banking crisis that focused on contagion among banks[3]. Allen and Gale (2000) show that an unforeseen liquidity shock could generate the

[1] Syed Ali (2007, page 12).

[2] Financial contagion between banks occurs when the bankruptcy of one bank causes the bankruptcy of a second bank or groups of banks (Allen and Gale, 2000).

[3] The term "liquidity" is linked here to the concept of "funding liquidity" which refers to the availability of funds and the ability of a solvent bank (a financial institution in general) to perform its intermediation function. The other concept is "market liquidity" which refers to the ease with which positions may be traded without significantly affecting their corresponding asset price (Crockett, A., 2008; Hesse, H. et al., 2008).

bankruptcy of the entire banking system under different configurations of the interbank market structure. In Allen and Gale (2000), banks belonging to different regions are cross-holding deposits on the interbank market. The authors show that the completeness of the latter (each bank is connected to all banks) reduces the likelihood of transmission of a bankruptcy from one region to another. The opposite happens when the interbank is incomplete (each bank is connected to small number of banks). This is because in the first configuration each bank of the different untroubled regions liquidates a small amount of its long asset and responds to the liquidity needs of the troubled bank without suffering a bankruptcy.

In Freixas et al (2000) a liquidity shock hitting one important bank may prompt the depositors to run on solvent banks if they fear that there is insufficient liquidity in the banking system. However, as stated by Adrian and Shin (2008) "the domino model (of contagion) paints a picture of passive financial institutions that stand by and do nothing as the sequence of defaults unfolds." In practice, financial distresses are more likely to be triggered through the impact of price changes on the banks' balance sheets. Mishkin (1998) affirms that the deterioration in bank balance sheets could result from excessive risk-taking due to inadequate bank regulation and supervision or because of negative shocks such as interest rate rises, stock market crashes, among other factors. This is confirmed by the 2007 subprime crisis which originated as a relatively small credit default event[4]. During this crisis the difficulty to access funding was coupled with the decline of the prices of the structured mortgage assets deteriorating the balance sheets of many banks exposed to the U.S. asset-backed securities (Hesse et al., 2008). As a consequence, major investments banks like Lehman Brothers bankrupted and other banks as Northern Rock and Bear Stearns were rescued.

This paper builds a theoretical model that analyses the effect of an Islamic bank presence on the propagation of bankruptcy across two conventional banks. We consider that the conventional banks belong to two different regions and that the Islamic bank belongs to a third region. Each region is characterized by different investment opportunities available in different industries and populated by a continuum of consumers (depositors) of mass 1. The interconnectedness between the regions takes the form of direct investments (purchasing of equity shares) by banks of one region in the investment projects of the other region. When a bankruptcy occurs in one region, the other region is impacted due to the premature termination (liquidation) of the investment projects financed by the bankrupted bank. We consider that this bankruptcy could generate a negative externality in the form of reduced return of the remaining long investment projects. In practice, in case of a banking crisis,

[4] "A 1 percent gain or loss in the US stock market is about the same order of magnitude of the likely subprime mortgage losses that will be gradually realized over the next few years." (Adrian and Shin, 2010, p. 2)

banks are likely to cut on their lending in order to shrink their asset base and restore their capital ratio. Since all businesses rely on finance to function the economic activity will slow[5] and the economies of scale are likely to decline reducing the return of the investment activities. In our model we enable different levels of the negative externality affecting the return of the long investment projects. This externality affects the return of the Islamic bank without causing its bankruptcy due to the amortizing effect of the Profit and Loss Sharing investment accounts. In addition, we show that the IB is incited to use its liquid funds to reduce the premature liquidation of the long investment projects by acquiring - at a discount - the shares of the investment projects initially owned by the bankrupted conventional bank. This in turn limits the negative effect on the bankruptcy externality on the second conventional bank and reduces the likelihood of its financial contagion. In our knowledge, this is the first theoretical attempt that analyzes this type of interaction between the IBs and CBs.

The rest of the paper is organized as follows: Section II develops a model of financial contagion within a banking system comprising only two conventional banks. Section III assesses the effect of the IBs' presence on the propagation of bankruptcy among the conventional banks. Finally section IV summarizes the main results.

2. A model of simultaneous bankruptcy of two conventional banks

In this section we develop a theoretical model based on Allen and Gale (2000) to analyse how unexpected shocks affecting the banking assets in one region can generate simultaneous bankruptcy of the conventional banks across two different regions. Many features distinguish our model from the above mentioned ones. For example we consider inter-banking linkages through the financing of common investment projects instead of cross-holding of deposits through the interbank money market as it is the case in Allen and Gale (2000). Besides, the source of the financial fragility is an unexpected productivity shock[6] affecting the investment project instead of the liquidity shock. Finally, we show that a decrease in the cost of premature termination of the investment project (liquidation cost) increases the vulnerability to simultaneous bankruptcy of the banks in the two regions. This result is opposite to that obtained by Allen and Gale (2000) in the context of a liquidity shock that propagates through the interbank deposit market.

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[5] The investment spending and economic activity might remain depressed for a long period in industrialized countries due to the "debt deflation" (Mishkin, 1998; Bhattacharya et al., 1998). For example this was the case of Japan in the early 1990s and U.S. in 1930-1933.

[6] This unforeseen exogenous productivity could be triggered by a political instability which worsens the macroeconomic environment.

2.1 The economic environment

There are three dates $t = 0, 1, 2$. There is a single consumption good and the economy is divided in two regions labeled A and B which can be interpreted as geographical regions with particular specialized sectors within a country. Each region contains a competitive banking sector which could be represented by a single bank and a continuum [0,1] of identical consumers (depositors).

Investment opportunities

In each region banks have two types of investment opportunities. First, there is a storage that yields a unit safe return. Indeed, one unit of the consumption good invested in this short investment at date t produces one unit of the consumption good at date $t+1$. Second, there is a long term investment project that has a higher expected return but matures after two periods and yields the following stochastic gross return over the two periods

$$R = \begin{cases} R_H & \text{with probability } 1/2 \\ R_L & \text{with probability } 1/2 \end{cases} \tag{1}$$

with $R_H > 1 > R_L$ and $E[R] = R_a = (R_L + R_H)/2 > 1$ signifying that the long asset is more productive in average than the short asset. If R_H occurs the economy is in the high state of the nature S_H otherwise it is in the low state S_L.

Definition 1. *The "liquidation cost" is the cost of premature termination of the investment project*.

If the owner of the investment project is asked by his financiers to repay their capital while the production process has not completed it is natural that this will generate additional costs for the firm (e.g. the project owner will be obliged to borrow or to sell a part of its raw material/equipment) and generate lower cash flow. Another justification of the liquidation cost rests on the fact that in presence of economies of scale (which is the case of many industries) the premature liquidation of a part of the investment project reduces its size and causes the decreasing of its productivity. Therefore, the long asset is not completely illiquid and each unit of the long asset can be prematurely liquidated to produce rR units of the consumption good at *date 1 (*where R is given by 1*)* such that $r<1$ and $rR_a < rR_H < 1$.

Economic signals

All the uncertainty is resolved at date 1 when banks and depositors observe a signal $S \in \{S_1, S_2\}$ that predicts with perfect accuracy the state of the nature that will take place in regions A and B at date 2. As in Allen and Gale (1998) this signal can be thought as a leading economic indicator that predicts the value of the investment projects cash-flows in each region. Although, there is an uncertainty at date *0* regarding the states of the nature that will occur at date 1, its distribution is known by all banks and depositors and is given by table 1.

Table 1. Distribution of the State of Nature across the two regions

	A	*B*	*Probability*
S_1	S_H	S_L	1/2
S_2	S_L	S_H	1/2

Hence in region *i=A, B* there is a probability ½ that the state of the nature S_H occurs and a probability ½ that the state S_L occurs. It is clear that given this distribution of the states of nature, the average gross return of the investment opportunity across the economy (comprising the two regions) is certain and equals R_a in each state[7].

Banks' inter-linkages

In order to diversify their assets banks will invest in the short investment as well as in the two regions' long term investment projects. Banks invest by acquiring equity shares issued by the projects' owners. This direct investment in the region was discussed in Allen and Gale (2000) when they argued that their model could be extended to the case of risky long asset[8]. This diversification reduces the risk of each bank long term investment since it guarantees it a certain average gross return of R_a. In the absence of this diversification the gross return is random and may be equal to R_H or R_L depending on the state of the nature that occurs in the bank's region.

In addition, it is possible that the liquidation of the long investment project by one bank reduces the return of the entire investment projects even if they are continued till their

[7] In each region there is an investment of 1 unit of the consumption good. Therefore the total investment across the economy (the two regions) is equal to 2 units. In case of S_1 or S_2 the total output across the two regions is certain and equal to $R_H + R_L$. The uncertainty (which is released at t = 1) concerns which region will contribute with R_H and which one will contribute with R_L. Therefore for two units invested at t = 0 the economy produces $R_H + R_L$ at date t = 2. It follows that the average return is $(R_H + R_L)/2 = R_a$.

[8] The authors argued that their results will remain the same if the financial interconnectedness between the regions takes the form of claims held by banks in one region on banks in another region. However, they mentioned that "*If, instead of holding claims on banks in other regions, banks were to invest directly in the long assets of that region, there would be a spillover effect, but it would be much weaker*" (Allen and Gale, 2000; page 31).

maturity. In this case, the return of the long term investment project becomes φR instead of R where $r < \varphi \leq 1$. If φ equals one there is no negative externality. In conclusion, the liquidated investment project generates no cash flows at date 2 and *rR* at date 1. Whereas, the remaining investment project of the region generates the following cash-flows at date 2

$$\varphi R = \begin{cases} \varphi R_H & \text{with probability } 1/2 \\ \varphi R_L & \text{with probability } 1/2 \end{cases}$$

<u>Depositors (consumers)</u>

Each region contains a continuum of mass 1 of ex ante identical consumers (depositors). A consumer has an endowment equal to one unit of the consumption good at date 0 and nothing at dates 1 and 2. Consumers are initially uncertain about their time preferences. At date 1 each consumer knows whether he is an *early consumer* who only want to consume at date 1 or *late consumer* who only want to consume at date 2. In addition, this is a private information of the consumer which is not observable by banks. Hence, late consumers can pretend to be early consumers and withdraw their deposits at date 1 if they will obtain higher return than withdrawing at date 2. It is only in section 2.2 that we assume that a social planner can identify the type of each consumer. At date 0 each consumer has a probability γ to be an early consumers and a probability 1 - γ to be a late consumer. Therefore, the ex-ante preferences of a consumer could be represented by

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability } \gamma \\ \delta u(c_2) & \text{with probability } 1\text{-}\gamma \end{cases}$$

Where c_t denotes consumption at date t =1, 2 and $\delta < 1$ is the discount factor. The utility function *u(.)* is assumed to be twice continuously differentiable, increasing, and strictly concave. In ex-ante terms the expected utility of a consumer is

$$EU = \gamma Eu(c_1) + (1-\gamma)\delta Eu(c_2) \tag{2}$$

2.2 Autarcy

We start by the simplest case where each consumer chooses independently the quantity y that he invests in the short asset and $x = 1 - y$ that he invests in the long investment project. If he has to consume early then the quantity x should be liquidated at date 1. Therefore, we have

$$\begin{aligned} c_1 &= y + rRx \\ c_2 &= y + Rx \end{aligned} \tag{3}$$

Where R is the random cash-flow of the investment project given by (1) which is revealed at date 1. Initially at date 0 the consumer chooses the allocation (x^a, y^a) so as to maximize its expected utility EU under the constraints (3). At date 1 all the uncertainties are released and we have the following cases

	Early Consumer	Late Consumer
High return R_H	$c_1^a = y^a + rR_H x^a < 1$ $c_2^a = 0$	$c_1^a = 0$ $c_2^a = y^a + R_H x^a$ with $y < c_2^a < R_H$
Low return R_L	$c_1^a = y^a + rR_L x^a < 1$ $c_2^a = 0$	$c_1^a = 0$ $c_2^a = y^a + R_L x^a < 1$

Since $rR_L < rR_H < 1$ and $R_L < 1$ the allocation (x^a, y^a) will be ex post (at date 1) suboptimal in all the cases. Indeed, if the consumer is an early one the optimal decision is $(x, y) = (0,1)$ and the consumption is $c_1 = 1 > c_1^a$. While, if the consumer is of a late type and the return is high the optimal decision is $(x, y) = (1,0)$ and the consumption is $c_2 = R_H > c_2^a$. When the consumer is of a late type and the return is low the optimal decision is $(x, y) = (0,1)$ and the consumption is $c_2 = 1 > c_2^a$. This inefficiency can be mitigated by a social planner who maximizes the expected utility of the entire population of consumers overs the economy (two regions).

2.3 *The optimal (symmetric) allocation*

We now consider that there is a social planner that collects the two units of consumption good endowment of the consumers across the two regions. He invests $2y$ units in the short asset and x in the long investment project of region A and x in the long investment project of region B. Therefore, the total quantity of the consumption good available at date 1 is $2y$ whereas it is $R_H x + R_L x = 2R_a x$ at date 2. The social planner maximizes the following social expected utility $EU = 2\gamma Eu(c_1) + 2(1-\gamma)\delta Eu(c_2)$ which is the sum of the consumers expected utilities. The parameter γ (respectively 1 - γ) represents the probability for an individual to be an early (late) consumer. Since the total mass of consumer in the economy is equal to 2 and using the law of large numbers, the probability 2γ (respectively $2(1-\gamma)$) represents also the fraction of early (respectively late) consumers in the economy. Therefore, the social planner program is given by

$$
\begin{cases}
\underset{(x,y)}{Max}\ EU = 2\gamma Eu(c_1) + 2(1-\gamma)\delta Eu(c_2) & (4) \\
s.t. & \\
& (5) \\
2x + 2y = 2 & \\
2\gamma c_1 \le 2y & (6) \\
2(1-\gamma)c_2 = 2R_a x + (2y - 2\gamma c_1) & (7)
\end{cases}
$$

Recalling equation (2) it is clear that maximizing the objective function (4) is equivalent to maximize the expected utility of each individual consumer in the economy belonging to region A or B (which are ex-ante identical). Constraint (5) means that the value of the total investment equals to the available funds. Constraint (6) signifies that consumption needs of the early consumers 2γ are covered by the investment $2y$ in the short asset. Constraint (7) signifies that the output $R_a x$ from the investment projects plus the residual quantity from the short investment after the payment of early consumers equals the consumption needs of the late consumers. It is simple to show[9] that the solution of the planning problem is characterized by the following conditions

$$
\begin{cases}
u'(c_1^*) = \delta R_a u'(c_2^*) & (8) \\
c_1^* = y^* / \gamma & \\
c_2^* = R_a x^* / (1-\gamma) & (9)
\end{cases}
$$

Since the utility function u is concave the late consumer obtain higher amount than early consumer $c_2^* \ge c_1^*$ if and only if [10]

$$\delta R_a \ge 1 \tag{10}$$

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[9] Constraints (6) and (7) hold with equality since it Is not optimal to invest in the short asset above the consumption needs at date 1, i.e. $y = \gamma c_1$. This is because it is possible to invest in the investment projects across the economy and obtain a higher certain gross return Ra>1. Hence, the constraint (7) becomes $c_2 = R_a x / (1-\gamma)$. Using equation (5) we obtain $c_2 = R_a(1-\gamma c_1)/(1-\gamma)$. After replacing c_2 by the previous expression in the objective function (4) and calculating the derivative relatively to c_1 we obtain the first first-order condition $Eu'(c_1) = \delta R_a Eu'(c_2)$ which gives us equation (8) because c_1 and c_2 are deterministic contrarily to the case of autarky.

[10] $c_2^* \ge c_1^* \Leftrightarrow u'(c_2^*) \le u'(c_1^*)$ since u' is a decreasing function. Using (8) we obtain $u'(c_2^*) \le \delta R_a u'(c_2^*)$ or equivalently condition (10).

Under this condition a late consumer has no incentive to declare he is an early one to obtain c_1 and store it to consume at date 2. Therefore, even if the planner cannot observe the consumers' types the latter will correctly reveal it. Thus, the above characterized optimal allocation can be achieved in this more general case.

Lemma 1.

The optimal allocation dominates the autarky allocation.

Proof. See the appendix.

By pooling the deposits of a large number of consumers the social planner can offer to them insurance against their uncertain consumption needs. This is done by providing early consumers some of the high yielding risky asset without exposing them to the volatility of the investment projects in their region.

2.4 *Decentralization of the optimal allocation*

In each region there is a continuum of banks constituting a competitive banking system. Banks are assumed to be identical and adopt the same behavior which simplifies the analysis by considering a representative bank for each region. Each consumer deposits his endowment of one unit of the consumption good in the representative bank of his region in exchange of a contract (c_1^*, c_2^*) (which is the solution of the planning problem) allowing him to withdraw either c_1^* units of consumption at date 1 or c_2^* units consumption at date 2. In the rest of the paper we denote (c_1^*, c_2^*) by (c_1, c_2). Since the deposit contract is not contingent on the state of nature that will occur in regions A and B the question that arises is how banks of the two regions could perform the role played by the social planner in the previous section. This is done through the investment in the long term investment projects in regions A and B[11]. Indeed, the bank belonging to region *i=A,B* invests its one unit of deposit in a portfolio (x^i, y^i, z^i) where x^i and y^i represent respectively the amount invested in the investment project and the short asset of region *i* and and z^i the investment in the investment project of region j≠ i. Therefore, given the distribution of states of the nature across the two regions (given by table 1), the portfolio (x^i, y^i, z^i) of bank *i=A,B* satisfies the following conditions:

$$x^i + y^i + z^i = 1 \tag{11}$$

$$y^i = \gamma c_1 \tag{12}$$

[11] In Allen and Gale (2000) the decentralization of the first best is realized through the interbank deposit market.

$$R_H x^i + R_L z^i = (1-\gamma)c_2 \tag{13}$$

$$R_L x^i + R_H z^i = (1-\gamma)c_2 \tag{14}$$

where condition (12) says that the liabilities of the bank at date 1 are covered by the short asset. Conditions (13) and (14) signify that the output of the long term investment enable each bank to pay its depositors a constant amount c_2 whatever the state of the nature it takes place. Indeed, the liabilities of the bank at date 2 are covered by the sum of the output of the long assets across the two regions. It is simple to show[12] that each bank holds the same investment in the long term projects of the two regions:

$$x^i = z^i = \frac{1-\gamma c_1}{2} \tag{15}$$

Hence by diversifying their long term investment across the two regions, banks are able to satisfy their budget constraints in each state of the nature and at each date *t = 0, 1, 2* while providing their depositors with the optimal consumption allocation through a standard deposit contract.

2.5 Productivity shock and bankruptcy

We perturb the decentralized first best allocation by introducing a new state S_C where an unexpected shock affects the productivity of the investment projects in region A. Table 2 presents the characteristics of the different states of the economy in terms of the investment projects' return.

Table 2. Distribution of the long asset's return with the perturbation

	A	*B*	*Probability*
S_1	R_H	R_L	1/2
S_2	R_L	R_H	1/2
S_C	$R_H - \varepsilon$	R_L	0

Hence in region *i=A,B* there is a probability ½ that the state of the nature S_H occurs, a probability ½ that S_L occurs and banks assign a zero probability to the state S_C at date 0.

[12] From (13) and (14) we have $R_H x^i + R_L z^i = R_L x^i + R_H z^i \Leftrightarrow (R_H - R_L)x^i = (R_H - R_L)z^i \Leftrightarrow x^i = z^i$

Thus, the sum of the probabilities of the states of nature equals 1. Since banks did not expect the realization of the state S_C, contracts and investment decisions at date *0* are the same as before. If S_1 or S_2 occurs, the first best allocation is realized. However, if the third state S_C occurs, the unexpected shock ε reduces the high return of the investment project in region A. We showed in section 2.4 that the the diversification of bank's assets across the two regions enables the decentralization of the optimal allocation. However, this strategy exposes the banks of region B to the negative unexpected shock happening in region A.

When state S_C occurs the average return of the investment projects across the economy is lower than expected. As illustrated in table 2 region B has the same return R_L but region A faces an unexpected productivity shock lowering the return R_H by ε. At date *1* all the depositors observe a signal (an economic indicator) revealing perfectly that the state S_C will occur at date *2*. Therefore, late depositors may withdraw their deposit prematurely at date 1 claiming they are early depositors and causing a bank run.

Definition 2. *There is a bankruptcy when the bank run deplete all the assets of a bank at date 1 and the latter cannot honor its engagement as specified in the demand deposit contract.*

The following proposition presents the condition of bankruptcy of banks in regions A and B.

Proposition 1.

i) *If* $\varepsilon \leq \varepsilon_m(r)$ *then there is no bankruptcy.*

ii) *If* $\varepsilon > \varepsilon_m(r)$ *then banks A and B are bankrupt.*

with

$$\varepsilon_m(r) = 2\mathrm{R}_a - \frac{(1-\gamma)\, y}{(1-\mathrm{r}(1-\gamma))x^B}$$

Proof. See the appendix □ Figure 1 represents the results of proposition 1 in the diagram (r,ε) where r is the liquidity cost defined in section 2.1.

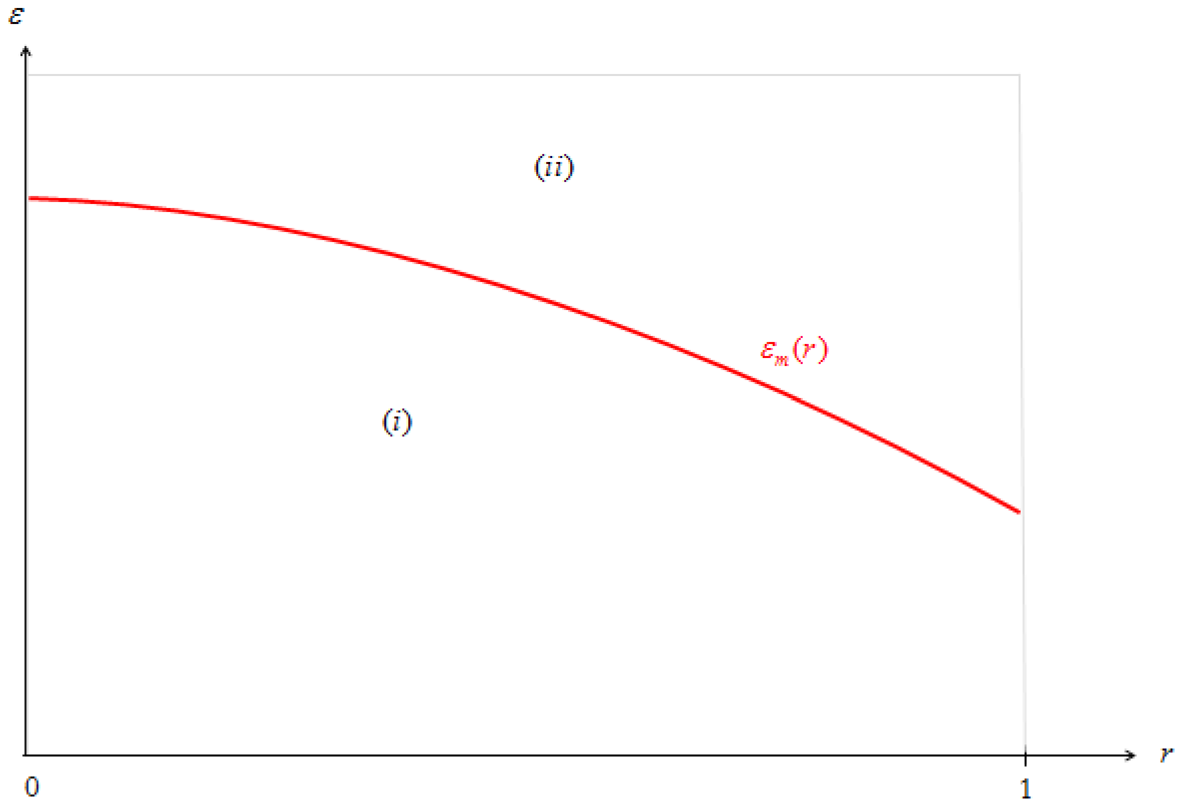

***Figure 1**. The bankruptcy regions in the diagram* (r,ε)

Region (i) represents the different combinations of (r,ε) for which neither bank A nor B are bankrupt. It is clear that for the bankruptcy to take place it is necessary that the productivity shock ε affecting the productivity of the long investment project in region A assets exceeds the minimum threshold $\varepsilon_m(r)$. Another interesting result to note is the fact that the economy becomes more vulnerable to the bankruptcy of banks A and B as the liquidation cost (1– r) decreases. This result is opposite to that obtained by Allen and Gale (2000) in the context of a liquidity shock that propagates through the interbank deposit market. It seems counterintuitive but in our case this is due to the fact that late consumers have lower incentive to early withdraw their deposit when the liquidation cost is higher. In Allen and Gale (2000) the liquidation of the long asset is not an option but always happen for the bank that faces the liquidity shock.

2.6 Partial diversification and bankruptcy of bank B

Until now there is a complete symmetry between banks A and B. Let's now assume that bank B partially diversify its portfolio and chooses the following portfolio $((1+\rho)x^B, y^B, (1-\rho)z^B)$ where y^B is defined by (12) and (x^B, z^B) are the optimal solution of the first best decentralization problem given by (15). This means that bank of region B over-invests in the long-term investment projects of its region and under-invest in the projects of region A. Naturally, this undermines the final remuneration of its late depositors who obtain $c_2^{'}$ verifying $c_1 < c_2^{'} \leq c_2$. Instead of equation (13) and (14) we have the following conditions for bank B:

$$R_H(1+\rho)x^B + R_L(1-\rho)z^B = (1-\gamma)c_2^{'} + \pi_\rho \qquad (16)$$

$$R_L(1+\rho)x^B + R_H(1-\rho)z^B = (1-\gamma)c_2^{'} \qquad (17)$$

where π_ρ represents an additional revenue for bank B which could be justified as a reserve requirement in case of good performance of the region B. Although this portfolio allocation does not permit to realize the first-best allocation, it reduces the exposure of bank B to the unexpected productivity shock that takes place in the state S_C. The following proposition gives the new regions of bankruptcy across regions A and B.

Proposition 2. *Under the condition that the negative externality of bank A bankruptcy is such that $\varphi > \varphi_m$, then bank B is less vulnerable to the bankruptcy triggered by the negative productivity shock relatively to the case of full diversification. Indeed, the bankruptcy of bank B takes place if $\varepsilon > \varepsilon_m^B(\varphi, r) > \varepsilon_m(r)$ with*

$$\varepsilon_m^B(\varphi, r) = 2\mathrm{R}_a + \frac{2\rho \mathrm{R}_L x^B - (1-\gamma)y}{(\varphi - \mathrm{r}(1-\gamma))x^B} \qquad \cancel{(17)}(18$$

$$\varphi_m = 1 - \frac{2\rho \mathrm{R}_L x^B}{(1-\gamma)y}\left(1 - r(1-\gamma)\right) \qquad \cancel{(18)}(19$$

Proof. See the appendix □

Definition 3. *There is a financial contagion from bank A to bank B when the bankruptcy of the latter is due to the negative externality of the former's bankruptcy.*

Lemma 2. *For a given (φ, r), financial contagion occurs when the productivity shock belongs to the region $\left[\varepsilon_m^B(\varphi, r), \varepsilon_m^B(1, r)\right]$.*

Proof. Noting that by assumption $\varphi_m > r$ it is simple to show from (18) and (19) that $\partial \varepsilon_m^B(\varphi, r) / \partial r < 0$ and that $\partial \varepsilon_m^B(\varphi, r) / \partial \varphi > 0$. Figure 2 represents the results of proposition2 in the diagram (r, ε) for two values $\varphi = 1$ and $\varphi < 1$. Compared to the results of proposition1 (illustrated by figure 1) there is now two intermediate regions (iii) and (iv). In region (iii) the bankruptcy occurs only for bank A. In this region bank B is partially affected by the negative shock reducing the productivity of the long term investment in region A as well as by the premature liquidation of the investment projects financed by bank A across the two regions. In region (iv) bank B is also bankrupted. However, this bankruptcy do not take place if there is no externality from the bankruptcy of bank A in the form of reduction of long term investment project return (i.e. if $\varphi = 1$). In other words, if bank A is (exogenously) rescued the bank B will not go in bankruptcy since the productivity shock in region (iv) is inferior to the threshold $\varepsilon_m^B(1,r)$. Therefore, we could qualify the region $\left[\varepsilon_m^B(\varphi,r), \varepsilon_m^B(1,r)\right]$ as the region of financial contagion which disappears if $\varphi = 1$.

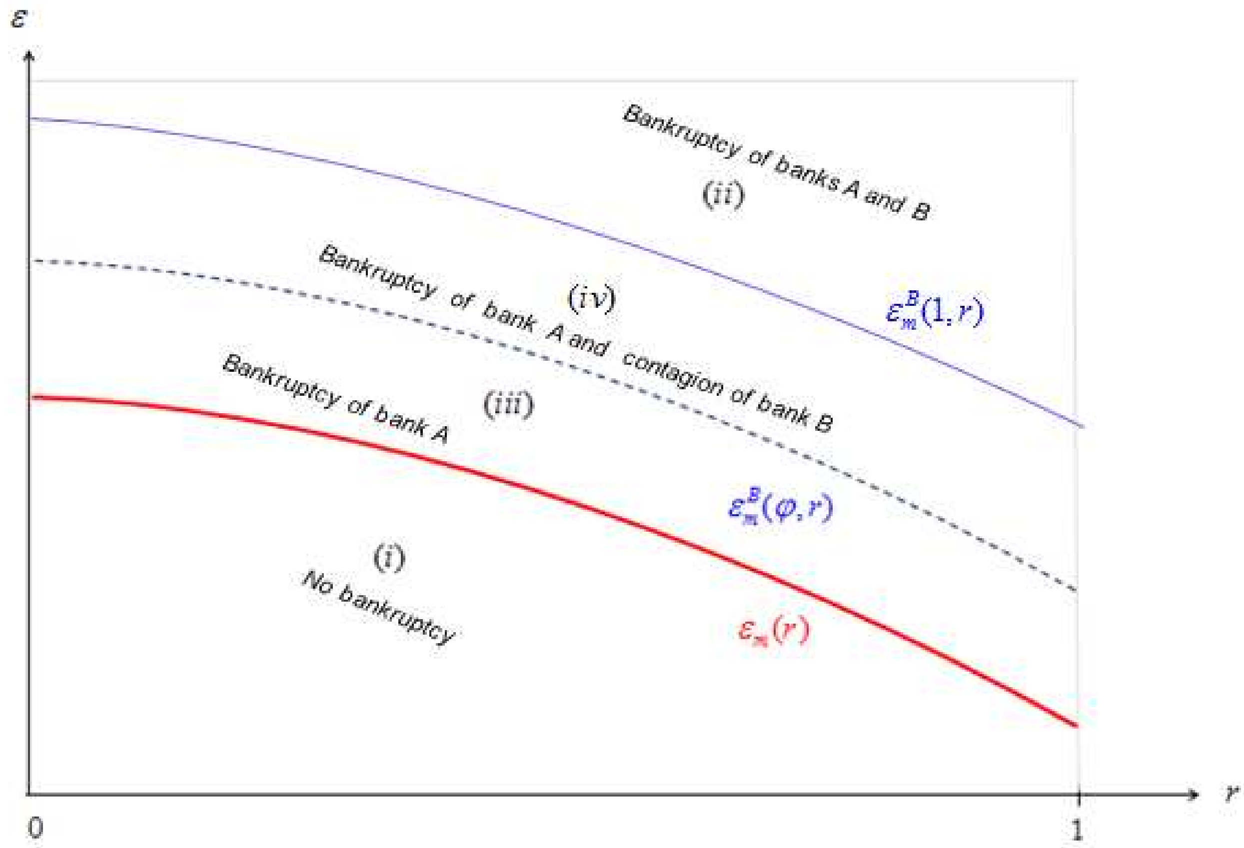

Figure 2*. The bankruptcy regions in case of partial diversification of bank B*

3. The presence of an Islamic bank and financial contagion

Allen and Gale (2000) conclude that an interbank market for one-period loans opening at the second period could limit the contagion to other banks but cannot avoid the bankruptcy of the bank faced with a liquidity shock. In this section, we analyze if the presence of an Islamic bank in a third region C could avoid the bankruptcy of the conventional bank in region B.

3.1 Characterization of the Islamic bank

A representative Islamic bank is located in region C which contains a continuum [0,1] of consumers (depositors). Each consumer deposits his endowment of one unit of consumption good in one of the two deposit contracts offered by the Islamic bank: a *demand deposit contract* or a *Profit Sharing Investment account*. I assume that a fraction β of the consumers hold a demand deposit contract enabling them to withdraw one unit of consumption at date 1 or at date 2 conditionally on the liquidity shock they face at date *1*. Hence, they do not share the bank's long asset risk but only want to keep their deposit intact in order to pay their expenditures. At date *1*, only the fraction $\gamma\beta$ early consumers will withdraw their deposit and the IB will carry the remaining amount of deposit $(1 - \gamma)\beta$ to date *2*. The fraction *1* - β of depositors hold a Profit Sharing Investment account which enable them to withdraw their deposit only at date 2. The investment holders accept to be paid an amount contingent on the long asset return. For simplicity, we assume that the IB sector is competitive so that the share of the investment cash-flows that goes to the Profit Sharing Investment account holders is $\mu = 1$ and the share of the IB is $1-\mu = 0$. Therefore, the payoff of the Profit Sharing Investment account is given by

$$c_2^{is} = \begin{cases} R_H & \text{with a probability } 1/2 \\ R_L & \text{with a probability } 1/2 \end{cases} \tag{20}$$

3.2 The IB diversified portfolio

We assume that the distribution of the long assets' return in region C is ex-ante symmetrically correlated with those in regions A and B. Table 3 shows that we have the same distribution of return for regions A and B as in table 2.

Table 3. Distribution of the long asset's return in the presence of an IB.

	A	*B*	*C*	*Probability*
S_1	R_H	R_L	R_H	1/4
			R_L	1/4
S_2	R_L	R_H	R_H	1/4
			R_L	1/4
S_C	$R_H - \varepsilon$	R_L	$R_{H,L}$	0

The only difference concerns the distribution of the return in region C where the IB exists which is detailed in the following. There is an equal probability of ¼ for the high return and the low return to take place conditioned on the realization of state S_1 or S_2. Therefore, the sum of the probabilities of all the possibilities for the IB is equal to 1. Let's also note that the IB like the CBs do not expect the realization of the state S_C which explains that the probability initially assigned to this state is zero. Let's now show that the IB could reduce the risk of the profit sharing investment account while holding the same expected return by diversifying its portfolio across the three regions by investing $(1-\beta)/2$ in region C, $(1-\beta)/4$ in region A and $(1-\beta)/4$ in region B. This strategy will provide the investment account holders with the following remuneration which replaces that presented in (20):

$$\hat{c}_2^{is} = \begin{cases} \frac{3}{4}R_H + \frac{1}{4}R_L & \text{with probability } \frac{1}{2} \\ \frac{3}{4}R_L + \frac{1}{4}R_H & \text{with probability } \frac{1}{2} \end{cases} \tag{21}$$

From (20) and (21) we obtain $E(\hat{c}_2^{is}) = E(c_2^{is})$ while $\mathrm{var}(\hat{c}_2^{is}) < \mathrm{var}(c_2^{is})$. This diversification strategy will however expose the IBs to the negative effect of the unpredictable crisis state S_C. The following section delves with the reaction of the IB in this state.

3.3 Could the presence of an Islamic bank reduce the vulnerability to financial contagion?

When the investment account holders observe the negative shock affecting the assets of the IB they have no incentive to early withdrawal their investment. This is not only because the contract stipulates that withdrawal is only possible at the maturity of the long asset but also because (contrarily to the conventional banks' late consumers) there is no additional benefit from doing this. Therefore, the negative shock on the asset side could be entirely passed-through to the liability side of the IBs. However, fearing a confidence crisis that pushes its investment account holders to switch to other competing banks the IB may not remain passive particularly if there is simultaneous bankruptcy of banks A and B which results in the liquidation of a high proportion of the long assets in regions A and B. According to Syed Ali (2007) *Ihlas Finance House* allowed withdrawals from its Investment Accounts during the Turkish financial crisis of 2000-2001 to advertise its financial strength relatively to its competitors or to cool down the confidence crisis. This strategy which appeared to be a strategic error and led to the closure of *Ihlas Finance House* intended initially to keep the confidence of the clients. In our model, the IB adopts the strategy of the competitors of *Ihlas Finance House* who survived the crisis and refused to pay prematurely their investment account holders (Syed Ali, 2007). In addition, we assumed that the Islamic banking sector is competitive in region C thus our representative IB will try to reduce the pass-through of the CB's bankruptcy to its investment account holders. This will be clarified in the rest of the section.

<u>Assumption 1</u>

The negative externality affecting the return of the long investment projects due to bankruptcy is increasing with the proportion l of liquidated investment project.

$$\partial(1-\varphi)/\partial l > 0 \tag{22}$$

Proposition 3.

i) The presence of the Islamic bank in region C reduces the vulnerability of bank B to Contagion.

ii) The higher the liquidity available for the Islamic bank the lower is the vulnerability of bank B to Contagion.

Proof. If the crisis state S_C takes place the return of the investment account will be the following

$$\hat{c}_2^{is,C} = \begin{cases} \frac{1}{4}R_{H,L} + \frac{1}{4}(R_H - \varepsilon) + \frac{1}{4}R_L & \text{if no bankruptcy} \\ \frac{1}{4}R_{H,L} + \frac{1}{4}\varphi(R_H - \varepsilon) + \frac{1}{4}\varphi R_L & \text{if banks A and/or B is bankrupt} \end{cases} \tag{23}$$

The only solution for the IB in region C to limit the deterioration of its assets in this situation is to ensure the continuing financing of the maximum proportion of the long investment projects in region B. For this to happen the IB should be able to use its available liquidity at date 1 which is equal to $(1-\gamma)\beta$ (corresponding to the late demand deposits) to purchase the maximum proportion m of projects financed by bank A in region B. The IB should pay at least the unitary price rR_L which bank A could otherwise obtained. The purchased asset will provide the IB a payoff R_L generating an additional profit of $mR_L - mrR_L$. This operation will also reduce the negative externality affecting the return of the long investment projects which is captured through the new value of the parameter $\varphi' > \varphi$. Therefore, the IB will remunerate its investment account holders $\hat{\hat{c}}_2^{is,C} > \hat{c}_2^{is,C}$ given by

$$\hat{\hat{c}}_2^{is,C} = \left(\frac{1}{4}R_{H,L} + \frac{1}{4}\varphi'(R_H - \varepsilon) + \varphi'\frac{1}{4}R_L\right) + \frac{R_L(1-r)m}{1\text{-}\beta} \tag{24}$$

Using the results of lemma 2, it is clear that the region of bankruptcy of bank B is reduced by the above described behavior of the IB. Indeed, the latter by acquiring a proportion of the long term investment project of bank A is also reducing the exposure of bank B to the negative externality resulting from the bankruptcy of A □ Figure 3 illustrates the effect of the presence of the Islamic bank on the region of financial contagion which shrinks from $\left[\varepsilon_m^B(\varphi,r),\varepsilon_m^B(1,r)\right]$ to $\left[\varepsilon_m^B(\varphi',r),\varepsilon_m^B(1,r)\right]$.

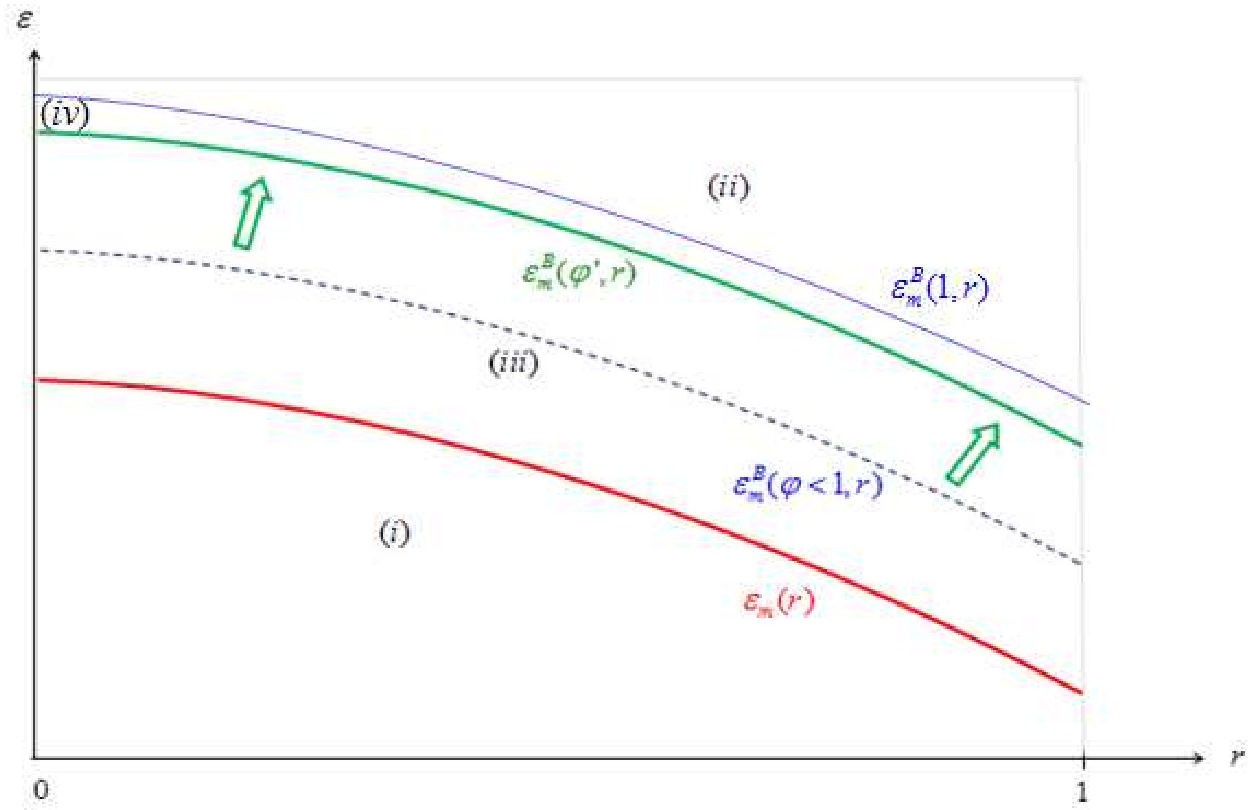

Figure 3: The bankruptcy regions in the diagram (φ,ε) in presence of an Islamic bank

Figure 3 shows that the region (iv) of contagion is now reduced compared to that in figure 2. The presence of the Islamic bank enlarged the region (iii) of non-bankruptcy of bank B in the case of bankruptcy of bank A. Hence, the IB's presence generates in this region the same effect on bank B as would do a lender of last resort.

5. Conclusion

The share of Islamic banks in the banking system of many countries is growing. This transformation generated new challenges for the design of regulatory and supervisory frameworks by central banks and motivated many research studies aiming to compare the behavior of IBs relatively to CBs. This paper shed light on the optimal behavior of an IB when bankruptcy occurs in the conventional banking sector. To this end we develop a theoretical model inspired from Allen and Gale (2000). In this model an unexpected shock affects the banking assets in one region and generates financial contagion among the conventional banking sector. We show that the presence in a third region of an Islamic banking sector (offering demand deposit accounts as well as Profit and Loss Sharing investment accounts) reduces the vulnerability to financial contagion.

Appendix

Proof of Lemma 1.

Let's define a function f as following

$$f(y) = EU(y) = \gamma u(y/\gamma) + (1-\gamma)\delta u(R_a(1-y)/(1-\gamma)) \quad \text{(A1)}$$

The first derivative of f is given by

$$f'(y) = u'(y/\gamma) - \delta R_a u'\left[R_a(1-y)/(1-\gamma)\right] \quad \text{(A2)}$$

Given the conditions (8) and (9) we have

$$f'(y^*) = 0 \quad \text{(A3)}$$

It is easy to calculate the second derivative of f and to find that it is strictly negative given the assumption that the utility function u is strictly concave.

$$f''(y) = \frac{1}{\gamma}u''(y/\gamma) + \frac{\delta}{1-\gamma}u''(R_a(1-y)/(1-\gamma))$$

Therefore the function f is strictly concave with a maximum in y^*. Therefore we have

$$f(y) \le f(y^*) \text{ with equality } \Leftrightarrow y = y^* \quad \text{(A4)}$$

This is true in particular for $y = \gamma$

$$f(\gamma) = \gamma u(1) + (1-\gamma)\delta u(R_a) \le f(y^*) \quad \text{(A5)}$$

Let's now turn to the autarky allocation. From equations (3) we have the following relations

$$\begin{aligned} E(c_1^a) &= y^a + rR_a x^a = y^a + rR_a(1-y^a) < 1 \\ E(c_2^a) &= y^a + R_a x^a = (1-x^a) + R_a x^a < R_a \end{aligned} \quad \text{(A6)}$$

From equation (2) we obtain the following expression of the expected utility in autarky

$$EU^a = \gamma Eu(c_1^a) + (1-\gamma)\delta Eu(c_2^a) \quad \text{(A7)}$$

Since the utility function is strictly concave we have the following Jensen inequality $Eu(.) < uE(.)$ which enables us to obtain from (A7) and (A6)

$$EU^a < \gamma u\left[E(c_1^a)\right] + (1-\gamma)\delta u\left[E(c_2^a)\right] < \gamma u(1) + (1-\gamma)\delta u(R_a) \tag{A8}$$

Finally the proof is completed by combining (A8) and (A5) since we have

$$EU^a < \gamma u(1) + (1-\gamma)\delta u(R_a) \le f(y^*) = EU^* \tag{A9}$$

Where EU^* represents the expected utility of a consumer in the presence of the social planner.

Proof of Proposition 1. Late consumers of banks A have an incentive to withdraw their deposit prematurely at date 1 (and store it for consumption at date 2) if they obtain a larger payment than waiting until date 2. The value of bank A total assets at date 2 is $(\mathrm{R_H} - \varepsilon)\mathrm{x^A} + \mathrm{R_L z^A}$ then late consumers will receive at date 2 a payment $\tilde{c}_{2A}$ given by:

$$\tilde{c}_{2A} = \frac{(\mathrm{R_H} - \varepsilon)\mathrm{x^A} + \mathrm{R_L z^A}}{1-\gamma} \tag{A10}$$

However, if they decide to withdraw their deposit at date 1 late consumers will trigger a bank run constraining bank A to liquidate its long asset and retrieve its investment in the long term project of region B. They will receive at date 1 a payment $\hat{c}_{2A}$ given by:

$$\hat{c}_{2A} = y + r(\mathrm{R_H} - \varepsilon)\mathrm{x^A} + \mathrm{rR_L z^A} \tag{A11}$$

Since there is a symmetry of the problem relatively to banks A and B, the value of bank B total assets at date 2 is $\mathrm{R}_L\mathrm{x}^B + (\mathrm{R}_H\text{-}\varepsilon)\mathrm{z}^B$ and late consumers will receive at date 2 a payment $\tilde{c}_{2B}$ given by:

$$\tilde{c}_{2B} = \frac{\mathrm{R}_L\mathrm{x}^B + (\mathrm{R}_H\text{-}\varepsilon)\mathrm{z}^B}{1-\gamma} \tag{A12}$$

However, if they decide to withdraw their deposit at date t = 1 late consumers will trigger a bank run on B which liquidates its long asset and pays its depositors $\hat{c}_{2B}$ given by:

$$\hat{c}_{2B} = y + r\mathrm{R}_L\mathrm{x}^B + \mathrm{r}(\mathrm{R}_H\text{-}\varepsilon)\mathrm{z}^B \tag{A13}$$

Late consumers of bank i=A,B have an incentive to run on their bank if $\hat{c}_{2i} > \tilde{c}_{2i}$ which is equivalent after using equations (15), (A13) and (A14) to

$$\varepsilon > \varepsilon_m(r) = 2\mathrm{R}_a\text{-}\frac{\left(1-\gamma\right)y}{(1-\mathrm{r}(1-\gamma))x^B} \tag{A14}$$

Proof of Proposition2. Late consumers of banks B have an incentive to withdraw their deposit prematurely at date 1if they obtain a larger payment than waiting until date 2. The value of bank B total assets at date 2 if bank A defaults ($\varepsilon > \varepsilon_m(r)$) is $\varphi R_L(1+\rho)x^B + \varphi(R_H - \varepsilon)(1-\rho)z^B$ then late consumers will receive at date 2 a payment $\tilde{\tilde{c}}_{2B}$ given by:

$$\tilde{\tilde{c}}_{2B} = \frac{\varphi R_L(1+\rho)x^B + \varphi(R_H - \varepsilon)(1-\rho)z^B}{1-\gamma} \tag{A15}$$

However, if they decide to withdraw their deposit at date 1 late consumers will trigger a bank run constraining bank B to liquidate its long asset and retrieve its investment in the long term project of region B. They will receive at date 1 a payment $\hat{\hat{c}}_{2B}$ given by:

$$\hat{\hat{c}}_{2B} = y + rR_L(1+\rho)x^B + r(R_H - \varepsilon)(1-\rho)z^B \tag{A16}$$

Late consumers of bank B have an incentive to run on their bank if $\hat{\hat{c}}_{2B} > \tilde{\tilde{c}}_{2B}$ which is equivalent after using equations (15), (A15) and (A16) to

$$\varepsilon > \varepsilon_m^B(\varphi, r) = 2\mathrm{R}_a + \frac{2\rho \mathrm{R}_L x^B - (1-\gamma)\,y}{(\varphi - \mathrm{r}(1-\gamma))x^B} \tag{A17}$$

Since we are in the case $\varepsilon > \varepsilon_m(r)$ then we should have the following condition on φ

$$\varphi > \varphi_m = 1 - \frac{2\rho \mathrm{R}_L x^B}{(1-\gamma)\,y}\big(1 - r(1-\gamma)\big)$$

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