Profit Sharing, Income Inequality and Capital Accumulation

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The relationship between economic development and income inequality isn't neutral vis-à-vis the role of the financial system in responding to the needs of different categories of agents. Indeed, as shown by the literature of the persistent inequality (e.g. Banarjee and Newman, 1993; Piketty, 1997), taking in account the asymmetric impact of the financial imperfections on wealthy and poor agents changes the pace of the Kuznets (1955) relationship between economic development and income inequality. In this paper we try to analyze the effect of introducing profit-sharing financial contract between banks and entrepreneurs on the evolution of the capital accumulation/income inequality relationship. It is interestingly shown that income inequality disappears when the economy reaches a second stage of development.

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1. Introduction

In recent decades the income inequality in many developed countries has widen. For example, in OECD countries the inequality of incomes has increased since mid-1980s such that the income of the richest 10% of people was nearly nine times that of the poorest 10% in 2005 (OECD, 2008). As mentioned by OECD (2008) this situation could degenerate in social unrest fueled by the confining of the political power by a few wealthy citizens. Therefore, economists should provide urgent and innovative solutions to the unequal growth which constituted also one of the triggering ingredients of the recent Arab spring revolution namely in Tunisia and Egypt.

The analysis of the relationship between economic development and income inequality has been debated by economists for a long time since the seminal work of Kuznets (1955). This study argued that an inverted U-shaped relationship exists between the two variables: income inequality increases in a first stage of development before decreasing. More recent studies (e.g. Persson and Tabellini, 1994; Aghion and Bolton, 1997; Perotti, 1996) confirm the result of Kuznets (1955). However, taking in account the asymmetric impact of the financial imperfections on wealthy and poor agents changes the pace of the relationship between economic development and income inequality as illustrated by the Kuznets curve. Indeed, another branch of the literature called the theory of persistent inequality (e.g. Banarjee and Newman, 1993; Piketty, 1997) shows that if credit markets are imperfect then poor dynasties face limited investment opportunities and the catching up of wealthy dynasties isn't always possible even at advanced stage of development. Consequently, it has been shown that an improvement in financial markets, contracts, and intermediaries reduces income inequality (Demirgüç-Kunt and Levine, 2009). Thus, it is clear that the relationship between economic development and income inequality isn't neutral vis-à-vis the role of the financial system in responding to the needs of different categories of agents. According to Demirgüç-Kunt and Levine (2009, p.2) « The financial system influences who can start a business and who cannot, who can pay for education and who cannot, who can attempt to realize one's economic aspirations and who cannot. Thus, finance can shape the gap between the rich and the poor and the degree to which that gap persists across generations." Clarke et al. (2003) argues that policy makers should know whether "finance" can be used as an instrument to affect income inequality.

Yet, observing the recent widening of income inequality in many countries we might conclude that the influence of policy makers on "finance" through taxation and monetary policy is ineffective. It is therefore legitimate to imagine a new financial mechanism leading to lower inequality while insuring sustainable growth. According to Chapra (1985) economic growth isn't by itself of prime importance if not accompanied by the reduction of inequalities, full employment and broad-based economic well-being. In order to achieve these goals he advocates for the establishment of equitable money and banking system. Criticizing the traditional capitalism he argues that "high or low interest rates are the result of restrictive or liberal monetary policies adopted in the larger national interest. There is no reason why the entrepreneur or the financier should be the only one to suffer or benefit from such polices. Why shouldn't the gains or the losses be equitably distributed between them?" (Chapra, 1985, p217). In order to achieve what he called equitable distribution of gains and loss between the entrepreneur and the financier, he suggest considering a profit-and-loss sharing mechanism which guarantees the convergence of the two parties' interest while interacting with the economic conditions as the interest rate do.

In this paper we try to analyze the effect of introducing profit-sharing financial contract between banks and entrepreneurs on the evolution of the capital accumulation/income inequality relationship. Many studies analyzed the microeconomic and macroeconomics effects of substituting interest-fixed contracts by profit-sharing contracts. Ahmed (2000) develops a model analyzing the incentive to an entrepreneur to under-report his profit in the case he should share it with a bank at an agreed ratio. He proposes an incentive-compatible profit-sharing contract that reduces the moral hazard problem. The suggested incentive mechanism is based on a reward/punishment mechanism involving collateral and random audit. Ul Haque and Mirakhor (1986) develop an IS-LM-like model with profit-sharing contracts and show that the economy behaves as an economy with debt contracts when information is perfect and the environment is certain. However, in presence of uncertainty they show that the level of investment may increase under certain conditions. The reason behind this is that profit-sharing contracts allow greater utilization of capital and higher profitability. Khan (1987) develops a model to analyze the effect of the substitution of interest by PLS on the market of loanable funds. He shows that using profit shares as an instrument of monetary policy would be inefficient. In addition he points out that the PLS contracts finance the more profitable projects. However, it is well-known that the problem with profit-sharing contracts is the excessive cost of monitoring required to enforce them (Khan, 1987). There is a large literature showing that debt dominates profit sharing (equity contracts) in presence of information problems and costly monitoring. For example, in Townsend (1979) and

Gale and Hellwig (1985) debt is optimal because it minimizes monitoring costs. However, these studies show also that the debt contract is not ex post efficient because underinvestment can occur.

Our concern in this paper is about the effect of profit-sharing financial contracts on the evolution of the "income inequality" with capital accumulation process. To this end we will start from a modified version of Aghion and Bolton (1997) model where we integrate two new features. The first feature is costly contract enforcement as a second type of credit market imperfection in addition to the moral hazard problem. The second feature is enabling, contrarily to Aghion and Bolton (1997), wealthy agents to undertake larger projects. It is interestingly shown that income inequality disappears in a second stage of development.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 describes the financial contract. Sections 4 and 5 characterize respectively entrepreneurs and banks. Section 6 is devoted to the analysis of occupational choices and credit market equilibrium. I investigate the evolution of the wealth inequality in section 7 and illustrate the results through a numerical example in section 8. Finally, section 9 concludes.

2. The model

The economy is closed and contains a sequence of one-period-lived overlapping generations. An initial generation of old entrepreneurs coexists with young agents at date $t = 0$. Each generation is composed of a continuum of mass 1 agents indexed by *i*. Each agent has one offspring and works or invests. Agents are risk-neutral and their utility depends only on consumption and bequest. Hence, an agent divides the income he receives between consumption and bequest. The only source of heterogeneity among agents is their inherited wealth w_t^i. Each agent *i* is endowed with one unit of effort (labour) $(l^i = 1)$. He may choose to undertake a project requiring a minimum fixed investment of $\widetilde{w} > 1$ that generates an uncertain revenue $\kappa^i(w^i)$ from an investment $w^i \geq \widetilde{w}$ which is given by

$$\kappa^i(w) = \begin{cases} aw^i & \text{with probability } p^i \\ 0 & \text{with probability } 1 - p^i \end{cases}$$

where $a > 1$ and $p^i = l^i$ denotes the probability of success which is equal to 1 if the agent supplies his entire effort. We assume that there is an effort cost $C(l^i) = \frac{a\widetilde{w}}{2}(l^i)^2$. At the beginning of his life an agent decides the effort to supply and how to invest his inherited wealth w_t^i. At the end of his lifetime, the individual allocates his net final wealth between consumption and bequest. As in Aghion and Bolton (1997) agents are assumed to have Leontieff preferences over consumption and bequest. Therefore, the optimal bequest is a linear function of end of period wealth w_{t+}^i and is given by $b_{t+1}^i = w_{t+1}^i = (1 - \delta)w_{t+}^i$ where $1 - \delta$ is the saving propensity of individuals. We assume that at date t = 0, a proportion π of the young agents has a low inherited wealth $\underline{w}_0 < \widetilde{w}$ (resulting from the initial old generation's bequests) and constitutes the class $i = l$. The remainder proportion $1 - \pi$ which constitutes the class $i = h$ has a higher inherited wealth $\overline{w}_0$ verifying $\underline{w}_0 < \overline{w}_0 < \widetilde{w}$. An agent of class $i = l, h$ born at date $t \geq 0$ with an initial wealth w_t^i such that $w_t^i \geq \widetilde{w}$ could self-finance his project but may have an incentive to ask for a bank loan in order to enlarge it. Even if his project succeeds, an agent may default on the loan if it is more profitable to do so. In this case, the bank seizes a fraction $\lambda \in [\frac{1}{2}, 1]$ of the produced output. The unseized fraction $1 - \lambda$ corresponds to an enforcing repayment cost which could be related to the efficiency of the legal system[1]. For an agent with an inherited wealth $w_t^i \leq \widetilde{w}$ who isn't granted a loan there is no choice but depositing his wealth in a bank.

3. Financial contract

An agent of class $i = l, h$ can self-finance a project if his inherited wealth w_t^i is superior to the minimum fixed investment $\widetilde{w}$. He may also ask for a bank loan di t for an additional investment. We deviate from the debt contract considered in Aghion and Bolton (1997) and Nabi (2009) and we consider the following contingent financial contract (γ_t^i, d_t^i) where γ_t^i denotes the share of the output that will be paid back to the bank. Hence the repayment is

$$R_t^i = \begin{cases} \gamma_t^i a(w_t^i + d_t^i) & \text{with probability } p_t^i \\ 0 & \text{with probability } 1 - p_t^i \end{cases} \tag{1}$$

where the probability of success is given by $p_t^i = l_t^i$.

4. Entrepreneurs

An agent with an inherited wealth $w_t^i < \widetilde{w}$ is obliged to ask for a loan of an amount $d_t^i \geq \widetilde{w} - w_t^i$ in order to undertake a project. If the agent's inherited wealth is sufficient to self-finance a project $w_t^i \geq \widetilde{w}$ he will ask for a loan d_t^i to enlarge the project if he could recuperate higher revenue relatively to a self-financed project: $(1-\gamma_t^i)\, p_t^i a(w_t^i + d_t^i) > p_t^i a w_t^i$ which imposes the following constraint on the loan's amount

$$d_t^i > \underline{d}_t^i = w_t^i \left(\frac{1 - \gamma_t^i}{\gamma_t^i} \right) \tag{2}$$

In the two previous cases, the final revenue of the entrepreneur is increasing with the loan's amount. We will discuss later the maximum amount that he will be grant. Hence, an agent who undertakes a project whether through self-financing or banking funding will choose the effort l^i to supply in order to maximize his expected revenue net of both repayment and effort costs

$$(\wp): \underset{0<l_t^i\leq 1}{Max} U^i = \begin{cases} (1-\gamma_t^i)\, l_t^i a(w_t^i + d_t^i) - C(l_t^i) & \text{if } \begin{cases} w_t^i < \widetilde{w} \\ \text{or } w_t^i \geq \widetilde{w} \text{ and } d_t^i \geq \underline{d}_t^i \end{cases} \\ l_t^i a(w_t^i) - C(l_t^i) & \text{if } w_t^i \geq \widetilde{w} \text{ and } d_t^i < \underline{d}_t^i \end{cases} \tag{3}$$

[1] Krasa et al (2008) has also considered an enforcement problem in the legal system which rests on two key parameters. The first one is the efficiency of enforcement captured by the cost paid to secure rights in court. The second one is the percentage of total assets that a court can seize.

Note that for the case $w_t^i \geq \widetilde{w}$ and $d_t^i < \underline{d}_t^i$ the agent will self-finance his project. The solution to problem $(\wp)$ is presented in the following proposition.

Proposition 1

i) For a given financial contract (γ_t^i, d_t^i), the probability of success of the project undertaken by an agent i is given by

$$p_t^i = \begin{cases} \dfrac{(1-\gamma_t^i)(w_t^i + d_t^i)}{\widetilde{w}} & \text{if} \quad w_t^i < \widetilde{w} \text{ and } d_t^i < \widetilde{d}_t^i \\ 1 & \text{if} \begin{cases} w_t^i < \widetilde{w} & \text{and} \quad d_t^i \geq \widetilde{d}_t^i \\ \text{or} \\ w_t^i \geq \widetilde{w} & \text{and} \quad d_t^i \geq \underline{d}_t^i \end{cases} \end{cases} \tag{4}$$

where $\widetilde{d}_t^i = \dfrac{\widetilde{w}}{1-\gamma_t^i} - w_t^i$

ii) When an entrepreneur self-finance his project $(w_t^i \geq \widetilde{w})$ he devotes the maximum effort: $p_t^i = 1$.

Proof: See the appendix.

It is clear from (4) that the higher the bank's share γ_t^i in the output the lesser the agent's effort devoted to increase the probability of success of his project when his wealth is inferior to the threshold $\widetilde{w}$ and the external financing is inferior to $\widetilde{d}_t^i$. However, when the latter reaches the threshold $\widetilde{d}_t^i$ the probability of success becomes 1. Note that when the agent self-finances his project he devotes the maximum effort (case *ii*).

5. Banks

In order to prevent the borrower's default, the repayment should be at most equal to the default's cost. The default cost is equal to the output the bank is willing to seize in case of success of the project which is $\lambda a(w_t^i + d_t^i)$. Hence, we should impose $R_t^i \leq \lambda a(w_t^i + d_t^i)$ which gives us using equation (1) the following condition: $\gamma_t^i \leq \lambda$. Therefore, the bank's share shouldn't exceed the fraction that could be seized. In this case, and given that the economy comprises at each date *t* a continuum of agents belonging to the two classes (the class of low inheriting wealth and the high inheriting one) and that the random returns on risky projects are independently and identically distributed the proportion of successful projects in class *i* is p_t^i. Hence, the aggregate repayment banks receive from class *i = l, h* are deterministic and respectively given by $\phi^l \pi p_t^l \gamma_t^l a(w_t^l + d_t^l)$ and $\phi^h (1-\pi) p_t^h \gamma_t^h a(w_t^h + d_t^h)$ where ϕ^i denotes the proportion of agents of class i = l, h who are granted a loan. Hence, it is possible to define r_t^i the aggregate deterministic gross return on loans for class *i = l, h* as follows $p_t^i \gamma_t^i a(w_t^i + d_t^i) = r_t^i d_t^i$ or equivalently

$$r_t^i = \gamma_t^i p_t^i a(w_t^i + d_t^i)/d_t^i \quad (5)$$

where p_t^i is given by (4). At this stage, we should make the following assumption regarding the amount of loans banks are willing to provide for an agent of class $i = l, h$.

Assumption 1

Competitive banks are willing to grant loans to agents offering them the higher profit sharing fraction. In case of equality of the profit-sharing ratio, their lending strategy is characterized by the following:

i) If $w_t^l < \widetilde{w}$ and $w_t^h < \widetilde{w}$ then the amount of loan banks are willing to grant to an agent of class i is given by $d_t^i = \widetilde{w} - w_t^i$.

ii) If $w_t^l < \widetilde{w}$ and $w_t^h \geq \widetilde{w}$ then $d_t^h = d_t^l = \widetilde{w} - w_t^l$ if the two types of agents ask for loans. If only agents "h" ask for loans then the amount d_t^h is determined by the credit market equilibrium.

iii) If $w_t^l \geq \widetilde{w}$ and $w_t^h \geq \widetilde{w}$ then $d_t^h = d_t^l = d_t$ where d_t is to be determined by the credit market equilibrium.

Hence, when the two types of agents can't undertake a project without external financing (case *i*), banks provide no more than the missing amount to reach an overall financing of $\widetilde{w}$. Therefore, banks are willing to finance the maximum of projects. In case ii, when only agents of class *l* needs external financing and agents of class *h* could be interested in enlarging their projects than banks are willing to offer the latter the same amount as agents of class *l*. Finally, in case *iii* banks also provide the two categories of agents with the same amount which is determined by the equilibrium of the credit market.

6. Occupational choice and credit market equilibrium

6.1 Occupational choice

An agent $i = l, h$ of generation *t* with an initial wealth w_t^i will prefer to undertake a project rather than deposit his wealth in a bank if it procures him higher expected revenue.

$$U^i > r_t^j w_t^i \quad (6)$$

where U^i is defined by (3) and proposition 1 and $r_t^j w_t$ represents his final wealth which will finance his consumption and bequest. The occupation of agent *i* depends not only on hid choice but also on the choice of agents belonging to the other class $j \neq i$. The following proposition presents the possible occupational choices for agents of the two classes $i = l, h$.

Proposition 2

Under assumption 1 and given a financial contract $(\gamma_t^i, d_t^i)_{i=l,h}$

i) If $w_t^l < \widetilde{w}$ and $w_t^h < \widetilde{w}$ then agents of class $i = l, h$ prefer undertaking projects if $\gamma_t^i < \widehat{\gamma}_t^i$. Otherwise, they prefer deposit their wealth in a bank.

ii) If $w_t^l < \widetilde{w}$ and $w_t^h \geq \widetilde{w}$

ii-1) If $\gamma_t^h \leq \underline{\gamma}_t$ then agents of class h prefer becoming entrepreneurs in the following cases: *ii-1a)* $w_t^l < \overline{w}_t^l$ and $w_t^h > \widetilde{w}$; *ii-1b)* $w_t^l \geq \overline{w}_t^l$, $w_t^h > \overline{w}_t^h$ and *ii-1c)* $w_t^l \geq \overline{w}_t^l$ and $\widetilde{w} \leq w_t^h \leq \overline{w}_t^h$ and $\gamma_t^h < \Omega_t$

Meanwhile, agents of class l prefer becoming depositors rather than entrepreneurs if $\gamma_t^h > \Delta_t$

ii-2) If $\gamma_t^h > \underline{\gamma}_t$ then agents of class h deposit their wealth in the bank if $\left(\gamma_t^l, w_t^l, w_t^h\right) \in F$ otherwise they prefer selffinance their projects.

iii) If $w_t^l \geq \widetilde{w}$ and $w_t^h \geq \widetilde{w}$

iii-1) If $\gamma_t^i < \widetilde{\gamma}_t^i = \frac{d_t}{w_t^i + d_t}$ then agents of class $i = l, h$ prefer becoming entrepreneurs.

iii-2) If $\gamma_t^i \geq \widetilde{\gamma}_t^i$ and $\gamma_t^j \leq \widetilde{\gamma}_t^j$ then agents of class i prefer becoming depositors if $\gamma_t^j \in]\overline{\overline{\gamma}}_t^j, 1]$ otherwise they prefer selffinance a project.

where

$$
\begin{aligned}
\widehat{\gamma}_t^i &= 1 - \sqrt{\frac{2(1-\gamma_t^j)\gamma_t^j w_t^i}{\widetilde{w} - w_t^j}}, \quad \underline{\gamma}_t = \frac{\widetilde{w} - w_t^l}{\widetilde{w} - w_t^l + w_t^h} \\
\Omega_t &= 1 - \frac{\widetilde{w}}{\widetilde{w} + w_t^h - w_t^l}\left(\frac{1}{2} + \frac{(1-\gamma_t^l)\gamma_t^l w_t^h}{\widetilde{w} - w_t^l}\right) < \underline{\gamma}_t, \quad \Delta_t = \frac{\left(\widetilde{w} - w_t^l\right)\widetilde{w}(1-\gamma_t^l)^2}{2w_t^l\left(\widetilde{w} + w_t^h - w_t^l\right)} \\
\overline{w}_t^l &= \widetilde{w}\left(1 - 2(1-\gamma_t^l)\gamma_t^l\right) < \widetilde{w}, \quad \overline{w}_t^h = \widetilde{w} / \left[1 - \frac{w_t^l - \overline{w}_t^l}{\widetilde{w} - w_t^l}\right] \\
\overline{\overline{w}}_t^h &= \frac{\frac{\widetilde{w}}{2}\left(\widetilde{w} - w_t^l\right)}{\frac{3}{4}\widetilde{w} - w_t^l}, \quad A_t = \frac{\left(\widetilde{w} - w_t^l\right)\left(w_t^h - \frac{\widetilde{w}}{2}\right)}{\widetilde{w} w_t^h} \\
\overline{\overline{\gamma}}_t^j &= \frac{d_t\left(w_t^i - \frac{\widetilde{w}}{2}\right)}{w_t^i\left(w_t^j + d_t\right)} < 1 \\
F &= \left]\frac{1}{2} - \sqrt{\frac{1}{4} - A_t}, \frac{1}{2} + \sqrt{\frac{1}{4} - A_t}\right[\times \left]\frac{\widetilde{w}}{2}, \frac{3\widetilde{w}}{4}\right[\times \left]\widetilde{w}, \overline{\overline{w}}_t^h\right[
\end{aligned}
$$

Proof: See the appendix.

The different configurations presented in proposition 2 share a common feature: when the bank's share exceeds a determined threshold then the agents' occupational choice will shift from entrepreneurship to depositing or self-financing.

6.2 Credit Market Equilibrium

Banks finance loans by the deposits they collect from agents having chosen depositing as an occupation and/or from those preferring entrepreneurship but who are credit rationed. It is the competition between agents belonging to each class to obtain a loan and competition between banks on the assets as well as on the liabilities sides which lead to the equilibrium of the credit market. Competition between agents is driven by the bank's share γ_t^i they are willing to accept. Below a determined threshold, the higher this share the greater the expected return banks could obtain from granting loans to agents of class i. The latter will compete in order to obtain a loan until a proportion of them become indifferent between undertaking a project or depositing their wealth in a bank. Therefore, at the equilibrium the banks' optimal share γ_t^* equals the minimum threshold that makes one of the two class of agents indifferent between depositing and entrepreneurship. Due to competition between banks on the asset side this share is fixed even for agents who still prefer strictly entrepreneurship rather than depositing. On the liabilities side, the return on deposits is equal to the aggregate return on loans. The following proposition characterizes the equilibrium of the credit market.

Proposition 3

Under assumption 1 the credit market equilibrium is characterized by

i) If $w_t^l < \widetilde{w}$ and $w_t^h < \widetilde{w}$

i-1.a) If $w_t^l + w_t^h < \widetilde{w}$ and $w_t^h \leq \left(\widetilde{w} - w_t^l\right)/2$ then $\gamma_t^* = 1/2$ and agents of class "h" and "l" prefer strictly becoming entrepreneurs.

i-1.b) If $w_t^l + w_t^h < \widetilde{w}$ and $w_t^h > \left(\widetilde{w} - w_t^l\right)/2$ then $\gamma_t^* = \left(\widetilde{w} - w_t^l\right)/\left(2w_t^h + \widetilde{w} - w_t^l\right)$ and agents of class "h" are indifferent between depositing/entrepreneurship and those of class "l" prefer strictly becoming entrepreneurs.

i-2) If $w_t^l + w_t^h > \widetilde{w}$ then $\gamma_t^* = \left(\widetilde{w} - w_t^h\right)/\left(2w_t^l + \widetilde{w} - w_t^h\right)$ and agents of class "l" are indifferent between depositing/entrepreneurship and those of class "h" prefer strictly becoming entrepreneurs.

i-3) If $w_t^l + w_t^h = \widetilde{w}$ then $\gamma_t^* = 1/3$ and the two types of agents are indifferent between depositing/entrepreneurship.

ii) If $w_t^l < \widetilde{w}$ and $w_t^h \geq \widetilde{w}$ then $\gamma_t^* = Min(\widehat{\gamma}_t^l, \underline{\gamma}_t, 1/2)$ where $\widehat{\gamma}_t^l$ is solution of $\Delta_t(\gamma_t^l) = \gamma_t^l$ and we have

ii-1) If $\gamma_t^* = \widehat{\gamma}_t^l$ than agents of class "l" are indifferent between depositing/entrepreneurship and those of class "h" prefer strictly becoming entrepreneurs.

ii-2) If $\gamma_t^* = Min(\underline{\gamma}_t, 1/2)$ than agents of classes "l" and "h" prefer strictly becoming entrepreneurs.

iii) If $w_t^l \geq \widetilde{w}$ and $w_t^h \geq \widetilde{w}$ then $\gamma_t^* = d_t/(w_t^h + d_t)$ and agents of class of "l" prefer strictly becoming entrepreneurs whereas those of class "h"prefer becoming depositors and we have $d_t = (1-\pi)\, w_t^h/\pi$.

Proof.

Let's begin by prooving case i) where the wealth of both classes of agents are not sufficient to finance their projects. In this case, as shown by proposition 2, they are incited to ask for loans whenever banks are willing to offer them a profit sharing contract $(\gamma_t^i, \widetilde{w} - w_t^i)$ such that $\gamma_t^i < \widehat{\gamma}_t^i$.

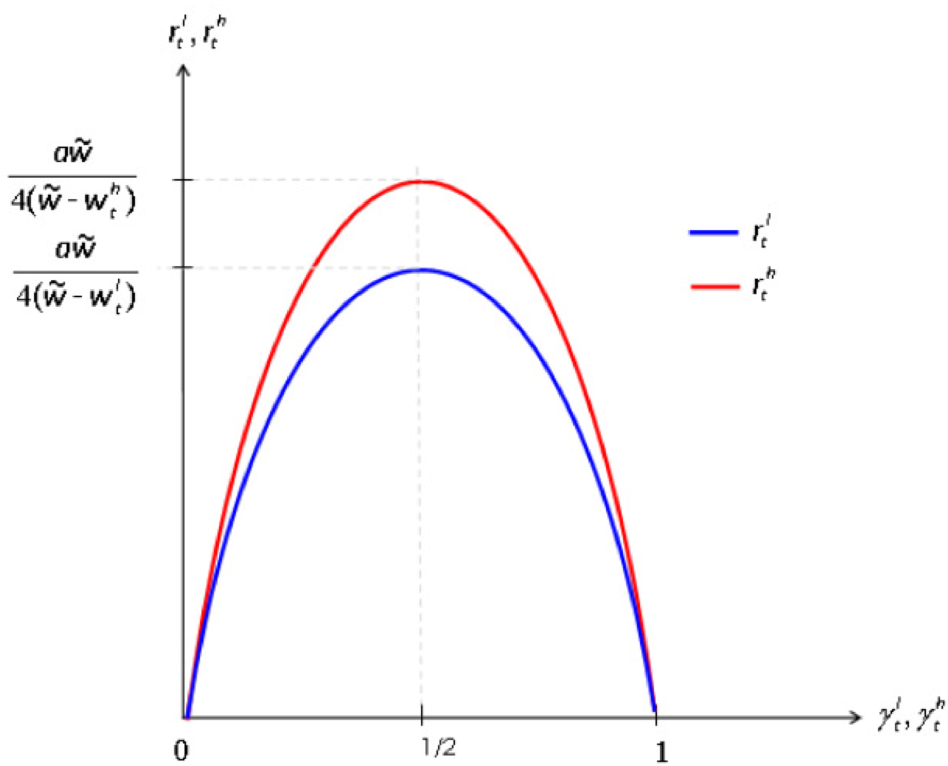

Fig. 1: Expected gross return on loans to the two classes of agents (case *i*)

As shown by figure 1 the expected return r_t^i on loan granted to an agent of class I increases as the share γ_t^i of the bank increases from 0 to 1/2 where it reaches its maximum before declining from 1/2 to 1. This nonlinear relationship is due to the conjunction of two opposite effects. In one hand, the bank profits from higher proportion of the entrepreneur's output in case of success of the project. In the other hand, the entrepreneur reduces his effort and therefore the probability of success as his share in the output decreases. In this case, his expected net revenue given by $U^i = \frac{a\widetilde{w}}{2}\left[1-\gamma_t^i\right]^2$ decreases as the bank's share increases. As far as the expected revenue from entrepreneurship is superior to the final wealth he obtains from deposit, the agent competes to obtain a loan. From proposition 2 (case *i*) it is clear that this competition leads agent *i* to increse γ_t^i until $\gamma_t^i < \widehat{\gamma}_t^i$ or $\gamma_t^i = 1/2$. Using the expression $\widehat{\gamma}_t^i$ from proposition 2 and noting that $r_t^i = \left(1-\gamma_t^i\right)\gamma_t^i a\widetilde{w}/\left(\widetilde{w}-w_t^i\right)$ it is clear that $\widehat{\gamma}_t^i = 1 - \sqrt{2r_t^j w_t^i/a\widetilde{w}}$. Note that there are two forces pushing toward the equilibrium. The first force is the agent's own strategy which consists in offering the bank's a share γ_t^i superior to γ_t^j offered by agent $j \neq i$. The second force is the effect of the agent *j*'s strategy which consists in increasing γ_t^j (eventually to 1/2 which maximizes the banks' gross expected return) to offer the bank higher expected return r_t^j which in the same time reduces the threshold $\widehat{\gamma}_t^i$ of the competing agent since $\partial\widehat{\gamma}_t^i/\partial r_t^j \leq 0$. Figure 3 illustrates the convergence to the equilibrium E which could be characterized by one of the three possible configurations $\gamma_t^l = \widehat{\gamma}_t^l$ or $\gamma_t^h = \widehat{\gamma}_t^h$, $\gamma_t^l = \gamma_t^h = 1/2$.

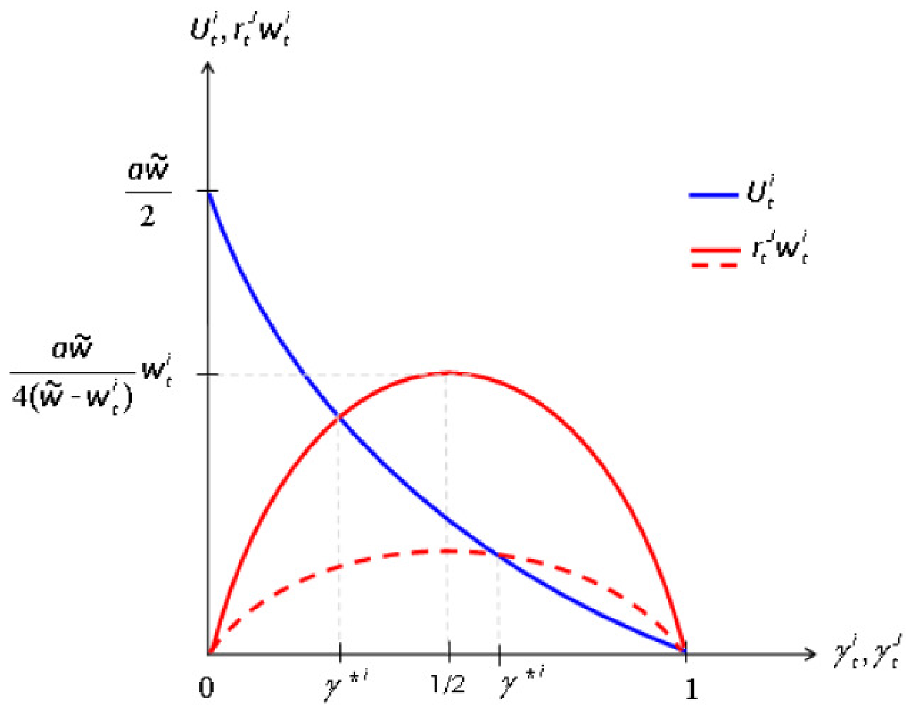

Fig. 2: Utilities procured from entrepreneurship and depositing

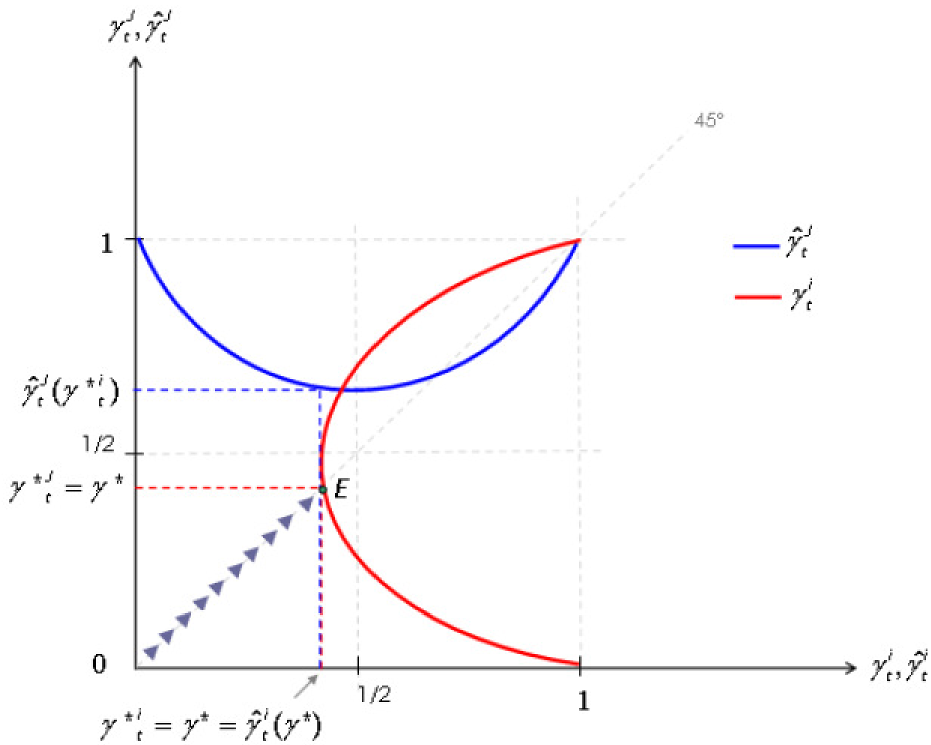

Fig. 3: Interaction between agents and convergence to the equilibrium (case i)

From figure 3 it is clear that the share γ_t^* at the equilibrium equals $Min(\gamma_t^{*l}, \gamma_t^{*h}, 1/2)$ where γ_t^{*i} is the solution to the equation $\widehat{\gamma}_t^i(x) = 1 - \sqrt{2(1-x)xw_t^i / \left(\widetilde{w} - w_t^j\right)} = x$. Therefore, we obtain $\gamma_t^{*i} = \left(\widetilde{w} - w_t^j\right) / \left(2w_t^i + \widetilde{w} - w_t^j\right)$ and it can be easily shown that

$$Min(\gamma_t^{*l}, \gamma_t^{*h}, 1/2) = \begin{cases} 1/2 & \text{if} \quad w_t^h + w_t^l < \widetilde{w} \text{ and } w_t^h < \frac{\widetilde{w} - w_t^l}{2} \\ \gamma_t^{*h} & \text{if} \quad w_t^h + w_t^l < \widetilde{w} \text{ and } \frac{\widetilde{w} - w_t^l}{2} \leq w_t^h < \widetilde{w} \\ 1/3 & \text{if} \quad w_t^h + w_t^l = \widetilde{w} \\ \gamma_t^{*l} & \text{if} \quad w_t^h + w_t^l > \widetilde{w} \end{cases}$$

Note that there are two cases characterized by $\gamma_t^* = \gamma_t^{*l} = \gamma_t^{*h}$ where the two types of agents compete and become indifferent between the two occupations at the equilibrium. Let's now analyze case *ii)* where $w_t^l < \widetilde{w}$ and $w_t^h \geq \widetilde{w}$. The following figure presents the evolution of the three possible configurations of the equilibrium resulting from proposition 2.

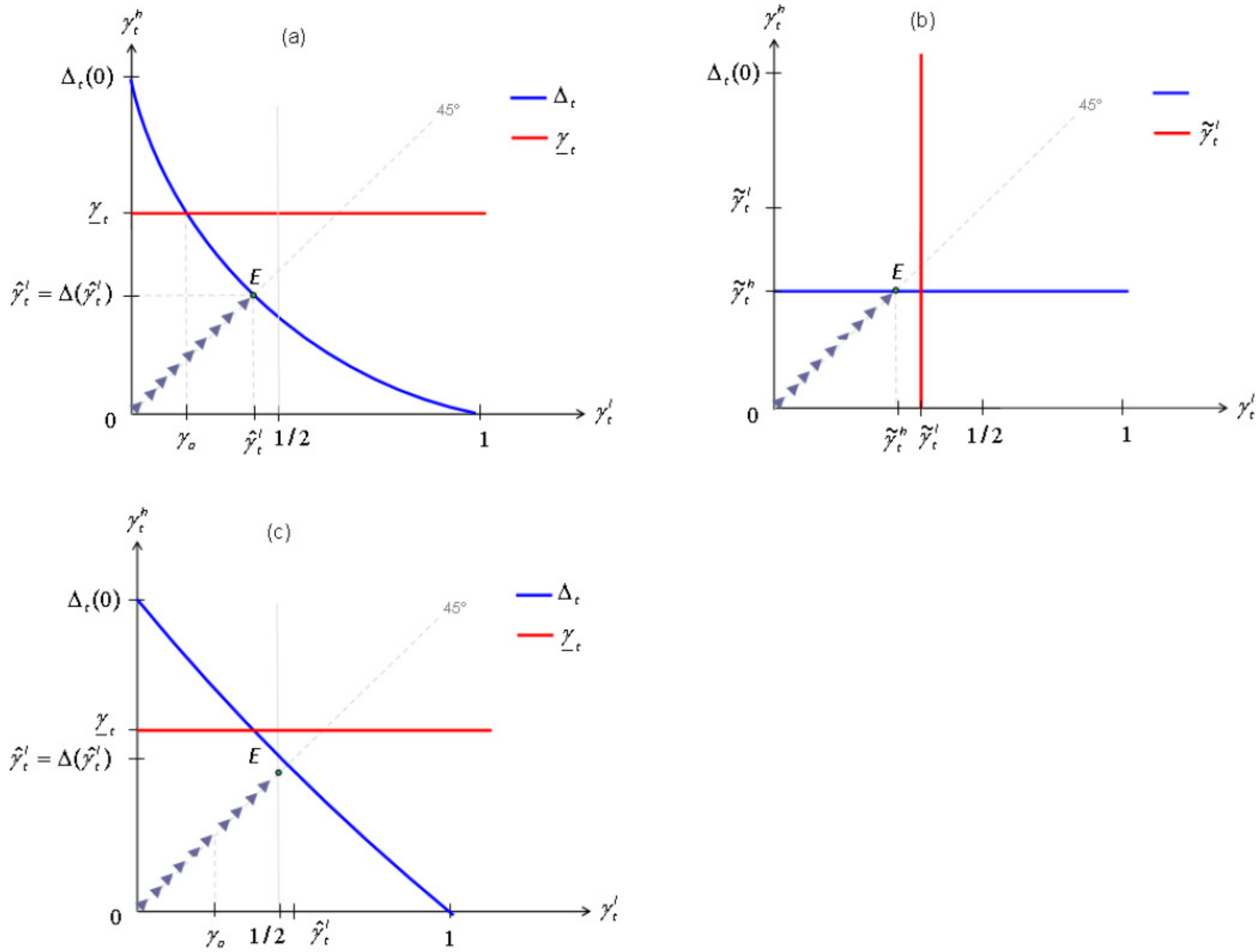

Fig. 4: The possible configurations of the equilibrium in the case $w_t^l < \widetilde{w}$ and $w_t^h \geq \widetilde{w}$

In configuration (a) we have $\gamma_t^* = \Delta_t(\gamma_t^*)$ and $\gamma_t^* < \underline{\gamma}_t$ which implies that agents "*l*" are indifferent between depositing and entrepreneurship and agents "*h*" prefer strictly becoming entrepreneurs. Configurations (b) and (c) verify respectively $\gamma_t^* = \underline{\gamma}_t$ and $\gamma_t^* < \underline{\gamma}_t$ as well as $\gamma_t^* < \Delta_t(\gamma_t^*)$ which imply that agents "*l*" and "*h*" prefer strictly becoming entrepreneurs in configuration (c). However, agents "*h*" are indifferent between the depositing and the entrepreneurship occupations. Although in case *ii-1*) of proposition 2 the threshold Ω_t matters in the occupational choice of agents "*h*", this is not the case for the equilibrium. It is due to the fact that $\Omega_t > \Delta_t$. Indeed, since Ω_t is concave in γ_t^l with a minimum at $\gamma_t^l = 1/2$. It is easy to show that $\Delta_t(0) < \Omega_t(0)$and $\Delta_t(1/2) = \frac{1}{4}\Delta_t(0) < \Omega_t(1/2)$ which enables us to conclude that $\Omega_t > \Delta_t$. Let's note that case *ii-2)* of proposition 2 couldn't take place in the equilibrium. Indeed, from the previous analysis the only interaction between agents that could lead to $\gamma_t^h > \underline{\gamma}_t$ is configuration (b). However, once $\gamma_t^h = \underline{\gamma}_t$ agents "*h*" become indifferent between depositing and asking for a bank loan.

Hence, there is no further force that will push γ_t^h above $\underline{\gamma}_t$. Finally, we have to determine the equilibrium for case *iii)* where $w_t^l \geq \widetilde{w}$and $w_t^h \geq \widetilde{w}$. From proposition 2, it is clear that the interaction between the two types of agents stops when $\gamma_t^* = \gamma_t^l = \gamma_t^h = d_t/(w_t^h + d_t)$ such that agents "*h*" are no more interested in loans to enlarge their projects and agents "*l*" prefer becoming entrepreneurs. Since $\gamma_t^* = \frac{d_t}{w_t^h+d_t} > \overline{\overline{\gamma}}_t^l = \frac{d_t(w_t^l-\widetilde{w}/2)}{w_t^l(w_t^h+d_t)}$ then we conclude from proposition 2 that agents "*h*" become depositors and agents "*l*" become entrepreneurs. Since the total deposits of agents "*h*" is $(1-\pi)\,w_t^h$ and the proportion of agents "*l*" is π then we conclude that the loan's amount is $d_t = (1-\pi)\,w_t^h/\pi$ ■

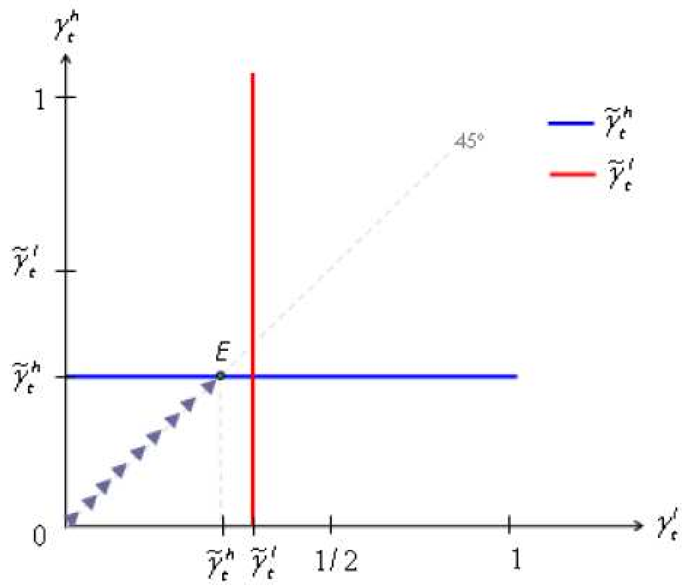

Fig. 5: Interaction between agents and convergence to the equilibrium (case iii)

Proposition 3 indicates the occupational choices of the two types of agents. Agents preferring strictly becoming entrepreneurs could be credit rationed if the total amount of deposits banks collect is less than the total amount of requested loans. The following lemma recapitulates the possible configurations.

Lemma 1

Under assumption 1 the credit market equilibrium is characterized by

i) If $w_t^l < \widetilde{w}$ and $w_t^h < \widetilde{w}$

i-1.a) If $w_t^l + w_t^h < \widetilde{w}$ and $w_t^h \leq \frac{\widetilde{w}-w_t^l}{2}$ then a proportion ϕ_t of agents $i = l, h$ are entrepreneurs and a proportion $1-\phi_t$ of them are depositors.

i-1.b) If $w_t^l + w_t^h < \widetilde{w}$ and $\frac{\widetilde{w}-w_t^l}{2} < w_t^h \leq \max(\frac{\widetilde{w}-w_t^l}{2}, w_t^{\pi})$ then agents "h" are depositors and a proportion $1-\phi_t^l$ of agents "l" are credit rationed and become depositors.

i-2) If $w_t^l + w_t^h < \widetilde{w}$ and $w_t^h > \max(\frac{\widetilde{w}-w_t^l}{2}, w_t^{\pi})$ then agents "l" are entrepreneurs and a proportion ϕ_t^h of agents "h" are entrepreneurs.

i-3) If $w_t^l + w_t^h > \widetilde{w}$ and $w_t^l \leq \widetilde{w}_t^\pi$ then agents "l" are depositors and a proportion $1 - \widetilde{\phi}_t^h$ of agents "h" are credit rationed and become depositors.

i-4) If $w_t^l + w_t^h > \widetilde{w}$ and $w_t^l > \widetilde{w}_t^\pi$ then agents "h" are entrepreneurs and a proportion $\widetilde{\phi}_t^l$ of agents "l" are entrepreneurs.

i-5) If $w_t^l + w_t^h = \widetilde{w}$ then a proportion ϕ_t of agents $i = l, h$ are entrepreneurs and a proportion $1 - \phi_t$ of them are depositors.

ii) If $w_t^l < \widetilde{w}$ and $w_t^h \geq \widetilde{w}$

ii-1) If $\gamma_t^* = \widehat{\gamma}_t^l$ then agents "l" are depositors and those of class "h" are entrepreneurs.

ii-2) If $\gamma_t^* = Min(\underline{\gamma}_t, 1/2)$ then a proportion $\overline{\overline{\phi}}_t$ from agents "h" and "l" are entrepreneurs.

iii) If $w_t^l \geq \widetilde{w}$ and $w_t^h \geq \widetilde{w}$ then agents "h" are depositors and agents "l" are entrepreneurs.

where γ_t^* and $\overline{\overline{\gamma}}_t^l$ are defined in proposition 3 and $w_t^\pi = \frac{\pi}{1-\pi}\left(\widetilde{w} - w_t^l\right)$, $\phi_t^l = \frac{1-\pi}{\pi}\left(\frac{w_t^\pi - w_t^h}{\widetilde{w}}\right)$, $\phi_t^h = \frac{w_t^h - w_t^\pi}{\widetilde{w}}$, $\widetilde{w}_t^\pi = \frac{1-\pi}{\pi}\left(\widetilde{w} - w_t^h\right)$, $\widetilde{\phi}_t^h = 1 - \frac{\pi}{1-\pi}\left(\frac{\widetilde{w}_t^\pi - w_t^l}{\widetilde{w}}\right)$, $\widetilde{\phi}_t^l = \frac{w_t^l - \widetilde{w}_t^\pi}{\widetilde{w}}$, $\phi_t = \frac{\pi w_t^l + (1-\pi) w_t^h}{\widetilde{w}}$, $\overline{\overline{\phi}}_t = 1/\left(1 + \frac{\widetilde{w} - w_t^l}{\pi w_t^l + (1-\pi) w_t^h}\right)$

Proof. See the appendix.

7. Wealth dynamic and income inequality

The dynamic of wealth accumulation is given by

$$w_{t+1}^i = (1 - \delta) w_{t+}^i$$

where $\delta \in]0, 1[$ is the consumption fraction and w_{t+}^i the wealth of an agent $i = l, h$ at the end of is life. If the agent is a depositor we have $w_{t+}^i = r_t^i w_t^i$. If he is an entrepreneur who obtained a loan then his final wealth is given by

$$w_{t+}^i = \begin{cases} (1 - \gamma_t^*)\, a(w_t^i + d_t^i) & \text{with probability} \quad p_t^i = l_t^i \\ 0 & \text{with probability} \quad 1 - p_t^i \end{cases}$$

To analyze the evolution of the income inequality from date *t* to *t + 1* between the two classes of agents we have to compare $\omega_{t+1}^{h}/\omega_{t+1}^{l}$ to $\omega_{t}^{h}/\omega_{t}^{l}$ where ω_{t+1}^{i} denotes the per capita wealth income of class *i = l, h*.

Proposition 4

The wealth inequality between the two classes of agents alternatively widens and shrinks in a first stage of development: $(w_t^l, w_t^h) \in [w_0^l, \widetilde{w}[\times [w_0^h, +\infty[$. However, it decreases in the region $[\widetilde{w}, +\infty[\times [\widetilde{w}, +\infty[$.

Proof. See the appendix.

Therefore, it is clear that the profit sharing contract changes the dynamic of wealth dynamic toward income equality. This result is totally different from that obtained in Nabi (2009) who considered the same theoretical framework except the financial contract which is the classic debt one. The intuition of this result is the following: when the wealth of the rich class exceeds the threshold $\widetilde{w}$ they prefer deposit it in the bank and profit from the higher expected return on deposits rather than competing with the less wealthy agents who are willing to offer banks higher profit-sharing ratio. Consequently, the entrepreneurship enables the latter to catch-up the initially wealth class. The following figure recapitulates the different configurations presented in Lemma 1 while précising for each corresponding region whether the wealth inequality decreases (-), increases (+) or remains constant (0).

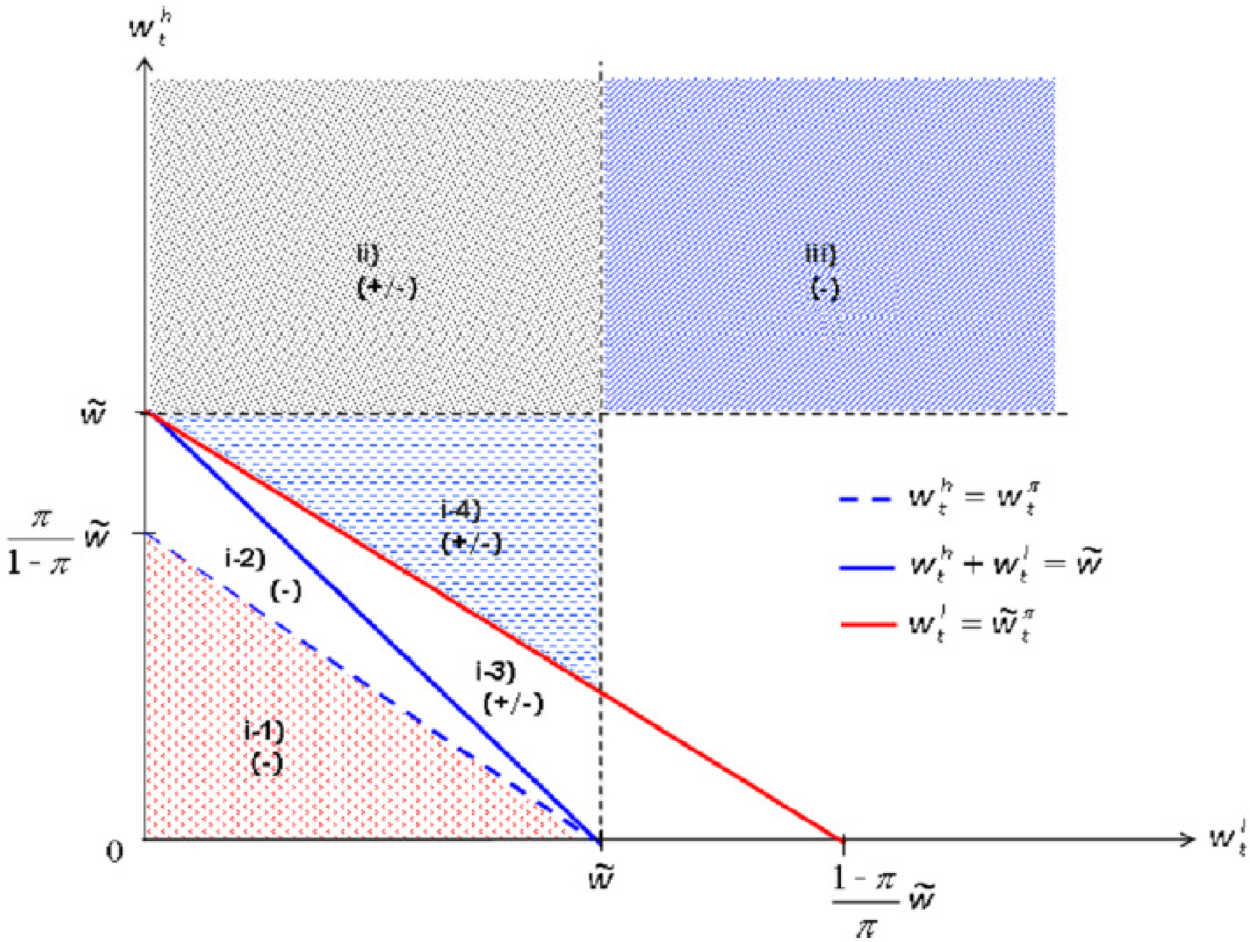

Fig. 6: Evolution of the wealth inequality across the development regions

8. Numerical example

In this section we simulate numerically the evolution of the wealth inequality (measured by the GINI index) for different initial conditions. The expression of the GINI index at date t could be derived from the figure 7 and is given by $G_t = A/(A + \sum_{i=1}^{3} B_i)$ which is equivalent to the following expression

$$G_t = \pi - \frac{1}{1 + \frac{1-\pi}{\pi}\frac{w_t^h}{w_t^l}}$$

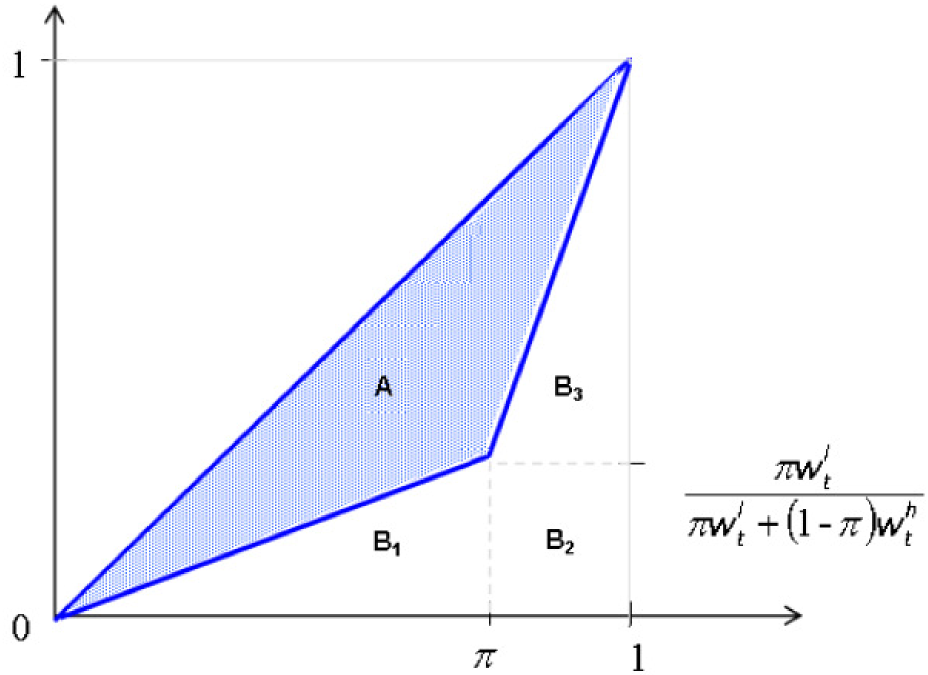

Fig. 7: Gini Index Construction

It is easy to verify that perfect equality of wealth distribution $G_t = 0$ occurs when $w_t^h = w_t^h$. For the numerical illustration of the results, I consider the following parameters' values: $\widetilde{w} = 2; \pi = 0.8; a = 1.9$ and $w_0^l = 0.25 w_0^h = 0.025\widetilde{w}$. This means that initially agents "*h*" and "*l*" are respectively endowed with 10% and 2.5% of the investment project's minimal size Figures 7 shows the dynamic of the GINI index and that of the banks' share. It illustrates clearly the proposition 4 showing that the wealth inequality increases or decreases in a first stage and then disappears *(Gt = 0)*. Besides, the banks' share is initially equal to the maximum level 50% and stabilizes at 20%.

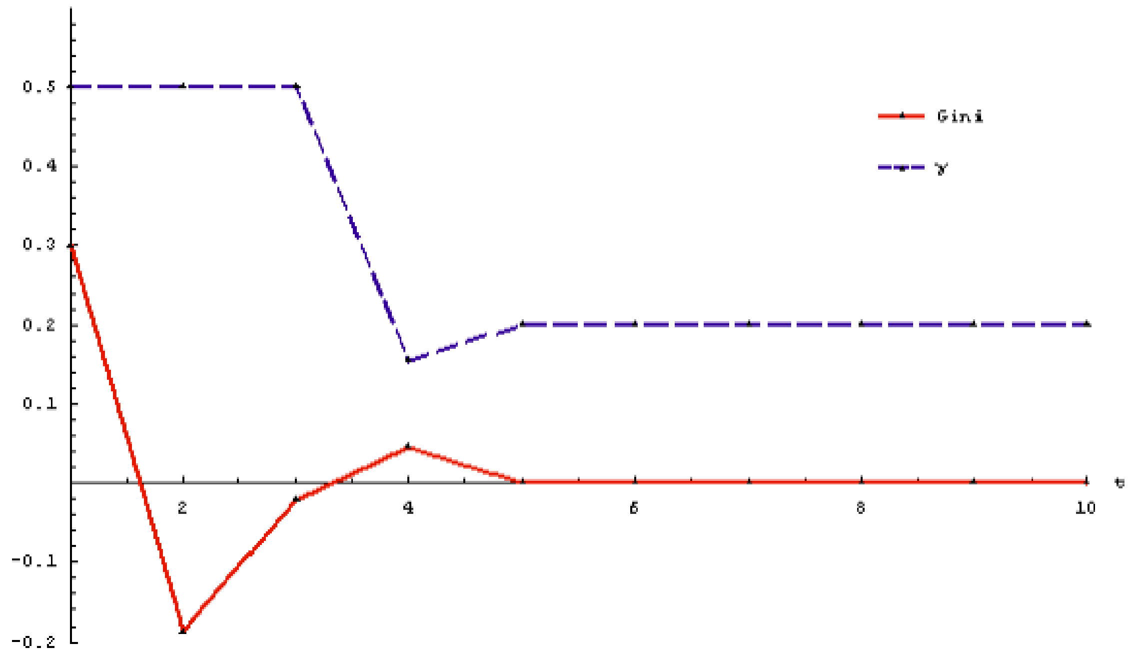

Fig. 7: Evolution of the GINI index and of the banks' share

9. Conclusion

This paper belongs to the literature analyzing how "finance" affects the relationship between economic development and income inequality. Particularly, I analyzed the effect of introducing profit-sharing financial contract between banks and entrepreneurs on the relationship between capital accumulation and income inequality. To this end I departed from the theoretical model of Nabi (2009) and substituted the debt contract with a profit-sharing contract. The intuition behind this was the possible superiority of the latter from an "income inequality" perspective. Indeed, the optimal financial contract literature (e.g. Townsend,1979 and Gale and Hellwig, 1985) recognized the possible ex-post inefficiency of the debt contract (in term of underinvestment) while showing its optimality in term of reducing monitoring costs. Interestingly, I show that substituting the debt contract by a profit-sharing contract modifies the relationship between capital accumulation and income inequality. Indeed, contrarily to Nabi (2009) I showed that income inequality disappears in a second stage of development.

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Annex

Proof of Proposition 1

Using the general expression of U^i from equation (3) we can distinguish the two following cases.

Case 1:

Let's consider $w_t^i < \widetilde{w}$ or $w_t^i \geq \widetilde{w}$ and $d_t^i \geq \underline{d}_t^i$ then the agent's expected net revenue is given by

$$U^i = \left(1 - \gamma_t^i\right) a(w_t^i + d_t^i) l_t^i - \frac{a\widetilde{w}}{2}(l_t^i)^2$$

The solution l_t^i which maximizes U^i is simply derived by setting $(\partial U^i / \partial l_t^i) = 0$ since U^i is a concave function in l_t^i. Hence,we obtain $l_t^i = p_t^i = (1 - \gamma_t^i)(w_t^i + d_t^i)/\widetilde{w}$.

Case 2:

Let's consider $w_t^i \geq \widetilde{w}$ and $d_t^i < \underline{d}_t^i$ then the agent's expected net revenue is given by

$$U^i = l_t^i a(w_t^i) - \frac{a\widetilde{w}}{2}(l_t^i)^2$$

Therfore, the solution l_t^i which maximizes U^i is given by $l_t^i = p_t^i = 1$.

Proof of Proposition 2

The agent expected net revenue could be deduced from equation (3) and proposition 1

$$U^i = \begin{cases} \dfrac{a}{2\widetilde{w}}\left[(1-\gamma_t^i)(w_t^i + d_t^i)\right]^2 & \text{if} \quad w_t^i < \widetilde{w} \text{ and } d_t^i < \widetilde{d}_t^i \\ (1-\gamma_t^i)(w_t^i + d_t^i)a - \dfrac{a\widetilde{w}}{2} & \text{if} \quad \begin{cases} w_t^i < \widetilde{w} & \text{and} \quad d_t^i \geq \widetilde{d}_t^i \\ \text{or} & \\ w_t^i \geq \widetilde{w} & \text{and} \quad d_t^i \geq \underline{d}_t^i \end{cases} \\ a w_t^i - \dfrac{a\widetilde{w}}{2} & \text{if} \quad w_t^i \geq \widetilde{w} \text{ and } d_t^i < \underline{d}_t^i \end{cases} \tag{7}$$

In order to determine the occupations of agents at the equilibrium we should distinguish the three cases presented in assumption 1.

***Case 1*: If $w_t^l < \widetilde{w}$ and $w_t^h < \widetilde{w}$**

Recalling assumption 1, in this case the amounts of loans are given by $d_t^l = \widetilde{w} - w_t^l$ and $d_t^h = \widetilde{w} - w_t^h$ and the condition $d_t^i < \widetilde{d}_t^i$ (where $\widetilde{d}_t^i$ is defined in proposition 1 and equal $\widetilde{w}/(1-\gamma_t^i) - w_t^i$) is clearly verified. Therefore, the agent $i = l, h$ expected net revenue is given by

$$U^i = \frac{a\widetilde{w}}{2}\left[1-\gamma_t^i\right]^2 \tag{8}$$

Besides, from (5) and proposition 1 the gross return on loans is given by

$$r_t^j = \frac{p_t^j \gamma_t^j a\widetilde{w}}{\widetilde{w} - w_t^j} = \frac{(1-\gamma_t^j)\gamma_t^j}{\widetilde{w} - w_t^j} a\widetilde{w}$$

Hence, the condition $U^i > r_t^j w_t$ becomes

$$\gamma_t^i < \widehat{\gamma}_t^i = 1 - \sqrt{\frac{2(1-\gamma_t^j)\gamma_t^j w_t^i}{\widetilde{w} - w_t^j}} \tag{9}$$

Hence, if the bank's share in the output is inferior to $\widehat{\gamma}_t$ then undertaking a project is preferable to becoming a depositor.

***Case 2:* If $w_t^l < \widetilde{w}$ and $w_t^h \geq \widetilde{w}$**

In this case and according to assumption 1, banks are willing to grant agents from each category the same amount of loan $d_t^h = d_t^l = \widetilde{w} - w_t^l$. According to proposition 1, the success probability of potential projects that could be undetaken by agents of class l is $p_t^l = (1-\gamma_t^l)$ since $w_t^l < \widetilde{w}$ and $d_t^l < \widetilde{d}_t^l$. Whereas, potential projects that could be undetaken by agents of class h will be successful with certainty ($p_t^h = 1$) but would be partially financed by banks loans only if $d_t^h = \widetilde{w} - w_t^l \geq \underline{d}_t^h = w_t^h\left(\frac{\gamma_t^h}{1-\gamma_t^h}\right)$. Therefore, we have the following constraint on the financial contract $(\gamma_t^h, d_t^h = \widetilde{w} - w_t^l)$ that could be provided to agents of class h

$$\gamma_t^h \leq \underline{\gamma}_t = \frac{\widetilde{w} - w_t^l}{\widetilde{w} - w_t^l + w_t^h} \tag{10}$$

Case 2.1: If $\gamma_t^h \leq \underline{\gamma}_t$. From (7) we obtain the expected net revenue that each type of agent derives from undertaking a project

$$U^l = \frac{a\widetilde{w}}{2}\left[(1-\gamma l)\right]^2 \tag{11}$$

$$U^h = (1-\gamma_t^h)(w_t^h + d_t^h)a - \frac{a\widetilde{w}}{2} \tag{12}$$

i) Agents of class h prefer becoming entrepreneurs rather than depositors if

$$U^h > r_t^l w_t^h \tag{13}$$

where r_t represents the gross return on loans granted to the depositors which should be in this case the class l agents. From (5) and proposition 1 we obtain

$$r_t^l = \frac{p_t^l \gamma_t^l a \widetilde{w}}{\widetilde{w} - w_t^l} = \frac{(1-\gamma_t^l)\gamma_t^l}{\widetilde{w} - w_t^l} a \widetilde{w} \tag{14}$$

Using (12) and (14) it is simple to show that condition (13) becomes

$$\gamma_t^h < \Omega_t = 1 - \frac{\widetilde{w}}{\widetilde{w} + w_t^h - w_t^l}\left(\frac{1}{2} + \frac{(1-\gamma_t^l)\gamma_t^l w_t^h}{\widetilde{w} - w_t^l}\right)$$

Hence, agents of class h prefer becoming entrepreneurs when $\gamma_t^h < Min(\Omega_t, \underline{\gamma}_t)$. The comparaison between Ω_t^h and $\underline{\gamma}_t$ enables us to find the following

$$Min(\Omega_t, \underline{\gamma}_t) = \begin{cases} \underline{\gamma}_t & \text{if } \begin{cases} w_t^l < \overline{w}_t^l \quad \text{and} \quad w_t^h > \widetilde{w} \\ \text{or} \\ w_t^l \geq \overline{w}_t^l \text{ and } w_t^h > \overline{w}_t^h \end{cases} \\ \Omega_t & \text{if } \; w_t^l \geq \overline{w}_t^l \text{ and } \widetilde{w} \leq w_t^h \leq \overline{w}_t^h \end{cases}$$

where $\overline{w}_t^l = \widetilde{w}\left(1 - 2(1-\gamma_t^l)\gamma_t^l\right) < \widetilde{w}$ and $\overline{w}_t^h = \dfrac{\widetilde{w}}{1 - \dfrac{w_t^l - \overline{w}_t^l}{\widetilde{w} - w_t^l}}$

ii) Equivalently, agents of class l prefer becoming depositors rather than entrepreneurs if

$$U^l < r_t^h w_t^l \tag{15}$$

where r_t represents the gross return on loans granted to the depositors which should be in this case the class h agents. From (5) and proposition 1 we obtain

$$r_t^h = \frac{p_t^h \gamma_t^h a(w_t^h + d_t^h)}{d_t^h} = \frac{\gamma_t^h a(w_t^h + d_t^h)}{\widetilde{w} - w_t^l} \tag{16}$$

Using (11) and (16) it is simple to show that condition (15) becomes

$$\gamma_t^h > \Delta_t = \frac{\left(\widetilde{w} - w_t^l\right)\widetilde{w}(1-\gamma_t^l)^2}{2w_t^l\left(\widetilde{w} + w_t^h - w_t^l\right)}$$

Case 2.2: If $\gamma_t^h > \underline{\gamma}_t$. In this case, agents of class h prefer selfinance their project. However, they may also choose to deposit their wealth in a bank. This is the case if the follwing condition is satisfied

$$U^h = w_t^h a - \frac{a\widetilde{w}}{2} < \left(\frac{(1-\gamma_t^l)\gamma_t^l}{\widetilde{w} - w_t^l} a\widetilde{w} \right) w_t^h$$

which is equivalent to

$$(1-\gamma_t^l)\gamma_t^l > A_t = \frac{\left(\widetilde{w} - w_t^l\right)\left(w_t^h - \frac{\widetilde{w}}{2}\right)}{\widetilde{w} w_t^h}$$

It is easy to show that this condition is verified if and only if $\left(\gamma_t^l, w_t^l, w_t^h\right) \in \left]\frac{1}{2} - \sqrt{\frac{1}{4} - A_t}, \frac{1}{2} + \sqrt{\frac{1}{4} - A_t}\right[\times \left]\frac{\widetilde{w}}{2}, \frac{3\widetilde{w}}{4}\right[\times \left]\widetilde{w}, \frac{\frac{\widetilde{w}}{2}\left(\widetilde{w} - w_t^l\right)}{\frac{3}{4}\widetilde{w} - w_t^l}\right[$.

***Case 3:* If $w_t^l \geq \widetilde{w}$ and $w_t^h \geq \widetilde{w}$**

In this case and according to assumption 1, banks are willing to grant agents from each category the same amount of loan $d_t^h = d_t^l = d_t$ where d_t is to be determined by the credit market equilibrium. According to proposition 1, the success probability of potential projects that could be undetaken by agents of class $i = l, h$ is $p_t^i = 1$ since $w_t^i \geq \widetilde{w}$. However, agents of class i will self-finance their projects if banks are not willing to offer them loans superior to $\underline{d}_t^i = w_t^i \left(\frac{\gamma_t^i}{1-\gamma_t^i}\right)$. Therefore, we have the following constraint on the financial contract (γ_t^i, d_t) that could be provided to agents of class h

$$\gamma_t^i \leq \widetilde{\gamma}_t^i = \frac{d_t}{w_t^i + d_t} \tag{17}$$

Case 3.1: $\gamma_t^i \leq \widetilde{\gamma}_t^i$. From (7) we obtain the expected net revenue that each type of agent derives from undertaking a project

$$U^l = (1-\gamma_t^l)(w_t^l + d_t)a - \frac{a\widetilde{w}}{2} \tag{18}$$

$$U^h = (1-\gamma_t^h)(w_t^h + d_t)a - \frac{a\widetilde{w}}{2} \tag{19}$$

Agents of class $i = l, h$ prefer becoming entrepreneurs rather than depositors if

$$U^i > r_t^j w_t^i \tag{20}$$

where r_t^j represents the gross return on loans granted to the depositors which should be in this case the agents of class $j \neq i$. From (5) and proposition 1 we obtain

$$r_t^j = \frac{p_t^j \gamma_t^j a(w_t^j + d_t)}{d_t} = \frac{\gamma_t^j a(w_t^j + d_t)}{d_t} \tag{21}$$

Using (12) and (14) it is simple to show that condition (13) becomes

$$\gamma_t^i < \Phi_t^i = 1 - \frac{\frac{\widetilde{w} d_t}{2} + \gamma_t^j (w_t^j + d_t) w_t^i}{\widetilde{w} + w_t^h - w_t^l}$$

Hence, agents of class i prefer becoming entrepreneurs when $\gamma_t^i < Min(\Phi_t^i, \widetilde{\gamma}_t^i)$. The comparaison between Φ_t^i and $\widetilde{\gamma}_t^i$ enables us to find straightforward that $Min(\Phi_t^i, \widetilde{\gamma}_t^i) = \widetilde{\gamma}_t^i$

Case 3.2: $\gamma_t^i > \widetilde{\gamma}_t^i$ **and** $\gamma_t^j \leq \widetilde{\gamma}_t^j$. In this case, agents of class i prefer selfinance their project. However, they may also choose to deposit their wealth in a bank. This is the case if the follwing condition is satisfied

$$U^i = w_t^i a - \frac{a\widetilde{w}}{2} < \left(\frac{\gamma_t^j a(w_t^j + d_t)}{d_t} \right) w_t^i$$

Proof of Lemma 1

The equality between total deposits and total loans could be written in general as following

$$\left(1 - \phi_t^l\right) \pi w_t^l + \left(1 - \phi_t^h\right) (1 - \pi) w_t^h = \phi_t^l \pi d_t^l + \phi_t^h (1 - \pi) d_t^h \tag{22}$$

where ϕ_t^i denotes the proportion of agents that are granted a loan. Let's now apply (22) for the different configurations specified by proposition 3.

Case *i-1.a)*

Since agents of class "h" and "l" prefer strictly becoming entrepreneurs. Hence, we could set in (22) $\phi_t^h = \phi_t^l = \phi_t$, $d_t^l = \widetilde{w} - w_t^l$ and $d_t^h = \widetilde{w} - w_t^h$ which gives us

$$\begin{aligned} (1 - \phi_t) \pi w_t^l + (1 - \phi_t)(1 - \pi) w_t^h &= \phi_t \left(\pi \left(\widetilde{w} - w_t^l\right) + (1 - \pi)\left(\widetilde{w} - w_t^h\right)\right) \\ \phi_t &= \frac{\pi w_t^l + (1 - \pi) w_t^h}{\widetilde{w}} \end{aligned}$$

where $w_t^{\pi} = \frac{\pi}{1-\pi}\left(\widetilde{w} - w_t^l\right)$. Since $\phi_t^l \leq 1$ then this configuration occurs if $\left(\pi w_t^l + (1-\pi) w_t^h\right)/\pi\widetilde{w} \leq$ 1 or equivalently $w_t^h \leq w_t^{\pi}$.

Case *i-1.b)*

Since agents of class "h" are indifferent between depositing/entrepreneurship and those of class "l" prefer strictly becoming entrepreneurs. Hence, we could set in (22) $\phi_t^h = 0$ and $\phi_t^l > 0$ with $d_t^l = \widetilde{w} - w_t^l$ which gives us

$$\begin{aligned}
\left(1 - \phi_t^l\right)\pi w_t^l + (1-\pi) w_t^h &= \phi_t^l \pi \left(\widetilde{w} - w_t^l\right) \\
\phi_t^l &= \left(\pi w_t^l + (1-\pi) w_t^h\right)/\pi\widetilde{w} \\
&= \frac{1-\pi}{\pi}\left(w_t^{\pi} - w_t^h\right)/\widetilde{w}
\end{aligned}$$

where $w_t^{\pi} = \frac{\pi}{1-\pi}\left(\widetilde{w} - w_t^l\right)$. Since $\phi_t^l \leq 1$ then this configuration occurs if $\left(\pi w_t^l + (1-\pi) w_t^h\right)/\pi\widetilde{w} \leq$ 1 or equivalently $w_t^h \leq w_t^{\pi}$. But, we are in the case $w_t^h \geq \frac{\widetilde{w} - w_t^l}{2}$. Hence, this case corresponds to the region $\left[\frac{\widetilde{w} - w_t^l}{2}, \max(\frac{\widetilde{w} - w_t^l}{2}, w_t^{\pi})\right]$

Case *i-2)*

When $w_t^h > \max(\frac{\widetilde{w} - w_t^l}{2}, w_t^{\pi}) \geq w_t^{\pi}$ the deposits that could be collected from agents "h" exceeds the maximum amounts of loans that could be granted to agents "l". Since agents "h" are indifferent between depositing and entrepreneurship, a proportion ϕ_t^h become entrepreneurs. Equation (22) becomes after setting $\phi_t^l = 1$, $d_t^l = \widetilde{w} - w_t^l$ and $d_t^h = \widetilde{w} - w_t^h$

$$\begin{aligned}
\left(1 - \phi_t^h\right)(1-\pi) w_t^h &= \pi\left(\widetilde{w} - w_t^l\right) + \phi_t^h (1-\pi)\left(\widetilde{w} - w_t^h\right) \\
\phi_t^h &= \left(w_t^h - w_t^{\pi}\right)/\widetilde{w}
\end{aligned}$$

Case *i-3), i-4) and i-5)* the same reasoning applies.

Case *ii-1)*

Since agents "l" are indifferent between depositing/entrepreneurship and those of class "h" prefer strictly becoming entrepreneurs. Then, agents "l" become depositors and agents "h" become entrepreneurs profiting from loans to enlarge their projects. Equation (22) becomes

$$\begin{aligned}
\pi w_t^l &= (1-\pi) d_t^h \\
d_t^h &= \frac{\pi w_t^l}{1-\pi}
\end{aligned}$$

Case *ii-2)*

If $\gamma_t^* = Min(\underline{\gamma}_t, 1/2)$ than agents of classes "l" and "h" prefer strictly becoming entrepreneurs. In this case, Banks ration a proportion $\overline{\overline{\phi}}_t$ of the two categories. According to assumption 1 we have $d_t^l = d_t^h = d_t = \widetilde{w} - w_t^l$. Thus, equation (22) becomes

$$\begin{aligned}\left(1-\overline{\overline{\phi}}_t\right)\left(\pi w_t^l + (1-\pi)\, w_t^h\right) &= \overline{\overline{\phi}}_t\left(\pi d_t + (1-\pi)\, d_t\right) = \overline{\overline{\phi}}\left(\widetilde{w} - w_t^l\right)\\ \overline{\overline{\phi}}_t &= 1/\left(1 + \frac{\widetilde{w}-w_t^l}{\pi w_t^l + (1-\pi) w_t^h}\right)\end{aligned}$$

Case *iii)*

The proof was showed in proposition 3.

Proof of Proposition 4

Let's denote W_t^i the total wealth income of class $i = l, h$ then we have

$$W_t^h = (1-\pi)\, w_t^h \text{ and} W_{t+1}^h = \pi_{h_1} w_{t+1}^{h1} + \pi_{h_2} w_{t+1}^{h2}$$

$$W_t^l = \pi w_t^l \text{ and} W_{t+1}^l = \pi_{l_1} w_{t+1}^{l_1} + \pi_{l_2} w_{t+1}^{l_2}$$

where π_{i_1} and π_{i_2} represent respectively the proportion of class i agents becoming entrepreneurs and those becoming depositors ($\pi_{i_1} + \pi_{i_2} = \pi_i$ where $\pi_l = \pi$ and $\pi_h = 1-\pi$). Therefore, the per capita wealth income could be written as following

$$\omega_t^h = w_t^h \;\text{ and }\; \omega_{t+1}^h = \frac{W_{t+1}^h}{1-\pi}$$

$$\omega_t^l = w_t^l \;\text{ and }\; \omega_{t+1}^l = \frac{W_{t+1}^l}{\pi}$$

The following table presents for each class of agents the proportion of successful projects as well as the proportion of depositors.

	π_{h_1} ("h" Succ. Ent.)	π_{h_2} ("h" Dep.)	π_{l_1} ("l" Succ. Ent.)	π_{l_2} ("l" Dep.)
i-1.a)	$(1-\pi)\,\phi_t$	$(1-\pi)\,(1-\phi_t)$	$\pi\phi_t$	$\pi\,(1-\phi_t)$
i-1.b)	0	$1-\pi$	$\pi\phi_t^l$	$\pi\left(1-\phi_t^l\right)$
i-2)	$(1-\pi)\,\phi_t^h$	$(1-\pi)\left(1-\phi_t^h\right)$	π	0
i-3)	$(1-\pi)\,\widetilde{\phi}_t^h$	$(1-\pi)\,(1-\widetilde{\phi}_t^h)$	0	π
i-4)	$1-\pi$	0	$\pi\widetilde{\phi}_t^l$	$\pi(1-\widetilde{\phi}_t^l)$
i-5)	$(1-\pi)\,\phi_t$	$(1-\pi)\,(1-\phi_t)$	$\pi\phi_t$	$\pi\,(1-\phi_t)$
ii-1)	$1-\pi$	0	0	π
ii-2)	$(1-\pi)\,\overline{\overline{\phi}}_t$	$(1-\pi)\,(1-\overline{\overline{\phi}}_t)$	$\pi\overline{\overline{\phi}}_t$	$\pi(1-\overline{\overline{\phi}}_t)$
iii-1)	0	$1-\pi$	π	0
iii-2)	$1-\pi$	0	π	0

Therefore, we can deduce the per capita wealth for each class of agents.

	ω_{t+1}^h	ω_{t+1}^l
i-1.a)	$\phi_t\,(1/2)\,a\widetilde{w}+(1-\phi_t)\,r_t w_t^h$	$\phi_t\,(1/2)\,a\widetilde{w}+(1/2)\,r_t w_t^l$
i-1.b)	$r_t^l w_t^h$	$\phi_t^l\,(1-\gamma_t^*)\,a\widetilde{w}+\left(1-\phi_t^l\right)r_t^l w_t^l$
i-2)	$\phi_t^h\,(1-\gamma_t^*)\,a\widetilde{w}+\left(1-\phi_t^h\right)r_t^l w_t^h$	$(1-\gamma_t^*)\,a\widetilde{w}$
i-3)	$\widetilde{\phi}_t^h\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\widetilde{\phi}_t^h)r_t^h w_t^h$	$r_t^h w_t^l$
i-4)	$(1-\gamma_t^*)\,a\widetilde{w}$	$\widetilde{\phi}_t^l\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\widetilde{\phi}_t^l)r_t^h w_t^l$
i-5)	$\phi_t\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\phi_t)\,r_t w_t^h$	$\phi_t\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\phi_t)\,r_t w_t^l$
ii-1)	$(1-\gamma_t^*)\,a(w_t^h+\widetilde{w}-w_t^l)$	$r_t^h w_t^l$
ii-2)	$\overline{\overline{\phi}}_t\,(1-\gamma_t^*)\,a(w_t^h+\widetilde{w}-w_t^l)+\left(1-\overline{\overline{\phi}}_t\right)r_t w_t^h$	$\overline{\overline{\phi}}_t\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\overline{\overline{\phi}}_t)r_t w_t^l$
iii)	$r_t^l w_t^h$	$(1-\gamma_t^*)\,a\left(w_t^l+d_t\right)$

	$\omega_{t+1}^h/\omega_{t+1}^l$
i-1.a)	$\left(\phi_t\,(1/2)\,a\widetilde{w}+(1-\phi_t)\,r_t w_t^h\right)/\left[\phi_t\,(1/2)\,a\widetilde{w}+(1-\phi_t)\,r_t w_t^l\right]$
i-1.b)	$r_t^l w_t^h/\left[\phi_t^l\,(1-\gamma_t^*)\,a\widetilde{w}+\left(1-\phi_t^l\right)r_t^l w_t^l\right]$
i-2)	$\left[\phi_t^h\,(1-\gamma_t^*)\,a\widetilde{w}+\left(1-\phi_t^h\right)r_t^l w_t^h\right]/(1-\gamma_t^*)\,a\widetilde{w}$
i-3)	$\left[\widetilde{\phi}_t^h\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\widetilde{\phi}_t^h)r_t^h w_t^h\right]/r_t^h w_t^l$
i-4)	$\left[(1-\gamma_t^*)\,a\widetilde{w}\right]/\left[\widetilde{\phi}_t^l\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\widetilde{\phi}_t^l)r_t^h w_t^l\right]$
i-5)	$\left[\phi_t\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\phi_t)\,r_t w_t^h\right]/\left[\phi_t\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\phi_t)\,r_t w_t^l\right]$
ii-1)	$(1-\gamma_t^*)\,a(w_t^h+\widetilde{w}-w_t^l)/r_t^h w_t^l$
ii-2)	$\left[\overline{\overline{\phi}}_t\,(1-\gamma_t^*)\,a(w_t^h+\widetilde{w}-w_t^l)+\left(1-\overline{\overline{\phi}}_t\right)r_t w_t^h\right]/\left[\overline{\overline{\phi}}_t\,(1-\gamma_t^*)\,a\widetilde{w}+(1-\overline{\overline{\phi}}_t)r_t w_t^l\right]$
iii)	$r_t^l w_t^h/(1-\gamma_t^*)\,a\left(w_t^l+d_t\right)=1$since $r_t^l=\frac{a\gamma_t^*\left(w_t^l+d_t\right)}{d_t}$ and $\gamma_t^*=\frac{d_t}{w_t^h+d_t}$

	$\left(\omega^h_{t+1}/\omega^l_{t+1}\right) / \left(\omega^h_t/\omega^l_t\right)$		*Justification*
i-1.a)	$\dfrac{\phi_t\,(1/2)\,\dfrac{a\widetilde{w}}{w^h_t} + (1/2)\,r_t}{\phi_t\,(1/2)\,\dfrac{a\widetilde{w}}{w^l_t} + (1/2)\,r_t}$	< 1	$\dfrac{a\widetilde{w}}{w^h_t} < \dfrac{a\widetilde{w}}{w^l_t}$
i-1.b)	$\dfrac{r^l_t}{\phi^l_t\,(1-\gamma^*_t)\,\dfrac{a\widetilde{w}}{w^l_t} + (1-\phi^l_t)\,r^l_t}$	< 1	Agents "l" prefer strictly entrepreneurship: $(1-\gamma^*_t)\,a\widetilde{w} > r^h_t w^l_t > r^l_t w^l_t$
i-2)	$\dfrac{\phi^h_t\,(1-\gamma^*_t)\,a\widetilde{w}\dfrac{w^l_t}{w^h_t} + (1-\phi^h_t)\,r^l_t w^l_t}{(1-\gamma^*_t)\,a\widetilde{w}}$	< 1	$\dfrac{w^l_t}{w^h_t} < 1$ and agents "l" prefer strictly entrepreneurship : : $(1-\gamma^*_t)\,a\widetilde{w} > r^h_t w^l_t > r^l_t w^l_t$
i-3)	$\dfrac{\widetilde{\phi}^h_t\,(1-\gamma^*_t)\,\dfrac{a\widetilde{w}}{w^h_t} + (1-\widetilde{\phi}^h_t) r^h_t}{r^h_t}$	> 1 or < 1	Agents "h" prefer strictly entrepreneurship : $(1-\gamma^*_t)\,a\widetilde{w} > r^l_t w^h_t$ but we haven't necessary $(1-\gamma^*_t)\,a\widetilde{w} > r^h_t w^h_t$
i-4)	$\dfrac{(1-\gamma^*_t)\,\dfrac{a\widetilde{w}}{w^h_t}}{\widetilde{\phi}^l_t\,(1-\gamma^*_t)\,\dfrac{a\widetilde{w}}{w^l_t} + (1-\widetilde{\phi}^l_t) r^h_t}$		
i-5)	$\dfrac{\phi_t\,(1-\gamma^*_t)\,\dfrac{a\widetilde{w}}{w^h_t} + (1-\phi_t)\,r_t}{\phi_t\,(1-\gamma^*_t)\,\dfrac{a\widetilde{w}}{w^l_t} + (1-\phi_t)\,r_t}$	< 1	$\dfrac{a\widetilde{w}}{w^h_t} < \dfrac{a\widetilde{w}}{w^l_t}$

ii-1)	$\dfrac{(1-\gamma_t^*)\,a(1+\dfrac{\widetilde{w}-w_t^l}{w_t^h})}{r_t^h}$	>1 or <1	Agents "h" prefer strictly entrepreneurship : $(1-\gamma_t^*)\,a(w_t^h+\widetilde{w}-w_t^l)>r_t^l w_t^h$ but we haven't necessary $(1-\gamma_t^*)\,a(w_t^h+\widetilde{w}-w_t^l)>r_t^h w_t^h$
ii-2)	$\dfrac{\overline{\overline{\phi}}_t\,(1-\gamma_t^*)\,a(1+\dfrac{\widetilde{w}-w_t^l}{w_t^h})+\left(1-\overline{\overline{\phi}}_t\right)r_t}{\overline{\overline{\phi}}_t\,(1-\gamma_t^*)\,\dfrac{a\widetilde{w}}{w_t^l}+(1-\overline{\overline{\phi}}_t)r_t}$	<1	$a(1+\dfrac{\widetilde{w}-w_t^l}{w_t^h})<\dfrac{a\widetilde{w}}{w_t^l}$
iii)	$\dfrac{r_t^l w_t^h}{(1-\gamma_t^*)\,a\,\left(w_t^l+d_t\right)}$	$=1$	