Contingent Profit and Loss Sharing (C-Pls) Contracts

Nabi, Mahmoud Sami

Islamic Research and Training Institute, LEGI-Tunisia Polytechnic School, ERF

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Mahmoud Sami Nabi

Senior Researcher Economist
Islamic Research and Training Institute, IDB Group
&
Research Fellow
Economic Research Forum, Cairo

Abstract

This paper analyses the feasibility of profit and loss sharing (PLS) contracts in presence of moral hazard between the principal (financier) and the agent (entrepreneur). It shows that introducing a rule for sharing profits and losses in contingence with the outcome of the entrepreneur's project enlarges the feasibility region of the PLS contracts by enabling more entrepreneurs to access external financing. More specifically, in the contingent profit and loss sharing contract (C-PLS) losses are shared according to the capital participation of each party. But, in case of success, the entrepreneur receives a higher share of the profit which is endogenously determined according to the opportunity cost of capital of the financier (risk-free rate of return) and the characteristics of the project. It is also interestingly shown that having two subsequent C-PLS contracts where the second-period expected profit of the entrepreneur is positively correlated to the first-period probability of success, has an even more incentive effect when the entrepreneur is sufficiently fore-sighted and the size of the project inferior to a determined threshold. Finally, the developed framework sets the rationale for a list of policy recommendations to enhance the use of the C-PLS contracts.

JEL Codes: D82, D86
Keywords: Profit sharing, moral hazard.

1. Introduction

It is well known that one of the explanations of the low use by the Islamic banks of the profit sharing modes of finance is the difficulty to deal with the agency problems (moral hazard and adverse selection). According to Ul Haque and Mirakhor (1987, p161) "bankers ascribe the problem of moral hazard or asymmetric information to be an important explanation for individual preference for short-term liquidity." The agency problems inherent to the equity contracts cannot be implemented without the costly monitoring of the agent. This is the main rationale for the dominance of debt over equity. Indeed, the classical literature on debt contract optimality under costly state verification (e.g. Townsend, 1979 and Gale and Hellwig, 1985), debt contracts minimize the monitoring costs. But this result is due to the assumption of deterministic monitoring in case of equity contracts. In Al-Suwailem (2005), the asymmetric information regarding the realized output of the entrepreneur's project is revealed by the financier through a random auditing strategy. This strategy reduces the higher monitoring cost of the equity contract. Consequently, equity contract Pareto-dominates the debt contract for a determined range of the financier's opportunity cost. The range of this Pareto dominance increases with the project's probability of success as well as with the bankruptcy cost. Trester (1998) develops a four-period model trying to explain why venture capitalists use equity rather than debt to finance entrepreneurial projects. The author considers a framework where information is initially symmetric between the risk-neutral entrepreneur and the venture capitalist. However, subsequent asymmetric information about the payoff of the project arises with a non-null probability which is the case when the entrepreneur learns the status of the project one period before the venture capitalist. In this case, it is shown that debt contract may be infeasible and leads to the use of preferred equity contracts. This is mainly due to the fact that the foreclosure option of the debt contract may incentivize the entrepreneur to behave opportunistically which reduces the expected return of the venture capitalist.

The objective of this paper is to suggest a new type of profit and loss sharing contracts - we call it Contingent Profit and Loss Sharing contracts (C-PLS) – in order to enhance their feasibility and endow the financial institutions with a new direction of product development to increase their offer of equity finance. The C-PLS is based on a simple intuition which is to link the share of the entrepreneur in the project's payoff to the ex-post realized performance. The C-PLS contract initially signed by the two parties (financier and entrepreneur) stipulates that losses are shared according to the capital participation of each party. But, in case of success the entrepreneur receives a higher share of the profit. The characteristics of the contract are endogenously determined (depending on the opportunity cost of capital of the financier - risk-free rate of return - and the characteristics of the entrepreneur's project). We show that introducing a rule for sharing profits and losses contingent on the outcome of the entrepreneurs' projects enlarges the feasibility region of the classical PLS contracts by enabling more entrepreneurs to access external financing. The framework considers the presence of moral hazard which manifests as the hidden effort undertaken by the entrepreneur.

Although the idea of conditioning the shares of the PLS contract on the project's performance is not new per se and has been authorized by the Sharia scholars[1], it has not attracted the attention of the researchers in Islamic economics. In our knowledge, this is the first attempt to apply this rule in a theoretical model and assess its impact on the feasibility of

the PLS contract in presence of moral hazard. In addition to that, the paper is distinguished from the existing studies by analyzing the C-PLS contract in a general context along two dimensions. The first dimension is enabling the internal funds provided by the entrepreneur to vary between null and a level just inferior to 100%. The justification of considering this more general dimension is the intuition that the opportunistic behavior of the entrepreneur is less likely to happen when his own investment is higher. The second dimension is the extension of the time horizon of the financial relationship to two periods.

I show that the C-PLS contract have a larger feasibility region then the PLS contract since it enables the access to finance to more entrepreneurs. But still there are minimum required internal funds to be invested by the entrepreneur for his project to be financed. This threshold depends on the characteristics of the project (size, payoffs, and probability of success/failure) and the opportunity cost of the financier. But the level of the required internal funds is lower for the C-PLS contract relatively to the necessary level under the PLS one and is further lower in the case of a two-period relationship if the entrepreneur is sufficiently foresighted. Indeed, in this case the incentive for the entrepreneur to undertake the higher effort during the first period is even much more important since he is not only concerned by the cash-flow generated in the first-period, but also by increasing his share in the second period project. The higher the cash-flow generated in the first period, the higher its share during the second period. In the policy recommendation section, I discuss the ability of taxing the "risk-free" (or debt-like) financial operations and subsidizing the "higher effort" of the insufficiently-capitalized entrepreneurs on favoring the financial access.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 characterizes the C-PLS contract in the context of a one-period relationship. Section 4 characterizes the C-PLS contract in the context of a two-period relationship. Section 5 provides numerical examples that illustrate the theoretical results. In section 6, I discuss some policy implications of the model. Finally, section 7 concludes.

2. The model

I analyze the feasibility of building the contracting relationship between the risk-neutral financier and entrepreneur based on the C-PLS contract. This is considered in the presence of moral hazard due to the inability of the principal (financier) to observe the effort undertaken by the agent (entrepreneur) after the signature of the C-PLS contract. The entrepreneur is endowed with internal funds but needs complementary external funding to undertake the investment project. In section 3, I study the relationship in the context of one-period whereas a two-period relationship is considered in section 4.

2.1 Economic environment

A risk-neutral entrepreneur operates a firm which generates a stochastic output: a high level $\overline{\pi}$ and a low level $\underline{\pi}$. The probability θ_e of realization of the high output depends on the effort level e undertaken by the entrepreneur. This effort is a private information of the entrepreneur and cannot be observed by the risk-neutral financier. However, the latter can observe perfectly the output of the firm. The relationship between the entrepreneur and the

financier is analyzed in the context of sharing (equity) contract or more precisely the new suggested C-PLS contract.

2.2 Entrepreneur

A risk-neutral entrepreneur is endowed with a technology that produces a stochastic output according to the following distribution

$$\pi = \begin{cases} \bar{\pi} & \text{with a probability of } \theta_e \\ \underline{\pi} & \text{with a probability of } 1-\theta_e \end{cases} \tag{1}$$

where $\bar{\pi}$ and $\underline{\pi}$ represent respectively the high level and low level of output respectively verifying $0 \le \underline{\pi} < \bar{\pi}$. The probability θ_e is depending on the level of effort $e \in \{h,l\}$ undertaken by the entrepreneur such that the realization of the higher profit is more likely when the high level of effort h takes place: $1 > \theta_h > \theta_l > 0$. The disutility of the effort is captured through the costs c_h and c_l verifying $c_h > c_l \ge 0$ which means that the higher the effort the higher the cost for the entrepreneur. The investment funds needed to operate the firm are represented by F. The entrepreneur is initially endowed with an amount $f \in [0, F[$ which means that he needs complementary external funds of $F - f$ in order to operate the firm. We denote by $x_0 = (F - f)/F$ the share of capital provided by the financier. The remainder share provided by the entrepreneur is therefore $1 - x_0$. We assume that the expected output of the firm is superior to the investment F only in case of higher effort:

$$\begin{aligned} E_\pi^h &= \theta_h \bar{\pi} + (1-\theta_h)\underline{\pi} > F \\ E_\pi^l &= \theta_l \bar{\pi} + (1-\theta_l)\underline{\pi} < F \end{aligned} \tag{2}$$

Since the entrepreneur is assumed to be risk-neutral he increases his utility by maximizing his end of period output after payment of the financier's share and supporting the cost of effort.

Assumption1. *If the entrepreneur could self-finance the firm then he would chose the higher level of effort. This is equivalent to the following condition*

$$E_\pi^h - E_\pi^l = (\theta_h - \theta_l)(\bar{\pi} - \underline{\pi}) > c_h - c_l \tag{3}$$

which signifies that the additional expected revenue resulting from the higher effort exceeds the additional cost.

2.3 Financier

A risk-neutral financier requires an expected rate of return equal to $0 < \rho < 1$ which is the available return on "risk-free" (or debt-like) financial operations. Therefore, the expected payoff of the financier, generated from the investment project, should be equal to $(1+\rho)x_0 F$.

While the financier cannot observe the entrepreneur's effort, he observes without cost the firm's output and can infer the entrepreneur's effort choice from the latter's utility maximization problem.

Assumption2. *The expected return of the investment project is higher than the risk-free return if and only if the higher level of effort is undertaken, i.e.:*

$$\frac{E_\pi^l}{F} < 1+\rho \le \frac{E_\pi^h}{F} \tag{4}$$

3. A one-period C-PLS contract

I consider a one-period relationship which begins at date $t = 0$ and finishes at date $t = 1$. The entrepreneur and the financier agree on a partnership contract $(x_0 F, \alpha, \beta = x_0)$ whereby the entrepreneur commits to undertake the high level of effort $(e = h)$ and invests an amount $f = (1-x_0)F$ whereas the financier finances the firm by providing an amount $F - f = x_0 F$. The contract stipulates also that the financier receives a share α ($\beta = x_0$) of the output and the entrepreneur receives a share is $1-\alpha$ $(1-\beta)$ in case of success (failure) of the project. We are looking for a sharing contract such that $\alpha \le \beta$ which means that the financier's share in case of success is lower than its share in case of failure of the project. Given this specification, it is clear that in case of failure of the project (realization of the low level of the output $\underline{\pi}$) the financier (entrepreneur) receives a share x_0 $(1-x_0)$ equal to its initial participation to the capital. For the C-PLS contract to be fully characterized we need to determine the share α.

3.1. Symmetric information

In case of symmetric information, the financier observes the effort of the entrepreneur who cannot deviate from its contractual engagement to undertake the high level of effort. In this case, the share α^* that procures the financier an expected rate of return of ρ is given by:

$$\mathrm{E}(\mathrm{W}^{inv}*) = \theta_h \alpha^* \bar{\pi} + (1-\theta_h)\beta^* \underline{\pi} = (1+\rho)x_0 F \tag{5}$$

Or equivalently

$$\begin{aligned} \alpha^*(x_0) &= x_0[1 - \frac{E_\pi^h - (1+\rho)F}{\theta_h \bar{\pi}}] \\ &= \frac{x_0}{\theta_h \bar{\pi}}\left[(1+\rho)F - (1-\theta_h)\underline{\pi}\right] \\ &\le \beta^* = x_0 \end{aligned} \tag{6}$$

This expression shows that if the high expected return E_π^h / F of the project equals the risk-free return $1+\rho$ then $\alpha^* = \beta^* = x_0$. Otherwise, if the high expected return exceeds the risk-free return then the financier accepts a lower share $\alpha^* < \beta^*$ in case of success of the

project. It is also clear that the share α^* in case of success (realization of the high output $\bar{\pi}$) increases with the amount of the external financing $x_0 F$. It also increases with the risk-free rate of return ρ. However, the share of the financier decreases when the project become safer. Indeed, it decreases with E_π^h, the higher expected return of the project as well as with θ_h, the probability of success in case of high effort. The entrepreneur utility is given by

$$\mathrm{E}(\mathrm{W}^{ent}*) = \theta_h\left(1-\alpha^*\right)\bar{\pi} + (1-\theta_h)\left(1-\beta^*\right)\underline{\pi} - c_h \tag{7}$$

3.2. Asymmetric information

Assume that the financier offers to the entrepreneur the contract $(x_0 F, \alpha^*, \beta)$ but the entrepreneur deviates from its commitment to undertake the higher effort ($e = h$) and performs the lower effort $(e = l)$. In case of asymmetric information, this deviation is not observable by the financier and could not be inferred from the observation of the output. Indeed, the lower output $\underline{\pi}$ could occur even in the case of higher effort with a probability $1-\theta_h$. Undertaking the lower effort just increases the probability of failure from $1-\theta_h$ to $1-\theta_l$.

Assumption 4. *If the entrepreneur is indifferent between the lower and higher effort then he will fulfills his commitment and will undertake the higher effort* $(e = h)$.

Let's now analyze in which case the deviation of the entrepreneur could occur.

Lemma 1. *The entrepreneur has an incentive to undertake the higher effort* $(e = l)$ *if his capital participation in the firm* $(1-x_0)$ *exceeds* $(1-\underline{x})$ *where* $\underline{x}$ *is given by*

$$x > \underline{x} = \theta_h \frac{\left(E_\pi^h - E_\pi^l\right) - \left(c_h - c_l\right)}{\left((1+\rho)F - \underline{\pi}\right)(\theta_h - \theta_l)} \tag{8}$$

Proof. See the appendix.

This lemma signifies that the additional cost of effort borne by the entrepreneur ($c_h - c_l$) is lower than his additional expected revenues if he invests at least $(1-\underline{x})F$. Otherwise, (i.e. $1-x_0 < 1-\underline{x}$) the sharing contract $(x_0 F, \alpha^*, \beta)$ is not an incentive-compatible contract since the entrepreneur will be incited to shirk and undertake the lower effort $(e = l)$. Figure 1 shows that the C-PLS contract is not feasible in case of total external financing of the project $(x_0 = 1)$. However, it becomes feasible when the internal funds of the entrepreneur are sufficiently higher $\mathbf{1 - x > 1 - \underline{x}}$.

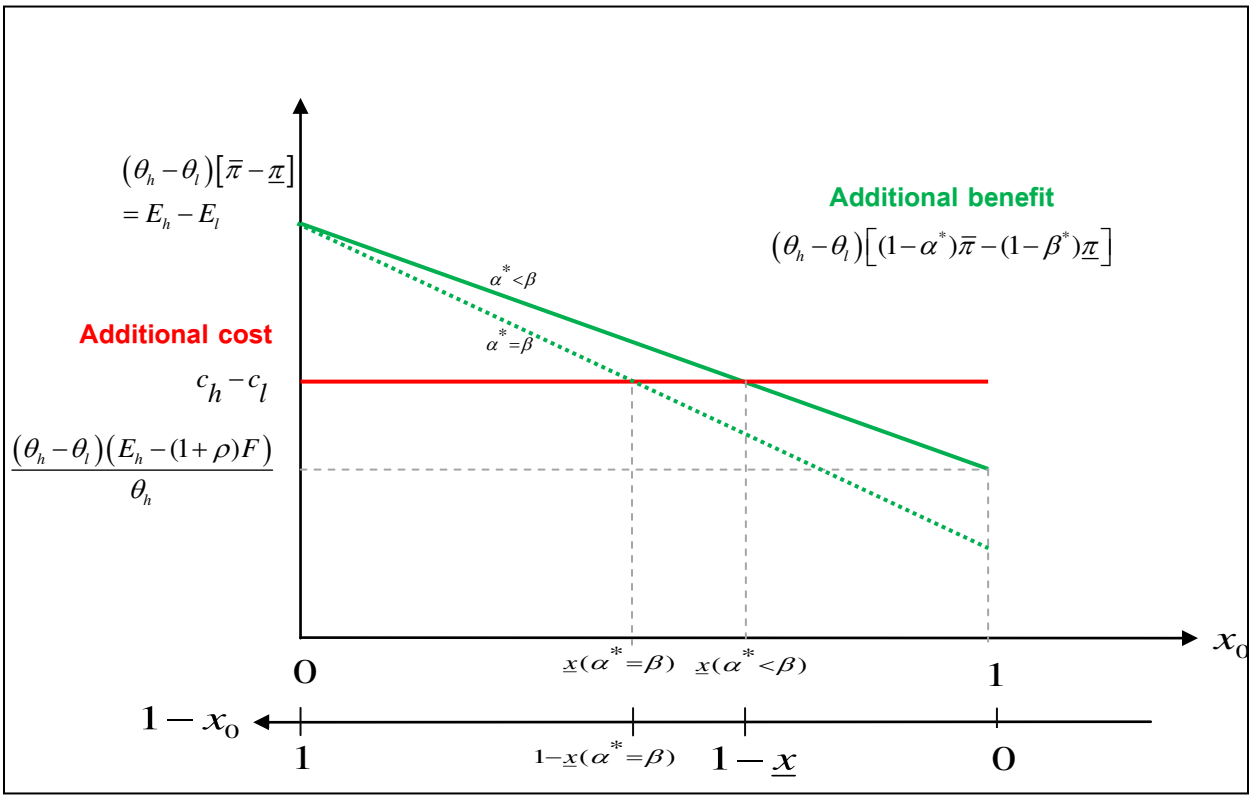

Figure 1. Additional benefit and cost resulting from undertaking higher effort

Figure 1 illustrates also that differentiating the financier's share by fixing a lower share in case of success of the project $(\alpha^* < \beta)$ enlarges the region of feasibility of the C-PLS contract relatively to the classical PLS contract which corresponds to the particular case $(\alpha^* = \beta)$. In other words, the C-PLS contract has a higher feasibility than the classical PLS contract. The justification of this result rests naturally on the additional incentive to the entrepreneurs endowed with insufficient internal resources to undertake the higher effort.

Proposition 1.

i) *If* $x_0 \le \underline{x}$ *then the C-PLS contract* $(x_0 F, \alpha^*, \beta)$ *provides the financier with an expected rate of return of* ρ.

ii) *If* $x_0 > \underline{x}$ *then*

ii-1) Under the C-PLS contract $(x_0 F, \alpha^*, \beta)$ *the entrepreneur chooses the lower effort and the financier's expected rate of return is* $\tilde{\rho} < \rho$.

ii-2) If $\underline{x} < x_0 \le \max(\underline{x}, \hat{x})$ *then the C-PLS contract which provides the financier with an expected rate of return* ρ *is* $(x_0 F, \hat{\alpha}, \beta)$ *where* $\hat{\alpha}$ *verifies:*

$$\alpha^* < \hat{\alpha} = \alpha^* \left(1 + \frac{(1+\rho)F}{\theta_l \bar{\pi}} \right) < \beta = x_0 \qquad (9)$$

$$\hat{x} = \frac{\theta_l \bar{\pi}}{\theta_l \bar{\pi} + (1+\rho) F} \tag{10}$$

$$\tilde{\rho} = \rho - \left(\frac{\theta_h - \theta_l}{x_0 F} \right) \left(\alpha^* \bar{\pi} - \beta \underline{\pi} \right) \tag{11}$$

Proof. See the appendix.

Figure 2 presents the possible payoffs of the financier under the two C-PLS contracts $(x_0 F, \alpha^*, \beta)$ and $(x_0 F, \hat{\alpha}, \beta)$. The payoff of the entrepreneur could be derived geometrically as the difference between the 45^o line and the payoff of the financier.

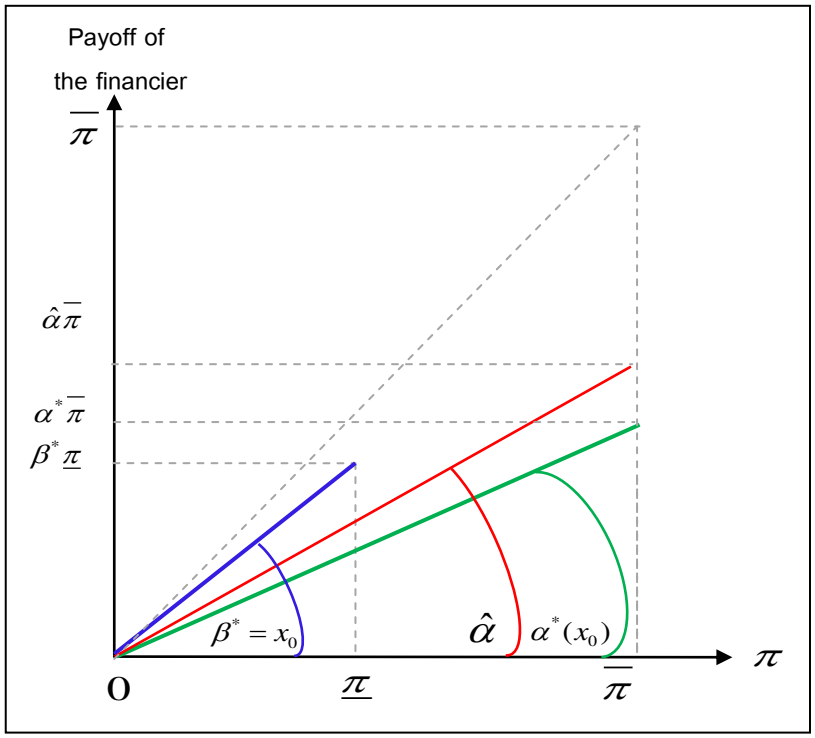

Figure 2. Payoff of the financier according to the three feasible C-PLS contracts

Figure 3 illustrates the different regions presented in proposition 1 under the condition that the financier requires an expected rate of return of ρ. In the region $[0, \underline{x}]$ the entrepreneur does not require large external financing and has an incentive to undertake the higher effort. In this case, the share of the financier is α^*. However, for $x_0 > \underline{x}$ the amount of entrepreneur's financing is not higher enough and the entrepreneur has an incentive to shirk and undertakes the lower effort.

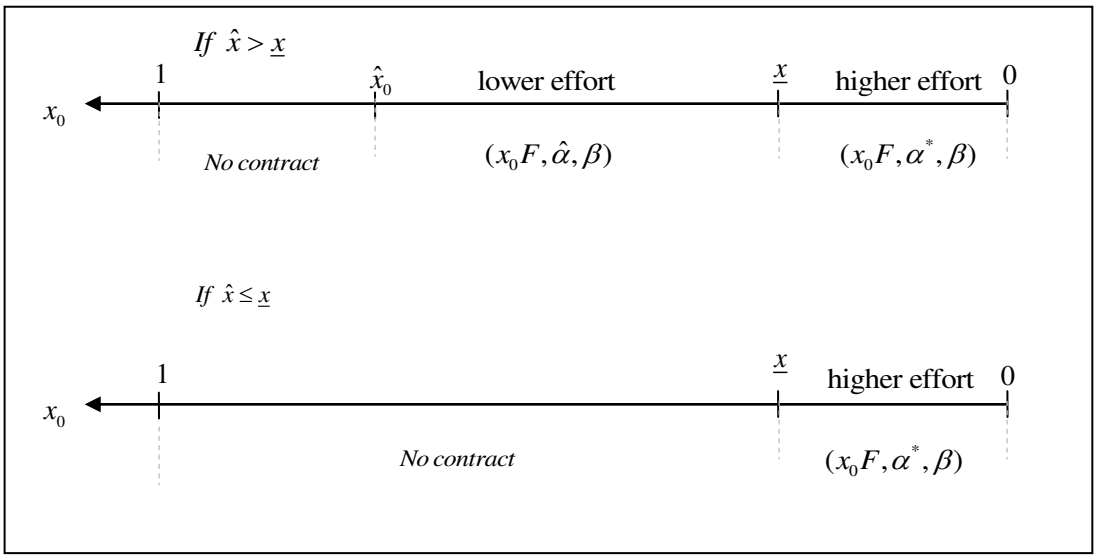

Figure 3. Feasibility and characteristics of the C-PLS contract according to the external financing needs

For this reason, the financier offers in the region $]\underline{x}, \max(\underline{x}, \hat{x})]$ the C-PLS contract $(x_0F, \hat{\alpha}, \beta)$ whereby he requires a higher share ($\hat{\alpha} > \alpha^*$) of the payoff in case of success of the project. This result is intuitive since in order for the financier to keep constant its expected return at ρ he has to overcome the decreasing of the probability of success (from θ_h to θ_l) by increasing its share in the outcome from α^* to $\hat{\alpha}$. In the region $]\max(\underline{x}, \hat{x}), 1]$ the external financing required by the entrepreneur is extremely higher so that no C-PLS contract is feasible due to the inability of the financier to incentivize the entrepreneur to undertake the higher effort, or to increase its share to the level that generates an expected return equal to the risk-free rate of return.

4. C-PLS contracts in a two-period relationship

The characteristics of the entrepreneur are identical to those described in section 2. He is initially ($t = 0$) endowed with an amount of capital equal to $f = (1 - x_0)F$. At the beginning of the second period he reinvests any cash-flow $(1 - x_1)F$ generated from the first-period project in the new project. It is particularly interesting to analyze if extending the horizon of the relationship to two periods will incentivize the entrepreneur to undertake the higher effort and consequently, if it enlarges the region of financial access. Indeed, as illustrated by figure 3, if the initial amount $f = (1 - x_0)F$ of capital endowment of the entrepreneur is inferior to $(1 - \max(\underline{x}, \hat{x}))F$ or equivalently if the required external financing is high and verifies the following condition

$$x_0 \in]\max(\underline{x}, \hat{x}), 1] \tag{15}$$

then the entrepreneur could not benefit from external funding in the context of a one-period relationship and therefore he could not undertake his project. I will analyze if the extension of

the time horizon to two periods and having two subsequent C-PLS contracts will enable the access to finance with lower constraint in regards to the initial funding. Hence, the partnership between the financier and the entrepreneur covers now two periods. At the initial date $t = 0$ the two parties agree on two separate partnership contracts. At the beginning of the first period $(t = 0)$ the entrepreneur and the financier agree on a partnership contract $(x_0 F, \widehat{\alpha}_1, \widehat{\beta}_1 = x_0)$ whereby the entrepreneur commits to undertake the high level of effort $(e = h)$ and participates with an amount of capital of $f = (1 - x_0) F$ whereas the financier finances the firm by providing an amount $F - f = x_0 F$. The entrepreneur commits to reinvest his share of the payoff during the second period so that the second partnership contract becomes $(x_1 F, \widehat{\alpha}_2, \widehat{\beta}_2 = x_1)$ where

$$(1 - x_1) F = \begin{cases} (1 - \widehat{\alpha}_1) \overline{\pi} & \text{with prob } \theta_e \\ (1 - \widehat{\beta}_1) \underline{\pi} & \text{with prob } 1\text{-}\theta_e \end{cases} \tag{16}$$

Condition (16) states that the self-financed capital of the entrepreneur during the second period arises from the payoff he receives at the end of the first period. We have also to ensure that even in the case the entrepreneur is incentivized to undertake the higher effort, the expected wealth of the financier in each period is equal to wealth he would obtain if he invests his capital in a risk-free asset. Therefore, the following conditions should hold:

$$\mathrm{E}(\widehat{\mathrm{W}}_1^{inv}) = \theta_h \widehat{\alpha}_1 \overline{\pi} + (1 - \theta_h) \widehat{\beta}_1 \underline{\pi} = (1 + \rho) x_0 F \tag{17}$$

$$\mathrm{E}(\widehat{\mathrm{W}}_2^{inv}) = \theta_h \widehat{\alpha}_2 \overline{\pi} + (1\text{-}\theta_h) \widehat{\beta}_2 \underline{\pi} = (1 + \rho) x_1 F \tag{18}$$

Therefore, we obtain the following expressions of the shares that characterize the two C-PLS contracts:

$$\widehat{\alpha}_1 = \alpha^*(x_0) \tag{19}$$

$$\widehat{\alpha}_2 = \alpha^*(x_1) \tag{20}$$

Where $\alpha^*(x_0)$ is given by (6). Let's note that at date $t = 0$ the share $\widehat{\alpha}_2$ that characterizes the second-period partnership contract is not fully determined. Only its variation with the payoff $(1 - x_1) F$ according to (20) and (6) is known. Using (20), (16) and (6) we obtain the ex-ante (at $t = 0$) expression of the share $\widehat{\alpha}_2 (e_1 = e)$ which depends on the state of the nature that takes place at $t = 1$ and the effort undertaken by the entrepreneur during the first period:

$$\widehat{\alpha}_2(e_1=e)=\begin{cases}\widehat{\alpha}_2^{up}=\dfrac{F-(1-\widehat{\alpha}_1)\overline{\pi}}{\theta_h\overline{\pi}F}\left((1+\rho)F-(1-\theta_h)\underline{\pi}\right) & \text{with prob } \theta_e \\ \widehat{\alpha}_2^{down}=\dfrac{F-(1-x_0)\underline{\pi}}{\theta_h\overline{\pi}F}\left((1+\rho)F-(1-\theta_h)\underline{\pi}\right) & \text{with prob 1-}\theta_e\end{cases} \tag{21}$$

Given that the share $\widehat{\alpha}_2$ is decreasing with the payoff ($\overline{\pi}$ or $\underline{\pi}$) that takes place during the first period, it is possible that the entrepreneur will be incentivized to undertake the higher effort during the first period in order to maximize its second period payoff. Therefore, the decision of the entrepreneur should be analyzed by considering its expected inter-temporal discounted wealth

$$EW^{ent}(e_1,e_2)=-c_{e_1}+\delta EW_2^{ent}(e_1,e_2) \tag{22}$$

where $\delta<1$ is the entrepreneur's discount factor. Equation (22) shows that the effort $e_1\in\{l,h\}$ undertaken by the entrepreneur requires a cost c_{e_1} and affects the expected wealth $EW_2^{ent}(e_1,e_2)$ during the second period since it impacts the probabilities of success and failure of the project and therefore the revenues that will be reinvested during the second period. Consequently, the effort undertaken by the entrepreneur during the first period impacts also the probabilities of the second's period shares $\widehat{\alpha}_2^{up}$ or $\widehat{\alpha}_2^{down}$.

Proposition 2. *The partnership over two periods enlarges the region of financial access to* $x_0\in\left[\max(\underline{x},\hat{x}),\tilde{x}\right]$ *and incentivizes the entrepreneur to undertake the higher effort* $(e_1=h,e_2=h)$ *if the latter is sufficiently foresighted and the size of the project inferior to a determined threshold:*

$$\delta>\begin{cases}\tilde{\delta} & \text{If } F<\underline{F} \\ \dfrac{\underline{\underline{x}}-\underline{x}}{\underline{\underline{x}}-\hat{x}}\tilde{\delta} & \text{If } \underline{F}\le F\le\max(\underline{F},\underline{\underline{F}})\end{cases}$$

where the thresholds $\tilde{\delta},\tilde{x},\underline{\underline{x}},\underline{F}$ and $\underline{\underline{F}}$ *are defined in the appendix.*

Proof. See the appendix.

This result is intuitive signaling that if the entrepreneur does not put sufficient importance on the payoffs that he will obtain during the second period, then no additional incentive will result from the larger horizon of the relationship financier/entrepreneur. Meanwhile, even if the entrepreneur is sufficiently foresighted but the project's size exceeds $\max(\underline{F},\underline{\underline{F}})$ then the financier will not provide financing for the projects that are not financially feasible over a one-period relationship.

5. Numerical Simulations

I showed in proposition 1 that the C-PLS contract is feasible when the internal funds (provided by the entrepreneur) are superior to a determined threshold (see also figure 3). This threshold depends on the characteristics of the project (size, payoffs, and probability of success/failure) and the opportunity cost of the financier. In this section, I generate numerical simulations to illustrate the theoretical results and analyze the effect on the financial access of different parameters of the problem.

5.1. Effect of the variation of the probability of success

I consider a project of size F =200 which generate a high payoff $\overline{\pi} = 1.98F$ in case of success and low payoff $\underline{\pi} = 0.22F$ in case of failure. The probability of success in case of low effort is $\theta_l = 0.46$ and I vary the probability of success in case of high effort as follows $\theta_h \in [0.55;1]$. I consider that the risk-free rate of return is $\rho = 5\%$ and the discount factor of the entrepreneur is $\delta = 0.95$. Moreover, the cost of low effort is $c_l = 0.1(E_\pi^h - E_\pi^l)$ whereas the cost of high effort is $c_h = 0.4(E_\pi^h - E_\pi^l)$ which ensures that condition (3) is satisfied. In addition to the variation of the probability of success θ_h in case of high effort I also vary the proportion x_0 of external financing needs in the range $[0.1\%,100\%]$. The following table summarizes the different values of the parameters while figures 6.a, 6.b, 7.a and 7.b illustrate the results of the simulations.

$\rho = 5\%$	$F = 200$	$\underline{\pi} = 0.22F$	$c_l = 0.1(E_\pi^h - E_\pi^l)$	$\theta_h \in [0.55;1]$
$\delta = 0.95$	$\theta_l = 0.46$	$\overline{\pi} = 1.98F$	$c_h = 0.4(E_\pi^h - E_\pi^l)$	$x_0 \in [0.1\%,100\%]$

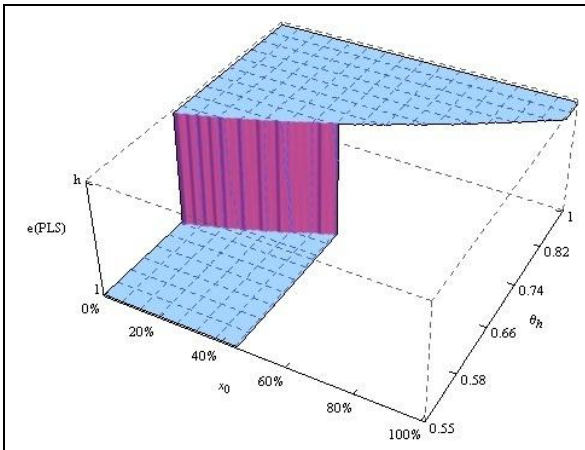

Figure 6. Effort, financial access and the probability of success in case of one-period relationship.

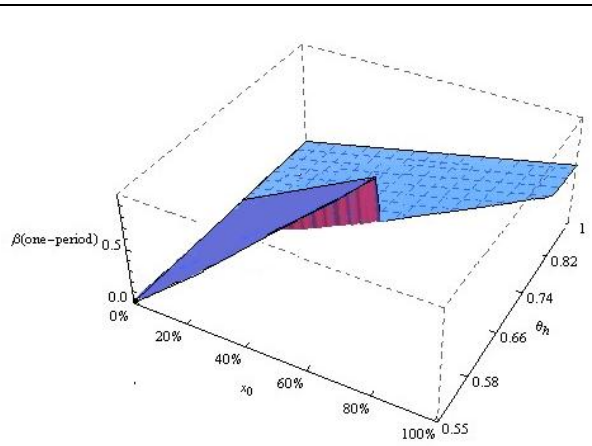

Figure 7. Effect of the external financing and the probability of success on the share ratio of the financier in case of one-period relationship.

Figure 6 shows that C-PLS contract is not feasible when the external financial needs are superior to 45% of the cost of the project and the probability of success in case of higher effort is inferior to 0.8. In addition, it is clear that the higher effort is undertaken by the entrepreneur only for sufficiently level of the higher probability of success. This threshold is itself increasing with the external financing need which means that higher external needs may induce the entrepreneur to choose the lower effort whereas for the same type of projects another entrepreneur with higher internal funds chooses the higher effort. Figure 7 illustrates the profit sharing ratio β.

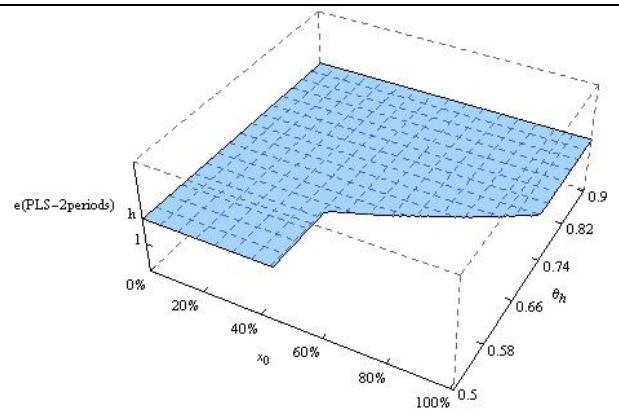

Figure 8. Effort, financial access and the probability of success in case of two-period relationship.

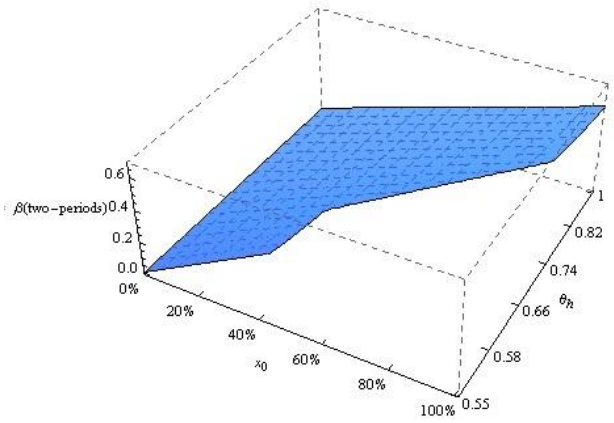

Figure 9. Effect of the external financing and the probability of success on the share ratio of the financier in case of two-period relationship.

Comparing figures 6 and 8 it is clear that the relationship between the financier and the entrepreneur over two periods enlarges the region of feasibility of the C-PLS contract. It is also interesting to note that this longer relationship horizon induces the entrepreneur to choose the higher effort. Figure 9 shows that the financing cost of the C-PLS contract is reduced mainly due to the incentivizing effect of the larger horizon of the financial relationship pushing the entrepreneur to undertake the higher effort.

5.2. Effect of the variation of the risk-free return

I consider a project of size F =3000 which generate a high payoff $\overline{\pi} = 2.09F$ in case of success and low payoff $\underline{\pi} = 0.001F$ in case of failure. The probability of success in case of low effort is $\theta_l = 0.45$ and the probability of success in case of high effort is $\theta_h = 0.8$. I vary the risk-free rate of return ρ in the interval $[1\%;30\%]$. The discount factor of the entrepreneur is $\delta = 0.95$. Moreover, the cost of low effort is $c_l = 0.1(E_\pi^h - E_\pi^l)$ whereas the cost of high effort is $c_h = 0.8(E_\pi^h - E_\pi^l)$ which ensures that condition (3) is satisfied. In addition to the variation of the risk-free rate of return ρ, I also vary the proportion x_0 of external financing needs in the range $[0.1\%,100\%]$. The following table summarizes the different values of the parameters while figures 10. and 11. illustrate the results of the simulations.

$F = 3000$	$\theta_l = 0.45$	$\underline{\pi} = 0.001F$	$c_l = 0.1(E_\pi^h - E_\pi^l)$	$\rho \in [1\%; 30\%]$
$\delta = 0.95$	$\theta_h = 0.8$	$\bar{\pi} = 2.09F$	$c_h = 0.8(E_\pi^h - E_\pi^l)$	$x_0 \in [0.1\%, 100\%]$

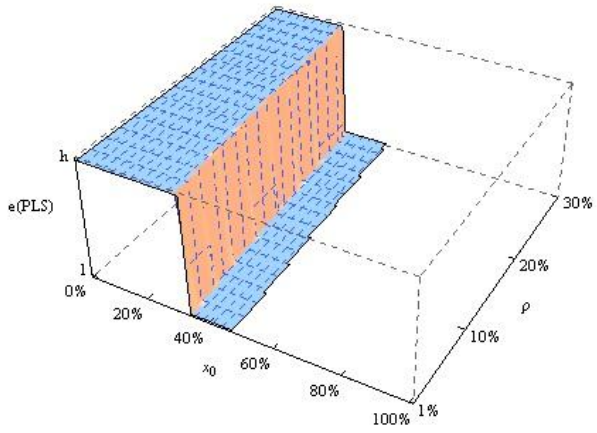

Figure 10.. Effort, financial access and the risk-free return in case of one-period relationship.

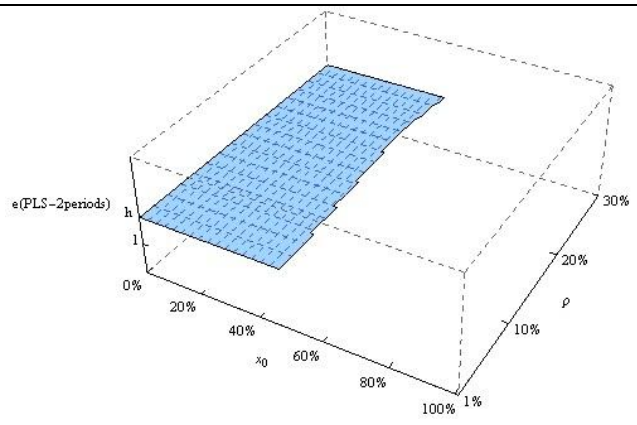

Figure 11. Effort, financial access and the risk-free return in case of two-period relationship.

Again, figures 10 and 11 confirm the the positive effect of the longer horizon on the incentive of the entrepreneur to undertake higher effort and consequently on the enlargement of the region of access to finance.

6. Policy implications

From the results obtained in the context of one-period financial relationship (section 3) it is easy to note that it is possible to extend the use of the C-PLS contract by widening the financial access to the entrepreneurs through two policies or a mix of them. The first policy consists in subsidizing the cost of higher effort (The higher effort should be interpreted broadly and could reflect the use of a modern technology by the entrepreneur which necessitates additional expenses and learning process):

P1) *A special governmental fund subsidizing the undertaking of higher effort by the entrepreneurs endowed with low internal resources (case of MSMEs) could be one of the tools to implement this policy.*

Figure 12 illustrates the effect of this policy which reduces the minimum internal resources required for the entrepreneur to access external financing. These minimum resources decreases from $1 - \underline{x}$ to $1 - \underline{x}^s$. The second policy consists in reducing the opportunity cost of the financier which equals the risk-free rate of return ρ .

P2) *In order to encourage the C-PLS contract a higher taxation of the financier's revenues generated through the "risk-free" financial operations could be adopted.*

Figure 13 illustrates the effect of taxation on the additional benefit of higher effort. The tax's effect has been chosen (for explanation reason) to generate the same effect as the subsidizing policy, which is decreasing the minimum resources required for financial access from $1-\underline{x}$ to $1-\underline{x}^s$.

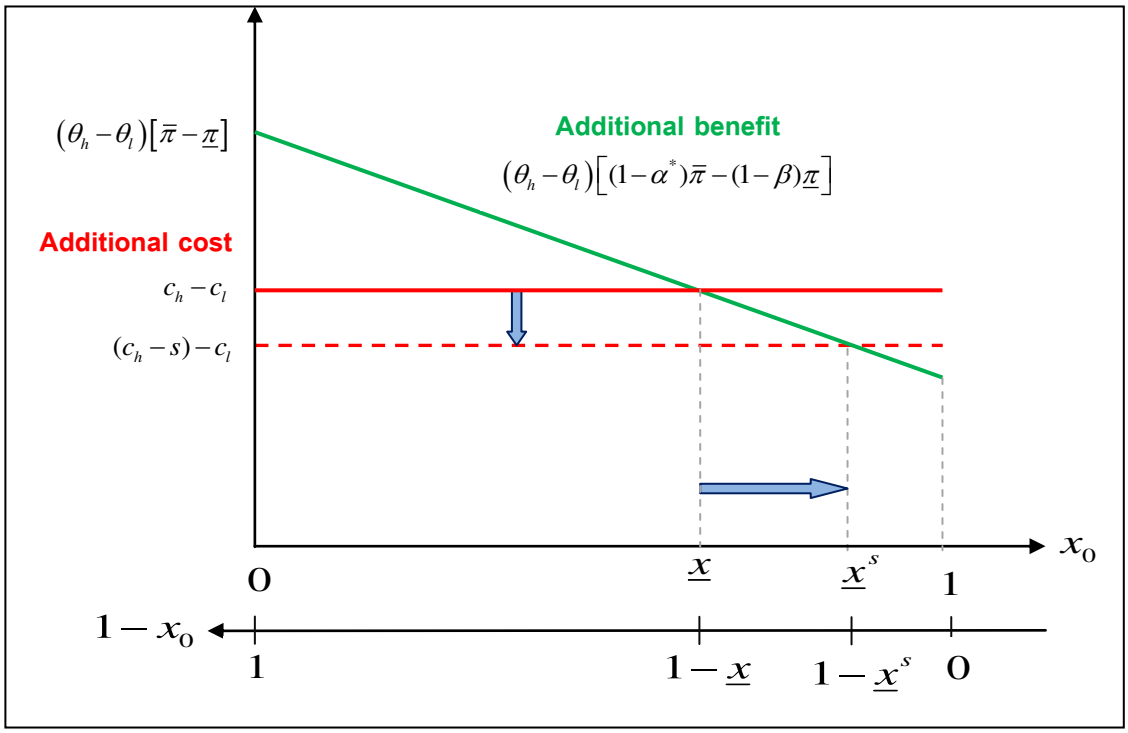

Figure 12. Effect of subsidizing the cost of higher effort

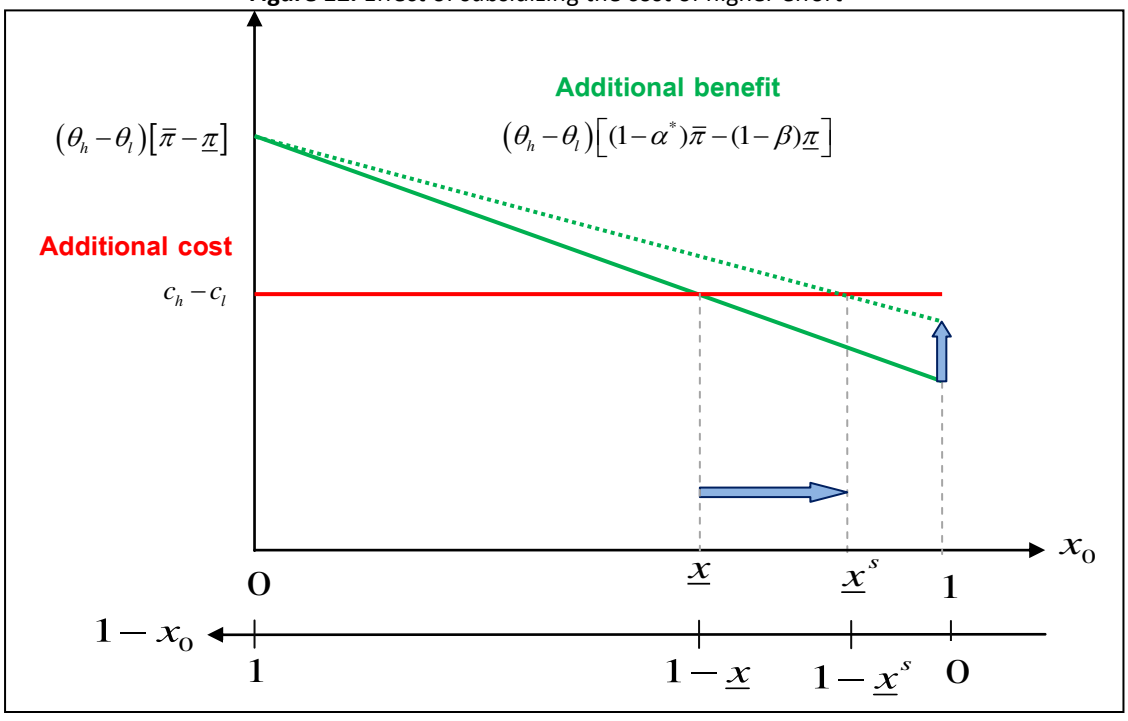

Figure 13. Effect of reducing the risk-free rate of return (through taxation for example)

A policy-mix consisting in a combination of taxing the "risk-free" financial operations and subsidizing of the "higher effort" has the advantage to reduce the amplitude of the adjustment made through each policy. As illustrated in figure 14, a combination of lower subvention of the higher effort compared to figure 12 and lower taxation compared to figure 13 generates the same effect (relatively to individual policy) on the enlargement of the region of financial access.

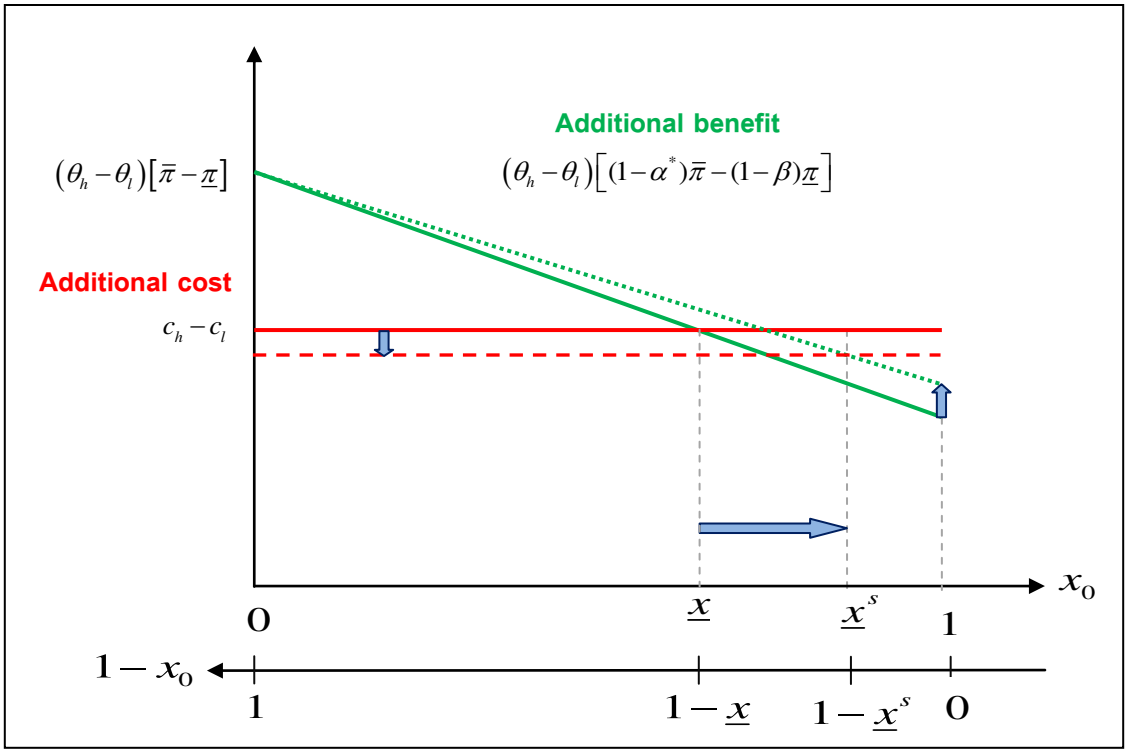

Figure 14. Effect of a mix-policy of subsidizing and taxation

The results obtained in the context of two-period financial relationship (section 4) show that it is possible to enlarge the region of financial access through extending the horizon of financial relationship from one to two periods. It has also been shown that for the larger horizon to have a positive impact, the entrepreneurs need to be sufficiently foresighted. In this context, a third policy recommendation can be done which is the following:

P3) *A governmental agency that accompanies the entrepreneurs with low financial resources, by providing them with economic incentives on a larger horizon is expected to have a positive externality on the financier-entrepreneur relationship.*

7. Conclusion

This paper builds a theoretical model accompanied by numerical illustration setting the rationale for the development of new type of profit and loss sharing contract. The suggested Contingent Profit and Loss Sharing (C-PLS) contract includes an endogenous incentive mechanism which enable to reduce the negative impact of the moral hazard problem between the financier and the entrepreneur in a situation where the former cannot observe the effort

undertaken by the latter. In the contingent profit and loss sharing contract (C-PLS) losses are shared according to the capital participation of each party. But, in case of success, the entrepreneur receives a higher share of the profit which is endogenously determined according the opportunity cost of capital of the financier (risk-free rate of return) and the characteristics of the project. This variety of the profit sharing contract although approved from a sharia compliance perspective many years ago has not been analyzed in any theoretical model dealing with the agency problems between a financier and an entrepreneur. Another originality of this paper relatively to the existing studies consists in enabling the internal funds provided by the entrepreneur to vary between null and a level just inferior to 100%. This dimension is important because the entrepreneur's (financier's) share in the project's payoff increases (decreases) with the internal funds. The paper showed that the C-PLS contract is feasible if the internal funds of the entrepreneur are superior to a minimum determined threshold. This threshold depends on the characteristics of the project (size, payoffs, and probability of success/failure) and the opportunity cost of the financier. I have also shown that the enlargement of the financial-relationship to two periods has an incentivizing effect on the entrepreneur and may increase the feasibility of the C-PLS contract enabling a larger access to finance. This is the case if the entrepreneur is sufficiently foresighted so that he gives a minimum level of importance to his profits during the second period, and the size of the project does not exceed a determined threshold. In the policy recommendation section, I discussed the effects of taxing the "risk-free" (debt-like) financial operations and subsidizing the "higher effort" of the insufficiently-capitalized entrepreneurs and show that they can enlarge of the financial access and enhance the feasibility of the C-PLS contracts.

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Appendix

Proof of Lemma 1

The entrepreneur undertakes the lower level of effort after signing the contract if this increases his utility. This is the case if the following condition holds

$$\mathrm{E}(\tilde{\mathrm{W}}^{ent}) = \theta_l\left(1-\alpha^*\right)\bar{\pi} + (1\text{-}\theta_l)\left(1-\beta^*\right)\underline{\pi} - c_l > \mathrm{E}(\mathrm{W}^{ent}*) \tag{A1}$$

Using (7) condition (A1) becomes

$$(c_h - c_l) > \left(\theta_h - \theta_l\right)\left[(1-\alpha^*)\bar{\pi} - (1-\beta^*)\underline{\pi}\right] \tag{A2}$$

Injecting the expression (6) of α^* in (A2) and noting that $\beta^* = x$

$$\frac{(c_h - c_l)}{\left(\theta_h - \theta_l\right)} > \bar{\pi} - \underline{\pi} + \beta^*\underline{\pi} - \alpha^*\bar{\pi}$$

$$\frac{(c_h - c_l)}{\left(\theta_h - \theta_l\right)} - \left(\bar{\pi} - \underline{\pi}\right) > \beta^*\underline{\pi} - \alpha^*\bar{\pi} \tag{A3}$$

Let's show that $\alpha^*\bar{\pi} > \beta^*\underline{\pi}$. For that assume that $\alpha^*\bar{\pi} \le \beta^*\underline{\pi}$ and let's show that this is not possible. Multiplying the two members of this inequality by θ_h and adding $(1\text{-}\theta_h)\beta^*\underline{\pi}$ we obtain $\theta_h\alpha^*\bar{\pi} + (1-\theta_h)\beta^*\underline{\pi} \le \beta^*\underline{\pi}$. Recalling that equation (5) gives $\theta_h\alpha^*\bar{\pi} + (1-\theta_h)\beta^*\underline{\pi} = (1+\rho)xF$ we obtain that $(1+\rho)xF \le \beta^*\underline{\pi}$ which becomes $(1+\rho)F \le \underline{\pi}$ since $\beta^* = x$. This is clearly in contradiction with the assumption that $F > \underline{\pi}$. Therefore, we can now return to (A3) to obtain

$$\frac{\left(E_\pi^h - E_\pi^l\right) - (c_h - c_l)}{\left(\theta_h - \theta_l\right)} < \alpha^*\bar{\pi} - \beta^*\underline{\pi} = x\left[\bar{\pi} - \underline{\pi} - \frac{E_\pi^h - \left(1+\rho\right)F}{\theta_h}\right] \text{ or equivalently}$$

$$x > \underline{x} = \frac{(E_\pi^h - E_\pi^l) - (c_h - c_l)}{\left(1-\theta_l/\theta_h\right)\left[\theta_h(\bar{\pi} - \underline{\pi}) + \left(1+\rho\right)F - E_\pi^h\right]} = \theta_h\frac{\bar{\pi} - \underline{\pi} - \left(c_h - c_l\right)/(\theta_h - \theta_l)}{\left(1+\rho\right)F - \underline{\pi}} = \theta_h\frac{(\theta_h - \theta_l)\left(\bar{\pi} - \underline{\pi}\right) - \left(c_h - c_l\right)}{\left(\left(1+\rho\right)F - \underline{\pi}\right)(\theta_h - \theta_l)}$$

$$x > \underline{x} == \theta_h\frac{\left(E_\pi^h - E_\pi^l\right) - \left(c_h - c_l\right)}{\left(\left(1+\rho\right)F - \underline{\pi}\right)(\theta_h - \theta_l)}$$

Proof of Proposition 1

i) From Lemma 1 we know that if $(1-x) \geq (1-\underline{x})$ then the entrepreneur will undertake the higher effort. In this case, the entrepreneur's expected rate of return is identical to the case of symmetric information and equals ρ.

ii-1) From Lemma 1 we know that if $(1-x) < (1-\underline{x})$ then the entrepreneur chooses the lower effort if he is offered the partnership contract $(e = h, xF, \alpha^*, \beta^*)$. The expected rate of return of the financier is determined by the following equation

$$\mathrm{E}(\tilde{\mathrm{W}}^{inv}) = \theta_l \alpha^* \overline{\pi} + (1-\theta_l)\beta^* \underline{\pi} = (1+\tilde{\rho})x\ F \tag{A4}$$

Which gives us using (6)

$$\tilde{\rho} = \rho - \left(\frac{\theta_h - \theta_l}{xF}\right)\left(\alpha^* \overline{\pi} - \beta^* \underline{\pi}\right) \tag{A5}$$

ii-2) For the expected rate of return of the financier to equal ρ although the entrepreneur chooses the lower effort we should have the following equation

$$\mathrm{E}(\hat{\mathrm{W}}^{inv}) = \theta_l \hat{\alpha} \overline{\pi} + (1-\theta_l)\beta^* \underline{\pi} = (1+\rho)x\ F \tag{A6}$$

Recalling equation (5):

$$\mathrm{E}(\mathrm{W}^{inv}*) = \theta_h \alpha^* \overline{\pi} + (1-\theta_h)\beta^* \underline{\pi} = (1+\rho)xF$$

It is easy to show that

$$\hat{\alpha} = \frac{\theta_h}{\theta_l}\alpha^* + \frac{\theta_l - \theta_h}{\theta_l}\beta^* \frac{\underline{\pi}}{\overline{\pi}} = \frac{\theta_h \overline{\pi}\alpha^* + (\theta_l - \theta_h)x\underline{\pi}}{\theta_l \overline{\pi}} \tag{A7}$$

From equation (6) we have $\alpha^* = x\left[1 - \frac{E_\pi^h - (1+\rho)F}{\theta_h \overline{\pi}}\right] = x\left[\frac{\theta_h \overline{\pi} - (\theta_h - \theta_l)(\overline{\pi} - \underline{\pi}) + (1+\rho)F}{\theta_h \overline{\pi}}\right]$

Injecting this expression of α^* in (A7) we found

$$\hat{\alpha} = x\frac{\theta_h \overline{\pi} - (\theta_h - \theta_l)(\overline{\pi} - \underline{\pi}) + (1+\rho)F + (\theta_l - \theta_h)\underline{\pi}}{\theta_l \overline{\pi}} \tag{A8}$$

$$= x\left(1 + \frac{(1+\rho)F}{\theta_l \overline{\pi}}\right) = \alpha^*\left(1 + \frac{(1+\rho)F}{\theta_l \overline{\pi}}\right) > \alpha^*$$

However, we should ensure that $\hat{\alpha} \leq 1$ which is the case only if $x \leq \hat{x} = \frac{\theta_l \overline{\pi}}{\theta_l \overline{\pi} + (1+\rho)F} < 1$. Since we are under the case $x > \underline{x}$ the possibility for the financier to offer this contract is conditioned by the following condition $x \in \left]\underline{x}, \max(\underline{x}, \hat{x})\right]$. From Lemma 1 we know that if $(1-x) < (1-\underline{x})$ then the entrepreneur chooses the lower effort if he is offered the partnership contract $(e = h, xF, \alpha^*, \beta^*)$ since we have the inequality (A2) and given that $\hat{\alpha} > \alpha^*$ the same inequality (A2) holds for $\hat{\alpha}$ and the entrepreneur chooses the lower effort.

Proof of Proposition 2

Before proving the results we need the following intermediary result which is easy to show using the equations (8) and (10).

Lemma1.

$$\max(\underline{x},\hat{x}) = \begin{cases} \underline{x} & \text{if } F < \underline{F} \\ \hat{x} & \text{if } F \geq \underline{F} \end{cases} \quad with$$

$$\underline{F} = \left(\frac{\overline{\pi}\theta_l}{1+\rho}\right)\left[\left(\frac{\overline{\pi}\theta_l}{\theta_h\left(\overline{\pi}-\underline{\pi}-A\right)}\right)-1\right]^{-1} \quad and \;\; A = \left(c_h - c_l\right)/(\theta_h - \theta_l) \tag{A14}$$

i) For the entrepreneur to undertake the higher effort during the first period,

$$\mathrm{E}(\mathrm{W}^{ent} \setminus e_1 = h, e_2) \geq \mathrm{E}(\mathrm{W}^{ent} \setminus e_1 = l, e_2) \tag{A15}$$

with

$$\mathrm{E}(\mathrm{W}^{ent} \setminus e_1 = h, e_2) = -c_h + \delta\left(\theta_{e_2} E\left(\left(1-\hat{\alpha}_2\right) \setminus e_1 = h\right)\overline{\pi} + \left(1-\theta_{e_2}\right)E(1-x_1)\underline{\pi} - c_{e_2}\right) \tag{A16}$$

$$\mathrm{E}(\mathrm{W}^{ent} \setminus e_1 = l, e_2) = -c_l + \delta\left(\theta_{e_2} E\left(\left(1-\hat{\alpha}_2\right) \setminus e_1 = l\right)\overline{\pi} + \left(1-\theta_{e_2}\right)E(1-x_1)\underline{\pi} - c_{e_2}\right) \tag{A17}$$

For the entrepreneur to choose the higher effort during the first period we have to obtain
Using (A16) and (A17) the condition (A15) becomes

$$c_h - c_l \leq \delta\theta_{e_2}\overline{\pi}\left[E(1-\hat{\alpha}_2 \setminus e_1 = h) - E(1-\hat{\alpha}_2 \setminus e_1 = l)\right] \tag{A18}$$

(A17) signifies that the additional effort cost should be more than compensated by the increase in the expected discounted wealth. From equation (22) we have

$$E(1-\hat{\alpha}_2 \setminus e_1 = h) - E(1-\hat{\alpha}_2 \setminus e_1 = l) = \left(\theta_h - \theta_l\right)\left((1+\rho)F - (1-\theta_h)\underline{\pi}\right)\frac{\left(1-\hat{\alpha}_1\right)\overline{\pi} - \left(1-x_0\right)\underline{\pi}}{\theta_h F}$$

Injecting the last equation in (A17) we obtain

$$\frac{c_h - c_l}{\theta_h - \theta_l} \leq \delta\theta_{e_2}\frac{\left(1-\hat{\alpha}_1\right)\overline{\pi} - \left(1-x_0\right)\underline{\pi}}{\theta_h F}\left[(1+\rho)F - (1-\theta_h)\underline{\pi}\right]$$

$$\frac{c_h - c_l}{\theta_h - \theta_l} \leq \frac{\delta\theta_{e_2}}{\theta_h F}\left[(1+\rho)F - (1-\theta_h)\underline{\pi}\right]\left[\left(1-\hat{\alpha}_1\right)\overline{\pi} - \left(1-x_0\right)\underline{\pi}\right]$$

$$\frac{c_h - c_l}{\theta_h - \theta_l} \leq \frac{\delta\theta_{e_2}}{\theta_h F}\left[(1+\rho)F - (1-\theta_h)\underline{\pi}\right]\left[\overline{\pi} - \underline{\pi} + x_0\underline{\pi} - \hat{\alpha}_1\overline{\pi}\right]$$

According to (20) we have $\hat{\alpha}_1\overline{\pi} = x_0\left(\frac{(1+\rho)F - (1-\theta_h)\underline{\pi}}{\theta_h}\right)$. Therefore (A16) is also equivalent to

$$\frac{c_h - c_l}{\theta_h - \theta_l} \leq \frac{\delta\theta_{e_2}}{\theta_h F}\left[(1+\rho)F - (1-\theta_h)\underline{\pi}\right]\left[\overline{\pi} - \underline{\pi} - \frac{(1+\rho)F - \underline{\pi}}{\theta_h}x_0\right]$$

We consider that the incentive for the entrepreneur to undertake the higher effort during the second period holds ($e_2 = h$) therefore we obtain

$$\left(\frac{(1+\rho)F-\underline{\pi}}{\theta_h}\right)x_0 \le \bar{\pi}-\underline{\pi}-\frac{F}{\delta}\frac{\frac{c_h-c_l}{\theta_h-\theta_l}}{\left[(1+\rho)F-(1-\theta_h)\underline{\pi}\right]}$$

which becomes

$$x_0 \le \tilde{x} = \underline{\underline{x}} - \frac{\tilde{\delta}}{\delta}\left(\underline{\underline{x}}-\underline{x}\right) \tag{A19}$$

with $\underline{x}$ given by (8) and the rest of the variables are

$$\underline{\underline{x}} = \frac{\theta_h\left(\bar{\pi}-\underline{\pi}\right)}{(1+\rho)F-\underline{\pi}} > \underline{x} \tag{A20}$$

$$\tilde{\delta} = \frac{F}{\left[(1+\rho)F-(1-\theta_h)\underline{\pi}\right]} \tag{A21}$$

Therefore, the partnership contract over two periods enlarges the region of financial access if $\tilde{x} > m\max(\underline{x},\hat{x})$. Using lemma 2 this is equivalent to the following conditions

$$\begin{cases}\tilde{x}>\underline{x} & \text{if } F<\underline{F}\\ \tilde{x}>\hat{x} & \text{if } F\ge\underline{F}\end{cases} \Leftrightarrow \begin{cases}\underline{\underline{x}}-\frac{\tilde{\delta}}{\delta}\left(\underline{\underline{x}}-\underline{x}\right)>\underline{x} & \text{if } F<\underline{F}\\ \underline{\underline{x}}-\frac{\tilde{\delta}}{\delta}\left(\underline{\underline{x}}-\underline{x}\right)>\hat{x} & \text{if } F\ge\underline{F}\end{cases} \Leftrightarrow \begin{cases}\delta>\tilde{\delta} & \text{if } F<\underline{F}\\ \delta\frac{\underline{\underline{x}}-\hat{x}}{\underline{\underline{x}}-\underline{x}}>\tilde{\delta} & \text{if } F\ge\underline{F}\end{cases} \tag{A22}$$

Using (A20) and (10) it is easy to show that $\underline{\underline{x}}-\hat{x}>0$ is possible if and only if

$$F<\underline{\underline{F}}=\frac{\bar{\pi}\theta_h\theta_l}{1+\rho}\frac{\bar{\pi}-\left(1+1/\theta_h\right)\underline{\pi}}{\left(\theta_h-\theta_l\right)\bar{\pi}+\theta_h\underline{\pi}} \tag{A23}$$

Therefore (A22) becomes

$$\begin{cases}\delta>\tilde{\delta} & \text{if } F<\underline{F}\\ \delta>\frac{\underline{\underline{x}}-\underline{x}}{\underline{\underline{x}}-\hat{x}}\tilde{\delta} & \text{if } \underline{F}\le F\le\max(\underline{F},\underline{\underline{F}})\end{cases} \tag{A24}$$

ii) Now, for the entrepreneur to choose the higher effort during the second period we have to ensure that the following condition holds:

$$W_2^{ent}(e_1=h,e_2=h)\ge W_2^{ent}(e_1=h,e_2=l)$$

$$\begin{aligned}&\theta_h\left(1-\hat{\alpha}_2\right)\bar{\pi}+\left(1-\theta_h\right)(1-x_1)\underline{\pi}-c_h \ge \theta_l\left(1-\hat{\beta}_{h2}\right)\bar{\pi}+\left(1-\theta_l\right)\left(1-x_1\right)\underline{\pi}-c_l\\ &\left(\theta_h-\theta_l\right)\left[\left(1-\hat{\alpha}_2\right)\bar{\pi}-\left(1-x_1\right)\underline{\pi}\right]\ge c_h-c_l\\ &\left(\theta_h-\theta_l\right)\left[\bar{\pi}-\underline{\pi}+x_1\underline{\pi}-\hat{\alpha}_2\bar{\pi}\right]\ge c_h-c_l\end{aligned} \tag{A25}$$

Let's recall that equation (21) gives us

$$\hat{\alpha}_2\bar{\pi}=x_1\left(\frac{(1+\rho)F-\underline{\underline{\pi}}}{\theta_h}+\underline{\pi}\right)$$

Therefore (A24) becomes

$$\bar{\pi}-\underline{\pi}-x_1\left(\frac{(1+\rho)F-\underline{\pi}}{\theta_h}\right)\geq\frac{c_h-c_l}{\theta_h-\theta_l}$$

$$\bar{\pi}-\underline{\pi}-\frac{c_h-c_l}{\theta_h-\theta_l}\geq x_1\left(\frac{(1+\rho)F-\underline{\pi}}{\theta_h}\right)$$

Or equivalently $x_1\leq\bar{x}=\left(\theta_h-\theta_l\right)\underline{x}$ where x_1 is defined by (17) and (20) and given by

$$x_1=\begin{cases}1-\dfrac{x_0\underline{\pi}}{F}-\dfrac{x_0}{\theta_h}\left((1+\rho)-\dfrac{\underline{\pi}}{F}\right) & \text{with prob } \theta_h\\ 1-\dfrac{x_0\underline{\pi}}{F} & \text{with prob } 1-\theta_h\end{cases}$$

Therefore the condition for the entrepreneur to undertake the higher effort at the second period (whatever the state of the nature that occurs at the end of the first period) is the following

$\bar{x}\geq 1-\dfrac{x_0\underline{\pi}}{F}$ or equivalently $x_0\geq\bar{x}'=\dfrac{F}{\underline{\pi}}\left(1-\left(\theta_h-\theta_l\right)\underline{x}\right)$

Let's now note that $\bar{x}'\leq 1$ if and only if $F\geq\bar{F}=\dfrac{\theta_h\left(E_\pi^h-E_\pi^l-\left(c_h-c_l\right)\right)}{(1+\rho)\left(\theta_h-\theta_l\right)}+\dfrac{\underline{\pi}}{(1+\rho)}$

In other words if $F<\bar{F}$ then the entrepreneur undertakes the higher effort uncondionally. However, if $F\geq\bar{F}$ then the entrepreneur undertakes the higher effort only if $x_0\geq\bar{x}'$. Finally, the different thresholds presented in proposition 3 are summarized in the following:

$$\tilde{\delta}=\frac{F}{\left[(1+\rho)F-(1-\theta_h)\underline{\pi}\right]}\ ;\ \tilde{x}=\underline{\underline{x}}\left(1-\frac{\tilde{\delta}}{\delta}\right)+\frac{\tilde{\delta}}{\delta}\underline{x}\ ;\ \underline{\underline{x}}=\frac{\theta_h\left(\bar{\pi}-\underline{\pi}\right)}{(1+\rho)F-\underline{\pi}}$$

$$\underline{F}=\left(\frac{\bar{\pi}\theta_l}{1+\rho}\right)\left[\left(\frac{\bar{\pi}\theta_l}{\theta_h\left(\bar{\pi}-\underline{\pi}-\left(c_h-c_l\right)/\left(\theta_h-\theta_l\right)\right)}\right)-1\right]^{-1} \qquad \text{(A26)}$$

$$\underline{\underline{F}}=\frac{\bar{\pi}\theta_h\theta_l}{1+\rho}\frac{\bar{\pi}-\left(1+1/\theta_h\right)\underline{\pi}}{\left(\theta_h-\theta_l\right)\bar{\pi}+\theta_h\underline{\pi}}$$

[i] From the Shari'ah compliance point of view, the practice of incentivizing the agent in case of Mudharabah by increasing his share in the payoffs in case of success of the project, has been approved in the 2nd Islamic Finance Conference in Kuwait and the fourth Shari'ah opinion of the first Al-Barakah Conference. For the Musharakah contract it is known that the sharing of the losses should be proportional to the capital participation of each party but the sharing of the profits could be different according to an initial agreement.