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The effects of exchange rate volatility on international trade flows: evidence from panel data analysis and fuzzy approach[*1]

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Abstract

The aim of this paper is to analyze the effects of exchange rate volatility on international trade flows by using two different approaches, the panel data analysis and fuzzy logic, and to compare the results. To a panel with the cross-section dimension of 91 pairs of EU15 countries and with time ranging from 1964 to 2003, an extended gravity model of trade is applied in order to determine the effects of exchange rate volatility on bilateral trade flows of EU15 countries. The estimated impact is clearly negative, which indicates that exchange rate volatility has a negative influence on bilateral trade flows. Then, this traditional panel approach is contrasted with an alternative investigation based on fuzzy logic. The key elements of the fuzzy approach are to set fuzzy decision rules and to assign membership functions to the fuzzy sets intuitively based on experience. Both approaches yield very similar results and fuzzy approach is recommended to be used as a complement to statistical methods.

Key words: *linguistic modeling, fuzzy relations, exchange rate volatility, bilateral trade, gravity model*

JEL classification: *C23, F14*

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1. Introduction

The objective of this study is to investigate the effects of exchange rate volatility on bilateral trade flows across European countries from 1964 to 2003 by employing panel data analysis and a fuzzy approach. The paper uses the gravity model to analyze bilateral trade flows among EU-15 countries. Firstly, statistical methods are used to identify the determinants of international trade flows and to quantify their effects. The interest focuses especially on the effects of exchange rate volatility on bilateral trade flows. After finding the individual effect of exchange rates on trade flows, we use the fuzzy approach to see the effect of exchange rate volatility on trade flows between EU-15 countries.

The gravity model applied to panel data has already been proven to be a successful and reliable tool in international trade literature. It serves to reveal the factors that influence trade flows, and the individual impact of each factor on trade flows. Moreover, it also reports to the user the overall explanatory power of the model in explaining trade flows.

Contrasting statistical methods with artificial intelligence methods in the same application allows a detailed comparison of results. In econometric analysis, a large data set and a strong model is needed to obtain reliable results. However, there might be some cases in which it is difficult to obtain a sufficiently large data set to get reliable results, or there may be some missing data which affect the reliability of results. In these cases, combining the fuzzy approach with the expertise in the topic studied could be a good solution to get first approximate results. To this aim, our study compares the results obtained by panel data analysis to the ones given by fuzzy logic. While we have a large data set and thus it can be argued that fuzzy logic is not really necessary, we propose to use this approach as a robustness check on traditional modeling. Once fuzzy logic proves to be a good alternative, it can also be used in cases of data problems that impair the validity of traditional methods.

Our hypothesis in this study **is** that the fuzzy logic can approximate the effects of exchange rate volatility on trade flows and it can be used as a complement to statistical models. Especially in the cases where there is missing data or no data, fuzzy logic can give the user first approximate results if there is some background information about the topic studied.

The structure of the paper is as follows. Section 2 reviews the literature on this topic. Section 3 introduces the modified gravity model of total trade and gives the basics of the fuzzy approach. Section 4 reports the results obtained by panel data analysis and fuzzy approach. Finally, section 5 concludes. Appendix provides information about fuzzy calculation.

2. Literature review

The history of international trade shows that different exchange rate regimes were preferred at different periods. Recent decades have seen a tendency towards purely fixed or purely floating exchange rate regimes. Fischer (2001) indicates that most countries have abandoned intermediate exchange rate regimes and instead prefer a purely floating or a purely fixed exchange rate. From 1991 to 1999, the share of fixed exchange rate regimes increased from 16% to 24% and the share of floating exchange rate regimes from 23% to 42%. By contrast, the intermediate regimes declined sharply from 62% to 34%. According to Fischer (2001), this movement from the intermediate regimes is towards currency boards, dollarization or currency unions on the hard peg side, and towards a variety of floating exchange rate regimes on the other side. The main reason suggested for this change is that "soft pegs are crisis-prone and not viable over long periods". Moreover, Bubula and Otker-Robe (2003) provide some support for the proponents of the bipolar view. They find that, during 1990–2001, intermediate regimes were more frequently subject to crises as compared with purely fixed and floating ones, while even the latter have not been totally free of pressures.

The choice of exchange rate regime gives a country the freedom to use macroeconomic policies to manipulate the economy and enables it to fight recessions, crises etc. Furthermore, exchange rates influence the level of international trade as well. Therefore, the effects of volatility in exchange rates and of exchange rate regimes on the economy and on international trade have long been studied. There are two sides in the literature. One side claims that exchange rate uncertainty/volatility/variability does not have any impact on trade while there is another side which tries to prove the opposite. Hooper and Kohlhagen (1978) analyze the impact of exchange rate uncertainty on the volume of the US – German trade between 1965 and 1975 and conclude that there is no statistically significant effect. Gotur (1985) reaches the same conclusion by analyzing the effects of exchange rate volatility on the volume of trade among the US, Germany, France, Japan and the UK. A famous IMF study (1984) summarizes that the large majority of empirical studies could not find a significant relationship between exchange rate variability and the volume of trade either on aggregated or bilateral basis. More recently, this view was supported by Bacchetta and van Wincoop (2000), who find that exchange rate uncertainty, or different exchange rate systems do not have any impact on trade.

On the other hand, Ethier (1973) analyzes the effects of exchange rate uncertainty on the level of trade and finds that uncertainty in future exchange rates reduces trade. Cushman (1983) estimates fourteen bilateral trade flows among industrialized countries and finds a significant negative effect of exchange risk on trade. Akhtar and Hilton (1984) establish a significant negative effect of nominal exchange rate uncertainty on bilateral trade between Germany and the US. Kenen and Rodrik (1986) analyze the effects of volatility in real exchange rates on trade and conclude that volatility depresses the volume of trade. De Grauwe and De Bellefroid (1987) employ cross sectional techniques for the European Economic Community countries for 1960-1969 and 1973-1984, and investigate the effects of variability in real exchange rates on trade. They find significant negative effects. Lane and Milesi-Ferretti (2002) examine the effects of appreciation and depreciation of exchange rates on trade and conclude that in the long run, larger trade surpluses are to be expected with more depreciated real exchange rates. Viane and de Vries (1992) study this issue from a different perspective, by analyzing the effects of exchange rate volatility on exports and imports separately and find that exporters and importers are affected differently by the changes in exchange rates, because they are on opposite sides of the forward market.

The theory of fuzzy sets has been applied first to engineering fields and then spread to a wide range of areas such as economics, management, artificial intelligence, psychology, linguistics, information retrieval, medicine etc. (Fu and Yao, 1980). In the last decade, artificial intelligence methods such as neural networks and fuzzy logic have been employed in econometric studies especially in time series analysis. Tseng et al. (2001) propose a fuzzy model and apply it to forecast foreign exchange rates. Lee and Wong (2007) use an artificial neural network and fuzzy reasoning to improve the decision making under foreign currency risk and analyze the effect of trading strategy on the changes in exchange rates. They use fuzzy logic because they claim that it is capable to perform text reasoning of macroeconomic news. Moreover, Bencina (2007) introduce fuzzy logic in coordinating investment projects in two Slovenian municipalities, while Oyuk et al. (2007) analyze the effects of exchange rates on international trade flows by using fuzzy logic. Their panel results show that changes in real exchange rates affect bilateral trade flows in a negative way and by -0.60%. They find very similar ---0.61%--- effect of exchange rate volatility on trade flows using the fuzzy approach. In this paper, we think that “volatility of exchange rates” can explain bilateral trade flows better than “changes in real exchange rates”. Therefore, we estimate the effects of exchange rate volatility on bilateral trade flows and compare the results given by panel data analysis with the ones given by the fuzzy approach.

3. Methodology

3.1. The gravity model

According to the Gravity Model, trade flows between two countries depend on their income positively and on the distances between them negatively as shown in Equation 1

$$T_{ij} = C \frac{Y_i \times Y_j}{D_{ij}} \tag{1}$$

where C is a constant term, T_{ij} is the value of trade between country i and country j, Y_i and Y_j denote the real GDP of countries i and j, respectively, and D_{ij} is the distance between countries i and j (see also Krugman and Obstfeld, 2006).

The gravity model says that large economies are expected to spend more on imports and exports; so, the higher the GDP of a country, the higher its total trade. The gravity model can be extended to catch other effects – such as population, exchange rates, having a common language and common border or being in the same trade union – that promote bilateral trade.

In this study, the gravity model is extended with additional variables, namely the population of exporting and importing country and exchange rate volatility. Another difference from the original model is that incomes of country i and j are not taken as products with the same coefficient but as separate variables. The same approach applies to the population, where we have different coefficients for each country. The proposed model that is used to capture the effects of exchange rate volatility on bilateral trade is:

$$\ln T_{ijt} = \alpha + \beta_1 \ln D_{ij} + \beta_2 \ln Y_{it} + \beta_3 \ln Y_{jt} + \beta_4 \ln Pop_{it} + \beta_5 \ln Pop_{jt} + \beta_6 VolXR_{ijt} + \varepsilon_{ijt} \qquad (2)$$

where T_{ijt} represents total bilateral trade flows between country i and country j during time t which is calculated as the sum of exports from country i to country j and imports from country j to country i. Exports and imports are measured in nominal terms and then are converted to the volumes by using GDP deflators for each country at time t. D_{ij} is the distance between capital cities of country i and country j that is measured in kilometers. Two basic variables of the gravity model are Y_{it} and Y_{jt}, real GDP of country i and j respectively. Pop_{it} and Pop_{jt} are the populations of country i and country j in time t.

$Vol(xr_{ijt})$ is the volatility of nominal exchange rate between exporter and importer country in year t which is calculated as the moving average of standard deviations of the first difference of logarithms of quarterly nominal bilateral exchange rates (Kowalski, 2006). $Vol(xr_{ijt})$ is the 5-year ("t-4,...,t") average of standard deviations from the average quarter-on-quarter percentage change in bilateral nominal exchange rate calculated over the last 4 quarters, given by the following formula:

$$Volxr_{ijt} = \frac{1}{20} \sum_{q}^{q-19} \delta_q , \qquad (3)$$

where q is the last quarter in year t and

$$\delta_q = \sqrt{\sum_{q}^{q-3} \frac{1}{3} \left(de_q - \sum_{q}^{q-3} \frac{1}{4} de_q \right)^2} \quad . \qquad (4)$$

δ_q is a standard deviation from the average quarter-on-quarter percentage change in bilateral nominal exchange rate calculated over the last 4 quarters where $de_q = e_q - e_{q-1}$ and e_q is a logarithm of bilateral exchange rate at the end of quarter q.

3.2. The fuzzy approach

3.2.1. What is Fuzzy Set Theory?

The difference between conventional dual logic and fuzzy set theory is that in conventional dual logic a statement can be either true or false; in set theory, an element can be either a member of a set or not. However, real situations are very often uncertain. Lack of information, for instance, may cause the future state of the system to be unknown. This type of uncertainty has been handled by statistics and probability theory. Fuzziness can be found in many areas of life such as meteorology, medicine, engineering, manufacturing etc. In daily life, the meaning of words is often vague. When we say "tall man", "beautiful women", "successful company" the meaning of a word may change from person to person or from culture to culture. Fuzzy set theory provides a mathematical framework to study vague phenomena precisely. It is defined as a modeling language for fuzzy relations, criteria and situation (Zimmermann, 2001).

In fuzzy set theory, normal sets are called crisp sets to be differentiated from fuzzy sets (Driankov et al., 1996). Let C be a crisp set and F a fuzzy set defined on the universe U. For any element u of U, either $u \in C$ or $u \notin C$. However, in fuzzy set theory it is not necessary that either $u \in F$ or $u \notin F$. In fuzzy set theory, a membership function μ_F assigns a value to every $u \in U$ from the unit interval [0, 1], instead from the two element set {0, 1} as is done in crisp sets. A fuzzy set is defined on the basis of a membership function.

According to Zimmermann (2001), major goals of fuzzy set theory are the modeling of uncertainty and the generalization of classical methods based on dual logic from dichotomous to gradual features. Moreover, it aims to reduce the complexity of data to an acceptable degree by means of linguistic variables. Computational units (see Figure 4) process these linguistic expressions, use membership functions of fuzzy sets and finally retranslate the fuzzified result into the words via linguistic approximation which is explained in the following section.

3.2.2. Linguistic variables in Fuzzy Set Theory

Zadeh (1975) defines a linguistic variable as a variable whose values are words or sentences in a natural or artificial language. For example, age is a linguistic variable when it is defined as "young, very young, old, not very old" instead of 18, 15, 60 or 40.

The following framework, cited from Driankov et al. (1996), explains the notions of linguistic variable, linguistic value, actual physical domain and semantic function:

$$(X, \mathcal{L}(X), \mathcal{X}, Mx).$$

Here, X represents the symbolic name of a linguistic variable, for example age, temperature, error, weight, etc. In section 3, instead of X we have *"A"* and *"B"* which are the linguistic variables representing "exchange rate volatility" and "total trade" respectively.

$\mathcal{L}(X)$ denotes the set of linguistic values that X can take. Again, in our case (A) = {high, medium, low}. $\mathcal{L}(X)$ can also be called the term-set or the reference-set of X.

Furthermore, $\mathcal{X}$ is the actual physical domain over which the linguistic variable X can take its quantitative values. In the case of the linguistic variable "exchange rate volatility", $\mathcal{X}$ is the interval [0%, 1%] with 0.1 increments.

Mx is a semantic function which gives a quantitative interpretation of a linguistic value from the interval $\mathcal{X}$ and is defined as

$$Mx\colon \mathcal{L}(X) \rightarrow \widetilde{L(X)}$$

where $\widetilde{L(X)}$ is a denotation for a fuzzy set defined over $\mathcal{X}$. Put differently, Mx returns the meaning of a word into the fuzzy terms. Instead of $\widetilde{L(X)}$ it is also possible to use μ_{LX} which is the membership function.

The symbolic translation of natural language in terms of linguistic variables is explained by Driankov et al. (1996) as follows. The symbolic representation of the natural language expression "Error has the property of being negative-big" is written as "E is NB" and called an atomic fuzzy proposition.[4] The interpretation of this atomic representation is defined by the fuzzy set $\widetilde{\mathrm{NB}}$ or the membership function μ_{NB} on the normalized physical domain ε=[–6,6] of the physical variable "error", $\forall e \in \varepsilon$: $\widetilde{\mathrm{NB}} = \mu_{NB}$ = "membership function"

where μ_{NB} shows the degree to which a specific quantitative crisp value of the physical variable error, e, belongs to the set $\widetilde{\mathrm{NB}}$. For example, the degree of membership of -3.2 to the fuzzy set of negative big is $\mu_{NB}(-3.2) = 0.7$. This degree of membership shows the degree to which the symbolic expression "E is *NB*" is satisfied given the following circumstances: *NB* is interpreted as μ_{NB} and E takes the value -3.2.

4 The symbol E denotes the physical variable "error" and *NB* the particular value "negative big" of error.

3.2.3. Fuzzy if-then statements

A fuzzy conditional or a fuzzy if-then statement describes the relationship between process state (which contains a description of the process output) and control output variables (which describe the control output that should be produced given the particular process output).

The meaning of the expression

if X is *A*, *then Y* is *B*

is represented as a fuzzy relation defined on $\mathcal{X}\times\mathcal{Y}$ where $\mathcal{X}$ and $\mathcal{Y}$ are the physical domains of the linguistic variables *X* and *Y*. The meaning of "*X* is *A*" is called the *rule antecedent* and represented by the fuzzy set $\tilde{A} = \int_{x} \frac{\mu_A(x)}{x}$.

The meaning of "*Y* is *B*" is called the *rule consequent* and represented by the fuzzy set $\tilde{B} = \int_{y} \frac{\mu_B(y)}{y}$.[5]

Then, the meaning of the fuzzy conditional is a fuzzy relation μ_R such that

$$\forall\, x \in \mathcal{X} \;\; \forall\, y \in \mathcal{Y}\colon\;\; \mu_R(x, y) = \mu_A(x) * \mu_B(y).$$

where "$*$" can be either Cartesian product or any fuzzy implication operator (Driankov et al., 1996).

To give an example, first of the three *if-then* rules used in section 4.2 is:

if <increase in exchange rate volatility is high> *then*
<decrease in total trade is medium>

represented by $\tilde{A}_1 \times \tilde{B}_1$ in Table 4. Cartesian product is used to process the relation between the variable "exchange rate volatility" and "total trade".

3.2.4. Fuzzy Set mathematics

The following definitions except Definition 2 and 6 will be cited from Zimmermann (2001).

Definition 1: If *X* is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ is a set of ordered pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}}\ (x) \mid x \in X\}$

[5] This fuzzy set was in error in the mentioned source, therefore it was corrected by the author of this article.

where $\mu_{\tilde{A}}$ is called the membership function of x in $\tilde{A}$ that maps X to the membership space M. The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. However, as a matter of convenience it is assumed that fuzzy sets are normalized to the range [0, 1].

The membership function is the fundamental part of a fuzzy set. Therefore, operations with fuzzy sets are defined through their membership functions. It is defined by Driankov et al. (1996) as follows:

Definition 2: The membership function μ_F of a fuzzy set F is a function $\mu_F : U \rightarrow [0,1]$.

So, each element u from U (universe) has a membership degree $\mu_F(u) \in [0,1]$.

F is completely determined by the set of tuples $F = \{(u, \mu_F(u)) | u \epsilon U\}$.

Definition 3: The membership function $\mu_{\tilde{C}}(x)$ of the intersection $\tilde{C} = \tilde{A} \cap \tilde{B}$ is pointwise defined by

$$\mu_{\tilde{C}}(\mathbf{x}) = \min\left\{\mu_{\tilde{A}}(\mathbf{x}), \mu_{\tilde{B}}(\mathbf{x})\right\}, \mathbf{x} \in \mathbf{X}.$$

Definition 4: The membership function $\mu_{\tilde{D}}(x)$ of the union $\tilde{D} = \tilde{A} \cup \tilde{B}$ is pointwise defined by $\mu_{\tilde{D}}(\mathbf{x}) = \max\left\{\mu_{\tilde{A}}(\mathbf{x}), \mu_{\tilde{B}}(\mathbf{x})\right\}, \mathbf{x} \in \mathbf{X}$.

Definition 5: The Cartesian product of fuzzy sets is defined as follows: Let $\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n$ be fuzzy sets in $X_1, ..., X_n$. The Cartesian product is then a fuzzy set in the product space $X_1 \times X_2 \times ..... \times X_n$ with the membership function

$$\mu_{(\tilde{A}_{1} \times \cdots \times \tilde{A}_{n})}(\mathbf{x}) = \min\left\{\mu_{\tilde{A}_i}(\mathbf{x}_i) \middle| \mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_n), \mathbf{x}_i \in \mathbf{X}_i\right\}.$$

Definition 6: Compositional Rule of Inference: If $\tilde{R}$ is a fuzzy relation from U to V, and $\tilde{x}$ is a fuzzy subset of U, then the fuzzy subset $\tilde{y}$ of V which is induced by $\tilde{x}$ is given by the composition of $\tilde{R}$ and $\tilde{x}$; that is $\tilde{y} = \tilde{x} \circ \tilde{R}$ in which $\tilde{x}$ plays the role of a unary relation (Zadeh, 1973).

4. Data and results

4.1. Results obtained by panel data analysis

This section employs panel data analysis to see the effects of exchange rate volatility on bilateral trade flows. The classical gravity model is extended by including population of exporting and importing countries and exchange rate volatility as explanatory variables. This model is estimated for a data set of EU15 countries from 1964 to 2003. The sample period covers 40 years. Belgium and Luxembourg are treated as one country because of data availability. From the data set of 14 countries, 91 bilateral trade flows are obtained during fixed, flexible and Euro periods which means that for 14 countries we obtain 91 unidirectional trade flows for 40 years, which makes 3640 observations. Since the data for Belgium is missing for the years 1991 and 1992, total number of observations is 3601. The sources for the data are World Bank`s World Development Indicators 2005, OECD`s International Trade by Commodity Statistics and IMF`s International Financial Statistics.

Instead of using only cross-sectional or time-series data alone, combining both might give econometrically more efficient results. In the cross-sectional part we have total trade flows between two countries, i.e. exports from Austria to Belgium plus imports from Belgium to Austria. In times series dimension, we have total trade flows for each country pair in each year between 1964 and 2003. We prefer using panel data in our estimates due to its advantages such as increasing degrees of freedom and reducing the collinearity between explanatory variables. Moreover, cross sectional data cannot give information about the dynamic effects. On the other hand, using time series data alone can only give information about one individual over a certain time period (Hsiao, 2003). In our estimates period fixed effects model has been used since it is suspected that during the 40 years period there might have been some period specific events which affect all countries in the sample in the same way.

Table 1 reports the results from estimating Equation 2 obtained by using the software package Eviews. According to the gravity theory, the income of a country is expected to affect its trade in a positive way. Table 1 shows that both income terms for countries i and j have the expected positive sign. The difference from previous studies is emphasized by the discrepancy in the two coefficients. The contributions by the income terms of each country to the bilateral trade are quite different. We find that a 1 percent increase in the income of the exporting country i entails a 0.09% higher bilateral trade. On the other hand, a 1% increase in the income of the importing country j boosts bilateral trade by 1.1%.

Moreover, population has a negative sign for the importing country. This negative coefficient, which is smaller than the positive coefficient of aggregate income, reflects the positive effect of per capita income *Y/Pop* and a net effect of population on imports. On the other hand, population of the exporting country has a positive effect on bilateral trade, which by the same token suggests a strong net effect of country size measured by population on exports.

One of the basic elements of the gravity model is the distance between countries, which is on the denominator of the gravity equation (Equation 1). For this reason, its expected sign is negative, as a larger distance tends to decrease international trade by increasing transportation costs and imposing other impediments to trading such as informational and psychological frictions (Huang, 2007). Finally, exchange rate volatility has a negative effect on bilateral trade. According to our results, an increase in volatility by 1% entails a percentage change in bilateral trade of -0.21%.

Table 1: Balanced panel estimates with period fixed effects for time range 1964-2003.

Explanatory variable	Coefficient	T-statistic	P-value
Intercept	-14.57	-20.38	0.00
Distance	-0.85	-41.18	0.00
Exporter GDP	0.09	2.30	0.02
Importer GDP	1.10	35.64	0.00
Exporter Population	0.67	16.50	0.00
Importer Population	-0.41	-12.11	0.00
Exchange-Rate Volatility	-0.21	-3.29	0.00

Note: Dependent variable is log of total bilateral trade; R^2 = 0.85, 3601 observations, 40 time periods, 91 cross sections.

Source: Estimation results obtained by using software package *"E-views"*

4.2. The application of a fuzzy approach to total trade

As shown in section 4.1, exchange rate volatility leads to fluctuations in the volume of trade. The objective of this study is to compare the results obtained by panel data analysis with the ones obtained by using fuzzy logic to see how close they are. If fuzzy rules are set appropriately, and if their intuition is realistic and in accordance with theory, fuzzy reasoning may give very similar results to panel data analysis without requiring a very large data set that is necessary in panel data analysis. For econometric methods, the data set is crucially important. When there is any problem in obtaining or processing the data or in the specification of the model, it is impossible to get reliable results. Moreover, if no sufficient data is available, conventional models cannot give reliable results. For these cases, fuzzy reasoning can be suggested as an alternative to get some approximate results.

In this section, the effects of exchange rate volatility on bilateral trade will be analyzed using fuzzy reasoning. Steps to be taken to apply a fuzzy approach to total trade are:

(i) setting the fuzzy decision table;

(ii) determining the change in total trade following a 1 percent increase in exchange rate volatility.

To start with, it is necessary to fuzzify exchange rate volatility and the decrease in bilateral trade. Describing process states by means of linguistic variables and using these variables as inputs is a very important step in the fuzzy approach. Table 2 shows the partitioning of the universe of exchange rate volatility into three fuzzy sets: 'high', 'medium' and 'low'. Table 3 shows an analogous partitioning of the universe of total trade. When defining these expressions, membership values are assigned to each state intuitively based on experience (McNeill and Thro, 1994). A fuzzy set is defined solely by its membership function (Zimmermann, 2001). Membership degrees lie between 0 and 1. If an object completely belongs to the fuzzy set it has a membership value of 1. If an object does not belong to the fuzzy set at all, it has a membership value of 0. Membership degrees of borderline cases lie between 0 and 1. The more an element is characteristic of a fuzzy set, the closer to 1 is its membership degree (Driankov et al., 1996).

According to Table 2, "high increase in exchange rate volatility" is meant to be a 1% increase in volatility. If the increase is 0.9%, this volatility is considered to be high with a membership value of 0.75. When the volatility increases by 0.8%, the membership value for a high volatility decreases to 0.5. $\tilde{A}_1$ in Table 2 is the fuzzy set that describes a high increase in exchange rate volatility. Furthermore, a 0.5% increase in exchange rate volatility is defined as being medium and therefore it is assigned a membership value of 1 in $\tilde{A}_2$, which is a fuzzy set that describes a medium increase in exchange rate volatility. Similarly, $\tilde{A}_3$ represents the fuzzy set that describes a low increase in exchange rate volatility. These three fuzzy sets are fully described by their membership functions. In fuzzy language, "high", "medium" and "low" (increase in exchange rate volatility) are called linguistic values. "*A*" in general is the linguistic variable that represents "exchange rate volatility".

On the other hand, $\tilde{B}_1$, $\tilde{B}_2$ and $\tilde{B}_3$ are the fuzzy sets that describe a "medium", "low-medium" and "low" decrease in total trade respectively. "*B*" in general is the linguistic variable that stands for "total trade".

Table 2: Increase in exchange rate volatility partitioning

Increase in exchange rate volatility (%)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	Fuzzy set
high	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.50	0.75	1	$\tilde{A}_1$
medium	0.00	0.00	0.25	0.50	0.75	1.00	0.75	0.50	0.25	0.00	0	$\tilde{A}_2$
low	0.50	0.75	1.00	0.75	0.50	0.25	0.00	0.00	0.00	0.00	0	$\tilde{A}_3$

Source: Authors`own description

Table 3: Decrease in total trade partitioning

Decrease in Total Trade (%)	0.00	0.025	0.05	0.075	0.10	0.125	0.15	0.175	0.20	0.225	0.25	Fuzzy set
medium	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.50	0.75	1.00	$\tilde{B}_1$
low-medium	0.00	0.00	0.25	0.50	0.75	1.00	0.75	0.50	0.25	0	0	$\tilde{B}_2$
low	0.50	0.75	1.00	0.75	0.50	0.25	0.00	0.00	0.00	0.00	0	$\tilde{B}_3$

Source: Authors`own description

We note the basic concepts of fuzzy set theory. A number is not necessarily high or low with a 100% certainty. If a value is closer to the target, its membership value is closer to 1. For example, in Table 2, a 0.7% increase in exchange rate volatility is categorized as a high increase with a 0.25 membership degree, while it is classified as a medium increase with a membership value of 0.5. The membership degree to the fuzzy set $\tilde{A}_2$ is higher than $\tilde{A}_1$ because 0.7% is closer to 0.5% than it is to 1%. By contrast, in crisp sets, variables are categorized into specific classes, and they can only belong to one class. If a number belongs to one class, it cannot be member of another.

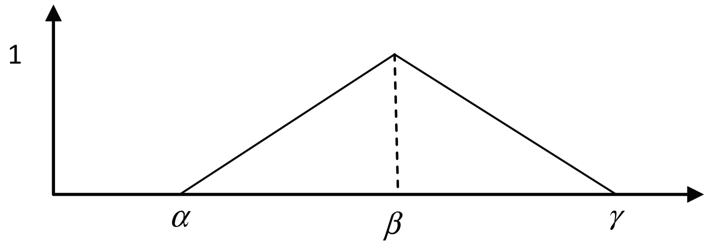
In this study, triangular membership functions are used due to computational efficiency (Figure 1). Alternative often used functions are the trapezoidal and the bell-shaped functions. The triangular membership function

$\Lambda : \mathbf{U} \rightarrow [\mathbf{0}, \quad \mathbf{1}]$ is defined by Driankov et al. (1996) as follows[6]:

$$\Lambda(u;\alpha,\beta,\gamma)=\begin{cases}0 & \text{for } u<\alpha\\ (u-\alpha)(\ \beta-\alpha) & \text{for } \alpha\le u\le\beta\\ (\gamma-u)(\ \gamma-\beta) & \text{for } \beta\le u\le\gamma\\ 0 & \text{for } u>\gamma\end{cases} \tag{5}$$

[6] The third line of this membership function differs from the mentioned source where it was in error.

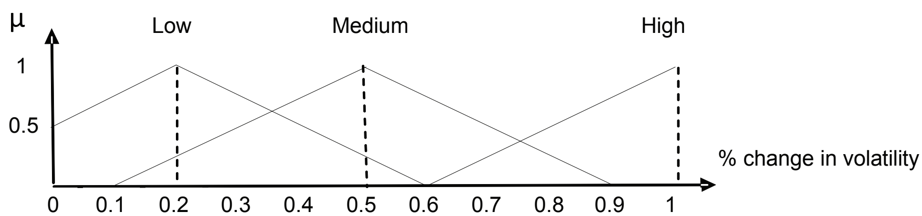
Figure 1: An example of a triangular function



Source: Driankov et al. (1996)

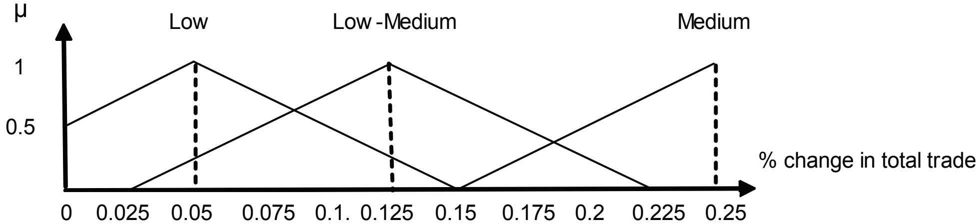
Figures 2 and 3 show the partitioning of the universe of exchange rate volatility and that of total trade into three fuzzy sets. These figures depict the information given in Tables 2 and 3 respectively. The linguistic variable “exchange rate volatility” in Figure 2 is described via 3 linguistic values which are “high”, “medium” and “low” increase in exchange rate volatility. Similarly, in Figure 3 the linguistic variable is “total trade” and linguistic values for it are “medium”, “low-medium” and “low” decrease in total trade.

Figure 2: Linguistic values for variable “exchange rate volatility”



Source: Authors`own drawing

Figure 3: Linguistic values for variable “total trade”



Source: Authors`own drawing

When dealing with fuzzy sets, the entire knowledge of the system is stored as rules in the knowledge base (Zimmermann, 2001). Thus, the rules play a very important role in fuzzy systems and therefore a considerable effort should be taken when defining the rules. Detailed information on the problem to be solved and experience are necessary to design a reliable fuzzy rule set and to obtain good results. If the designer does not have sufficient prior knowledge about the system or topic, it becomes impossible to develop a reliable fuzzy rule (Aliev et al., 2004). Fuzzy rules are the means that will translate inputs into the actual outputs (McNeill and Thro, 1994).

Under normal circumstances, traders do expect a stable economic environment and also no high volatility in exchange rates, as high volatility in exchange rates means high volatility in their revenues as well. When exchange rates fluctuate a lot, the impact of this change on total trade will be considerable, as economic agents do not expect enormous changes in exchange rate volatility. The construction of the fuzzy rule used in this study follows the assumption that, while any increase in exchange rate volatility will affect total trade, the amount of decrease in total trade will not be exactly by the same percentage but lower. According to the fuzzy rule used (see Table 4), a high increase in exchange rate volatility (1 percent) results in a medium (0.25 percent) decrease in bilateral trade, while a medium (0.5 percent) increase in exchange rate volatility leads to a low-medium (0.125 percent) decrease in bilateral trade. Furthermore, a low increase in exchange rate volatility causes a low decrease in bilateral trade.

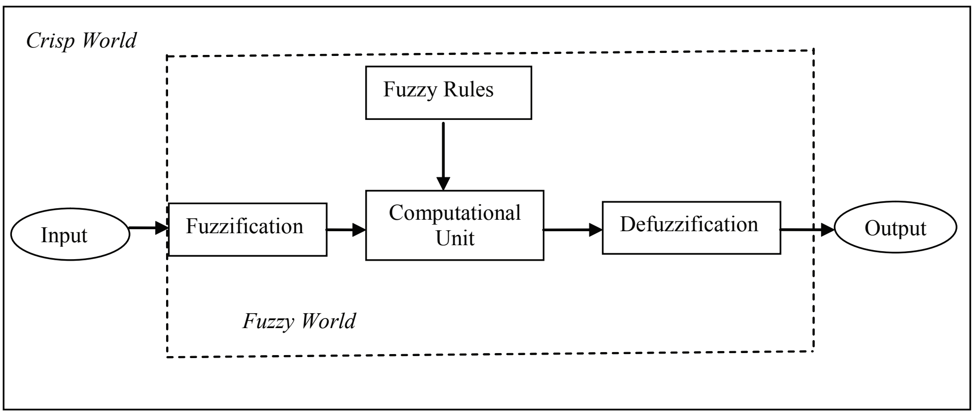
Table 4: Fuzzy rules for explaining the effects of increase in exchange rate volatility on bilateral trade

Fuzzy rules	Description of the fuzzy rules in matrix form
Rule 1: If increase in exchange rate volatility is high, then decrease in total trade is medium	$\tilde{A}_1 \times \tilde{B}_1$
Else	
Rule 2: If increase in exchange rate volatility is medium, then decrease in total trade is low-medium	$\tilde{A}_2 \times \tilde{B}_2$
Else	
Rule 3: If increase in exchange rate volatility is low, then decrease in total trade is low	$\tilde{A}_3 \times \tilde{B}_3$

Source: Authors`own description

After defining fuzzy rules, it is necessary to compute all rule-consequences (Zimmermann, 2001). In the fuzzification process, linguistic variables are described via linguistic values and quantitative values are assigned to these values. Then, possible consequences are defined for each possible input with 'if-then' rules (see Table 4), and the consequences are aggregated into a fuzzy set (see Figure 4). The last step is defuzzification, where one crisp value is generated from the fuzzy output set. The crisp value obtained after defuzzification enables the interpretation of the effect of a "1% increase in exchange rate volatility" on bilateral trade as a percentage value. Figure 4 shows how the whole process works.

Figure 4: The process of fuzzification and defuzzification



Source: Reconstructed from Zimmermann (2001) and Driankov et al. (1996)

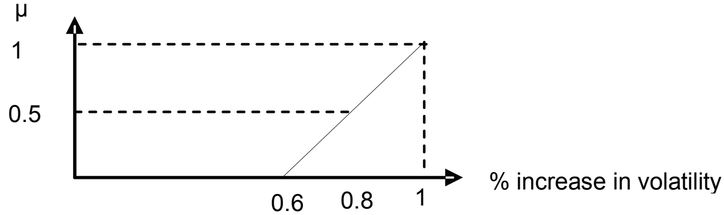
Given the conclusions obtained by individual fuzzy rules shown in Table 4, the overall fuzzy relation ($\tilde{R}$) is calculated by taking the union of all individual effects:

$$\tilde{R} = \bigcup_{i=1}^{3} \tilde{A}_i \times \tilde{B}_i = (\tilde{A}_1 \times \tilde{B}_1) \cup (\tilde{A}_2 \times \tilde{B}_2) \cup (\tilde{A}_3 \times \tilde{B}_3) \tag{6}$$

where $\tilde{A}_i$ and $\tilde{B}_i$ are fuzzy sets and "x" denotes the Cartesian product. The Cartesian product of $\tilde{A}_1$ and $\tilde{B}_1$ shows the impact of a high increase in exchange rate volatility on bilateral trade in a matrix form. Similarly, $\tilde{A}_2 \times \tilde{B}_2$ and $\tilde{A}_3 \times \tilde{B}_3$ depict the effect of a medium and a low increase in exchange rate volatility on bilateral trade respectively, again in a matrix form. The combination of these three individual effects is obtained by applying the union operator to these three matrices and the resultant matrix is $\tilde{R}$. Using this fuzzy relation ($\tilde{R}$) in matrix form, the impact of a "1 percent increase in exchange rate volatility" on bilateral trade will be determined (See Section 3.2.4 for fuzzy mathematics used and Appendix for the calculation of $\tilde{R}$).

To determine this effect we need to fuzzify "1% increase in exchange rate volatility". The fuzzy set $\tilde{\mathbf{C}}$, called "1% increase in exchange rate volatility", is described by the membership function illustrated in Figure 5[7].

Figure 5: Membership function for a "1% increase in volatility"



Source: Authors`own drawing

According to this membership function, a 1% increase in exchange rate volatility has a membership value of 1 to the fuzzy set $\tilde{C}$. When the increase in exchange rate volatility is nearer to 1%, for example 0.9%, its membership value is 0.75. A 0.8% increase in exchange rate volatility is the member of the fuzzy set of "1% increase in exchange rate volatility" with a degree of 0.5.

The effect of a 1 percent increase in exchange rate volatility on bilateral trade can be obtained by applying the compositional rule of inference to the fuzzy set $\tilde{C}$ and fuzzy relation $\tilde{R}$: (see A.4 Definition 6 for details about the compositional rule of inference and operator "$\circ$").

$$\tilde{B}` = \tilde{C} \circ \tilde{R}$$

$$\tilde{B}` = [0\ 0\ 0.25\ 0.25\ 0.25\ 0.25\ 0.25\ 0.25\ 0.5\ 0.75\ 1\,],$$

where $\tilde{C}$ = [0 0 0 0 0 0 0 0.25 0.5 0.75 1] as shown in Figure 5.

$\tilde{B}`$ is the fuzzified decrease in bilateral trade, where each number is a weight factor between 0 and 1, corresponding to the percentage values between 0 and 1 with an increment of 0.025 (see Table 3).

The last step requires the defuzzification process, which converts the overall fuzzy conclusion ($\tilde{\mathbf{B}}`$) into a real number that represents the decrease in bilateral

[7] Although it appears that the membership function of "1% increase in exchange rate volatility" corresponds to the fuzzy set which describes "high volatility in exchange rates" in Table 2 and also in Figure 2, it is just a coincidence. Different membership functions could also be used to define this fuzzy set such as $\tilde{C}$ = [0 0 0 0 0 0 0 0 0 0.75 1] or $\tilde{C}$ = [0 0 0 0 0 0 0 0 0 0 1]

trade following a 1% increase in exchange rate volatility. There are different defuzzification methods. Here, the centroid method—the center of the output membership function–is employed in the defuzzification process. This method uses a weighted average. Mathematically, it corresponds to the expected value of probability (Zimmermann, 2001). The centroid method yields that:

$$\%\text{ Change} = \frac{0x0+0.025x0+0.05x0.25+0.075x0.25+0.1x0.25+0.125x0.25+0.15x0.25+0.175x0.25+0.2x0.5+0.225x0.75+0.25x1}{0+0+0.25+0.25+0.25+0.25+0.25+0.25+0.5+0.75+1} = 0.183$$

In words, this means that a 1 percent increase in exchange rate volatility leads to a 0.18 percent decrease in bilateral trade. It is evident that this result is in accordance with the coefficient of 0.21 that was obtained by using panel data analysis with period fixed effects and is reported in Table 1.

5. Conclusion

The results prove the hypothesis that the fuzzy logic can approximate the effects of exchange rate volatility on trade flows and it can be used as a complement to statistical models. This paper contributes to international economics literature with the robustness check it made. On the one hand, it analyzes bilateral trade flows using a large data set and obtains results in line with the literature. On the other hand, it uses a totally different approach to analyze the same issue. It is quite reasonable to make this robustness check in a case where we have sufficient data to see what can be done in the absence of adequate data. In this study, the interest focuses especially on the effects of exchange rate volatility on bilateral trade flows. Therefore, only the effects of exchange rate volatility on trade flows are compared using two different approaches.

The study recommends the fuzzy approach to be used in economic analysis in the cases where the user does not have a large data set or has problems with the data set that affect the reliability of results. As it is well known, the problems about the data might lead to inefficient results in econometric analysis. For these cases, we think that the fuzzy approach could be used to get some approximate results since we obtained very similar results by comparing these two approaches. However, it should be emphasized that to use fuzzy reasoning needs expertise in the topic studied and the user should have a broad knowledge of the literature about the topic to be able to appropriately define fuzzy rules to solve the problem. According to our results, we recommend the fuzzy approach to be used as a complement to statistical methods. However, the use of fuzzy logic should be tested with further studies before generalizing this conclusion and making use of fuzzy reasoning more often in economics.

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Utjecaj volatilnosti tečaja na međunarodne trgovinske tijekove

Elif Nuroğlu[1], *Robert M. Kunst*[2]

Sažetak

Cilj ovog rada je analizirati utjecaj volatilnosti tečaja na međunarodne trgovinske tijekove pomoću dva različita pristupa i to panel analize podataka i fuzzy logike, te potom usporediti rezultate. Prema platformi presjeka dimenzija 91 par EU15 zemlje s vremenskim rasponom 1964–2003. godine primjenjuje se prošireni gravitacijski model trgovine kako bi se utvrdili utjecaji volatilnosti tečaja na bilateralne trgovinske tijekove u zemljama EU15. Procijenjeni utjecaj je jasno negativan, što znači da volatilnost tečaja ima negativan utjecaj na bilateralne trgovinske tijekove. Potom, ovaj tradicionalni platformni pristup je u suprotnosti s alternativnom istragom temeljenom na fuzzy pristupu. Ključni elementi fuzzy pristupa su intuitivno postaviti fuzzy pravila odlučivanja i dodijeliti funkcije članstva fuzzy skupovima temeljem iskustva. Vidljivo je da oba pristupa daju vrlo slične rezultate, te se fuzzy pristup preporuča kao dopuna postojećih statističkih metoda.

Ključne riječi: *jezično modeliranje, fuzzy odnos, volatilnost tečaja, bilateralna trgovina, gravitacijski model*

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Appendix

Calculation of Overall Fuzzy Relation

$$\tilde{R} = \bigcup_{i=1}^{3} \tilde{A}_i \times \tilde{B}_i = \left(\tilde{A}_1 \times \tilde{B}_1\right) \cup \left(\tilde{A}_2 \times \tilde{B}_2\right) \cup \left(\tilde{A}_3 \times \tilde{B}_3\right)$$

Table 1: Calculation of the decrease in total trade according to the Fuzzy Rule 1 in Table 4

$\tilde{A}_1 \times \tilde{B}_1$		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.25**	**0.50**	**0.75**	**1**
(High x medium)	**0**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25
	0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.50	0.50	0.5
	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.50	0.75	0.75
	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.50	0.75	1

Source: Authors' calculation according to the fuzzy rule described in Table 4

Table 2: Calculation of the decrease in total trade according to the Fuzzy Rule 2 in Table 4

$\tilde{A}_2 \times \tilde{B}_2$		**0.00**	**0.00**	**0.00**	**0.25**	**0.50**	**0.75**	**1.00**	**0.75**	**0.50**	**0.25**	**0**
(Medium x low-medium)	**0**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0.25	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0
	0.5	0.00	0.00	0.25	0.25	0.50	0.50	0.50	0.50	0.50	0.00	0
	0.75	0.00	0.00	0.25	0.25	0.50	0.75	0.75	0.75	0.50	0.00	0
	1	0.00	0.00	0.25	0.25	0.50	0.75	1.00	0.75	0.50	0.00	0
	0.75	0.00	0.00	0.25	0.25	0.50	0.75	0.75	0.75	0.50	0.00	0
	0.5	0.00	0.00	0.25	0.25	0.50	0.50	0.50	0.50	0.50	0.00	0
	0.25	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0
	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0

Source: Authors' calculation according to the fuzzy rule described in Table 4

Table 3: Calculation of the decrease in total trade according to the Fuzzy Rule 3 in Table 4

$\tilde{A}_3 \times \tilde{B}_3$	**0.50**	**0.75**	**1.00**	**0.75**	**0.50**	**0.25**	**0.00**	**0.00**	**0.00**	**0.00**	**0**
(Low x low) 0.5	0.50	0.50	0.50	0.50	0.50	0.25	0.00	0.00	0.00	0.00	0
0.75	0.50	0.75	0.75	0.75	0.50	0.25	0.00	0.00	0.00	0.00	0
1	0.50	0.75	1.00	0.75	0.50	0.25	0.00	0.00	0.00	0,00	0
0.75	0.50	0.75	0.75	0.75	0.50	0.25	0.00	0.00	0.00	0.00	0
0.5	0.50	0.50	0.50	0.50	0.50	0.25	0.00	0.00	0.00	0.00	0
0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0.00	0.00	0.00	0
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0

Source: Authors' calculation according to the fuzzy rule described in Table 4

Table 4: The overall fuzzy relation ($\tilde{R}$)

($\tilde{R}$)	0.5	0.5	0.5	0.5	0.5	0.25	0.00	0.00	0.00	0.00	0.00
	0.5	0.75	0.75	0.75	0.5	0.25	0.00	0.00	0.00	0.00	0.00
	0.5	0.75	1	0.75	0.5	0.25	0.25	0.25	0.25	0.00	0.00
	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.25	0.00	0.00
	0.5	0.5	0.5	0.5	0.75	0.75	0.75	0.5	0.25	0.00	0.00
	0.25	0.25	0.25	0.5	0.75	1	0.75	0.5	0.25	0.00	0.00
	0.00	0.00	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0.00	0.00
	0.00	0.00	0.25	0.5	0.5	0.5	0.5	0.5	0.25	0.25	0.25
	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.5	0.5	0.5
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.5	0.75	0.75
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.5	0.75	1

Source: Authors' calculation according to Equation 6.