Yield to Maturity Is Always Received as Promised:a Reply

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ABSTRACT

This note attempts to further spell out why it is a myth that YTM is viewed as only a promised but not really earned interest rate. It addresses some misconceptions in Shirnani and Wilbratte (2009, this issue) on what, between YTM and RCY, is a true rate of return of a coupon bond, why YTM is NOT just a "fictitious mathematical construct", and why YTM has nothing to do with yield curve.

Introduction

There has been a long-held myth in financial economics that the yield to maturity (YTM, hereafter) of a coupon bond is only promised and may not be actually earned unless coupon payments are *reinvested* at the same interest rate as the initial YTM (e.g., see Reilly and Brown, 1997, pp. 530-531, and Strong, 2004, p.70, among others). Recently, we have formally demonstrated why it is indeed a myth (Cebula and Yang, 2008). In that paper, we rigorously proved a proposition that the (initial) YTM when purchasing a coupon bond measures the interest rate actually earned from holding the bond until maturity (assuming no defaults); furthermore, the realized compounding yield (RCY hereafter) is in fact the YTM of a portfolio that holds the bond and reinvests some or all coupon payments as they are received.

Attempting to defend *the* long-held myth, Shirvani and Wilbratte (2009, this issue) criticize that the (initial) YTM "is based on the unrealistic assumption of a flat and unchanging yield curve", and hence it is only "a fictitious mathematical construct, never to be actually realized until and unless the bond coupons are all reinvested at the same unchanging current interest rate." Moreover, they propose that "the true rate of return on a coupon bond ... is ...the RCY." We understand that we should not expect everyone to accept a correct but unconventional point, in particular, since the conventional myth has been so deeply rooted in many textbooks as well as in professionals' hearts. However, we find that Shirvani and Wilbratte have not only missed the major point we have made in our paper, but also have made some incorrect assertions in their comment. In this reply, we endeavor to address these issues.

Is YTM Fictitious or Real? A Counter Example

The key difference between the long-held myth and our proposition lies in what is the correct measure of the interest rate from holding a coupon bond until maturity. We believe that it is the YTM, defined as the solution for the following equation:

$$P_0 = \sum_{t=1}^{N} \frac{C_t}{(1+YTM)^t} + \frac{F}{(1+YTM)^N} \qquad (1)$$

where P_0 = the purchase price of the coupon bond, C_t = coupon payment per period, F = face/par value of the bond, and N = term to maturity. Note that a bond contract, in theory and in practice, is completely

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characterized by parameters $\{F, C_t, N\}$, and that its purchase price is determined in the bond market by demand and supply. By definition, the YTM is internally determined by the foursome parameters $\{F, C_t, N; P_0\}$ at the time of purchase. Hence, as long as there is no default, interest is paid exactly on time, and the bond is held until maturity, the YTM is not only a promised, but also actually earned (annual) rate of return by the bond holder until maturity. In this definition, nothing is assumed about how coupon payments are used – reinvested or spent, and nothing is assumed how the *current* market interest rate will vary over time after purchase. What is more, nothing is claimed on how much ending worth in year N for the bondholder will be built up, which relies on how coupon payments are managed – an additional investment by the bondholder.

In contrast, Shervani and Wilbratte (2009) suggest that "the true rate of return on a coupon bond ... is a backward-looking concept measured by the RCY". Here, RCY stands for *realized compounding yield*, defined as the solution from the following equation:

$$P_0 (1 + RCY)^N = V_N \qquad (2)$$

given P_0 = the initial investment (i.e., the purchase price of the bond in this case), and V_N = the ending-worth value at the end of year N. By nature, RCY measures the annual rate of return for an investment account, rather than for a single investment. In this definition, it is implicitly assumed that there is no leakage from, or injection into, the account over the investment horizon. For example, if the investment is coupon bonds and when coupon payments are received, they are assumed to be reinvested at a *current* interest rate that the investor can obtain. Therefore, V_N and hence, RCY, will be determined by the interest rate of *reinvestment*. For convenience of presentation, technically, if the coupon payments are spent rather than reinvested, we interpret it as reinvesting at a rate of negative 100% (i.e., all coupon payments are completely lost in the reinvestment). In addition, we exclude the possibility that additional funds other than the coupon payments can be added to the account – a case of a *de facto* Ponzi scheme.

Since our first point in this note is to refute that "the true rate of return on a coupon bond ... is a backward-looking concept measured by the RCY" (Shervani and Wilbratte, 2009), we only need to give a counter example. In fact, we have already provided such an example in our paper (2008). Unfortunately, it was apparently overlooked. Hence, we cite it here.

Consider two investors, A and B. Each has bought the same coupon bond with F = \$1,000 and C_t = \$50 at a price of par value, i.e., P_0 = \$1,000. Besides, each has held the bond until maturity, e.g., N = 10 years. Clearly, both investors have put \$1,000 down and earned \$50 from it every year for 10 years. Clearly, YTM = C_t/F = 5% for both investors.

Moreover, investor A has somehow managed to reinvest all couple payments at an interest rate of 5% whenever received, whereas investor B has relied on the coupon payments to pay bills. As a result, investor A is able to have gathered an amount of $\$1{,}000(1.05)^{10}$ in her investment account by the end of year 10, while investor B can only have redeemed the principal of \$1,000. Hence, RCY = 5% for investor A, and it is zero for investor B.

So, what is the (annual) rate of return actually earned from holding the bond for each investor in this example? We would tell both investors that "you both have earned an interest rate of 5% from holding the bond, even though you have managed coupon payments differently." A possible question from investor A could be "if we both have earned the same interest rate, how come I have much more ending wealth accumulated than he does?" We would answer that it is because investor A has saved more (by reinvesting the coupon payments) than investor B does, but not because *the* bond bought by investor A yields a higher return than the one held by investor B.

Now, if Professors Shervani and Wilbratte told investor B that "the so-called 5% interest rate as promised by YTM is only a fictitious mathematical construct. In fact, you have earned *nothing* from holding this bond, because you have spent all coupon payments", investor B may question "What about the \$50 dollars I have received every year for 10 years from my \$1,000 investment? Isn't it real, or is it just fictitious?" We don't know how to answer this question if we were asked, but we would never tell investor B that he has earned nothing, especially in view of Cebula and Yang (2008).[3]

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[3] One of us must confess that he had taught *the* myth in his classes a couple of times until someday a student asked a similar question to one by investor B. Pretty often, we instructors could benefit a lot from "seemingly stupid" question that challenges a long-held convention. If we cannot answer them logically, it must be because either we are ignorant or something may be wrong in the long-held convention.

Obviously, there is something of a problem with using RCY as the measure of rate of return from holding a coupon bond. So, what is wrong? In our paper (2008), we have formally shown that the RCY does not measure the rate of return from holding a coupon bond *per se*; rather, it actually measures the YTM of a portfolio with two investments: holding the coupon bond until maturity *plus* investing all coupon payments when received in a time deposit compounded annually. Of course, the value built up at the end of year N will depend on how coupon payments are managed – reinvested or spent, and what interest rates are when coupon payments are reinvested. But the (initial) YTM is the same from holding the initial bond itself, no matter how coupon payments are managed after being dispersed to the bondholder. Intuitively, the RCY is a weighted average of the initial YTM and the interest rate of reinvestment; hence, not surprisingly, RCY = YTM *only if* the reinvestment interest rate equals to the initial YTM. The formal proof for this proposition can be found in our paper (2008). We must emphasize that the proposition we made in (Cebula and Yang, 2008) was not an "assertion", since we have rigorously proven it rather than simply claiming it without a proof.

YTM Has Nothing to Do with Yield Curve

In addition to the different perspectives on what, between YTM and RCY, correctly measures the interest rate from holding a coupon bond, there is another very basic misconception in Shervani and Wilbratte (2009), namely, that "the YTM of a coupon bond, which is based on the unrealistic assumption of a flat and unchanging yield curve, is a fictitious mathematical construct". In the previous section, we have employed a counter example to refute the assertion that "YTM is only a fictitious mathematical construct". Now, we clarify why the YTM has nothing to do with any assumptions on the *current* yield curve.

Recall that for a bondholder, the (initial) YTM at the time of purchase is defined in equation (1) and completely determined by parameters $\{F, C_t, N; P_0\}$. Once the bond is actually purchased, the resulting YTM is fixed *for the bondholder* until maturity, assuming that the bond is held until maturity without default. Indeed, after the bond is purchased, the *current* market YTM will continue to change over time along with the market value of the bond purchased. Also, the term "to maturity" of the bond purchased will get shorter and shorter as time goes on. However, the (initial) YTM has nothing to do with the *current* market interest rate *after* purchase, because it has been locked by $\{F, C_t, N; P_0\}$. This reasoning is the same as the fixed-term mortgage rate and the CD rate. The *current* market interest rates (of the mortgage and the CD) may change continually. Once a contract is signed, however, the rate becomes fixed for the persons who have signed the contract. We should not get confused between the *current* market interest rate, which is in change all the time, and the interest rate one actually receives/pays, which becomes fixed after signing the contract. A confusion between the two kinds of interest rates before and after a contract is signed is a very basic misconception.

Therefore, we don't need any additional assumptions on how the current market yield curve behaves when we define and calculate YTM. Likewise, we have not assumed whether the yield curve is flat and unchanging when we proved the proposition on how RCY is related to YTM in our paper (2008).

Concluding Remarks: More on YTM

YTM is an internally determined measure of (total) rate of return (IRR) of fixed-income investment instruments, by their cash in/out flows until maturity. By nature, it serves as a common yardstick when "an apple is compared to an orange".

One relevant question on bond investment is how to use coupon bonds to accumulate an ending-worth value of $P_0(1 + YTM)^N$ at time N, given an initial investment P_0 and the initial YTM of the coupon bond. Of course, it is not automatic to simply hold the bond; rather, it requires the investor or her account/portfolio managers to reinvest coupon payments when received. A sufficient condition is to invest all coupon payments in CDs or discount bonds at the initial YTM, compounded annually (Fabozzi and Modigliani, 2002, p. 364). But the reinvestment strategy has nothing to do with the initial YTM. Unfortunately, this real-world question and proposed solution for it have been misinterpreted as that the initial YTM of the coupon bond is only a promised "fictitious mathematical construct" unless the ending-worth value of $P_0(1 + YTM)^N$ can be accumulated. This misinterpretation is a problem for financial economics professionals, because it makes people confused between two different issues: What is the annual rate of return from holding a coupon bond until maturity? (*The* answer: It is the YTM), and how to accumulate an ending-worth value of $P_0(1 + YTM)^N$ by using coupon bonds with an initial YTM? (*An* answer: If one can reinvest coupon payments when received at the initial YTM).

Our paper (2008) has formally shown that the RCY is no other than the YTM of a portfolio. With a counter example this note has further refuted that the initial YTM of a coupon bond is NOT a "fictitious mathematical construct". We must acknowledge that we have not formally addressed what the initial YTM (of a coupon bond) actually measures. This issue is beyond the scope of this short reply. Nevertheless, our attention is drawn to another "hypothesis" in financial economics. Although many authors and researchers claim that the YTM "is often viewed as a measure of the average rate for return that will be earned on a bond if it is bought now and held until maturity" (Bodie, *et al*, 2002, p. 426), this statement remains an "assertion." To our knowledge, to date, it has not actually been formally proven in the published literature.[4]

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[4] A formal proof for how YTM and rate of return are related can be found in Cebula, *et al* (2009, available upon request.)